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Developing A Secure Cryptosystem with Rainbow Vertex Antimagic Coloring of Cycle Graph

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Abstrak. Pelabelan sisi pada graf G adalah fungsi g dari himpunan sisi dari graf G ke bilangan asli pertama sampai dengan kardinalitas himpunan sisi. Graf G mempunyai pewarnaan pelangi titik anti ajaib jika untuk setiap dua titik terdapat suatu lintasan dengan warna berbeda dari semua titik dalam. Warna titik dari graf G ditentukan oleh bobot titik. Bobot titik pada graf G diperoleh dengan menjumlahkan semua label sisi yang terkait dengan simpul tersebut. Nilai pewarnaan pelangi titik anti ajaib dari graf G, dilambangkan dengan $rvac(G)$ adalah jumlah minimum warna yang diinduksi oleh pewarnaan pelangi titik anti ajaib. Dalam makalah ini, kami menentukan batas atas nilai pewarnaan pelangi titik anti ajaib ($rvac$) pada graf siklus (C_n) dan mengembangkan modifikasi *Affine Cipher* dari pewarnaan pelangi titik anti ajaib untuk membuat kriptosistem yang aman.

Abstract. An edge labeling of graph G is a function g from the edge set of graph G to the first natural numbers up to the number of the edge set. Graph G admits a rainbow vertex antimagic coloring if, for any two vertices, there is a path with different colors of all internal vertices. The vertex color of graph G is assigned by vertex weight. The vertex weight of graph G is obtained by summing all edge labels that incident with that vertex. The rainbow vertex antimagic connection number of graph G, denoted by $rvac(G)$ is the smallest number of different colors induced by rainbow vertex antimagic coloring. In this research, we determine the upper bound of the rainbow vertex antimagic connection number ($rvac$) on a cycle graph (C_n) and create a secured cryptosystem using a modified Affine Cipher based on rainbow vertex antimagic coloring.

1. Introduction

Graph theory is one of the studies of mathematics that has applications in the field of mathematics. One of the popular graph theory studies is coloring theory. Coloring theory is developed from a Four-Color Theorem. The theorem states that each map can be colored using four colors, so that adjacent areas do not have the same color. The development of graph coloring theory covers vertex coloring, edge coloring, face coloring, r-dynamic coloring, and rainbow coloring.

In this paper, we discuss rainbow vertex antimagic coloring which begins with the following definition. The graph that is discussed in this study is a simple and connected graph, definitively it can be seen in [1]. The concept of rainbow connection has several interesting variants, one of them is rainbow vertex-connection. It was introduced by Krivelevich and Yuster [2]. Let G be a graph and $c: V(G) \rightarrow \{1, 2, \cdots, k\}$ be a vertex k-coloring, for some $k \in N$. A path P in G with a vertex k-coloring is said to be a rainbow vertex-path, if all internal vertices of P have distinct colors. The graph G is said to be a rainbow vertex connected, if for any two vertices u and v in $V(G)$ there is a rainbow vertex-path. A vertex k-coloring of G is said rainbow vertex coloring, if G rainbow vertex connected under c. The rainbow vertex connection number, denoted by $rvc(G)$, is the smallest positive integer k such that G has rainbow vertex k-coloring. Krivelevich and Yuster also gave the lower bound for a graph G, namely $rvc(G) \geq diam(G) - 1$, where $diam(G)$ is the diameter of the graph G [3]. Several other concepts as a result of developing the concept of rainbow vertex coloring can be seen in [4][5][6][7][8][9][10].

Septory et al. [11] developed rainbow coloring into rainbow antimagic coloring, while Marsidi et al. developed rainbow vertex coloring into rainbow vertex antimagic coloring. A formal definition related to rainbow vertex antimagic coloring can be seen in [12]. In this paper, we determine the upper bound of the rainbow vertex antimagic connection number of cycle graph ($rvac(C_n)$). Some other results on rainbow antimagic coloring of graphs can be seen in Antimagic labeling, rainbow coloring, rainbow antimagic coloring, and rainbow antimagic connection number[13][14].

Furthermore, we apply the concept of rainbow vertex antimagic coloring of graphs for developing a secured cryptosystem. Cryptography is a common approach for maintaining the secrecy of information dispersed across an insecure network [15]. A cryptosystem is a set of cryptographic algorithms used to implement a specific security service, such as confidentiality (encryption). The cryptosystem is a set of cryptographic techniques and infrastructure used to provide information security services. The cryptosystem is also known as a cipher system.

