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The Marginalization of 11th-Grade Urban African Students in Proof-Related Pedagogy: An Emancipatory Perspective

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The development of urban students' mathematical proving ability is a goal of several curricula frameworks, including some located in the southern hemisphere. However, in achieving this goal, most curriculum frameworks do so from a Western worldview, which is characterized by competition and the role of the individual. The purpose of this study was to use the emancipatory lens to critique the use of a quantitative methodology in favor of the Ubuntu worldview, a methodology grounded in indigenous African epistemologies, particularly storytelling. To this end, I analyzed data drawn from the administration of a survey questionnaire to a conveniently selected sample of 135 11th-grade students enrolled in three separate high schools from ethnically and socioeconomically diverse communities in the eThekwini metropolitan area of South Africa. The context for the argument in this study was provided by correlating students' understanding of functions of proof (verification, explanation, communication, discovery, and systematization) with their argumentation ability, two variables often considered as the key limiting factors for meaningful learning of mathematical proofs. The poor results obtained from a quantitative analysis of data using Western perspectives highlight the emerging need for finding postcolonial methodologies that are sensitive to ethnic issues in addition to language and gender issues. In addition, the inadequacy of the current mathematics curriculum to serve the linguistic and gender needs of urban African students became apparent. This increases the need for sub-Saharan instructors to have knowledge to pursue emancipatory instruction. The key contribution of this study to the field is that it sheds light on the marginalization of African students in learning mathematical proof and related concepts from Western perspectives rather than conducting instructional practices in the Global South's terms; the scope of the effort may explain why research efforts in this line of work have not been documented extensively in literature.

KEYWORDS: correlation, emancipatory perspectives, Ubuntu methodology, urban high school students, Western worldview

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The teaching and learning of mathematics takes place in different cultural contexts around the world. The South African mathematics curriculum framework, the setting of this study, does not, in Harris et al.'s (2020) terms, foreground the "different cultural ways of knowing" (p. 128), yet urban mathematics education is a rich research area with significant policy implications (Tate, 2008). The argument permeating this paper is that the failure to foreground the pedagogy of South African mathematics classrooms in Ubuntu philosophy engenders the perpetual marginalization of urban South African students' linguistic, cultural, and gender issues, especially so in relation to functional understanding of proof and argumentation ability. Given that the term *Ubuntu* has proven hard to define for academics because of the existence of partially corrupted versions of the term (Seehawer, 2018), a comprehensive philosophical exposition on Ubuntu is beyond the scope of this paper, save to say that in this study Ubuntu refers to an ontology and way of living that significantly differs from those of a Western worldview in that care for others and cooperation are valued more highly than competition and individual advancement (Brock-Utne, 2016; Keane, 2008).

Like in the United States (Matthews, 2008), there is not only a vacuum of urban scholarship in South Africa, but programs meant to address the issues of disadvantaged or underprepared students are fraught with contradictions. For instance, while official language education policies in the public school systems seek to accommodate disadvantaged students, the dominance of English has a substantial impact on linguistic minorities (Harper, 2011). In situating the study within literature on urban scholarship, I describe the economic, political, and racial conditions under which the study took place as important. This description is important because these conditions continue to affect the achievement of urban South African students in mathematics in general, and in proof education in particular, along racial lines and will put the results of this study into context. The results and implications of this study were numerous. Not only was the investigation of the relationship between functions of proof and argumentation important, but the actual research process formed part of the findings. Both these aspects of the research relate to the Ubuntu worldview.

At a broader level, Ubuntu is encapsulated by the principle that "Umuntu ngumuntu ngabantu," which in the IsiZulu[1] language means, "to be a human being is to affirm one's humanity by recognizing the humanity of others and, on that basis, establish humane relations with them" (Naicker, 2015, p. 3). The focus of the present study was on the relationship between students' functional understanding of proof and their argumentation ability, as well as to point to the importance of Ubuntu as an indigenous southern African research paradigm in the context of what a Eurocentric curriculum does to African students' performance in mathematical argumentation, in particular.

[1] IsiZulu is one of the indigenous languages of South Africa.

Urbanization and Urban Africans

To situate this study within the literature on urban mathematics education, I begin by attempting to define the term *urban* as viewed in the South African context. Although no consensus exists for the term *urbanization*, it is important to at least describe how it is understood for the purposes of this paper. Matthews (2008) lamented the relegation of the term "urban" to an umbrella term that indiscriminately denotes African American, Hispanic, immigrant, or low-income students. He provided a sanitized, tentative definition of "urban" as not only a term that considers geographical context but also encompasses "the lives of people within the multitude of cultural, social, and political spaces in which mathematics teaching and learning takes place" (Matthews, 2008, p. 2). For the context of this study, in South Africa when we say "urban students," we mean students schooling in metropolitan areas, cities, suburbs, towns, and townships, regardless of racial composition (Baffi et al., 2018).

I join Henslin (2017) in defining urbanization as a process that entails a movement of people into cities. The urban population in South Africa accounts for about 65% of the country's total population, making it one of the most urbanized countries on the African continent (Turok, 2014). African adults and their families are currently relocating from rural reservations to urban settings to have access to educational training and employment. The end result of this movement is the assimilation of African people into the dominant European culture thus rendering students unable to engage in a mathematics curriculum that reflects Ubuntu values and perspectives. Urban sprawl resulting from the relocation of African people from tribally controlled land (i.e., reservations) limits the opportunity for Ubuntu practices in an urban setting.

However, Ubuntu has its own problems, particularly in its inability to affirm women; the implications of Ubuntu on gender seems to be conveniently ignored. For instance, Manyonganise (2015) described Ubuntu as partially oppressing women in that, through the Ubuntu worldview, when a baby girl is born, it is tantamount to no human being having been born at all. In contrast, baby boys earn humanness from the moment of their birth. Nonetheless, Manyonganise (2015) goes on to point out that purely regarding Ubuntu as patriarchal would not only undermine its transformative potential but also its actualization "so that it ceases to be steeped in the past, especially in gender relations" (p. 6).

The urbanization of Africans in South Africa is fraught with slavery, cheap and docile labor, and racism, all intended to alienate them from their land (Vellem, 2014). In emphasizing the material issues underlying urbanization, Mabin (1992) argued that the urbanization of Africans in South Africa was a consequence of the discovery of gold and the resultant economic expansion. However, the trend continues to this day. The mushrooming of shantytowns on the periphery of South Africa's major cities is evidence of this claim. Møller (2007) pointed out that, for example, Cape Town doubled its size over the previous 20 years (1987–2007). He further pointed out that large-scale rural-to-urban migration reshaped the country's settlement patterns in the early twenty-first century. Rather than radically challenging the apartheid urban landscape, the neoliberal policies of the former "liberation movement," which is now the ruling party, the African National Congress, tend to reinforce race and class segregation (Maharaj, 2020). In fact, the financial circuits of capital exacerbate capitalism's intrinsic economic, social, and environmental inequalities in South Africa (Bond, 2013).

