Journal of Urban Mathematics Education
December 2014, Vol. 7, No. 2, pp. 21–30

PUBLIC STORIES OF MATHEMATICS EDUCATORS

Challenges and Affordances of Learning Mathematics in a Second Language

Mary P. Truxaw
University of Connecticut

Eliana D. Rojas
University of Connecticut

Prologue

In this public story, we explore challenges and affordances of learning mathematics when the learner's primary language is not the primary language of instruction. We, the authors, are Mary Truxaw, a predominantly monolingual (English, with limited Spanish) mathematics educator and Eliana Rojas, a bilingual (Spanish and English) bilingual/multicultural educator with expertise in mathematics education. We tell this story predominantly as a first-person narrative from Mary's point of view; however, cogenerative dialogue (Tobin & Roth, 2005) between us informs the story throughout.

Setting Up the Journey

I (Mary Truxaw) came to this journey with a research background focusing on language as a mediator of meaning (Vygotsky, 2002) in mathematics classrooms (e.g., Truxaw & DeFranco, 2008). In recent years, I had recognized that linguistic diversity was—and still is—growing in U.S. schools (National Clearinghouse for English Language Acquisition, 2011) and that there was evidence that U.S. schools, overall, are not adequately supporting English language learners (ELLs) (National Center for Education Statistics [NCES], 2012).[1] Further, I knew

[1] For example: The 2011 *National Assessment of Educational Progress (*NAEP) results reported that nationally only 14% of fourth-grade English language learners (ELLs) (as compared to 40%

MARY P. TRUXAW is an associate professor of Mathematics Education in the Department of Curriculum and Instruction in the Neag School of Education at the University of Connecticut, 249 Glenbrook Road, Unit 3033, Storrs, CT 06269; email: mary.truxaw@uconn.edu. Her research interests include discourse to enhance mathematical meaning making in linguistically diverse mathematics classrooms, pre-service and in-service mathematics teacher education, and mathematics teacher collaborative leadership.

ELIANA D. ROJAS is an assistant professor of Bilingual Education in the Department of Curriculum and Instruction in the Neag School of Education at the University of Connecticut, 249 Glenbrook Road Unit 3033, Storrs Connecticut 06269; email: eliana.rojas@uconn.edu. Her research and practice concentrate on the ecology of processes involved in teaching and learning mathematics within socio-culturally and -linguistically diverse contexts. She is the director and PI of Math-LEAD a professional development research grant that promotes configuring a transdisciplinary approach to the development of mathematics discourses.

that, other than English, Spanish is the language spoken most frequently in the United States (United States Census Bureau, 2013).

I had all of these things in mind when I had the opportunity to take a sabbatical from my university faculty position in 2012. I decided to use my sabbatical to enhance my understanding of linguistically diverse mathematics classrooms and, in particular, classrooms where some or all of the students had Spanish as a home language. I did not want to employ a deficit model for teaching mathematics to bilingual students, but I also recognized that, as an English-speaking person in an English-dominant culture, I could not fully appreciate the challenges or affordances involved in trying to learn mathematics in a second language.

I wanted to combine formal research with a more personal journey and, therefore, set up experiences including an intensive immersion Spanish language experience in Guatemala (in order to develop greater Spanish language fluency that would allow me to participate more actively in linguistically diverse schools for research and professional purposes). I also conducted observations and data collection in three types of mathematics classrooms: classes taught in Spanish in Guatemala, classes taught in Spanish in the western United States, and classes taught in English using strategies designed to support ELLs. The larger study (Truxaw, 2014) includes formal discourse analysis of dialogue within the linguistically diverse mathematics classrooms, but this article focuses on the story of the more personal journey to better understand challenges and affordances related to learning mathematics in a second language.

As I was planning these experiences, I had ongoing conversations with my colleague and co-author, Eliana Rojas, who has expertise in both bilingual education and mathematics education. Eliana provided professional support such as helping me to make connections with a member of the Guatemalan Ministry of Education in order to set up observations in schools. She also provided scholarly support related to bilingual education and shared personal perspectives from the point of view of someone whose first language is Spanish rather than English, who came to the United States as a graduate student, taught mathematics at college and high school levels, and, for years, has followed the educational experiences of mathematics teachers working with ELLs. Eliana encouraged me to do a self-study (Loughran, 2007) of my own experiences in Guatemala as an English-speaking mathematics educator immersing myself in a primarily Spanish-speaking culture and its schools. Related to this idea, she encouraged me to spend time in mathematics classrooms while my Spanish language was still very much developmental. This experience helped me to appreciate what it might feel like to try to learn mathematics in a second language. To support this process, Eliana

of non-ELL fourth-grade students) were at or above proficient levels for mathematics and that nationally only 5% of eighth-grade students (as compared to 35% of non-ELL eighth-grade students) were at or above proficient levels for mathematics (NCES, 2012).