The cryptosystem is typically formed by three algorithms: one for key generation, one for encryption, and one for decryption. The term cipher (or cypher) refers to a pair of algorithms, one for encryption and one for decryption. As a result, when the key generation algorithm is important, the term cryptosystem is most generally used. The term cryptosystem is commonly used to refer to public key techniques; however, for symmetric key techniques, both "cipher" and "cryptosystem" are used. The strength of cryptography protocols relies on the encryption-decryption keys management: how to protect the keys from disclose to unauthorized parties [16].

The Affine Cipher and rainbow vertex antimagic coloring concepts are combined in this article to form the modified Affine Cipher. The Affine Cipher is a monoalphabetic substitution cipher in which each letter of an alphabet is given a numeric representation, encrypted with a simple mathematical function, and then converted back to a letter. Because of the formula, each letter encrypts to and from another letter, implying that the cipher is basically a regular substitution cipher with a structured which letter goes to which. As a result, it suffers from the same issues as all substitution ciphers. Each letter is encoded using the function, where denotes the length of the shift. As encryption and

decryption keys, we use labeling of rainbow vertex antimagic coloring and rainbow vertex antimagic connection number.

2. Preliminaries

In determining the exact value of rainbow vertex antimagic connection number of graph G ($rvac(G)$), a lower bound is needed. The lower bound of $rvac(G)$ has been found by Marsidi, *et al*. For more details see [12]. The lower bound of $rvac(G)$ that found by Marsidi, *et al* is as follows.

Remark 1 [16]

Let G be a connected graph, $rvac(G) \geq rvc(G)$.

2.1 Cryptosystem

A simple model of a cryptographic system that ensures the confidentiality of transmitted data is illustrated in Figure 1. The illustration below depicts this fundamental model.
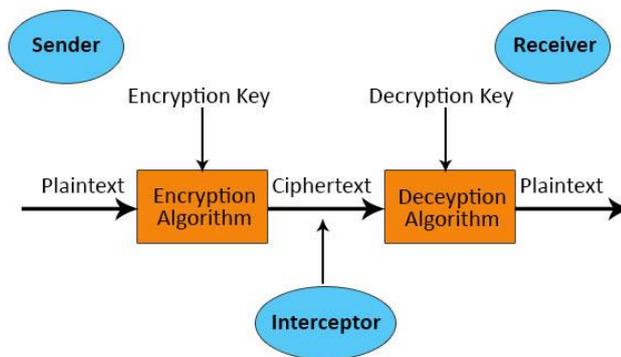
Figure 1. Basic Model of Cryptosystem.

Before the plaintext encryption process is carried out, it must be converted first with the Affine Cipher method. The plaintext conversion rules with Affine Cipher are presented in Table 1.

Table 1. The Conversion Rules on Affine Cipher

A	B	C	D	E	F	G	H	I	J
0	1	2	3	4	5	6	7	8	9
K	L	M	N	O	P	Q	R	S	T
10	11	12	13	14	15	16	17	18	19
U	V	W	X	Y	Z				
20	21	22	23	24	25				

Components of a Cryptosystem

The various components of a basic cryptosystem are as follows.

a) **Plaintext.**

It is the data that must be secured during transmission.

b) **Encryption Algorithm.**

Given a plaintext and an encryption key, it is a mathematical procedure that creates a ciphertext. It is a cryptographic technique that converts plaintext into ciphertext using an encryption key.

c) **Ciphertext.**
It is the scrambled version of the plaintext generated by the encryption algorithm when a specific encryption key is used.

d) **Decryption Algorithm.**
It is a mathematical process that creates a unique plaintext for each ciphertext and decryption key combination. It is a cryptographic method that takes in ciphertext and a decryption key and outputs plaintext. Because it reverses the encryption technique, the decryption algorithm is closely related to it.

e) **Encryption Key.**
The sender knows what it is. The sender inserts the encryption key and plaintext into the encryption algorithm to generate the ciphertext.

f) **Decryption Key.**
To the receiver, it is a known value. The encryption key and the decryption key are related, but not always identical. The receiver inputs the decryption key and the ciphertext into the decryption algorithm to retrieve the plaintext.