Similar to trends in indigenous Māori schools (Trinick & Stevenson, 2010), the academic achievement of urban African students who come from poorer socioeconomic backgrounds is lower than that of students from higher socioeconomic backgrounds, such as Whites, Indians, and Coloreds. Students from higher socioeconomic backgrounds come from families who can afford to send their children to elite private schools that charge exorbitant fees. These inequalities are perpetuated by a capitalist economic system inherited by the democratic government. Although South Africa achieved democracy in 1994 after the end of the apartheid (Afrikaans for "apartness," used to describe a system of separate, racial, and ethnic development based on a White supremacy stance) era, the economic system was not changed. As Schneider (2003) put it:

> Although the political environment in South Africa is vastly improved, economic apartheid still exists: the economic divisions along racial lines created by apartheid are still in place today. Despite these divisions, neoliberal economists continue to press for a largely unregulated market system, which is unlikely to improve the lives of most black [African] South Africans. (p. 23)

In South Africa, four official racial groups were declared: (Black) Africans, Whites, Indians, and Coloreds. "Africans" refers to Black people indigenous to South Africa, "Whites" refers to those born in this country but are descendants of immigrants from Europe, and "Indians" refers to those with origins from the country of India. The term "Coloreds," as explained by Petrus and Isaacs-Martin (2012), is used to identify a specific group in South Africa and is most often attributed to students popularly perceived as being of mixed racial and ethnic descent who, over time and due to specific historical, cultural, social, and other factors, have undergone various changes in their perceptions of their identity as Coloreds.

Although South Africa has indigenous nations—for example, the Koi Sans, AmaZulu, AmaXhosa, and so on—its urban classrooms are multicultural; they are not organized along lines of nationality. Put differently, there are no urban schools deliberately designated to attend to a specific indigenous community. As a consequence, all schools follow a neoliberal, capitalist urban system whose curriculum is structured along Western contexts and delivered in English. However, standard South

African English is not the native language of many African students. The absence of mathematics registers in the various indigenous languages of South Africa presents a huge challenge for teachers and teacher educators at a time when, as Warren et al. (2007) have pointed out, the discrepancy between home and school language directly impacts African students' achievement in mathematics in the long term. Although Ubuntu can be viewed as one way of understanding the struggles of urban African students in a Western-oriented mathematics curriculum, mathematics education research requires theories that explicitly address the urban African student. In this regard, I echo Tate's (2008) sentiment that "if urban mathematics education research is to be taken seriously, this kind of theoretical and empirical interaction should be the norm" (p. 6).

An Economic, Political, and Racial Context of the Study

The still unresolved idea of South Africa has a long history whose full account is beyond the focus of the present study. However, its brief description is, nonetheless, possible. The settlement of the Dutch (who later became known as the Afrikaner nation) at what is now known as the Western Cape was followed by an agreement with the British to construct South Africa into a White state in which Africans featured in the discussions as providers of cheap labor (Ndlovu-Gatsheni, 2018).

Worth mentioning here is that about 80% of the South African population is African, yet the Western worldview dominates the education system. As Ndlovu-Gatsheni (2018) asserted, race rather than class is still an invisible but active organizing principle of informing unchanging patterns of inequality, poverty, and a Eurocentric curriculum. Accordingly, in the Europeanization of Africans by Western education, the Ubuntu worldview has been deliberately suppressed, even under democratic rule. Put more emphatically, a predominantly African government has embraced European education and economic systems and demands them even in the postapartheid era.

The consequences of this has proven to be dire for the African child, especially in the learning of the concept of proof. Proving, like mathematics, is a social activity in which, for example, one of its functions is to serve as a means to communicate mathematical knowledge. The communication function of proof means that students must make sense of the arguments embedded in a proof, learn the mathematical language, and "transmit" this knowledge publicly (de Villiers, 1990). Clearly, they need to present arguments. This is where the problem lies for urban African students. For most of them, English is a second, third, or even fourth language. Certainly, this hampers their articulation of mathematical ideas and the elements of the proving process (e.g., patterning, conjecturing, exemplification, and generalization), which in turn affects their achievement in mathematics. An alternative, however, exists.

The alternative I am referring to in the preceding paragraph is Ubuntu, widely recognized as being the African's worldview (Church, 2012). According to Ndlovu-Gatsheni (2018), the use of Ubuntu has gained traction in academic literature over the past years, partly because of the move towards foregrounding African constructions about the world and unravelling of the legacy of colonialism. From a mathematical proof and argumentation perspective, urban students need to come together in mutually supportive and respectful ways as they make arguments to prove a theorem. However, I ask a more general research question: How can an urban African student engage in mathematical argumentation successfully when his or her indigenous language has been relegated to an inferior position and thus never been considered to be a suitable medium of instruction?

Probably the most sensible answer to this question is provided by Brock-Utne (2016), who argued that things will not change because the use of the former colonial language serves to keep a tiny minority at the top. Hence, she further argued, the majority of students who do not speak the colonial language "either drop out of school or sit there year after year learning nothing except self-contempt" (p. 33). A similar concern has been raised by Garcia-Olp et al. (2019), who argued that the teaching of mathematics to indigenous students through the European viewpoint rather than through an indigenous lens limits their participation in the mathematics field after high school.

Brock-Utne and Desai (2010) argued that the majority of school teachers and students prefer the so-called international (i.e., English) language to continue as the language of instruction even though they can hardly understand it. Although I tend to agree with Brock-Utne and Desai, the situation is different for students once they are confronted with more cognitively challenging concepts in mathematics. They yearn for their indigenous languages. From my own existential experience in my mathematics education classes, I would often hear students pleading to provide their answers or to express key ideas in IsiZulu: "Ngingayisho ngesiZulu?" (Can I say it in IsiZulu?). That said, I want to echo Brock-Utne's (2016) sentiments that for urban African students to achieve in mathematics, the indigenous languages these students speak must be brought into the classroom.

Writing From a Problematic Positionality

Learning mathematics is a social activity. As such, the social context in which its teaching and learning takes place shapes an individual's experiences. As is common in many African settings, the context in which communication between children and adults differs from that of their White counterparts. Therefore, children's learning experiences may also vary (Mercer & Littleton, 2007). However, this variety is impoverished because of the absence of literature on the functional understanding of proof and argumentation from Ubuntu perspectives. The present study is written from the perspective of a Western trained academic man (not by design though) who lives and works in the multicultural, sub-Saharan African continent. Perhaps this explains why I did not situate this study within an Ubuntu framework but have only done so retrospectively. As far as I can ascertain, this study is the first to incorporate elements of both Western and African views on the relationship between functional understanding of proof and argumentation.

The adoption of this approach to the study stems from a reminder that an honest investigation was the one that took into account not only my experiences as a South African but also the historical factors that continue to influence my research work. My Africanness and previously designed disadvantages have given me limited opportunities in life—malnutrition from infancy, poor health care, poor education, fewer opportunities to see other parts of the world, daily fear and anxiety, and so on. In the recent past, I have come to view the difficulties with which African students learn mathematics—the proof concept in particular—as more of a sociocultural issue than a cognitive matter. This article challenges existing theories to begin to rethink the teaching and learning of the concept of proof as a space of ontological struggle for urban students of African descent. Although the vast majority of African students live in urban environments today, what the mandatory curriculum designers ignored is the ontological orientation that such students bring with them to the classroom. As Harris et al. (2020) pointed out, this is a deficit approach in that it considers White, middle-class, monolingual English as the norm against which students of all backgrounds are to be measured.

As is the case with all researchers, my life experiences informed various aspects of this study. In this subsection I provide insights into the hypotheses derived from my experiences with the concept of proof. All these aspects influenced the research process (for example, research questions, sample, methods, interpretations, and so on). For instance, though some schools with predominately African students tend to achieve a 100% pass rate in their Grade 12 examinations, the quality of these passes tend to be weaker than those of higher socioeconomic schools with similar pass rates. Hence, I chose to compare the quality of learners' functional understanding of proof and argumentation with the economic, political, and racial contexts in mind.