generously offered to participate in "cogenerative dialogue" with me before, during, and after my time in Guatemala (usually via email). Cogenerative dialogue involves reflection where members refer to the same set of events and explanations are cogenerated, thus supporting reflection on experiences and co-generation of perspectives (Tobin & Roth, 2005).

The focusing question for my self-study and for this public story follows: *What can I, as a monolingual (English-speaking) mathematics educator, learn about relationships of language to mathematics education through an immersion experience where Spanish is the language of mathematics instruction?*

The Journey

To help me make sense of this question, I observed mathematics classrooms, audio or video recorded classroom dialogue, took field notes, kept journals documenting and reflecting on classroom observations and experiences, and participated in cogenerative dialogue with Eliana. Our cogenerative dialogue reinforced our contention that language is critical to understanding mathematics. Further, my experiences in Spanish-language classrooms, coupled with cogenerative dialogue with Eliana, helped me to gain greater appreciation of challenges and affordances inherent in learning mathematics in a second language.

To illustrate themes and issues of this journey, I highlight observations and reflections related predominantly to one mathematics lesson in a second grade classroom in an all-girls public school in Guatemala. There were 23 students, 1 teacher, and 6 *practicantes* (high school students practicing to be teachers; 2 men, 4 women). The students were seated in desks arranged in rows. As I entered the class, the students stood up and offered a choral greeting. The teacher introduced me and I briefly explained (in Spanish) that I was from the United States and was interested in seeing their classroom. The teacher showed me to a desk in the back of the room where I could observe. All dialogue took place in Spanish.

The lesson seemed to be in progress when I entered as the teacher referred to representations showing circles, lines, and numbers on a whiteboard (see Figure 1). The teacher asked students to come to the board to complete parts of the representation and to explain their work. I noticed that the verbal exchanges followed a typical triadic structure with the teacher initiating, the student responding, and the teacher evaluating (Cazden, 2001). It seemed that the class was reviewing previously learned skills. I sat in the back of the room (with notebook and audio recorder), trying to make sense of the language, the representations, and the mathematics.

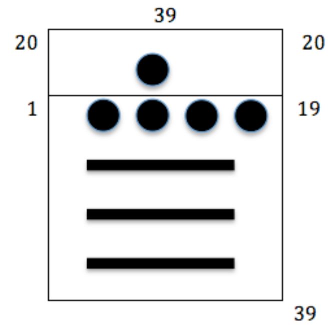

Figure 1
Representation from second-grade classroom in Guatemala.

Following are excerpts from my journal:

> Even with some Spanish language skills and, hopefully, math skills (for second-grade math!)…I did not know what was happening most of the class. It became a puzzle for me as I copied every example, as best I could, to see if I could figure out what was happening
>
> My initial thoughts as I copied example after example from the board was that they were doing some form of decomposition of numbers.

After reviewing several examples on the whiteboard, the students (with the help of the practicantes) were directed to cut out paper circles and strips. Also, they each folded a paper in two parts to create a "mat" for placing the circles and strips. Then, the teacher asked students to form numbers on their mats using the circles and strips—for example, "Forma el número cuarenta y siete" (see Figure 2).

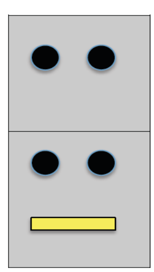

Figure 2
Example representation for "Forma el número cuarenta y siete" [Form the number 47].

As the students represented the numbers on their mats using the circles and strips, the practicantes walked around to correct and to help the students. For example, I heard a practicante count by 20s with a student (providing a hint that 20s were important). The following excerpt is from my journal:

> I kept thinking that I had figured it out, trying to guess the correct picture before looking at student work. Sometimes I'd get it right and sometimes wrong. One thing that I didn't figure out until the end was that the lines represented 5 times the place value. My scribbly notes include things like, "Boy am I lost…I don't understand… some kind of decomposition…I don't understand why they are using 20s instead of 10s."

