

Learning Mathematics in a Borderland Position: Students' Foregrounds and Intentionality in a Brazilian Favela¹

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In this article, the authors introduce a theoretical framework for discussing the relation between favela students' life conditions in relation to their educational experiences and opportunities. A group of five students from a favela in a large city in the interior of the state of São Paulo in Brazil was inter-viewed. The students were invited to look into their future and explore whether or not there could be learning motives relating mathematics in school and possible out-of-school practices, either in terms of possible future jobs or further studies. Four themes were identified: discrimination, escape, obscurity of mathematics, and uncertainty with respect to the future. Students in a favela could experience what the authors call a borderland position, a relational space where individuals meet their social environment and come to terms with the multiple choices that cultural and economic diversity make available to them.

KEYWORDS: Brazilian favela, borderland position, students' foregrounds

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The permanent growth of shanty towns (*favelas* in Brazil, *invasiones* in Colombia and Ecuador, *townships* in South Africa, or *grecekondu* in Turkey) is characteristic of the unequal growth of modern society in many countries in the world. The Brazilian word *favela* refers to an urban area formed when large groups of people moved from the rural areas to the big cities in search of work, took possession *de facto* of large empty extensions of land, and started constructing dwellings by putting together plastic, cardboard, wood, concrete, or whatever material could offer shelter from the inclemency of sun and rain. When the roof is completed, the house is finished. A favela is always transient and in permanent construction, even if time seems to have regularised it. The older red brick houses are rough, built side by side and on top of one other in layers that remind one of a fragile domino rack. The red bricks remain exposed and uncovered as they are; they never get dressed with cement, and the walls never get painted. The entrenched network of small, almost impenetrable streets is a labyrinth where vulnerable electricity installations meet flying water pipes and open sewers. For an outsider, a favela signals resignation.

The film *Cidade de Deus* (City of God) provides an impression of life, and of criminal life in particular, in one of the most famous favelas in Rio de Janeiro.² That is the picture that many people have when thinking of a favela. However, favelas in other cities in Brazil, with different conditions, look more like slums where disadvantaged people struggle to make a living.

The metropolis of today includes a patchwork of neighbourhoods and economic extremes. One finds squatter settlements beneath highway junctions where the passing of speedy, fashionable new cars almost blow poverty away. Rich neighbourhoods and favelas are separated by only a few streets. The patchwork of diversity is kept together by invisible threads that also maintain radical forms of separation. Rich *condominios* (gated communities) are surrounded by high walls topped with electric wires. A guarded gate separates the outer reality from the apparently protected, wealthy life inside a *condominio*, which looks more like a small city surrounded by a wall than a neighbourhood. Here, unlike most houses in Brazilian cities, no walls separate the houses and windows are not barred. Green lawns and gardens, crystal blue swimming pools, and well-dressed families certainly contrast with the air of messiness that emanates from a favela only a few streets away from the outer walls of the *condominios*.

That students coming from different neighbourhoods experience different educational opportunities is no new eye-opener in educational research. Many studies focusing on students' backgrounds and their influence on education have provided

² *Cidade de Deus* is an Oscar-nominated Brazilian film, released in its home country in 2002 and worldwide in 2003. It was adapted by Bráulio Mantovani from Paulo Lins's novel *City of God* (1997/2006), which is based on the true story of the parallel lives of two young men from a favela in Rio de Janeiro.

evidence of the fact that there is a strong relation between students' material and cultural life conditions and their experience in an educational system. It is beyond the scope of this article to provide a thorough account of research documenting this relationship because there have been many in different countries in the world. Researchers such as Cooper and Dunne (1999) in England, Zevenbergen (2001) in Australia, Vithal (2003) in South Africa, and Oakes and collaborators (2004) in the USA, have provided an analysis of this issue in operation in mathematics and science education.

Our intention in this article is to bring into the discussion a set of different theoretical tools to cast light on the relation between students' life conditions and their educational experiences and opportunities. Students coming from different neighbourhoods can experience and foresee very different life opportunities. Students belonging to disadvantaged and marginalised social groups are faced with the stark question of who they are and who they can become. Students' perceptions of their future life possibilities are full of conflicting experiences, realities, dreams, and hopes for the future. All of these can impact students' motives for engaging in schooling and learning in general, and in learning mathematics in particular. In what follows, we start by introducing the notions of *foreground*, *intentions for learning*, and *borderland position*. We explore the potentiality of these notions by relating them to a conversation with a group of Brazilian students in a favela. We highlight some of the issues that we see emerging from the interview in relation to the notions, and we conclude by discussing the potentialities of the concepts in relation to mathematics education.

Foregrounds, Intentions for Learning, and Borderland Position

We have been developing the notions of students' foregrounds and intentions for learning over a longer period of time, while only recently have we try to explore the notion of borderland position. We define a person's foreground as his or her interpretations of life opportunities in relation to what appears to be acceptable and available within the given socio-political context (see, e.g., Alrø & Skovsmose, 2002; Skovsmose, 1994, 2005a, 2005b). This notion emphasises that students' engagement in learning is deeply rooted in the meaning they attribute to learning with respect to their future life. In this sense, the intentions for learning might be connected not only to the "past" or the background of a student but also to his or her "future" or foreground. Seeing meaning in learning as related to the future, rather than to the past, emphasises that students' making sense of schooling in general, and of mathematics education in particular, is not only cognitive in nature but also

socio-political. Meaning given to learning is bounded by the learner's social, political, cultural, and economic conditions and how the learner interprets them.³

The notions of students' foregrounds and intentions for learning have been used in interpreting a variety of educational phenomena. Some educational research has located certain groups of students as having problems with mathematics. A grotesque example of such a stigmatization is found in the so-called white research on black education, conducted during the apartheid period in South Africa (see Khuzwayo, 2000, for a critical discussion of this research). This research identified black children as low achievers in mathematics and suggested an explanation of this observation in terms of a deficit discourse. This discourse could take different racist formats: the weak performances of black children are due to their biological origin; or: the weak performances are due to the structures of black families. However, do we consider the black children's foregrounds in apartheid South Africa; it simply appeared ruined due to the very apartheid regime. A socio-political and economic destruction of opportunities for a certain group of people is a tremendous obstacle for learning. Considering the students' foregrounds might reveal the limitations of deficit interpretations of school performances, and turn the attention to the socio-political and economic formatting of life opportunities, and, as a consequence, of conditions for learning.