Overall, despite centuries of engaging with proof, mathematicians, philosophers, educators, and scientists still find the notion of proof as nebulous and contested as ever. It can be argued that this is because proof is as much a philosophical construct as it is a mathematical one. Evidence of the nebulous nature of proof is found in difficulties in proof construction as encountered by both high school and undergraduate students, including preservice teachers. However, in proving, students need to construct arguments. These two constructs are important to study because they define the mathematical practice.

The Functions of Proof and Argumentation Under a Western Perspective

Attempts to teach proof to high school students (frequently during short periods of time) have been unsuccessful (Hadas et al., 2000; Pedemonte, 2007). Given that the "failure to teach proofs seems to be universal" (Hadas et al., 2000, p. 128), functional understanding of proof and argumentation—activities Edwards (1997) referred to as the "territory before proof" (p. 188)—need to be part of the mathematical instruction that precedes and supports the development of students' practice with proofs. Along this line, Marrades and Gutiérrez (2000) argued that it is vitally important for both teachers and researchers in the area of proof to determine students' understanding of functions of mathematical proof in order to gain insight into their attempts to solve proof problems. The general motivation for this study came from the need to measure students' understanding of the functions of proof in mathematics and argumentation quality because lack thereof contributes to difficulties with learning proofs meaningfully (e.g., de Villiers, 1990, Healy & Hoyles, 1998). As de Villiers (1994) put it:

> Extensive experience with children in interview and classroom contexts seems to indicate that many of their problems with mathematical content and processes often do not lie so much with poor instrumental proficiency nor inadequate relational or logical understanding as in a poor understanding of the usefulness or function thereof. (p. 11)

de Villiers' (1990) model describes five functions that proof performs in mathematics: verification, explanation, communication, discovery, and systematization. Thus, proving in the mathematics classroom includes not only engaging in its cognitive functions (explanation and discovery) but also in its social (verification and communication) and epistemological (systematization) ones. According to Hanna (2000), the explanatory function of proof helps to make mathematics meaningful and understandable. This "enlightening" or "illuminating" function brings argumentation into the arena. Support for this view comes from Hanna's (2007) statement that "an argument presented with sufficient rigor will enlighten and convince more students, who in turn may convince their peers" (p. 22). Noteworthy is that argumentation may be of low or high quality based on the absence or presence of rebuttals (i.e., counterexamples) in an argument (Osborne et al., 2004).

However, as Brock-Utne and Skattum (2009) pointed out, one other reason why learning and teaching of proof has been difficult, particularly for African students, which is often not given the attention it requires, is the student's indigenous language. They further pointed out that despite more than 40 years of emphasis by educational linguists that the use of a language in which students (and teachers) do not possess the necessary proficiency for cognitive academic development has disastrous consequences—as it does for the economic, social, and political development of a country—the schooling system has not placed a premium on this emphasis.

Thus, students' battle with optimal performance in proof and argumentation can be ascribed to either their poor understanding of the text or their difficulty expressing themselves clearly in English rather than to their cognitive ability. This calls for a restructuring of teaching and learning of mathematics, in general, according to the Ubuntu worldview. This worldview can be harnessed to improve the learning of proof functions and argumentation.

However, it is not clear to what extent students' functional understanding of proof affects the quality of argumentation made by them. In addition, prior research suggests that teaching approaches may vary across countries (Alexander, 2020; Mercer & Littleton, 2007). Moreover, the majority of the existing literature on mathematical proof and its functions as well as argumentation stems from studies conducted in European and North American countries, whereas systematic research on proof and argumentation across international contexts remains limited. Learning of proof is embedded in and influenced by socioeconomic factors (Mercer & Howe, 2012). Therefore, students' understanding of proof functions and argumentation ability identified in these countries may not be generalized to other educational contexts, especially urban settings. Still, research on the relationship between students' understanding of proof functions and argumentation ability is limited and, to my knowledge, no research exists on this hypothesized relationship on the African continent, especially in sub-Saharan Africa.

I am yet to find a study that takes into account the interplay among economic, political, and racial conditions under which students learn proof. The current study reported in this article presents an opportunity to test the universality of Western education research findings. It is against this background that this study presents a unique contribution to the field. In addition, by analyzing this relationship, the findings expand previous research to indicate how the lack of mathematics registers in indigenous languages impacts urban African students' understanding of functions of proof and, by extension, development of argumentation skills.

The remainder of this paper is presented in six parts. The first deals with a review of literature related to the functions of proof, argumentation, argumentation quality, and Ubuntu methodology that can allow the discipline to respond to social change in proof-related pedagogy while still retaining the ideals of scientific rigor. The next considers the emancipatory lens as a theoretical foundation of the study. What follows next is a discussion of the quantitative methodology. Then the results are presented and discussed with reference to the emancipatory framework. The article concludes with a reflection on the results.

Review of Related Literature

The review of literature is conducted for the reader to understand the background to the argument for a need for social change and is done so from the Western worldview. Put slightly differently, this approach is deliberately taken to provide the basis for arguing for a curriculum that acknowledges the importance of a pedagogy that foregrounds the linguistic and cultural uniqueness of the urban African student.

Functional Understanding of Proof

The use of the phrase "functional understanding of proof" is meant to refer to understanding the role, function, purpose, or value of proof in mathematics rather than to its application to the real world outside of the mathematics discipline. As far as I could ascertain, only Healy and Hoyles (1998) have attempted to capture students' functional understanding of proof. They used an open-ended survey questionnaire on which students were to write about everything they knew of proof and its functions in mathematics. Further, they investigated the influence of statutory instruction on the nature of proof following suggestions that such instruction could contribute to deeper understanding of the notion of proof itself and thus improve its didactic treatment in the classroom. They found that the function of proof as a means to verify was prevalent.

Hanna (1995) posited that learning about the functions of proof in mathematics is of primary importance to mathematicians. I contend that the value of understanding the functions of proof in mathematics needs to be reflected in the mathematics classroom itself if students are to gain insight into the nature of mathematics. However, as already mentioned, argumentation is a process that brings the explanatory power of proof to bear.

Argumentation

In distinguishing between an "argument" and "argumentation," like Blair (2012), I see the former as a "set of one or more reasons for doing something" (p. 72). Although Pedemonte (2007) correctly argued that there is no common definition for the concept of argumentation in the field of mathematics education, the current study adopted van Eemeren et al.'s (2013) definition of argumentation as "a verbal and social activity of reason aimed at increasing (or decreasing) the acceptability of a controversial standpoint" (p. 5), because it is compatible with classroom contexts advocated by reform statements (e.g., Common Core State Standards Initiative, 2010; Department of Basic Education [DBE], 2011; National Council of Teachers of Mathematics, 2009). Perhaps it is important to note from this distinction that an argument is the product of the process of argumentation. Further, argumentation is not used to refer to a debate, although debate is one form of argumentation, but rather to a process of thinking and dialogue in which students construct and critique each other's arguments (Nussbaum, 2011).

Toulmin (2003) presented a model, generally referred to as Toulmin's argument pattern (TAP), to describe the structure of an argument and how its elements are related. The TAP model consists of six interdependent components: claim, data, warrant, backings, qualifiers, and rebuttals. In addition, TAP is a model that has been extensively used in instructional practices as a tool to construct mathematically sound arguments (Osborne et al., 2004; Venville & Dawson, 2010). In the context of mathematics lessons, the use of TAP has mainly concentrated on the description of small group discussions among students (e.g., Inglis & Mejia-Ramos, 2009; Knipping, 2003; Krummheuer, 1995, 2000; Pedemonte, 2007).