It wasn't until the end of the class when homework was written on the board that I was able to figure out what they had been doing. The teacher wrote, "*Tarea: formar los siguientes números utilizando la numeración maya en su cuaderno"* [Homework: Form the following numbers using the Mayan numeration system in your notebook]

> "Aha!" I said to myself. "Mayan numbers!"…suddenly there was a context. There was a potential reason for using 20 as a place value—a different number system.

Having a context made a difference, but I still needed time to think (in English—my primary language), drawing on notes taken during class. Eventually, I was able to figure out the number system,[2] but not before recognizing the impact of language, representation, and context on my ability to learn and perform mathematically:

> I had to ask myself, if I, a university mathematics educator, was confused in a second-grade math class, how would a second-grade student in similar circumstances feel?

Cogenerative dialogue with Eliana helped to further unpack the experiences. Eliana suggested that "live" experiences (such as what I was doing) help to provide some sense of the challenges involved with learning and doing mathematics in a second language (for me, Spanish) and/or within an unfamiliar culture (e.g., Mayan number system). Eliana asked me to think about how experiences trying to learn mathematics in such a context might impact students' attitudes and "appre-

[2] In Figure 1, the numbers to the left show the place value. The top box has a place value of 20. One circle represents 1 x 20 = 20. The bottom box has a place value of 1. Four circles represent 4 times the place value or 4 x 1 = 4. A line represents 5 times the place value. Three lines represent 3 x 5 = 15. The total value is 20 + 4 + 15 = 39.

ciation of mathematics" (National Council of Teachers of Mathematics, 2000, p. 15) and, in turn, their self-efficacy[3] (Bandura, 1986). Eliana reminded me that successful (or unsuccessful) learning experiences could impact attitudes, self-efficacy, and performance. Eliana stressed that involvement in the mathematical activity requires a disposition where one "*dares* to do it" (*atreverse* in Spanish). She also noted that lack of language to communicate one's understanding might hinder attempts to solve a problem.

To push the point a bit more, Eliana asked me if I had been confronted with having to answer a mathematics question publicly in Spanish. I had been asked to introduce myself in Spanish, but that was a practiced recitation using everyday language. I had not been asked to answer any questions related to the mathematics lesson itself or to use academic language. With Eliana's question in mind, I imagined myself as a second-language student in this classroom. Although the lesson included strategies identified as helpful for teaching second language learners—for example, use of visuals, hands-on activities, and interactions (e.g., Echevarría, Vogt, & Short, 2010)—I could still picture myself as a student in this class trying to shrink down to avoid public participation. Understanding, self-esteem, and performance would have been issues for me if I had been a student in this classroom trying to learn in a second language. This classroom experience uncovered several challenges: basic (i.e., second-grade) academic language in a second language, lack of opportunity to ask questions or make sense in my first language, unfamiliar representations, unfamiliar mathematical content/concepts, and cultural differences.

In subsequent observations in this same classroom, I had some opportunity to see how decreasing one or more of the challenges could impact understanding of the classroom practices and the mathematics involved. For example, I found it easier to follow along during the next class that I observed. On that day, one of the practicantes was demonstrating adding one- and two-digit numbers using a base ten chart—a representation that was familiar to me. The vocabulary was in Spanish (e.g., "*centenas, decenas, y unidades*" instead "hundreds, tens, and ones"), but the numerals and the setup of the place value chart were familiar (see Figure 3).

[3] Self-efficacy is defined as "people's judgments of their capabilities to arrange and execute courses of action required to attain designated types of performances" (Bandura, 1986, p. xii). Self-efficacy impacts the things we do, our efforts toward them, and how long we persist in working out solutions to problems.

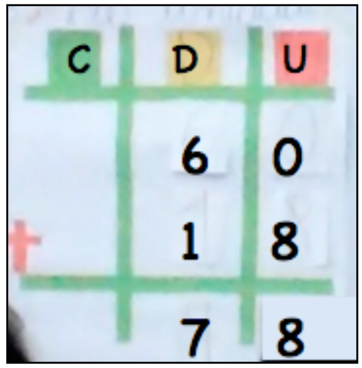

Figure 3
Place value chart used to represent addition. C = centenas (100s), D = decenas (10s), U = unidades (1s).