In previous studies, we have illustrated how the way students experience learning may relate to their foregrounds. In Alrø, Skovsmose, and Valero (in press) we interviewed 8th grade students in a multicultural school in Denmark. In one interview, Razia, an Iraqi refugee, clearly points out how, in her perception of her school mathematics experience and her hopes for the future, discrimination is present. Her reaction to this discrimination is incarnated in her head-scarf, a symbol of Muslim womanhood that she herself has decided to keep and defend fiercely as a way of showing who she is, where she comes from, and what she wants to become. Valero (2004) illustrates how the mathematical school experience of Colombian students in poor public schools is deeply rooted in the socio-political context where the students act as human beings. Escaping a harsh life might be a reason to learn, however, not powerful enough to give full meaning to school mathematics. In Skovsmose, Alrø, and Valero (2007), we have explored how a group of indigenous students in Brazil see their foregrounds, and the meaning they attribute to the experience of learning mathematics. The apparent lack of significance of mathematics is replaced mainly with an instrumental significance. Baber (2007) has studied how

³ Such a definition of foreground allows thinking about the similarities and differences with other powerful notions such as "identity," which has been increasingly used in educational research. The discussion of identity and foreground deserves an article on its own. Suffice it to say, here, we see similarities and differences with the notion of identity as presented by, for example, Sfard and Prusak (2006).

Pakistani families in Denmark see mathematics as playing a central role in their participation as citizens of the country, and he points to the uncertainty about the future that characterises their current situation.

We see the students' foregrounds and their intentions for learning as closely related. Furthermore, we find that both foregrounds and intentions are structured differently for different groups of students. Here, we will pay particular attention to the notion of borderland position, which refers to a position from where the individual can see his or her current life conditions in relation to other life possibilities. The "borderland" metaphor has been used in research dealing with cultural diversity to signal the vicinity and overlapping, as well as the conflict between people's participation in different cultural worlds.⁴ We see the borderland as a space of individual and social exchange where the meaning of difference is negotiated. A borderland position is a relational situation where individuals meet their social environment and come to terms with choices that diversity makes available for them, as well as with the many choices that are beyond reach.

Borderland positions exist for all people. For a person placed in a marginal position in relation to the dominant culture or establishment, however, the borderland position shows the sharp and clear contrast between his or her world and other worlds, particularly those belonging to the participants in the dominant culture. Being in a borderline position allows that person to experience social, cultural, and political differentiation and the stigmatization that operates through the stories that the dominant culture constructs about his or her life. Focusing on people in borderland positions allows us to have an insight into how exclusion/inclusion mechanisms operate and, more important, are experienced by those deeply affected by them.

We now turn to the streets, houses, and people in a Brazilian favela with the intention of illustrating the significance of the notions in relation to how a group of youngsters experience their mathematical learning.

Inter-viewing Students in a Brazilian Favela

In what follows, we will meet five students from a favela located in a large city in the interior of the state of São Paulo in Brazil. Pedro Paulo Scanduzzi has known them for some time and invited them to look into their future: How would they like to see themselves in the future? Could there be any "learning motives" relating mathematics in school and possible out-of-school practices, either in terms of possible future jobs or further studies? Ole Skovsmose has also met the five students and spoken with them. Paola Valero and Helle Alrø have never met the stu-

⁴ For further discussion of related notions see Chang, 1999, and MacDonald and Bernardo, 2005.

dents personally but have read Pedro Paulo's inter-view transcript and Ole's accounts of his meetings with the students and with Pedro Paulo.

The five students that Pedro Paulo inter-viewed⁵ were: Júlia, Mariana, Natália, Argel, and Tonino.⁶ Mariana was 14 years old at the time, while Júlia and Natália were 16 years old. The two boys, Argel and Tonino, were both 16 years old. Argel was eager to present what he wants in life, while Tonino remained quieter. Mariana and Natália talked rather freely, while Júlia was normally relatively quiet. But given that the inter-view took place in Júlia's house, she might have taken upon herself the responsibility of being a hostess, and in this respect, she participated eagerly.

Júlia, Mariana, Natália, and Argel attended a public school called Floriano Paixoto. This school is comprehensive, containing primary, secondary, and upper secondary levels. The school is surrounded by high walls. The gate of the school is locked and watched by a guard who ensures that only those who are supposed to enter, in fact do enter. In this city, even a poor school is in danger of being robbed. The walls might also help to protect the students when they are in school, as well as preventing them from escaping before they are allowed to leave. The school is located in a densely populated and rather poor area of the city. Part of the area includes the favela Cidade de São Pedro, where four of the students come from; Tonino is from a nearby favela. Tonino does not attend Floriano Paixoto but an agricultural school called Esperança Verde, which is located on the outskirts of Meia-dia, a neighbouring town. This school is surrounded by fields and has a variety of animals. The students have the opportunity to learn farming through the praxis of farming. The agricultural school applies an alternative educational programme, where students have to be at the school for 2 weeks, and then work at home for another 2 weeks. This alternating attendance ensures better possibilities for students from poor families to go to school, given that their financial support could be needed at home. In Esperança Verde, 5 hours per day are dedicated to regular school subjects, while 4 hours are reserved for practical subjects. Pedro Paulo and Ole visited the schools, Floriano Paixoto and Esperança Verde. The head of Floriano

⁵ Following previous studies where our task was the empirical exploration of students' foregrounds (see Skovsmose, Alrø, & Valero, 2007; Alrø, Skovsmose, & Valero, in press), Pedro Paulo orchestrated a conversation with the students where questions about their life, their imaginations for the future, their like for mathematics, and their perception of mathematics in their current and future life were discussed. We use the term inter-view, inspired by Kvale's (1996) concept of a semi-structured inter-view that develops as a conversation about selected topics. Thus, a semi-structured inter-view is "an interview whose purpose is to obtain descriptions of the life world of the interviewee with respect to interpreting the meaning of the described phenomena" (p. 5). This description also implies an active asking of questions and exploring of answers between inter-viewer and inter-viewee that emerge through the conversation.