Briefly, the basic idea of this modified model (Figure 1) is that a statement, claim, or conclusion is justified by providing a ground (as shared by the mathematical community). According to DeJarnette and González (2017), making and justifying a claim is a fundamental aspect of doing mathematics. For this study, "ground" refers to a datum, warrant, or backing provided by the interlocutor in justifying their claim. This stance finds support in Osborne et al.'s (2004) assertion that claims, rebuttals, and justifications are the salient features of argumentation that are critical for developing and evaluating practice with argumentation in the classroom. In addition, grouping these elements into "ground" circumvents the ambiguity embedded in them, as any claim, rebuttal, or justification is referred to simply as "ground" (Osborne et al., 2004).

A warrant is a proposition that connects a datum and claim. "Rebuttal" is taken to mean a statement that seeks to show the weakness in a ground. Worthy to note is that argumentations with rebuttals are of better quality than those without given that rebuttals make substantive challenges to the grounds as they refute their applicability (Osborne et al., 2004). Figure 1 depicts a typical argumentation in proving: "The sum of interior angles of a triangle is 180 degrees." TAP has been adapted by many in mathematics education research (e.g. Krummheuer, 1995; Rasmussen & Stephan, 2008; Yackel, 2001) as a lens to frame arguments and to analyze the quality of a specific mathematical argument.

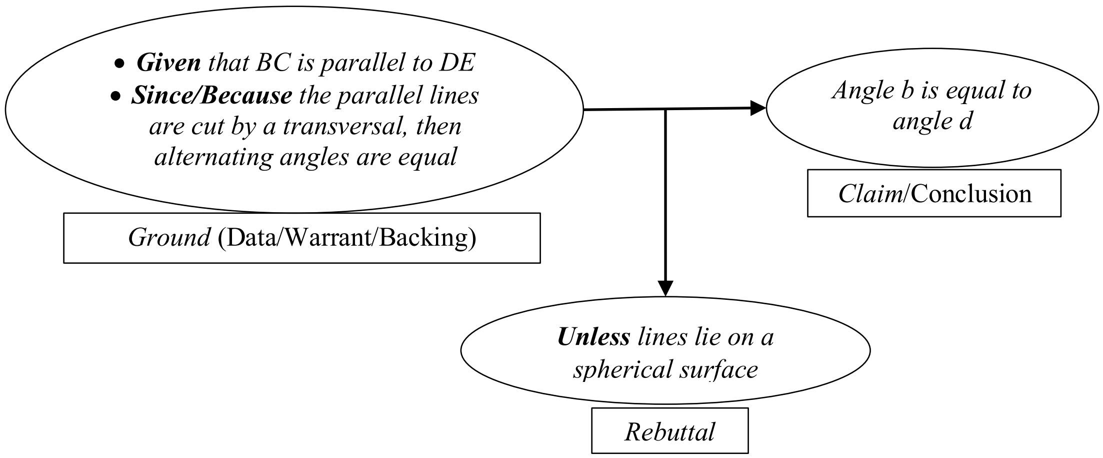

Figure 1. The Modified Toulmin's (2003) Argumentation Scheme

Not every one of these TAP components is used in every argument. For instance, given the tentative nature of mathematical knowledge and the fact that for students the knowledge being constructed is new, qualifying phrases such as "most probably" or "presumably" are omitted and therefore implied in a claim.

There is scarcity of empirical evidence on the influence of students' functional understanding of proof on the quality of argumentation. Knipping (2003) commented that it would be interesting if the relationship between functions of proof and argumentation structures were examined. Alibert and Thomas (1991) discussed the relationship between functional understanding of proof largely from a theoretical basis rather than conducting a systematic investigation. They believed that students' distorted understanding of the functions of proof is a direct consequence of instruction that presents proof as a finished product, an approach that deprives students of opportunities to be partners in mathematical knowledge construction. The present study, therefore, sought to expand on previous research on functional understanding of proof and argumentation by disaggregating functional understanding of proof and quality of argumentation into their association with a variety of indicators.

Argumentation Quality

Several studies on argumentation have focused on identifying, creating, and evaluating argument structures (Aberdein, 2013; Mariotti, 2006; Pedemonte, 2007). The underlying theme of the findings of these studies is that the argumentation process enables the shifting of mathematical authority and ownership from the textbook or teacher to the community of students, who become producers of mathematical knowledge (Bay-Williams et al., 2013; Rumsey & Langrall, 2016. In addition, the power of argumentation is that it bears resemblance to how mathematical knowledge is constructed in the practice of mathematicians. Although argumentation is seen by mathematics education documents and researchers in mathematics alike as vital in the learning of mathematics, little research has focused on measuring the quality of argumentation in students across the school grades. Given this background, the purpose of this study was to take a step towards addressing this scarcity by examining the quality of students' arguments along with the mathematics inherent in a proof-related task.

Recent mathematics curriculum reform statements have framed investigations as a key feature in the learning of mathematics in high schools (e.g., National Council of Teachers of Mathematics, 2009; DBE, 2011). These efforts are supported by research suggesting that formulating arguments supports learning of mathematics (Jahnke, 2008; Rumsey & Langrall, 2016). In addition, current research in learning, teaching, and assessment has repeatedly pointed to the importance of eliciting students' preconceptions in instruction (National Research Council, 1993). I argue that these approaches, which differ from the more dominant knowledge transmission method, are appropriate, as they seek to create classroom environments that resemble the practice of mathematicians (i.e., abstracting, conjecturing, proving, and seeking counterexamples).

The knowledge transmission method relates to the notion that the "expert" (teacher) is required to fill students' minds with information to be memorized and regurgitated when required (Thomas & Pedersen, 2003). Ricks (2010) bemoaned the character of school mathematics by pointing out that it deprives students of the natural socializing appeal of mathematical activity. In contrast, the methods advocated by curriculum reform efforts underscore investigation as a mathematical activity to reflect mathematics as a human activity. However, conducting mathematical investigations involves high levels of mathematical reasoning (Desforges & Cockburn, 1987). The benefit that accrues with investigations is much more than the sharing of mathematical ideas and strategies in that as students "prepare to present their work, and as they think about how they will communicate their work and anticipate their classmates' questions, their own understanding deepens" (Fosnot & Dolk, 2001, p. 3).

The Ubuntu Methodology in Mathematics Education

The preceding review highlights the Western epistemologies in mathematics. However, these ways of knowing are in sharp contrast to how an African child constructs their knowledge. Therefore, contemporary research methodologies that are used to investigate African students' knowledge cannot be grounded in the Western worldview. As the results section in this study will reveal, it is disingenuous to describe the work of an urban African child from this worldview. One of the aims of this study is to argue for Ubuntu as a viable methodology to study the African child's performance in proof-related concepts, at least.

As already mentioned, the essence of Ubuntu is that knowledge construction is a communal effort as expressed through indigenous languages. Ubuntu defines the individual in terms of their relationships with the community in which the individual exists; in this case, the individual student flourishes but assists, and interacts with, others (Muwanga-Zake, 2009). The intent of this paper is not to project an Ubuntu research methodology as categorically oppositional to conventional methodologies, but rather on using empirical evidence to advocate for an approach to research that is grounded in indigenous African epistemologies. I provide one essential instance to support the notion that the Western way of knowing is indeed incompatible with an Ubuntu methodology that can allow the discipline to respond to social change in proof-related pedagogy while still retaining the ideals of scientific rigor.