2.2 Affine Cipher

An m-letter alphabet's letters are first mapped to integers in the range $0, \cdots, m - 1$. The number that corresponds to each plaintext letter is then converted into another integer that corresponds to a ciphertext letter using modular arithmetic. The encryption function of a single letter is

$$E(x) = (ax + b) \bmod m$$

where modulus m is the alphabet size and a and b are the cipher keys. The value of a must be chosen so that a and m are coprime. The decryption of a function is

$$D(x) = a^{-1}(x - b) \bmod m$$

where a^{-1} is the modular multiplicative inverse of a modulo m. I.e., it satisfies the equation

$$1 = aa^{-1} \bmod m$$

The multiplicative inverse of a exists if a and m are coprime. Without the constraint on a, the decryption may be impossible. As shown below, the decryption function is the inverse of the encryption function.

$$D\big(E(x)\big) = a^{-1}(E(x) - b) \bmod m$$

$$= a^{-1}(((a(x) + b) \bmod m) - b) \, mod \, m$$

$$= a^{-1}(a(x) + b - b) \bmod m$$

$$= a^{-1}ax \bmod m$$

$$= x \bmod m$$

3. Results and Discussion

This research produces the upper bound of rainbow vertex antimagic connection number of cycle graph that has not been found by previous researchers, so the results of this study are new. In addition, researchers develop applications in cryptosystems that are encrypted and decrypted using the resulting rainbow vertex antimagic connection number of cycle graph.

3.1 Rainbow Vertex Antimagic Coloring of Cycle Graph
Theorem 2

If C_n is a cycle graph with orders n and $n \geq 3$, then

$$rvac(C_n) \leq \begin{cases} \dfrac{n}{2}, & if\ n \equiv 2(mod\ 4) \\ \dfrac{n}{2}+1, & if\ n \equiv 0(mod\ 4) \\ \left\lfloor\dfrac{n}{2}\right\rfloor+2, & if\ n \equiv 1(mod\ 4) \\ \left\lfloor\dfrac{n}{2}\right\rfloor+1, & if\ n \equiv 3(mod\ 4) \end{cases}$$

Proof. Let C_n be a cycle graph with vertex set $V(C_n) = \{x_i : 1 \leq i \leq n\}$ and edge set $E(C_n) = \{e_n = x_1 x_n;\ e_i = x_i x_{i+1} : 1 \leq i \leq n-1\}$. The diameter of C_n is $\left\lfloor\dfrac{n}{2}\right\rfloor$. We divide into four cases to prove the upper bound of $rvac(C_n)$ as follows.

Case 1. For $n \equiv 2(mod\ 4)$
The edge labels in the following functions are constructed to illustrate the upper bound of $rvac(C_n)$.
For $1 \leq i \leq \dfrac{n}{2}$, we have

$$g(e_i) = \begin{cases} 2i-1, & i \equiv 1(mod\ 2) \\ 2i, & i \equiv 0(mod\ 2) \end{cases}$$

For $\dfrac{n}{2}+1 \leq i \leq n$, we have

$$g(e_i) = \begin{cases} 2i-n, & i \equiv 0(mod\ 2) \\ 2i-n-1, & i \equiv 1(mod\ 2) \end{cases}$$

We have the following vertex weights based on the edge labels above.

$$w(x_1) = w\left(\frac{n}{2}+1\right) = n+1$$

$$w(x_i) \quad = \quad 4i-3 : 2 \leq i \leq \frac{n}{2}$$

$$w(x_i) \quad = \quad 4i-2n-3 : \frac{n}{2}+2 \leq i \leq n$$

From the vertex weight above, we can determine the different weight in the Table 2.

Table 2. The Different Weight on The Vertices of C_n.

i	$w(i)$	i	$w(i)$
1	$n+1$	$\frac{n}{2}+1$	$n+1$
2	5	$\frac{n}{2}+2$	5
3	9	$\frac{n}{2}+3$	9
4	13	$\frac{n}{2}+4$	13
5	17	$\frac{n}{2}+5$	17
⋮	⋮	⋮	⋮
$\frac{n}{2}$	$2n-3$	n	$2n-3$

We know that the number of different weights of C_n is $\frac{n}{2}$. It concludes that the upper bound of $rvac(C_n)$ is $\frac{n}{2}$. Furthermore, we prove that every pair of C_n vertices have rainbow vertex antimagic coloring. Suppose that $v \in V(C_n)$, Table 3 shows the rainbow vertex path in accordance with the vertex weight.

Table 3. The Path of Rainbow Vertex on Cycle of Order n.