⁶ All names of students, schools, and locations are pseudonyms.

Paixoto showed them around and told about the stressful life of directing a school. At Esperança Verde two students showed them around and talked about the organization of the school.

Pedro Paulo has had contact with people from Cidade de São Pedro for a long period of time. He knows many people there, and he is known by many. Because the neighbourhood is nearby the university where he works, and the university library is available for schools and students from the neighbourhood, Pedro Paulo has had the chance to help these students when they have needed a hand with homework or an activity in the University. In this way, Pedro Paulo has become a friend, a person who is allowed in the favela even though he does not live there. He has often visited Júlia's family, and Júlia was happy to invite her friends to her house for the inter-view. The inter-view was scheduled for the evening to make it possible for the students to participate.

In what follows, we turn to the inter-view and listen to how the students describe their situations and their expectations and hopes for the future, and their wishes for further education.⁷

What Do You Not Want to Do with Your Life?

The small room in Júlia's house accommodates enough chairs to seat everyone. Some of the chairs have seats made of braided plastic strings, originally of different bright colours. Time and use, however, have made them appear the same. Pedro Paulo breaks the ice and tells a bit about himself:

PEDRO PAULO (PP): When I was your age, 14 and 16, I studied in a public school in a town close to here [...] I went to school, played ball, went fishing, took small jobs, and dreamed of travelling, and that's why I studied a lot. I dreamed of attending good schools. And that was my life. I studied a lot. And afterwards, I left and went to work in Ubatuba⁸ as a mathematics teacher. Now I have returned, and I'm working here at the University [...] They say that I'm at the end of my life, being over 50. So, I'm getting to the end.

⁷ After Pedro Paulo conducted the inter-view, a transcript of the session was produced and translated into English. Readings of the transcript were discussed between Ole and Pedro Paulo, who provided additional information and contextualization about the students' ideas, based on Pedro Paulo's knowledge of them and their situation. All members of the research team discussed different interpretations of the students' words and of what seemed to be behind them. We do not use the inter-view as an empirical documentation of the students' actual thinking, motives, and intentions. We use what they express as a window into a reality that triggers our reflections on the concepts that we want to explore. The quotations from the inter-view are presented in the original order. Parts of the transcription, however, have been omitted.

⁸ Ubatuba is a town along the coast between São Paulo and Rio de Janeiro.

The first to be addressed is Argel, who is in his 2nd year of upper secondary school. In addition to his regular classes, he takes a course in electronics and a course to prepare for a military career.⁹ Such a career includes much competition, but Argel is ready to face the challenge. He says that he likes geography, history, and biology, and also art education, although less so. He also likes mathematics and Portuguese a little. He prefers physics and chemistry, however. Let us listen to his remarks about mathematics:

PEDRO PAULO (PP): What are you learning in mathematics?

ARGEL (A): Uh, I'm studying, now at this moment, matrixes; I'm studying matrixes and I'm studying all the definitions—reverse functions, inverse of the matrix [...]

PP: And what have you thought about doing with these matrixes?

A: Well, last year, other calculations appeared; this year for me, what am I going to do with this [...] another course that I'm taking is electronics. The matrixes I'm going to use—they have a binary sequence.

Argel is working with matrixes: their definition and formal properties. He also refers to possible connections between matrix calculations and the electronics course he is taking. The calculus of matrixes might well be included on the exam Argel needs to pass in order to get started on his military career. He tries to clarify connections between matrixes and binary numbers. It is obvious, however, that the possible applications of matrix calculus are not clear to Argel.

Crucial to Argel is his choice of career. He is interested in the military, and this priority provides meaning to many other activities in school:

A: I'm taking a preparatory course for the military.

PP: You want to be in the military?

A: I like it, the army or the naval air force.

PP: The army or the naval air force?

A: I'm not sure yet, where I'll go.

PP: Is that what you want to do with your life?

A: Yes.

PP: What do you *not* want to do with your life?

A: Hanging out here without doing anything, making a living doing what? I'm not going to keep depending on my parents for the rest of my life.

Argel has not made up his mind if he prefers the army or the naval air force. But his overall decision is made: he wants to pursue a military career. Argel certainly does not want to hang out in the neighbourhood doing nothing. And he does not want to be financially dependent on his parents.

Argel's comments touch upon the notion of meaning. Learning about matrixes might not be experienced as meaningful because of applications that he knows

⁹ The course prepares students for the entrance exams of the military schools.

about; but rather, it might have instrumental significance, if it is significant for passing tests crucial for his future career. In fact, students might be ready to accept an instrumental significance as a preliminary resource of meaning as they assume that what they learned could later turn out to be relevant.

To Escape from the City a Little

Tonino opted to study at Esperança Verde, near Meiadia. But why did he choose to do so?

TONINO (T): Ah, to escape from the city a little.

PP: Escape from the city a little? Are your parents agricultural workers?

T: My mother is a seamstress, and my father works in a factory.

PP: Yes, but were they agricultural workers before?

T: My mother lived in the country. I don't know about my father.

PP: What led you to want agricultural school?

T: Employment, you know—leave there with employment guaranteed.

Tonino wanted to escape from the city. However, he does not seem to have connections to rural life, except that he knows that his mother once lived in the country. It might not be the content of agricultural work that provides the main attraction for Tonino. It seems important to him to change location, and maybe, first of all, to be able to secure a job. This choice could provide stability in life, different from being a seamstress or a factory worker. Tonino seems to believe that an agricultural education would lead to “guaranteed employment.”

People from certain neighbourhoods in the city are not considered to be “reliable,” and they have difficulty getting a job. So, in order to get a job, it is not only important to get an education that could lead to a permanent job; it might also be important to change location, in order to get rid of the stigmatization that people from certain neighbourhoods, like Cidade de São Pedro, suffer.¹⁰

Which school subjects does Tonino like the best? Could his preferences in school have something to do with the choices he has made?