Although providing a rebuttal in argumentation is viewed as part of critical thinking, defined here as a set of cognitive skills aimed at strengthening a hypothesis or claim in decision making in complex situations (Lubben et al., 2010), it is inconsistent with African cultural practice. For instance, it is taboo for children to talk back in their conversations with adults. The Ubuntu worldview emphasizes the building of consensus and avoidance of confrontation by opting for utterances such as "I have another idea which I will explain" (Lubben et al., 2010). Noteworthy is that this approach to argumentation is not confined to students alone. Scholtz et al. (2008) found that teachers with an African cultural background avoid rebuttals and use alternative moves in their arguments, such as questions and seeking consensus. In addition, the lack of mathematics registers in indigenous languages perpetuates a mathematics field that adopts the notion that African students only know the functions of proof and argumentation when learning them from Western epistemological orientations, thereby masking the Ubuntu ways of knowing. This increases the need for sub-Saharan instructors to have knowledge to pursue emancipatory instruction.

The next section provides a lens through which I approached this paper. The struggles of the urban African student in mathematics classrooms in the context of functions of proof and argumentation are characterized as tantamount to an oppressive regime. As such, it is reasonable to argue for a need to seek emancipation for these students. The results will clearly support this stance.

The Emancipatory Framework

The emancipatory framework provided a lens through which the correlations between functional understanding of proof and argumentation quality after controlling for gender was explored and discussed. This lens was used throughout the study to argue for the inclusion of indigenous languages and the Ubuntu worldview in mathematics education as well as to critique and raise methodological issues associated with the use of a quantitative methodology.

The central feature of emancipatory research is its intent to challenge inequities and disrupt the status quo where necessary; it is responsive to social issues in that it has oppression as its central focus. Thus, the key objective of this framework is social change, fostering an ideology based on the paradigm that knowledge is constituted in a sociocultural context in which research is conducted (Rose & Glass, 2008).

The term *emancipation* defines a process of resistance aimed at the transformation of existing oppressive structures and the creation of alternative structures that are more progressive (Ali, 2002; Inglis, 1997). As Inglis (1997) put it, the process of emancipation involves "critically analyzing, resisting and challenging structures of power" (p. 4). It is incumbent upon the mathematics education research community to argue for social justice because, according to Gorelick (1996), the hidden nature of societal oppression can obscure a student's awareness of their marginalization. As Ali (2006) aptly noted:

> Self-report quantitative measures can also be problematic regarding the conclusions we may draw from the data they produce. For example, while it is presumed that participants' self-reports are truthful, the reporting in which participants engage may not fully capture their experience. (p. 32)

According to this lens, if oppression was more fully understood, necessary social change could be achieved through society taking political action (Ramazanoğlu, 2002). Thus, an emancipatory lens is important because it raises the consciousness of a people who are located in marginalized and oppressed positions. In the current study, the author's aim was to focus on the emancipatory potential of the Ubuntu methodology as a tool to challenge the current, dominant Western quantitative methodology, which is a mechanism for the perpetuation of economic exploitation and thus effectively deleting the identities of students of African origin in mathematics education.

Given this background, the purpose of this study was to use the emancipatory lens to critique the use of a quantitative methodology in favour of the Ubuntu worldview, a methodology grounded in indigenous African epistemologies, particularly storytelling. The context for the argument in this study was provided by correlating students' understanding of functions of proof (verification, explanation, communication, discovery, and systematization) with their argumentation ability, two variables often considered the key limiting factors for *meaningful learning*[2] of mathematical proofs. The following research questions guided the investigation of this relationship:

1. What does the Western method of analysis reveal about the relationship between students' functional understanding of proof and argumentation quality?

2. What is the extent to which the Western worldview marginalizes urban African students' performance?

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[2] According to Nafukho (2006), meaningful learning takes place in the languages that students speak. Thus, learning to prove and argue in a language in which students are not adequately proficient has negative consequences for them.

Methods

The quantitative study presented here is part of a larger research project that aimed to examine the interplay between self-efficacy in proving, functions of proof, proof construction, and argumentation in high school mathematics. The project began in 2015 and reached its finality in 2019.

Sample Design

The prediction study reported herein followed a correlational research design, which is important in understanding teaching and learning for several reasons. Correlational designs not only enable the preliminary identification of possible factors that influence students' ability to produce high-quality argumentation in the construction of mathematical proofs but also variables that require further investigations in the relationship (McMillan & Schumacher, 2010). Data were drawn from 135 students in Cambridge College, Ayanda High, and Tswelopele High (pseudonyms), randomly selected from a sample of 10 Dinaledi schools in ethnically and socioeconomically diverse communities in the urban district of the eThekwini metropolitan area.

In the pursuit of increasing the participation and performance in mathematics and physical sciences of historically disadvantaged students in South Africa, the DBE established the Dinaledi School Project in 2001 (DBE, 2009). The initiative involved selecting certain secondary schools for Dinaledi status that demonstrated their potential for increasing student participation and performance in mathematics and science (DBE, 2009). These schools were provided with resources (e.g., textbooks and laboratories) to improve the teaching and learning of mathematics and science. The ultimate intention was to improve mathematics and science results and thus increase the availability of key skills required in the South African economy (DBE, 2009).

The sample size of the current study was appropriate for a correlational study in that it was higher than Creswell's (2012) estimated threshold of 30 participants. The rationale for selecting Dinaledi schools for the investigation was that these schools were monitored by a team that included senior education department officials and individuals with an interest in educational research. Sample characteristics are in presented in Table 1. The purpose of collecting sociodemographic data was to be able to adequately describe the sample.

Table 1
Sociodemographic Characteristics of Sample

Characteristic		**Female** $n = 78$ (54.1%)	**Male** $n = 57$ (45.9%)	**Total**
Age				
	M	16.42	16.85	16.64
	SD	1.28	1.37	1.97
School Type				
	Fee-Paying	28	24	52
	No-Fee	44	39	83
Race				
	White	3	5	8
	African	44	39	83
	Indian	18	16	34
	Colored	7	3	10
Socioeconomic Status				
	Low	46	39	85
	Middle	23	19	42
	High	3	5	8

Measures

The independent variables that were hypothesized as influencing (predicting) the dependent variable were as follows: verification, explanation, communication, discovery, and systematization. Respondents' functional understanding of proof was assessed with the Learners' Functional Understanding of Proof (LFUP) scale, which was adequately developed and validated elsewhere (Mudaly & Shongwe, 2017). The LFUP is a 25-item Likert scale questionnaire whose first section contains items for gathering sociodemographic data, as shown in Table 2. Taking into account Kumar's (2005) guidelines for formulating questions, every effort was made to ensure that simple and everyday language in the questionnaire was used for two reasons.

First, English was not the home language of most of the participants. Second, there was no time allocated for explaining the questions to the participants. Hence, the language used was made appropriate because misunderstanding of the questions would have resulted in irrelevant responses. The overall Cronbach's alpha of the LFUP scale was .812.

Table 2
The Structure of the LFUP Questionnaire

Category	Description	Number of items
Sociodemographic	Code; Gender; Class; Home Language; School Type, Race; Socioeconomic Status	7
Verification function	Five-point Likert subscale assessing understanding of proof as a means to verify	3
Explanation function	Five-point Likert subscale assessing understanding of proof as a means to explain	5
Communication function	Five-point Likert subscale assessing understanding of proof as a means to communicate	5
Discovery function	Five-point Likert subscale assessing understanding of proof as a means to discover/invent	5
Systematization function	Five-point Likert subscale assessing understanding of proof as a means to systematize	7

The second section of the LFUP questionnaire consists of a 5-point scale (ranging from 1 = *strongly disagree* to 5 = *strongly agree*) used to judge students' functional understanding of proof. The scores on the LFUP scale were organized in interval categories, which was amenable to parametric statistical analyses (Creswell, 2012). Negatively worded items received a mean variance value of less than 2.5, neutral items received a mean variance value of 2.5 to less than 3.5, and positively worded items received a mean variance of value of 3.5 or higher. A sample of the items under the explanation function is shown in Table 3.