Case	u	v	Rainbow Vertex Coloring of $u - v$ Path
$i < j; j - i \leq \frac{n}{2}$	v_i	v_j	$v_i, v_{i+1}, v_{i+2}, v_{i+3}, \cdots, v_{j-1}, v_j$
$i < j; j - i > \frac{n}{2}$	v_i	v_j	$v_j, v_{j+1}, v_{j+2}, \cdots, v_n, v_1, \cdots, v_i$

Case 2. For $n \equiv 0 (mod\ 4)$
The edge labels in the following functions are constructed to illustrate the upper bound of $rvac(C_n)$.
For $1 \leq i \leq \frac{n}{2}$, we have

$$g(e_i) = \begin{cases} 2i - 1, & i \equiv 1 (mod\ 2) \\ 2i, & i \equiv 0 (mod\ 2) \end{cases}$$

For $\frac{n}{2} + 1 \leq i \leq n$, we have

$$g(e_i) = \begin{cases} 2i - n, & i \equiv 1 (mod\ 2) \\ 2i - n - 1, & i \equiv 0 (mod\ 2) \end{cases}$$

We have the following vertex weights based on the edge labels above.

$$\begin{aligned} w(x_1) &= n \\ w(x_i) &= 4i - 3 : 2 \leq i \leq \frac{n}{2} \\ w\left(\frac{n}{2} + 1\right) &= n + 2 \\ w(x_i) &= 4i - 2n - 3 : \frac{n}{2} + 2 \leq i \leq n \end{aligned}$$

From the vertex weight above, we can determine the different weight in the Table 4.

Table 4. The Different Weight on The Vertices of C_n.

i	$w(i)$	i	$w(i)$
1	n	$\frac{n}{2} + 1$	$n + 2$
2	5	$\frac{n}{2} + 2$	5
3	9	$\frac{n}{2} + 3$	9
4	13	$\frac{n}{2} + 4$	13
5	17	$\frac{n}{2} + 5$	17
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\frac{n}{2}$	$2n - 3$	n	$2n - 3$

We know that the number of different weights of C_n is $\frac{n}{2} + 1$. It concludes that the upper bound of $rvac(C_n)$ is $\frac{n}{2} + 1$. Furthermore, we prove that every pair of C_n vertices have rainbow vertex antimagic coloring. Suppose that $v \in V(C_n)$, Table 5 shows the rainbow vertex path in accordance with the vertex weight.

Table 5. The Path of Rainbow Vertex on Cycle of Order n.

Case	u	v	Rainbow Vertex Coloring of $u - v$ Path
$i < j; j - i \leq \dfrac{n}{2}$	v_i	v_j	$v_i, v_{i+1}, v_{i+2}, v_{i+3}, \cdots, v_{j-1}, v_j$
$i < j; j - i > \dfrac{n}{2}$	v_i	v_j	$v_j, v_{j+1}, v_{j+2}, \cdots, v_n, v_1, \cdots, v_i$

Case 3. For $n \equiv 1 (mod\ 4)$
The edge labels in the following functions are constructed to illustrate the upper bound of $rvac(C_n)$.
For $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$, we have
$$g(e_i) = \begin{cases} 2i - 1, & i \equiv 1(mod\ 2) \\ 2i, & i \equiv 0(mod\ 2) \end{cases}$$
$$g(e_{\left\lceil \frac{n}{2} \right\rceil}) = n$$

For $\left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n$, we have
$$g(e_i) = \begin{cases} 2i - n - 1, & i \equiv 0(mod\ 2) \\ 2i - n - 2, & i \equiv 1(mod\ 2) \end{cases}$$

We have the following vertex weights based on the edge labels above.
$$\begin{aligned} w(x_1) &= n - 1 \\ w(x_i) &= 4i - 3 : 2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ w\left(\left\lceil \frac{n}{2} \right\rceil + 1\right) &= n + 2 \\ w(x_i) &= 4i - 2n - 5 : \left\lceil \frac{n}{2} \right\rceil + 2 \leq i \leq n \end{aligned}$$

From the vertex weight above, we can determine the different weight in the Table 6.