PP: What are the courses you like the least?

T: History and Portuguese.

PP: Do you like mathematics?

T: More or less.

PP: What have you studied in mathematics?

T: I don't remember.

PP: You don't remember? What are you going to do with this subject matter that you don't remember?

T: I don't remember anything.

¹⁰ Students comment on this issue later in the inter-view.

Tonino might be referring to the mathematics from the secondary school. He might also be referring to mathematics at Esperança Verde. As mentioned before, the schedule in the school is organised around 2 weeks of work in school and 2 weeks of work at home. But mathematics, as well as any other school subjects, is out of Tonino's memory. He does not remember anything. Then the conversation includes Argel again:

PP: Argel plans to do what in the military, be a soldier?

A: Yeh, I suppose, you know [...] there you start out as a soldier and then you pass the exams, tests, to rise in rank to captain, sergeant [...] something to grow there within—but it's a course there. You go through a public exam, and if you pass, you have technical courses, and after you leave there, you can even work in a large factory. Because the highest salary is for colonel, retired earning double, in these industries.

PP: You want to work in a factory?

A: No, I want to take the course, because in addition to the military high school, I have technical courses in the morning, and in the afternoon, I practice and earn a salary like the ITA [Instituto Técnico da Aeronautica – Technical Institute in Aeronautics], the EsPCEX [Escola Preparatória de Cadetes do Exército – Preparatory School for the Army], or AMAN [Academia Militar das Agulhas Negras – Special Force Academy].

PP: You want to do one of those?

A: I want to do the ITA.

PP: Do you study a lot?

A: At least two hours a day; if not, I don't pass the tests.

PP: Two hours a day. You work, too, or just study?

A: Not me, I don't have time. I study during the week and on Saturday. I only have Sundays free.

Argel knows about career possibilities and about how to obtain them: studying, despite the fact that studying “at least two hours a day” seems to be considered a lot. Argel expresses his interest clearly. But what about Tonino? Is his interest limited to getting out of the city and getting a job? Are there more reasons for Argel to “remember” mathematics in light of his desire to enter the military, than for Tonino who wants only to get a stable job? It might well be that stronger desires for the future bring better reasons to want to remember school mathematics.

What Do You Remember?

At this moment, the girls enter the conversation; first Júlia, who is kind of a hostess. The subjects she likes include art education and physical education, while she does not like Portuguese, which she finds to be very difficult.

PP: Do you like mathematics?

JÚLIA (J): More or less.

PP: What are you studying in school right now in mathematics?

J: I'm reviewing the subject matter from the 3rd quarter for the test.

PP: What subject matter, do you remember?

J: Delta, sets, images, things like that.

PP: In the future, what do you plan to do with this mathematics that you are learning?

J: I don't know what I'm going to choose as a profession. I think it [mathematics] will help.

Júlia's first answer to what she is studying does not concern the mathematical content. She studies for the test. Asked directly about the subject matter, Júlia refers to topics like Delta, sets, and images (of functions). Delta is the expression $\Delta = b^2 - 4ac$ used when solving the 2nd-degree equation $ax^2 + bx + c = 0$. And what to make of this when one thinks of future education? Júlia, certainly a polite hostess, confirms that although she does not know what she will choose as profession, she thinks that mathematics will turn out to be helpful. Thus, Júlia also seems to believe in the instrumental significance of mathematics.

Later in the conversation, Júlia emphasises that she does not want to become a housewife and do housework. She does not want to stay at home preparing food for her husband. She says that she might want to study healthcare or medicine. These are ambitious wishes, and it might well be that Júlia knows that mathematics composes part of such studies, although she does not know in what way mathematics will be useful.

A Housewife, in My Opinion, is a Slave

Natália is 16 years old. She is in the 2nd year of the upper secondary school. Júlia and Natália are in the same grade, although they are not in the same class.

PP: But you're not in the same class? What are you studying in mathematics?

NATÁLIA (N): We're doing [...] seeing some things about 2nd-degree functions, the Delta. These 2nd-degree things.

PP: What do you like least about school?

N: The teachers.

Natália remembers the "2nd-degree things." She seems to remember more than Tonino, but a bit less than Júlia. Natália expresses clearly her dislike for teachers. Then she is asked what she would not like to be:

N: A housewife.

PP: You don't want to be a housewife?

N: A housewife, in my opinion, is a slave.

PP: Even if she owns her house?

N: Even if she owns her own house.

PP: Why do you think that?

N: Ah! Because everything you tell her to do, she does. She doesn't avoid doing it. Even if she doesn't want to, she does it. It's like being a slave; you're giving the order, and she's following it.

In Natália's view, a housewife is given orders and follows orders. This compliance is like the life of a slave, even if she is the owner of the place. Natália's words resonate with Júlia's. In the favela, girls have seen many women, starting with their own mothers, and they express their positive rejection of a life as a housewife. Studying and choosing a profession seems to be a way of escaping that frightening scenario. Natália therefore dreams of becoming a psychologist or a veterinarian. She likes animals very much, and she likes psychology because she likes to listen to people talk about their lives and to give them advice. When asked if mathematics has anything to do with veterinary medicine or psychology she answers:

N: Nothing.

PP: It has nothing to do with it? Júlia, Tonino, Argel, do you know what psychology and veterinary medicine would have to do with mathematics?

T: I don't have the faintest idea.

PP: No idea. So that means that what she's learning in mathematics will not be very useful to her?

N: I think it will, because when you go to a university, you have to study all the subjects.

Natália seems not to see the instrumental significance of mathematics with respect to psychology and veterinary medicine. She, however, sees clearly that when one gets to the university, one must "study all the subjects," including mathematics. That perspective might be reason enough to engage in school mathematics, above all, to avoid being a housewife.

Delta is Just a Formula

Mariana wanted to be the last to talk. She lives in a neighbourhood near by. She goes to the same school as Argel, Júlia, and Natália. Mariana is 14 years old and she is in the 8th grade, the last year of secondary school. She likes the school and the teachers, and she likes to study. But she does not like the school when there is much quarrelling and disorder.