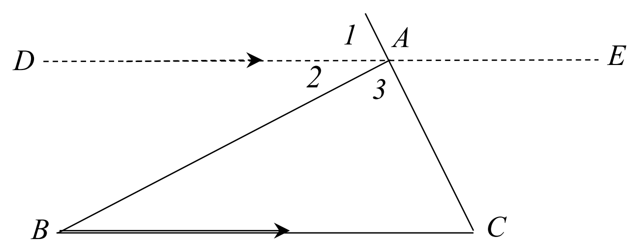
Table 3
An Extract of Items of the Explanation Subscale on the LFUP Instrument
(*n* = 135)

	Item	SD	D	N	A	SA
T4	A proof explains what a maths proposition means.	1	2	3	4	5
T5	A proof hides how a conclusion that a certain maths proposition is true is reached.	1	2	3	4	5
T6	Proof shows that maths is made of connected concepts and procedures.	1	2	3	4	5
T7	When I do a proof, I get a better understanding of mathematical thinking.	1	2	3	4	5
T8	Proving make me understand how I proceeded from the given propositions to the conclusion.	1	2	3	4	5

SD = strongly disagree; D = disagree; NO = no opinion; A = agree; SA = strongly agree.

Respondents' quality of argumentation was assessed with the Argumentation Frame in Euclidean Geometry (AFEG) using the mathematical statement that "The interior angles of a triangle sum up to 180º." The AFEG operationally is an index of an argument modelled after TAP designed to compare the quality of argumentation across individuals.

The duration of the questionnaire was 30 minutes. It consisted of prompts as shown in Figure 2. To successfully engage in proving, a student requires a variety of strategies for selecting, recalling, and connecting facts drawn from a rich knowledge base related to the specific geometric task (Magajna, 2011). The simplified version of the TAP model, like Webb and Webb's (2008), makes provisions for students to also think about possible rebuttals to their claims. Thus, the analysis of written argumentation enables the making of judgments of the quality of the arguments themselves, that is, determining what makes one argument better than the other. Students may then provide qualifiers, which is a way to show specific conditions in which the claim is true (Toulmin, 2003).



Please, make ANY statement or claim from the diagram and justify it. Please, think carefully as you argue your points using the guide provided below.

My statement is that ……………………………………………………………………. Claim

My reason for making this statement is that …………………………………………… Warrant

Arguments against my idea might be that ……………………………………………… Rebuttal

I will show the condition under which the claim is true by stating that ………………... Qualifier

Figure 1. The AFEG Questionnaire

The psychometric properties of this analytical tool were assessed to ensure that it was valid and reliable. To achieve content validity, a discussion on the constituent elements of the scheme, coding, and scoring system of respondents' data took place to develop an analytical tool. Two university experts in the field of argumentation who were from outside the university where the author was based were consulted. The internal consistency coefficient of the instrument was calculated as $\alpha = .81$.

Overall, the analytical framework was found to be sufficient for the purpose of the study. Further, these psychometric results suggest that this instrument can used as a reliable assessment and diagnostic tool in instructional practices and mathematics education research. I analyzed a student's written argumentation in which he or she engaged in a "social interaction" with himself or herself, in line with Aberdein's (2009) definition of argument as "an act of communication intended to lend support to a claim" (p. 1).

Analysis

The data contained no outliers, and there was no multicollinearity among the predictors. This was assessed by checking the correlation coefficients, the predictor variables, and argumentation quality (Table 4). In line with Nunnally and Bernstein's (1994) guidelines, correlations were acceptable if they exceeded .30. Some of the correlations were higher than .60. Then, the correlation matrix was further examined for multicollinearity. Although the items had to be intercorrelated, the correlations were not higher than Tabachnick and Fidell's (2013) threshold of .80, because multicollinearity makes the determination of the unique contribution of the items to a factor difficult (Field, 2009). As depicted in Table 4, the association was positive and linear, which means that low (or high) scores on functional understanding of proof were associated with low (or high) scores on argumentation quality.

Table 4

Correlation Matrix of LFUP and AFEG Scores

(n = 135)

Variables	1	2	3	4	5	6
1. ASV	—					
2. ASE	.589**					
3. ASC	.595**	.842**	—			
4. ASD	.439**	.639**	.785**	—		
5. ASS	.614**	.674**	.614**	.741**	—	
6. AFEG score	.386*	.479**	.592*	.376*	.383*	—

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

ASV = average of sum of construct with highest value being 5 where: ASV = verification; ASE = explanation; ASC = communication; ASD = discovery; ASS = systematization.

The scores on the LFUP scale were organized in interval categories, which was amenable to parametric statistical analyses. A five-tiered grading scale was used to assess and characterize students' functional understanding of proof (Table 5). Scores were characterized as naïve (belief that the only function of proof is verification), informed (beliefs about the functions of proof are consistent with those held by contemporary mathematicians), and hybrid (mix of naïve and informed beliefs).

Table 5
The Normative Map Based on LFUP Mean Acores

Classification	**General explanation**	**Mean score range**	
		From	**To**
Unencultured	Naïve	0	<1.5
Poorly encultured	Naïve	1.5	<2.5
Moderately encultured	Hybrid	2.5	<3.5
Highly encultured	Informed	3.5	<4.5
Extremely encultured	Informed	4.5	≤ 5

It is important to note that the task was one that Grade 10 and 11 learners with little exposure to formal proofs could understand. My goal was to make the task mathematically accessible to all participants to maximize learners' levels of response. Similar to other seatwork assessments routinely completed by students, this task may not be the most psychometrically sound assessment of student argumentation performance, but it is closely related to the realities of instruction and learning in most classrooms (Calfee, 1985).

Students' quality of argumentation was the dichotomous (binary; i.e., low or high) dependent variable whose values were to be predicted and therefore only contained data coded as 0, 1, 2, or 3. Table 6 describes how the quality of argumentation was assessed. Argumentation frame with a rebuttal was coded as high. The analysis was performed with the assistance of SPSS Version 24.0.

Table 6
Coding of Argument Components

Argument	Definition	Code Description	Score	Quality
My statement is that …	(Blank)/A *claim* (C) is a conclusion put forward publicly for general acceptance (Toulmin, 2003).	UC (Uncodifiable/ No reply).	0	Low
My statement is that …		C (Claim; conclusion)	1	Low
My reason is that …	A *warrant* is ground (G) provided in justifying the claim.	C+G (Providing ground for claim)	2	Low
Arguments against my idea might be that …	A *rebuttal* (R) is a statement that seeks to diminish the strength of a conclusion (Pollock, 2001).	C+G+R (Refutation of claim/ground)	3	High

Results

Descriptive analysis (Table 7) revealed that the scores on the verification function of proof were spread out about 0.53 above and below the mean. However, this result presented a rather bleak picture of functional understanding of proof among South African Grade 11 students in Dinaledi schools; only approximately 14% of respondents believed that proof has functions other than verification.

