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# Total Vertex Irregularity Strength of Disjoint Union of Ladder Rung Graph and Disjoint Union of Domino Graph

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**Abstrak**: Akan diselidiki pelabelan graf yang disebut total vertex irregularity strength (tvs(G)). tvs(G) adalah minimum $\boldsymbol{k}$ yang graf tersebut memenuhi pelabelan $\boldsymbol{k}$-total titik tidak teratur. Pada makalah ini, akan ditentukan pelabelan total tak reguler titik dari graf disjoint union of ladder rung dan graf disjoint union of domino graph.

***Kata kunci:*** *Graf; tvs; Ladder rung; Domino*

**Abstract**: We investigate a graph labeling called the total vertex irregularity strength (tvs(G)). A tvs(G) is minimum $\boldsymbol{k}$ for which graph has a vertex irregular total $\boldsymbol{k}$-labeling. In this paper, we determine the total vertex irregularity strength of disjoint union of ladder rung graph and disjoint union of domino graph.

***Keywords****: Graph*; *tvs; Ladder rung; Domino*

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## 1. Introduction

In 1735, graph theory was first introduced by Leonhard Euler to solve the problem of the Konigsberg bridge on the river Pregel, Russia [1]. Graph labeling is an interesting topic in graph theory so that various types of labeling are researched and developed [2]. Graph labeling is the assignment of labels to the graph elements such as edges or vertices, or both, from the graph [3].

Graph labeling can be applied to various fields including transportation systems, communication systems, geographical navigation, radar, and also security systems. For example, the design of the code for radar signals and missiles is equivalent to labeling a complete graph, where each point is connected to one side which has a label that is always different. This side label describes the distance between points, while the point label is the position at the time the signal is sent [4].

For a graph $G(V,E)$, a labeling $f: V(G) \cup E(G) \to \{1,2,\dots,k\}$ to be a vertex irregular total $k$-labeling if for any two different vertices x and y, their weights satisfy $wt_f(x) \neq wt_f(y)$ (see [5], [6]). The total vertex irregularity strength is minimum $k$ for which graph has a vertex irregular total $k$-labeling (see [6], [7]).

The total vertex irregularity strength problem has been investigated for trees [8], $C_n *_2 K_n$ graph [9], regular graph [10], forest graph [11], disjoint union of sun graph [5], wheel related graphs [12], graph obtained of a star [13], trees with maximum degree five [14], and comb product of two cycles and two stars [15]. In this paper we answer the open problem proposed by Baca, et.al [7]. In particular, we determine the total vertex irregularity strength of disjoint union of ladder rung graph and disjoint union of domino graph.

## 2. Preliminaries

Ball and Coxeter define the ladder graph $nP_2$, is $n$ copies of the path graph $P_2$ [4]. The ladder rung graph can be depicted as in Figure 1.

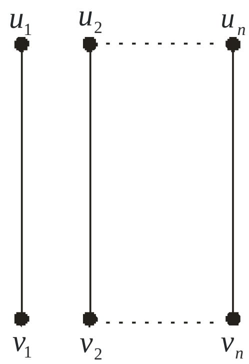


**Figure 1.** The ladder rung graph

**Theorem 1.** *The total vertex irregularity strength of disjoint union of ladder rung graph is* $tvs(tL_n) = nt + 1$, *for* $n \geq 1$, $t \geq 2$.

*Proof.* The disjoint union of ladder rung graph $tL_n$ has $2nt$ vertices. The smallest $wt(tL_n)$ must be 2 and the largest $wt(tL_n)$ is at least $nt + 1$. Because of every vertex has degree one, then $tvs(tL_n) \geq nt + 1$.

To show that $tvs(tL_n) \leq nt + 1$. The label of vertices of $tL_n$ are described in the following formulas:

$\lambda(u_i) = i$, for $i \in [1, nt]$,
$\lambda(v_i) = i$+1, for $i \in [1, nt]$.

Then, the label of edges of $tL_n$ are

$\lambda(v_i v_i) = i$+1, for $i \in [1, nt]$.
The weights of vertices $u_i$ and $v_i$ of $tL_n$ are:
$wt(u_i) = 2i + 1$, for $i \in [1, nt]$,
$wt(v_i) = 2(i + 1)$, for $i \in [1, nt]$.