Table 6. The Different Weight on The Vertices of C_n.

i	$w(i)$	i	$w(i)$
1	n-1	$\left\lceil \frac{n}{2} \right\rceil + 1$	$n + 2$
2	5	$\left\lceil \frac{n}{2} \right\rceil + 2$	5
3	9	$\left\lceil \frac{n}{2} \right\rceil + 3$	9
4	13	$\left\lceil \frac{n}{2} \right\rceil + 4$	13
5	17	$\left\lceil \frac{n}{2} \right\rceil + 5$	17
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\left\lceil \frac{n}{2} \right\rceil$	$2n - 1$	n	$2n - 5$

We know that the number of different weights of C_n is $\left\lfloor \frac{n}{2} \right\rfloor + 2$. It concludes that the upper bound of $rvac(C_n)$ is $\left\lfloor \frac{n}{2} \right\rfloor + 2$. Furthermore, we prove that every pair of C_n vertices have rainbow vertex antimagic coloring. Suppose that $v \in V(C_n)$, Table 7 shows the rainbow vertex path in accordance with the vertex weight.

Table 7. The Path of Rainbow Vertex on Cycle of Order n.

Case	u	v	Rainbow Vertex Coloring of $u - v$ Path
$i < j; j - i \le \left\lceil \frac{n}{2} \right\rceil$	v_i	v_j	$v_i, v_{i+1}, v_{i+2}, v_{i+3}, \cdots, v_{j-1}, v_j$
$i < j; j - i > \left\lceil \frac{n}{2} \right\rceil$	v_i	v_j	$v_j, v_{j+1}, v_{j+2}, \cdots, v_n, v_1, \cdots, v_i$

Case 4. For $n \equiv 3 (mod\ 4)$
The edge labels in the following functions are constructed to illustrate the upper bound of $rvac(C_n)$.
For $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$, we have

$$g(e_i) = \begin{cases} 2i - 1, & i \equiv 1(mod\ 2) \\ 2i, & i \equiv 0(mod\ 2) \end{cases}$$
$$g(e_{\left\lceil \frac{n}{2} \right\rceil}) = n$$

For $\left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n$, we have

$$g(e_i) = \begin{cases} 2i - n - 1, & i \equiv 1(mod\ 2) \\ 2i - n - 2, & i \equiv 0(mod\ 2) \end{cases}$$

We have the following vertex weights based on the edge labels above.

$$\begin{aligned}
w(x_1) &= n \\
w(x_i) &= 4i - 3 : 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\
w\left(\left\lceil \frac{n}{2} \right\rceil\right) &= 2n - 2 \\
w\left(\left\lceil \frac{n}{2} \right\rceil + 1\right) &= n + 2 \\
w(x_i) &= 4i - 2n - 5 : \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n
\end{aligned}$$

From the vertex weight above, we can determine the different weight in the Table 8.

Table 8. The Different Weight on The Vertices of C_n.

i	$w(i)$	i	$w(i)$
1	n	$\left\lceil \frac{n}{2} \right\rceil + 1$	$n + 2$
2	5	$\left\lceil \frac{n}{2} \right\rceil + 2$	5
3	9	$\left\lceil \frac{n}{2} \right\rceil + 3$	9
4	13	$\left\lceil \frac{n}{2} \right\rceil + 4$	13
5	17	$\left\lceil \frac{n}{2} \right\rceil + 5$	17
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\left\lceil \frac{n}{2} \right\rceil$	$2n - 2$	n	$2n - 5$

We know that the number of different weights of C_n is $\left\lfloor \frac{n}{2} \right\rfloor + 1$. It concludes that the upper bound of $rvac(C_n)$ is $\left\lfloor \frac{n}{2} \right\rfloor + 1$. Furthermore, we prove that every pair of C_n vertices have rainbow vertex antimagic coloring. Suppose that $v \in V(C_n)$, Table 9 shows the rainbow vertex path in accordance with the vertex weight.

Table 9. The Path of Rainbow Vertex on Cycle of Order n.

Case	u	v	Rainbow Vertex Coloring of $u - v$ Path
$i < j; j - i \leq \left\lceil \frac{n}{2} \right\rceil$	v_i	v_j	$v_i, v_{i+1}, v_{i+2}, v_{i+3}, \cdots, v_{j-1}, v_j$
$i < j; j - i > \left\lceil \frac{n}{2} \right\rceil$	v_i	v_j	$v_j, v_{j+1}, v_{j+2}, \cdots, v_n, v_1, \cdots, v_i$

As a result, the vertex coloring on C_n is rainbow vertex antimagic coloring. Thus, we obtain

$$rvac(C_n) \leq \begin{cases} \frac{n}{2}, & if\ n \equiv 2(mod\ 4) \\ \frac{n}{2} + 1, & if\ n \equiv 0(mod\ 4) \\ \left\lfloor \frac{n}{2} \right\rfloor + 2, & if\ n \equiv 1(mod\ 4) \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & if\ n \equiv 3(mod\ 4) \end{cases}. \qquad \blacksquare$$

3.2 Application

The encryption and decryption process can implement the result of labeling and the chromatic number of rainbow vertex coloring, which is used as the key for the encryption and decryption process. We developed a new algorithm to generate encryption and decryption keys with rainbow vertex antimagic chromatic numbers and labels.