In all of the classes observed, my limited Spanish language proficiency made it difficult to follow verbal directions. However, when the representations were familiar and when I already had background knowledge, it made it more possible for me to perform familiar arithmetic processes. An excerpt from a related journal entry follows:

> The math here was much more familiar to me than the Mayan numbers from the last observed class. Familiarity with the same system certainly helps. Also, having done the same math for 50+ years helps. In thinking about the Mayan numbers from the last time, I'm wondering if students unfamiliar with base 10 number system might be just as lost (or more lost) than I was last time. The Mayan system that I saw used just 2 symbols within each place value—1s and 5s. The base 10 system has 10 different digits. If a student didn't have the numerical background, it could be very confusing. If, however, the student had learned to add numerals in Spanish, then adding in English would be easy enough, I think—though the names for the numbers would take time to process. What would get lost, I think, is moving beyond computation to concepts. For example, if a student were asked in a second language to explain something related to zeros, they might be able to do so in their native language, but doing so in another language would be challenging. The how and why questions would be difficult.

As I compared experiences across the lessons, I found myself agreeing with theories and research suggesting the importance of opportunities to reason and make sense of new concepts in one's first language in order to support learning and engagement with a second language (e.g., Alanís & Rodríguez, 2008; Cummins, 2000, 2005).

Implications and Recommendations

Our personal experiences and cogenerative dialogue agree with research that suggests cognitive advantages of speaking more than one language including cognitive flexibility, better problem solving, higher order thinking skills, and linguistic resources for managing demands of group mathematics discussions (Anhalt & Rodriguez Pérez, 2013; Hakuta, 1986; Howard, Christian, & Genesee, 2004; Zahner & Moschkovich, 2011). However, we also agree that there are compound challenges when using a developmental second language while trying to learn mathematics—especially for a student who is a beginning speaker and if the mathematics and/or cultural representations are unfamiliar (Moshchkovich, 2007; Rojas, 2005, 2010).

Examples of issues and recommendations related to learning mathematics in a second language that emerged from my experiences, Eliana's experiences, and our cogenerative dialogue include:

- Academic language is much more challenging than conversational language; it is more abstract, more contextualized, more specific, and more culturally determined.
- Working to understand even basic academic instructions in a second language is challenging and exhausting.
- Asking or answering meaningful questions in a second language is intimidating and difficult; one may choose not to publicly participate when learning in a second language.
- One is likely to appear (and feel) less intelligent than one really is.
- Unfamiliar representations and contexts may cause confusion and present additional challenges.
- Second language learners may be ignored or called on less frequently than others in order to avoid communication challenges. Encouragement to speak in *either language* is important. Simple gestures like eye contact and smiling can make a difference.
- Visual representations can help, but are not sufficient—especially when the purposes of the representations are not clearly identified.
- Lack of opportunity to reason in one's primary language can hinder sense making.
- Providing opportunities to reason in one's primary language can support sense making.
- When purposely facilitated by the instructor, wait time to support reasoning in one's primary language can bolster self-esteem and support sense making.
- Personal experience as a second-language learner can be painful but enlightening.

The issues and recommendations presented are consistent with existing literature (e.g., Alanís & Rodríguez, 2008; Anhalt & Rodríguez Pérez, 2013; Cummins, 2000, 2005; Echevarría, et al., 2010; Moschkovich, 2002, 2007, 2013; Rojas, 2010; Thomas & Collier, 2002). However, they are more poignant when personally experienced.

There are potential implications for mathematics teaching and learning in a second language. Language is a mediator of meaning (Vygotsky, 2002) that is fundamental for learning mathematics (Moschkovich, 2002; Truxaw & DeFranco, 2008). Language comprehension may impact students' attitudes toward mathematics and, as a result, their self-efficacy. Because self-efficacy is associated with effort, persistence and resilience, it may, in turn, impact academic performance (Bandura, 1986, 2001). Considering relationships of learning, self-efficacy, and performance, teachers need to work to better understand both challenges and affordances inherent in trying to make sense of mathematics when language and/or representations and/or cultural contexts are unfamiliar. Through this public story, we encourage mathematics teachers to put themselves in positions where they experience affordances and challenges related to learning mathematics in a second language. Awareness is an important first step, but more needs to be done to make sound recommendations for supporting students whose first language is not the language of instruction. Issues of equity are at stake.

References

Alanís, I., & Rodríguez. M. A., (2008). Sustaining a dual language immersion program: Features of success. *Journal of Latinos and Education, 7*(4), 305–319.