Mariana intends to study law and become a lawyer, or maybe she wants to study medicine. And what about mathematics?

MARIANA (M): Ah! I'm in 2nd-degree Delta, these 2nd-degree things.

PP: And what will you do with these 2nd-degree things in medicine, or as a lawyer or a judge?

M: Ah! I think, for sure I'll need it to go to the university. I'll need it.

PP: To go to the university. In your profession, you don't think you'll use it?

M: Ah! I don't understand it a lot. But I don't think so. I don't know.

Like everyone else, except for Argel, Mariana does not know what to do with the Delta. Well, it might be necessary knowledge for entering the university or the faculty of medicine. Mathematics per se does not seem to be considered important.

Later in the inter-view, Mariana mentions that she does not like Portuguese, and grammar in particular: "What sense to make of issues like subordinate clause and punctuation?" Mariana does not think of Portuguese as being important for studying law. However, if it turns out that it is, she will be ready to study it. Then Pedro Paulo returns to mathematics, and the students comment again on the Delta formula.

A: [...] Delta is just a formula. But you use it for the rest of your life.

M: You keep deepening it, complicating it, more and more.

PP: The Delta gets complicated, just like life?

M: I think so.

J: More or less.

PP: More or less?

J: All is the same.

PP: As times goes it gets more complicated.

J: Yes.

M: In first grade you learn $2 + 2$ and then it gets more complicated, you learn to divide.

Delta is just a formula, but it seems to stick with you, as Argel emphasises: "you use it for the rest of your life." It will appear in more and more complex situations, as all mathematics do. You start with simple things like addition, but it always gets more and more difficult. But as things get complicated, it seems as if the meanings of mathematical expressions and techniques do not emerge in the context of learning. Their meaning might (or might not) be revealed later in school or in life. Students seem to be struggling with what we could call the "*Delta syndrome*," a weird kind of disease in which the patients are presented with some mathematical formula or technique, which they are supposed to master in order to get on with their education, but whose significance will not be revealed until later.

The inter-view then turns to a discussion of what the students' parents are doing. It is clear that Tonino, Argel, Júlia, Natália, and Mariana are hoping they will not become like their parents. Mariana does not want to become a maid or cleaning woman, a type of job that many women in the favela have, doing housework in other neighbourhoods. Mariana would like to become a housewife, however. She cannot follow Júlia and Natália who think that a housewife is a slave, even in her own house:

M: But a housewife, yes, because I like to do the housework at home. I'm the only one who does it because my mom and dad work.

Mariana's mother works in the butcher shop owned by Mariana's father. Natália's father works as a truck driver, and her mother is a seamstress. Júlia's father works as a driver for the local government. Part of his work is to assist in repairing the roads. Júlia's mother works as a kitchen assistant. Tonino's father is working in a furniture factory, while his mother is a seamstress. Argel's father has retired, and his mother is a housewife.

PP: And she likes being a housewife?

A: She likes it, because she didn't know her mother and father. She was raised by her aunts, so she was their slave. At our house, we tell her not to do stuff, but she ends up doing it. She likes to do things. I want to help her, but she doesn't let me.

Students' expressions of their future profession are far from being inspired by their parents' current occupations. Even when Júlia and Natália express their dislike of the housewife life, they seem to do so in relation to the situation of their own families and relatives. They hope for something different, probably better.

The Exams Are Very Complicated

The students come to talk about the possibility of realising their dreams. They believe it is possible to achieve what they hope for, but that there are many difficulties. One is the tuition at private universities; another is the cost of the preparatory courses for the college entrance exams. It seems particularly difficult for those who dream of enrolling in some of the most expensive programmes (such as medicine). For example, a driver like Natália's father earns about 800 Reais per month and one could expect that the study costs for Natália would be around 400 Reais per month; having children engaging in higher education puts a huge economic demand on a family. A student could do some work in addition to their studies, but a student's salary would cover only a minor part of the study costs. Only if one chooses to study at night and work during the day is it possible to make a reasonable amount of money. Another option is to enrol in shorter technical or vocational programmes; however, those programmes are less prestigious. It is also possible to get some kind of scholarship; but then one must be an exceptional student and have very good grades. Anyway, the cost of engaging in further studies is certainly a huge obstacle for making the students' dreams come true.

The public universities are free but very difficult to enter. In Brazil, each university applies its own entrance exam, which applicants are charged for. They can register (and pay) for as many exams at different universities as they want, which are typically administered during the months of December and January. The results, often published in early February, take the form of a ranking list of all students that participated in the test. On the Internet, one can see one's position and also where the cut-off for entry was made. Naturally, the most attractive public universities are

the most difficult to enter. Many hopeful applicants take the exams, and the most attractive universities need only to select the top 10% of applicants. If one does not succeed one year, one can pay for a one-year study programme to prepare for the next year's exams. And so on, until one enters, or until one gives up on the idea of doing further studies. Again, the need for good exam results seems to go against the realisation of their hopes and future expectations.

A: ... the exams are very complicated.

N: There aren't many people admitted, either.

Students from a public school like Floriano Paixoto are unlikely to be as well-prepared for the college entry exams as students from private schools. Brazil has a large number of private elementary and high schools, always better equipped than the public schools, and usually more focussed on ensuring their students good possibilities for pursuing further studies. So, the private schools provide the very best preparation for students entering the attractive public universities. The situation could be very different with respect to the public school, as Argel explains:

A: The classes they give, they are the same in the private high schools, and in the public schools it's the same. But the teachers are slow. They're not too concerned. Some are concerned; others don't even care about you. You, who are from the public or municipal school.

N: In the public school, the teacher doesn't care about what he does.

A: In public universities, it's very difficult to find people like us who studied in the public schools. In the public universities, they only have daddy's little kids going there. They are in no need of going to public universities.

PP: So, what are you going to do? You are in the public schools. You depend on a salary, and the salary isn't high. You have the wish to get into a good program. What are you going to do? Are you going to say, like, we're just going to stop here?

A: We have to study, to fight.

N: We have to make an effort.