The weights calculated at vertices are distinct. So, $tvs(tL_n) = nt + 1, for\ n \geq 1, t \geq 2$.

## 3. Results

The domino graph is (2,3)-grid graph [16]. The disjoint union of domino graph can be depicted as in Figure 2.

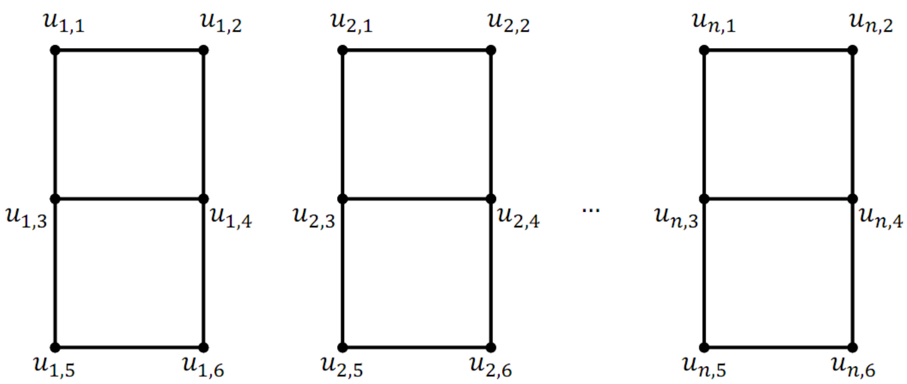


**Figure 2.** The disjoint union of domino graph

**Theorem 2.** *The total vertex irregularity strength of disjoint union of domino graph is* $tvs(nD) = n + 1$.

*Proof*. The disjoint union of ladder rung graph $nD$ has $6n$ vertices. The smallest $wt(nD)$ must be 3 and the largest $wt(nD)$at least $3n + 5$. It easy to see that $tvs(nD) \geq n + 1$.

To show that $tvs(nD) \leq n + 1$. The label of vertices of $nD$ are:

$\lambda(u_{i,1}) = i$, for $i \in [1, n]$,
$\lambda(u_{i,2}) = i$, for $i \in [1, n]$,
$\lambda(u_{i,3}) = 2$, for $i \in [1, n]$,
$\lambda(u_{i,4}) = 2$, for $i \in [1, n]$,
$\lambda(u_{i,5}) = i$, for $i \in [1, n]$,
$\lambda(u_{i,6}) = i + 1$, for $i \in [1, n]$.

Then, the label of edges of $nD$ are

$\lambda(u_{i,1}u_{i,2}) = i$, for $i \in [1, n]$,
$\lambda(u_{i,1}u_{i,3}) = i$, for $i \in [1, n]$,
$\lambda(u_{i,2}u_{i,3}) = i + 1$, for $i \in [1, n]$,
$\lambda(u_{i,3}u_{i,4}) = i + 1$, for $i \in [1, n]$,
$\lambda(u_{i,3}u_{i,5}) = i + 1$, for $i \in [1, n]$,
$\lambda(u_{i,4}u_{i,6}) = i + 1$, for $i \in [1, n]$,
$\lambda(u_{i,5}u_{i,6}) = i + 1$, for $i \in [1, n]$.

The weights of vertices $u_i$ and $v_i$ of $tL_n$ (respectively) are:

$wt(u_{i,1}) = 3n,$ for $i \in [1, n],$
$wt(u_{i,2}) = 3n + 1,$ for $i \in [1, n],$
$wt(u_{i,3}) = 3n + 4,$ for $i \in [1, n],$
$wt(u_{i,4}) = 3n + 5,$ for $i \in [1, n],$
$wt(u_{i,5}) = 3n + 2,$ for $i \in [1, n],$
$wt(u_{i,6}) = 3(n + 1),$ for $i \in [1, n].$

The weights calculated at vertices are distinct. So, $tvs(nD) = n + 1$.

## 4. Conclusions

The total vertex irregularity strength of disjoint union of ladder rung graph is $tvs(tL_n) = nt + 1$, for $n \geq 1$, $t \geq 2$ and the total vertex irregularity strength of disjoint union of domino graph is $tvs(nD) = n + 1$.

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