Table 7
Descriptive Statistics

	Mean	Std. Deviation	*N*
ASV	2.430	0.530	135
ASE	2.684	0.863	135
ASC	2.835	0.916	135
ASD	3.286	0.656	135
ASS	2.713	1.108	135
AFEG score	1.415	0.901	135

Attempts to interpret the correlation between functional understanding of proof and argumentation quality were hampered by the possible existence of a third variable that may influence the relationship between the two variables. I used partial correlations technique to statistically control and thus nullify the effects of gender (Wilson & MacLean, 2011) as the third or secondary variable on the relationship between the primary variables, namely, functional understanding of proof and argumentation quality. The partialling out of gender was informed by research (e.g., Geary, 1999; Healy & Hoyles, 2000) that suggests that student performance in mathematics tends to be a function of gender.

Because the zero-order correlations have already been analysed above, I considered the section with the partial correlations in Table 8. In the previous section, the significant relationship between functional understanding of proof and gender seemed to suggest that gender has influence in explaining the functional understanding of proof-argumentation quality association. However, the partial correlations section shows that controlling for gender further weakens the strength of the significant relationship between functional understanding of proof and argumentation ability (R = .214, p = .013). Clearly, controlling for gender was justified given that gender was, as shown in Table 8, one secondary variable that seemed to influence the relationship between the two primary variables.

Table 8

Assessing the Influence of Functional Understanding of Proof on Argumentation, Controlling for Gender

Control Variables			LFUP Score	AFEG Score	Gender
-none-[a]	LFUP score	Correlation	—		
		Significance (2-tailed)			
	AFEG score	Correlation	.225	—	
		Significance (2-tailed)	.009	.	
	Gender	Correlation	.171	.089	—
		Significance (2-tailed)	.047	.302	
Gender	LFUP score	Correlation	—		
		Significance (2-tailed)			
	AFEG score	Correlation	.214	—	
		Significance (2-tailed)	.013	.	

[a.] Cells contain zero-order (Pearson) correlations.

Because statistical significance conflates results as it is affected by sample size, effect size was determined. In addition, relying on the size of the effect of a relationship rather than its statistical significance promotes a more scientific approach to the accumulation of knowledge (Creswell, 2012). The multiple correlation coefficient between argumentation scores and covariates combined, *R*, was computed to determine the coefficient of determination (R^2), which is the square of the Pearson product moment correlation coefficient. This was performed to express the proportion of variability in argumentation that can be accounted for by fucntional understanding of proof (Table 9). According to Muijs' (2004) criteria, this model is of poor fit, as it meant that only 6.3 % of the variance in the argumentation scores was explained by the covariates.

Table 9

A Summary of the *R*, *R*-Squared, and Adjusted *R*-Squared in Analysis of LFUP and AFEG

					Change Statistics				
Model	***R***	***R*-Squared**	**Adjusted R-Squared**	**Std. Error of the Estimate**	***R*-Squared Change**	***F* Change**	**df1**	**df2**	**Sig. *F* Change**
1	.252[a]	.063	.056	.87517	.063	9.011	1	133	.003

a. Predictors: (Constant), T15.

b. Dependent Variable: AFEG score.

Multiple regression was run to tease out which of the functional understanding of proof variables were most closely associated with argumentation quality, adjusting for gender. Gaining insight into whether each of the functions explained a significant amount of variance in argumentation quality was necessary because it helps not only to explain argumentation quality but also to understand the significance of the relationship between functional understanding of proof and quality of argumentation. The beta (β) values in Table 10 provide interesting information about some of these factors with regard to their relative effects on argumentation. First, whereas knowing that proof explains had the strongest positive and statistically significant effect on argumentation where $\beta = .50$ and the level of significance was $p = .01$, knowing both that proof is a means to verify and discover had a nonsignificant impact on argumentation. Second, whereas knowing that proof is a means to systematize and communicate mathematical ideas yielded nonsignificant results, the former had the weakest negative effect ($\beta = -.07$) and the latter the strongest negative effect ($\beta = -.33$). Third, only knowing that proof systematizes had a statistically nonsignificant result at .174 ($p > .01$) effect on argumentation. The interesting conclusion here was that only having an understanding that proof as a means to explain can be used to predict students' argumentation ability.

Table 10
The Beta Coefficient in Regression Analysis

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	**Std. Error**	**β**	***t***	**Sig.**
1	(Constant)	.001	.569		0.001	.999
	Verification	.140	.182	.083	0.770	.443
	Explanation	.524	.186	.502	2.814	.006
	Communication	-.073	.228	-.074	-0.318	.751
	Discovery	.164	.187	.119	0.875	.383
	Systematization	-.266	.195	-.327	-1.368	.174

a. Dependent Variable: AFEG Score.

The analysis of the respondents' writing frames, as shown in Figure 3, revealed several noteworthy findings. First, the majority of arguments emerging from the data was at a low level (70%). Second, though only a small minority, 18% of these arguments included claims that were substantiated. Third, particularly discouraging was that only 2% of arguments developed by students were characterized as being of high quality because they consisted of rebuttals. What was important about these findings was that they provided deeper insights into students' difficulties with constructing and sustaining a mathematical argument. There were no qualifiers.

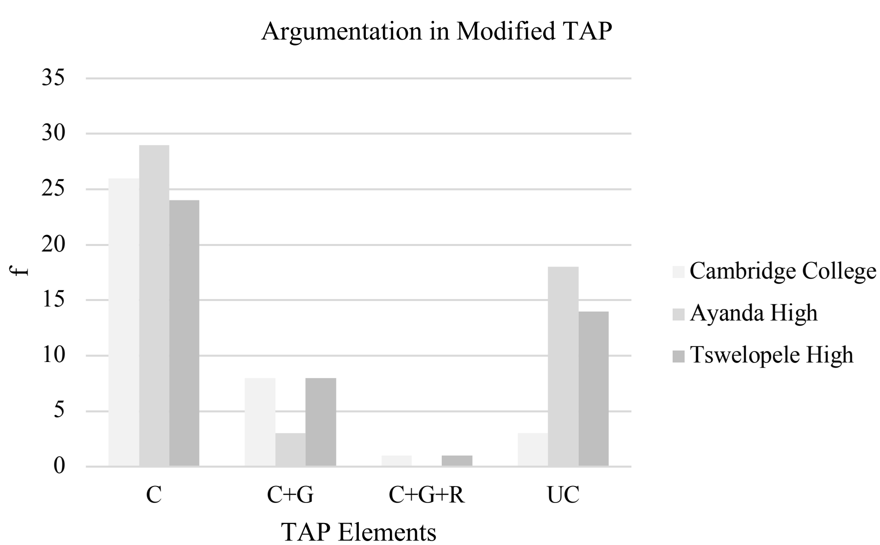


Figure 3. Distribution of Argumentation Elements Across the Three Schools

Although the data was analysed by two researchers, we used Cohen's (1968) kappa coefficient (κ) to determine the reliability. In addition, this coefficient was appropriate to use on the basis that we adopted a multicategory rubric comprising a ratio scale in which responses were classified into one of four categories. Cohen's kappa coefficients were calculated for each of the five responses using STATA, a statistical software that enables analysis, management, and graphical visualisation of data. The very few unanticipated responses received were fitted into the rubric such that the following kappa coefficients were obtained: content = .95 and argumentation = .97. As Altman (1991) suggested, these values indicated very good agreement between the raters.

Discussion

The discussion is organized according to the two research questions. The results of the first research question are discussed purely from its quantitative nature to provide a basis for arguing that the teaching and learning of functions of proof and argumentation from the Western worldview only serves to perpetuate the struggle of the urban African student. The next subsections discuss the results in turn.