The problem is clearly formulated by Argel: In public universities, there is no room for many students from public schools. It is mostly well-off students who manage to get in. The students really find their opportunities restricted by their economic situation. Some try to compensate by doing some extra courses. Thus, Júlia does extra studies in English, and Argel takes a course in electronics, including computation. The lack of access to computers at home is a problem, so it is important for students to take courses where they are able to get experience with computers. The situation at home does not facilitate any form of study. Most of the time there are not adequate resources to study; normally there are many people around, and it is difficult to find a quiet place to concentrate on studying. Besides, many other characteristics of life in a favela—such as violence, struggles related to drug trafficking,

and even sexual assaults—are not the most nurturing for youngsters who want peace of mind and who are probably in need of getting rid of the “Delta syndrome.”

We're Discriminated Against

It is not difficult to list obstacles that these students have to face in their life. But are they able to find reasons for optimism as well?

PP: Do you guys see this desire of yours with optimism/excitement or not?

J: Ah! I get pretty excited when I think about what I want to be.

PP: And you, Tonino?

T: You have to go after it.

PP: Argel?

A: You have to fight. And if you get discouraged, feel down—you can't get discouraged.

PP: Why do you get discouraged, Argel?

A: Well, it's kind of different when it's time to study there. I feel discriminated against.

J: Sometimes people fail; give up, too. You have to persist.

M: Because public schools, the teaching is weak. Not that it's weak, it's that the teachers don't care, and the students even less.

[...]

N: We keep getting left behind.

M: I have a friend, he studies in the SETA. He's in the 8th grade. He knows five times more than I do.

SETA (Sociedade Educacional Tristão de Andrade – Educational Society Tristão de Andrade) is an expensive private school, located in the city centre. According to Mariana and Júlia, who know people attending this school, the students there are far ahead of those who study in public schools, including those who attend Floriano Paixoto. For them, it is really necessary to fight. As emphasised by Argel, even during education, one gets discriminated against. They perceive schooling as a form of establishing and maintaining inequalities, rather than promoting equity.

PP: Argel, you said that you feel discriminated against sometimes. Why do you feel discriminated against?

A: Ah! Because they feel—they're better than us, you know?

PP: Who?

A: These people who are daddy's little kids and are protected by their parents. Then they want to give us the cold shoulder. They think they're better than we are.

The students experience discrimination, not only in terms of attitudes, like the “daddy's little children” who think they are better; they are also discriminated against in real measures. In the private school, there are better teachers with more commitment, and the students have better conditions for learning. Pedro Paulo, however, points to a fact that might serve as a counter-balance to their experience of being left behind:

PP: Did you know that in the universities a lot of people are entering that studied in public schools? And those students are getting into a good study habit, facing all those people who had college preparation courses.

A: They're the bigger schools there downtown, aren't they? [...] those schools downtown where the teachers are stricter.

J: They get after you more, demand more.

Pedro Paulo points out that one finds many students from public schools studying in the universities. Argel, however, stresses that they are from the bigger public schools in the city centre, where teachers are more strict, demand more and, therefore, prepare students more adequately for further studies than schools in a poor favela neighbourhood. Then Mariana and Argel add:

M: They [the students from downtown schools] don't have the needs that we have. They [the teachers] discourage us, too.

A: Because of two or three in the class, she discriminates against everyone [...] Everyone pays for it; everyone is a trouble-maker. This is not true. Just because of two or three that are like that, everyone gets into trouble.

Mariana emphasises that there are differences among students in public schools. Different students could have different needs. She indicates that teachers discourage students from poorer neighbourhoods to try to pursue further studies. Argel follows up by pointing to teachers who exercise discrimination and stereotyping. There might be some students from their neighbourhood who might cause trouble for the teacher, but "everyone pays for it" and all are discriminated against.

Then Pedro Paulo turns to Tonino who attends the agricultural school in Meiadia. How are things experienced in this place?

PP: Is it like that in Meiadia, too, Tonino?

T: We're discriminated against in Meiadia.

PP: You're discriminated against in Meiadia.

T: It's the agricultural school they talk about. Leaving to go to another city is difficult.

PP: And why did you choose a school that is discriminated against?

T: I didn't know, either, right—I arrived there believing it was a wonderful place.

PP: Ah! Did they take you to visit?

T: It was my mom who visited the school.

PP: What school there has a good reputation?

T: Ah! I don't know.

Now Tonino realises that Esperança Verde might be a school that is also regarded as having a very low status. It is a rural school, and according to Tonino, they are discriminated against in Meiadia. The same is the case for Floriano Paixoto, located in Cidade de São Pedro.

J: They think, like, that it's poor suburbs. Even we who live in the poor suburbs, we're discriminated against, if you look.

A: There when I arrive at home—Cidade de São Pedro is the worst neighbourhood in our city, a favela. To get a job, it depends on courses. You get there to enrol, and they're even afraid to meet you.

J: Because of two or three, we get it because of that. I already tried to get a job, and I didn't get one.

The students address not only the problem of being stigmatised by coming from the favela of Cidade de São Pedro, but also teachers might exercise discrimination. It might be difficult to get a job in other parts of the city. People in general might feel afraid of someone coming from Cidade de São Pedro, as Argel says. The stereotyping of favela life as portrayed in the media falls on all its inhabitants. Júlia expresses it clearly: a few people get in trouble, but not all; still, that affects her own possibilities for employment.

Issues of Life, Learning, and Mathematics in a Favela

The inter-view between Pedro Paulo and Argel, Júlia, Mariana, Natália, and Tonino reflects different aspects of the life conditions of students in a favela as they perceive and experience them. Let us highlight some themes that we see emerging from the inter-view. These themes are related to the students' foregrounds being in a borderland position and they seem to influence their motivation for learning mathematics.