Algorithm 1. Role of Extra Key
a) Define f as the function of graph element labels of graph G.
b) If f is a bijective function, do 3, and bring it back to 1 otherwise.
c) Define b as the rainbow vertex antimagic chromatic number.
d) To use the vertex weight, define $z_i^1: 1 \leq i \leq n$, where n is the number of vertexes.
e) Add z_i^1 and arrange the sequence according to the vertex notation $x_i: 1 \leq i \leq n$, where n is the number of vertices.
f) To use the edge label, define $z_j^2: 1 \leq j \leq m$, where m is the number of edges.
g) Add z_j^2 and arrange the sequence according to the edge notation $e_j: 1 \leq j \leq m$.
h) Set k as element of the z_i^1 and z_j^2 sequence.

In Algorithm 1, it will use the process of developing the key in the encryption and decryption process. Algorithm 2 developed the Affine Cipher based on the results of Algorithm 1, which includes an encryption and description process.

Algorithm 2. Modified Affine Cipher

a) Given that the plaintext and its conversion with Affine Cipher $P_i: 1 \leq i \leq n$.

b) Compute the ciphertext using Equation 1 and compute the plaintext blocks using Equation 2.

$$\Phi_i = \left((P_i + K_{1i}) + K_{2i}\right) \bmod 26 \tag{1}$$

$$P_i = \left((\Phi_i - K_{2i}) - K_{1i}\right) \bmod 26 \tag{2}$$

where P_i, K_{1i}, K_{2i}, and Φ_i are the i-th of conversion plaintext, key sequence, and conversion ciphertext, respectively. Note that, for $n = 1$, Φ_{n-1} is a null vector.

For an illustration how the algorithms are working, we give the following examples. Given that a plaintext $P = UNIPARJEMBER$, by mean the two algorithms above we have a ciphertext $\Phi = HWWKAYZMBVFX$. The cryptosystem process can be described in the following tables.

Table 9. Encryption process.

Plaintext	U	N	I	P	A	R	J	E	M	B	E	R
P_i	20	13	8	15	0	17	9	4	12	1	4	17
K_{1i}	12	5	9	13	17	21	14	5	9	13	17	21
$P_i + K_{1i}$	32	18	17	28	17	38	23	9	21	14	21	38
K_{2i}	1	4	5	8	9	12	2	3	6	7	10	11
$(P_i + K_{1i}) + K_{2i}$	33	22	22	36	26	50	25	12	27	21	31	49
Φ_i	7	22	22	10	0	24	25	12	1	21	5	23
Ciphertext	H	W	W	K	A	Y	Z	M	B	V	F	X

Table 10. Decryption process.

Ciphertext	H	W	W	K	A	Y	Z	M	B	V	F	X
Φ_i	7	22	22	10	0	24	25	12	1	21	5	23
K_{2i}	1	4	5	8	9	12	2	3	6	7	10	11
$\Phi_i - K_{2i}$	6	18	17	2	-9	12	23	9	-5	14	-5	12
K_{1i}	12	5	9	13	17	21	14	5	9	13	17	21
$(\Phi_i - K_{2i}) - K_{1i}$	-6	13	8	-11	-26	-9	9	4	-14	1	-22	-9
P_i	20	13	8	15	0	17	9	4	12	1	4	17
Plaintext	U	N	I	P	A	R	J	E	M	B	E	R

4. Conclusions

We have determined the upper bound of $rvac(C_n)$. Since determining the $rvac$ of a graph is considered a Non-Deterministic Polynomial Time-complete problem, the exact value of any graph G remains unsolved. As a result, we offer the following open problems.

a) Find the exact value of the cycle graph's rainbow vertex antimagic connection number.

b) Find the exact value of any graph's rainbow vertex antimagic connection number.

5. Acknowledgments

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