The Relationship Between Students' Functional Understanding of Proof and Argumentation Quality From a Western Methodology

The result on the extent to which functional understanding of proof is related to argumentation quality is encouraging for various reasons. First, this positive relationship was anticipated because its existence was primarily based on the author's hunches, as there were no prior studies that investigated it. Second, although the result suggested that the association was tenuously significant, the fact that a correlation existed was important. It is hoped that this result spurs mathematics education researchers to further conduct studies that seek to enhance our knowledge on the role that functional understanding of proof and argumentation can play in the meaningful construction of proof. Third, this finding empirically corroborates and strengthens Knipping's (2003) suggestion that this relationship is important if promotion of meaningful learning of proof were to be better understood. Fourth, the mathematics classroom is a centre of struggle for urban African students.

In addition, this study has demonstrated that the understanding of proof as a means to verify the truth of mathematical statements is prevalent. This finding is consistent with the results of numerous other studies (e.g., de Villiers, 1990; Harel & Sowder, 1998; Healy & Hoyles, 1998; Knuth, 2002). There is absolutely nothing wrong with understanding the function of proof as verification; however, students tend to view verification not only as the sole function that proof performs in mathematics but also as merely using a few cases as proof that a conjecture is true. The main point here is to note that verification of a mathematical proposition can take two forms: empirical or deductive; empirical by selecting a few cases and deductive by logically connecting a set of axioms to produce a new result. de Villiers (1990) argued that if students see proof only as a means "to make sure" through their own experimentation, then they will have little incentive to generate any kind of deductive proof. In contrast, seeing the function of proof as a means to explain can motivate students to generate a proof of a conjecture deductively (Hanna, 2000).

The investigation also included a focus on understanding which of the five functions of proof best predicted the quality of students' argumentation ability. Seeing proof as a means to explain why a mathematical conjecture is true not only significantly predicted students' argumentation quality but also powerfully contributed to the variability of scores on argumentation quality, and understanding this function of proof was more important for students than appreciating that proof is a means to verify the truth of mathematical statements or believing that proof is a means to communicate mathematical knowledge. These empirical results give further credence to the importance of focusing on functional understanding of proof and argumentation as components of the "territory before proof."

The multiple regression results helped in providing an explanation of the impact of functional understanding of proof on the quality of argumentation that students can generate from a South African perspective. However, the communication function of proof was statistically insignificant in influencing argumentation quality. One explanation of this observation was found in respondents' low quality of argumentation; communication of mathematical knowledge to peers requires the ability to put together sound and convincing arguments, which clearly the respondents were unable to formulate.

The results also provided insight into the proportion of variance in students' quality of argumentation that can be explained by functional understanding of proof. Overall, findings indicated that the regression model was of poor fit because the predictor variables only explained 6.3% of the variance in the argumentation scores. This means that functional understanding of proof explains only a small proportion of students' quality of argumentation. One explanation of this finding is that although in the design of the study gender was removed so that the relationship between functional understanding of proof and argumentation could be more clearly determined, gender could also be a mediating variable (influencing both functional understanding of proof and argumentation). Although the effect size was not, in Cohen's (1988) terms, "grossly perceptible and therefore large" (p. 27) to equate to the difference between the heights of 13-year-old and 18-year-old boys, dismissing it as being of little practical significance can be irrational. Confidence in the effectiveness of this relationship can only follow widespread investigations in different contexts and countries.

Support for this stance is found in Glass et al. (1981). In criticizing Cohen's (1988) convention of "small," "medium," and "large," Glass et al. (1981) argued that effectiveness of a relationship can only be interpreted in relation to an average of estimates of different effect sizes obtained from other studies and that the practical importance of an effect depends entirely on its relative costs and benefits. For instance, if it could be shown that making a small and inexpensive change in proof instruction by paying attention to the "territory before proof" raises students' ability to learn proof meaningfully by an effect size of even as little as .10, then this could be a very significant improvement in proof education. Thus, it is in this light that this result is viewed as being of educational significance. Still, it adds to our knowledge of the nature of the relationship between functional understanding of proof and argumentation. Having discussed the results, I consider their implication for the urban African student.

What Is the Extent to Which the Western Worldview Marginalizes the Urban African Students' Performance?

The disappointing but not unexpected results are contemporary realities of African students; that is, the exclusion of the languages, cultures, and identities of urban African students and teachers account for these results. It my firm belief that these results do any in any way suggest that African students are poor at understanding functions of proof and at making and supporting their claims. Rather, the problem seems, more than anything, to be the foreign language in which mathematics instruction takes place. Because Ubuntu emphasizes relation to each other through storytelling, then Ubuntu storytelling is a research methodology (Mucina, 2011) that can be utilized to understand the performance of these students when taught in their indigenous languages.

Very little can be said about some of the students' argumentation ability, especially African students. They produced incoherent statements in the AFEG questionnaire that can only be described as idiosyncratic in their nature. These results were ascribed to these students' poor command of the language of instruction, English, thus confirming the need to rethink the way the mathematics curriculum is organized for genuine achievement of our students. It is in this light that future research must use Ubuntu stories as methodology to encourage Ubuntu scholars to make their work not only accessible to our larger African communities but also make the teaching and learning of proof and mathematics in general in specific African languages a flourishing experience. An attempt such as this will go a long way toward addressing the colonial legacy of research conducted in many parts of the world in general and in the sub-Saharan context specifically.

Concluding Remarks

The positive correlation between functional understanding of proof and quality of argumentation was helpful in confirming the importance of understanding these two domain areas in mathematics. The analysis of the results paved the way for presenting arguments about the marginalization of the urban African student effected by the Western worldview, including conducting systematic investigations in sub-Saharan Africa. Specifically, whereas students' appreciation of proof as a means to explain why a mathematical proof is true was the most powerful predictor of quality of argumentation ability, the communication function of proof exerted the smallest statistically significant influence on argumentation quality. In other words, the explanatory function of proof was found to be the most important determinant of argumentation ability. The analysis provided an image of the urban African student as poorly performing, yet there are compounding factors (language and gender) contributing to their performance. Put broadly, the analysis brought to the fore the need for social change that can create conditions for the flourishing of urban African students in proof-related education.

Future studies will need to be sensitive to African students' difficulties and design investigations with the question, "How can the learning and teaching of proof and argumentation be conducted through the Ubuntu framework?" Gobo (2011) reminded us that quantitative surveys or qualitative in-depth interview methods are based on competition and the role of the individual participant. Perhaps future research may employ the participatory research paradigm (Lincoln et al., 2018) in critical participatory action research, that is, research undertaken with and by people to build knowledge for understanding and transforming current practice (Kemmis et al., 2014). This, of course, in my opinion, could be the only hope for a revamp of the ontological and epistemological stance of a "new" mathematics curriculum that transforms a Western academic method into a multicultural framework that is sensitive to ethnic issues in addition to language and gender issues.

In short, the results highlight that conducting research from a Western lens tends to contribute to the perpetuation of eliminable forms of marginalization. Such a lens obstructs the acquisition of proof education, which is, like mathematics, a universal human heritage that must be accessible to urban African students too. Thus, future research efforts need to design studies whose approach to research is grounded in indigenous African epistemologies (Seehawer, 2018). Thus, research in mathematics education must take account of the economic, cultural, political, and racial milieu that affect urban African students' learning of proof and acquisition of argumentation skills. This increases the need for sub-Saharan instructors to be empowered to pursue emancipatory proof-related instruction. The study reported in this paper adds to literature challenging the relationship between conventional, Western-oriented mathematics education and indigenous students who have a fundamental human right to have their worldview validated and present in education systems.

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