The first theme is *discrimination*. The students feel they are being discriminated against due to the fact that they come from a favela, a poor neighbourhood. There is no doubt that the socio-economic conditions strongly limit the possibilities for people from Cidade de São Pedro. Favela life is a life in poverty, and poverty stigmatises people. It affects many aspects of life: the clothes one is wearing and one's habits (young people from a favela do not go to the cinema, but they might hang out at a gas station convenience store). It affects possibilities of doing homework, of accessing books and other resources for doing homework, and of studying. However, poverty not only sets a range of life conditions; it also frames the way others look at one. Based on their experience, the students feel it is better not to reveal that they come from Cidade de São Pedro. They could be discriminated against, not only economically speaking but also in terms of attitude: people could look down on them, look at them as potential criminals. Somehow poverty also frames the way one looks at oneself.

The students fear being trapped in some stereotype, and there could be good reasons for this fear. A dominant theme of the news in Brazil is violence, often associated with the favelas, particularly the famous ones in Rio de Janeiro and São Paulo. As already mentioned, *Cidade de Deus* (The City of God) is both a name of

a favela in Rio de Janeiro and the title of a film about the meeting of life, crime, and violence in this favela. This violence includes the wars between gangs, the war to expand or keep the drug markets, and the war against the police. But it also includes the everyday life of thieves who systematically assault the trucks that deliver tanks of propane gas to the households in other areas of the city. It certainly includes the struggles of many workers like the students' parents to make a living, and the struggle of the students themselves to have a chance in the future. All such common "knowledge" about life in a favela is the basis for the construction of stereotypes that stigmatise favela inhabitants. So, when the students react to the possibility of being discriminated against, they might well have good reasons for doing so.

The second theme is *escape*. There is a strong motivation to begin a new life away from the favela. However, it is not clear to what extent this "new life" is experienced by the students as something they, realistically speaking, could work for, or as just something they dream about. There is a strong motive for escaping the neighbourhood. It could be taken in a strict sense as expressed by Tonino. But "escape from the city" could also be taken as a metaphor for getting out of the life conditions the students know all too well, such as Júlia's and Natália's reactions towards being a favela housewife. They all acknowledge that the best way to escape is through further education. Therefore, the discussion of tuition fees for entering the university becomes at the same time important and fatal.

A third theme concerns the *obscurity of mathematics*. It seems clear to everyone that education is relevant for ensuring a change in life. The role of mathematics in changing life, however, is less visible. Mathematics lessons do not provide any clue of how mathematics might function in this respect. One could see an instrumental significance of mathematics, while the content of mathematics in itself appears meaningless. The mathematics curriculum in Brazil is a manifest representation of the school mathematics tradition. This tradition defines the curriculum with strong references to mathematical ideas, notions, and structures. Everyday examples might be included, but mainly to illustrate mathematical conceptions, and not as situations to be explored in greater detail. The school mathematics tradition places a particular emphasis on the teacher's presentation of the mathematical content (and not on, say, communication among students about mathematical problems). Naturally, the teacher's presentation takes on a particular significance when the students have no textbooks, and have to rely only on the notes they take themselves. And in order to make reliable notes, what could be better than carefully copying down what the teacher writes on the blackboard? A nice pedagogical contract could be established between teacher and students. As long as the teacher makes a careful presentation and students copy down the presentation, then everybody has done their job properly, and good order can prevail in the classroom. Still the obscurity of mathematics prevails.

The inter-view indicated clearly that it was very difficult for the students to point to any relationship between mathematics and their future studies and work. Argel gave it a try but was not very successful. The only relationship they could openly express was instrumental: Mathematics is a necessary ingredient for passing required university entry examinations. At the same time, they did not deny that mathematics might turn out to be significant; they were just unable to see what this could be like. The “Delta syndrome” was part of their experience.

This brings us to a fourth theme, namely the *uncertainty with respect to the future*. The students are remarkably aware of what they do not want from the future: Argel does not want to hang out and be financially dependent on his parents, Tonino does not want to stay in the favela, and Natalia does not want to become a housewife. And they agree that education could be an entry point into another kind of future life. The students find that they might have difficulties in competing with privileged children. They find that the differences are established because of differences in schooling, their teachers and the resources available to them. If one considers the ranking of the different schools in Brazil, there is no doubt that wealthy private schools top the list with respect to ensuring their students’ access to private and public universities and colleges. Schools located in favelas are very seldom found on such lists. The students also felt that teachers might treat them as inferior; as someone who is not capable of completing further studies.

The students could easily formulate very optimistic but almost unattainable aspirations, while reality might set some heavy limitations. How, then, to get out of such uncertainty? One way of getting out is simply to stop dreaming and hoping, and instead become “realistic” and renounce one’s ambitions. One could simply face that one is doomed to a poor modest life. So, it may be better to get out of school and get a job, a permanent job, if possible. Leaving school, however, is not what the students want to do and actually seem to do: “You have to fight,” “you have to go after it,” “make an effort,” “persist” are all expressions of their feelings that they can influence their future life.

What is the significance of these issues of discrimination, escape, obscurity of mathematics, and uncertainty with respect to the future for understanding the way in which students decide to engage in learning mathematics? In what follows, we explore a bit further the notion of students’ foregrounds, intentions for learning, and borderland position in order to address this question.

Generating Intentions for Learning While Constructing Foregrounds in a Borderland Position

We consider learning as an act, and as such it requires intentional engagement on the part of the learner. This claim does not apply to all forms of learning; thus, many habits may be adopted without much intentional engagement, and some

forms of learning may be forced on people. When we see learning as action we have in mind forms of learning as they might take place in school, for example, learning mathematics. Students might get involved in solving mathematical problems or be engaged with mathematical investigations; but they might also find the classroom activities to be without meaning and occupy themselves with other things. A decision about being involved in the mathematical tasks, or not, is not simply the result of a conscious individual choice, but rather a decision that is strongly associated with the intricate relationship between the student, the teacher, and the context for learning in the social-political-cultural environment. The meeting between the individual and the social is a space where intentions for learning emerge and grow, or might be destroyed. In that space, the individual constantly constructs and re-interprets both previous personal experiences and actual life conditions in dynamic relation to his or her wishes for life and dreams for the future. In other words, the individual's consideration of his or her background in relation to his or her foreground is a powerful source of reasons and intentions to decide to engage in learning as well as a cause for giving-up to be engaged in learning.¹¹

While the notion of background has been central in much research trying to establish a connection between students' learning experiences and students' social environment, the notion of foreground is relatively new. We find that the notion of foreground has a close relationship with intentions for learning, which in turn represents the broader meaningfulness that students might associate to processes of learning. The students' foregrounds are constructed through different social processes. In a profound way they are constructed through economic conditions; thus, poverty is a highly influential factor. The construction of foreground, however, includes many other elements. In this article, our interest has been focused on students who are constructed by others, and even by themselves, as marginalized and excluded from dominant cultural practices and forms of life. When students experience discrimination, they perceive that it will be difficult, if not impossible, for them to cross the line and become part of the dominant culture. This experience strengthens their awareness of their own stigmatized position. We find discrimination to be a powerful social factor, which might ruin the foregrounds of certain groups of people.