Anhalt, C. O., & Rodríguez Pérez, M. E. (2013). K–8 teachers' concerns about teaching Latino/a students. *Journal of Urban Mathematics Education, 6*(2), 42–61. Retrieved from http://ed-osprey.gsu.edu/ojs/index.php/JUME/article/view/158

Bandura, A. (1986). *Social foundations of thought and action: A Social cognitive theory*. Englewood Cliffs, NJ: Prentice-Hall.

Bandura, A. (2001). Social cognitive theory: An agentive perspective. *Annual Review of Psychology, 52*, 1–26.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning* (2nd ed.). Portsmouth, United Kingdom: Heinemann.

Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire*. Buffalo, NY: Multilingual Matters.

Cummins, J. (2005, September). *Teaching for cross-language transfer in dual language education: Possibilities and pitfalls*. Paper presented at TESOL Symposium on Dual Language Education: Teaching and Learning Two Languages in the EFL Setting, Istanbul, Turkey. Retrieved from http://www.achievementseminars.com/seminar_series_2005_2006/readings/tesol.turkey.pdf

Echevarría, J., Vogt, M. E., & Short, D. (2010). *The SIOP model for teaching mathematics to English learners*. Boston, MA: Pearson Education.

Hakuta, K. (1986). *The mirror of language: The debate on bilingualism*. New York, NY: Basic Books.

Howard, E. R., Christian, D., & Genesee, F. (2004). *The development of bilingualism and biliteracy from grade 3 to 5: A summary of findings from the CAL/CREDE study of two-way immersion education* (Research Rep. No 13). Santa Cruz, CA: Center for Research on Education, Diversity & Excellence.

Loughran, J. (2007). Researching teacher education practices: Responding to the challenges, demands and expectations of self-study. *Journal of Teacher Education, 58*(1), 12–20.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4* (2-3), 189–212.

Moschkovich, J. (2007). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*(2), 121−144.

Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6*(1), 45−57. Retrieved from http://ed-osprey.gsu.edu/ojs/index.php/JUME/article/view/204

National Center for Education Statistics (2012). *The nation's report card: Mathematics 2011.* (NCES 2012-458). Washington, DC: United States Department of Education. Retrieved from http://nces.ed.gov/nationsreportcard/pubs/main2011/2012458.aspx

National Clearinghouse for English Language Acquisition. (2011). *The growing number of English learner students*. Retrieved from http://www.ncela.us/files/uploads/9/growing_EL_0910.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Rojas, E. (2005, April). Mathematics and science learning for emigrant children: The ecology of classroom discourse. Paper presented at the International Society of Language Studies, Montreal, Canada.

Rojas, E. (2010). Using mathematics as an equalizer for gifted Latino/a adolescent learners. In J. A. Castellano & A. D. Frazier (Eds.), *Special populations in gifted education. Understanding our most able students from diverse backgrounds* (pp. 353−382). Waco, TX: Prufrock Press.

Thomas, W. P., & Collier, V. (2002). *A national study of school effectiveness for language minority students' long-term academic achievement*. Santa Cruz, CA: Center for Research on Education, Diversity & Excellence. Retrieved from http://escholarship.org/uc/crede_finalrpts

Tobin, K., & Roth, W. M. (2005). Implementing coteaching and cogenerative dialoguing in urban science education. *School Science and Mathematics, 105*(6), 313–322.

Truxaw, M. P. (2014, April). *Lessons learned from linguistically diverse mathematics classrooms*. Paper presented at the 2014 annual meeting of the American Educational Research Association, Philadelphia, PA. Retrieved from http://www.aera.net/Publications/OnlinePaperRepository/AERAOnlinePaperRepository/tabid/12720/Owner/230321/Default.aspx

Truxaw, M. P., & DeFranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education*, *39*(5), 489–525.

United States Census Bureau. (2013). Language use in the United States: 2011. *American Community Survey Reports*. Retrieved from http://www.census.gov/prod/2013pubs/acs-22.pdf

Vygotsky, L. S. (2002). *Thought and language* (13th ed.). Cambridge, MA: MIT Press.

Zahner, W., & Moschkovich, J. (2011). Bilingual students using two languages during peer mathematics discussions: ¿Qué significa? Estudiantes bilingues usando dos idiomas en sus discusiones matemáticas: What does it mean? In K. Téllez, J. Moschkovich, & M. Civil (Eds.), *Latinos/as and mathematics education: Research on learning and teaching in classrooms and communities* (pp. 37–62). Charlotte, NC: Information Age.