For the five students from Cidade de São Pedro, Argel, Júlia, Mariana, Natália, and Tonino, their borderland position allows them to constantly weigh a set of favela-life opportunities against, for example, a set of "city-centre life" opportunities, or "condominio-life" opportunities. They can see what it would take for them and for their education to cross the line to enter other ways of life. One reaction to the experienced discrimination turns into a dream of escape. Education is clearly

¹¹ For an in-depth discussion of the notions intentionality in learning, background, and foreground see Skovsmose (1994, 2005); see also Alrø, Skovsmose, and Valero (in press).

one possible way of doing so and, therefore, learning (mathematics)—even if the reasons are purely instrumental—makes sense and represents a more or less meaningful investment in the future. At the same time, however, they can also see and experience the enormous barriers to a successful jump over the border. Their borderland position makes evident the harshness of social division, stratification, and stigmatization.

We could imagine a borderland school as the site of learning that provides an opening for radically different life opportunities. (It might also be that such a school would jail students in their current positions.) Borderland schools should be able to establish opportunities for a transition from one way of life to a different one. At least students in a borderland school might consider such transitions to be possible. What transitions, realistically speaking, a borderland school might be able to prepare for is another question. The obscurity of mathematics has a strong implication for the students' experience of the opportunities a school might provide. There seems to be some agreement among the five students from Cidade de São Pedro that mathematics might play a role in further education, but it is not clear to them what role mathematics in fact could play. This lack of clarity means that it is simply impossible for students to relate their activities in the mathematics classroom to any more specific features of their foregrounds. As students' foregrounds are associated to their construction of meaning, the activities in the mathematics classroom remain meaningless, or, as best, instrumental. This construction is a huge learning obstacle for students in a borderland position, who experience an uncertainty with respect to their future.

In previous studies, we analysed Brazilian, indigenous students' perceptions of their educational possibilities and priorities.¹² One student had made a clear choice: he wanted to study medicine. Completing such study would certainly establish a new life situation for him. However, his priority did not include a break with his indigenous background and life in the indigenous village. He wanted to study medicine with the particular aim of being able to return to the village and contribute to the effort to improve the health situation of the Indian community. Therefore, one has to be aware that possible transitions can be thought of in very different ways. When one talks about transitions, one should not assume any simplistic scale of preferences. For example, it should not be assumed that white, middle-class priorities and life opportunities are, by definition, "better" than some other forms of priorities. One should not assume that the scale of priorities reflects a scale of economic wealth. Nor should one try to romanticise poverty. We try to avoid assuming any simplistic scaling, and instead to listen to how priorities might be expressed, how students might think of possible transitions, and how they can be related to their learning motives.

¹² See Skovsmose, Alrø, and Valero in collaboration with Silvério and Scanduzzi (2007).

Postscript: The Fragility of Dreams

Almost three years after the inter-view, Pedro Paulo and Ole again visited Júlia's family. Júlia, Natalia, Argel, and Tonino were all there; Mariana had moved to another city. It was a nice evening. The four students told about what had happened to them during the past 3 years and about their current situations. They told about what had become of their dreams and aspirations.

Júlia's family had moved to a house in the countryside. Three dogs barked and wagged their tails welcoming the visitors, together with chickens and ducks, while cows grazed in the field next door. The garden had vegetables, and Pedro Paulo picked a small bag of lemons from a tree to bring home. There were also some other friends around in the house. Júlia's mother had cooked the food, and her father showed the guests around.

Júlia was not talking very much, and when her boyfriend arrived—he was even more taciturn than Júlia—they spent the rest of the evening holding hands. Júlia had stopped her studies, and she was now working as an assistant in a lawyer's office. She was considering starting studying again in order to become a nurse specialized in radiology, which is a programme that can be completed in only 2 years.

Tonino had left the agricultural school. He had become much more talkative, and now he wondered why he had started at the agricultural school at all. Farming was not really something he found interesting, which he expressed while pressing his long thin fingers firmly together. He liked the city, and he had found a job. He was working in a goldsmith's shop, and one of his jobs was to put together different components of the jewellery. Did he like the work? He was not sure. He said that he would like to become a policeman. He believed this profession would bring him better opportunities in life.

Natália had begun studying to become a nurse. She helped her mother with the housework. She also helped her mother in her work as a seamstress. Natália had entered a private institution, and she had to pay for her programme of study. She was receiving a small scholarship, but the largest portion of the money she needed came from her parents.

During the evening, Argel was the one who spoke the most. He had stopped his studies and was no longer considering a military career. He had arrived that evening with his wife and their small baby. It was a smiling baby who, in a good mood, said hello to everyone who wanted to touch and tickle him to make him smile, which he did. It was a happy family, and Argel took perfect care of his son. He was considering moving to a city in the neighbouring state of Minas Gerais where he saw some better opportunities for getting a job. He hoped to work with computers.

When one considers students' learning of mathematics in a borderland position, one sees many factors in operation. We have pointed to discrimination, escape, obscurity of mathematics, and uncertainty with respect to mathematics. Meaningfulness (or lack of meaningfulness) of learning cannot be analysed if one concentrates on particular elements of the situation. Intentions in learning have to be related to students' backgrounds as well as to their present situation and foregrounds. Argel, Júlia, Mariana, Natália, and Tonino are still on their way, seeking a better future. The complexity of the situation, however, renders their dreams fragile.

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