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Ethnomodeling as a Research Theoretical Framework on Ethnomathematics and Mathematical Modeling

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In this article, the authors discuss a pedagogical approach that connects the cultural aspects of mathematics with its academic aspects in which they refer to as ethnomodeling. Ethnomodeling is the process of translation and elaboration of problems and questions taken from systems that are part of the daily life of the members of any given cultural group. Here, the authors offer an alternative goal for research, which is the acquisition of both emic and etic forms of knowledge for the implementation of ethnomodeling. They also offer a third perspective on ethnomodeling research, which is the dialectical approach, which makes use of both emic and etic knowledge. Finally, the authors define ethnomodeling as the study of mathematical phenomena within a culture because it is a social construct and culturally bound.

KEYWORDS: dialectical approach, ethnomathematics, ethnomodeling, emic and etic approaches, mathematical modeling

When researchers investigate knowledge possessed by members of distinct cultural groups, they may be able to find unique mathematical ideas, characteristics, procedures, and practices that we consider ethnomathematics, which is used to express the relationship between culture and mathematics. In this regard, the prefix *ethno*- describes characteristics related to the cultural identity of a group such as language, codes, values, jargon, beliefs, food and dress, habits, and physical traits; while the term *mathematics* expresses a broad view of mathematics, which includes ciphering, arithmetic, classifying, ordering, inferring, and modeling (D'Ambrosio, 2001).

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However, an outsider's understanding of cultural traits is always an interpretation that may emphasize only their inessential features, which can be considered as the misinterpretation of this distinctly unique and culturally specific mathematical knowledge. The challenge that arises from this understanding is how culturally bound mathematical ideas can be extracted or understood without letting the culture of the researcher and investigator interfere with the culture of the members of the cultural group under study.

This lack of interference happens when the members of distinct cultural groups have the same interpretation of their culture, which is named an *emic* approach as opposed to an outsider's interpretation, which is named the *etic* approach. The concepts of emic and etic were first introduced by the linguist Pike (1954) who drew upon an analogy with two linguistic terms: phon***emic***—which is considered as the sounds used in a particular language—and phon***etic***—which is considered as the general aspects of vocal sounds and sound production in that language. In other words, all the possible sounds human beings can make constitute the phonetics of the language. However, when people actually speak a particular language, they do not hear all its possible sounds. In this regard, as modeled by linguists, not all sounds make a difference because they are locally significant. This means that they are the phonemics of that language.

Researchers, investigators, educators, and teachers who take on an emic perspective believe that many factors such as cultural and linguistic backgrounds, social norms, moral values, and lifestyles come into play when mathematical ideas, procedures, and practices are developed by the people of their own culture. Different cultural groups have developed different ways of doing mathematics in order to understand and comprehend their own cultural, social, political, economic, and natural environments (Rosa, 2010). Furthermore, every cultural group has developed unique and often distinct ways to *mathematize* their own realities (D'Ambrosio, 1990).

Mathematization is the process in which members from distinct cultural groups come up with different mathematical tools that can help them to organize, analyze, solve, and model specific problems located in the context of their own real-life situation (Rosa & Orey, 2006). These tools allow these members to identify and describe specific mathematical ideas, procedures, or practices in a general context by schematizing, formulating, and visualizing a problem in different ways and discovering relations and regularities. Frequently, local mathematical practices are simply analyzed from a Western view by translating daily problems to academic mathematics through mathematization (Eglash, Bennett, O'Donnell, Jennings, & Cintorino, 2006) without considering the cultural aspects of these practices.

It is important that researchers mathematize local mathematical practices because modeling techniques may be used to translate these practices into academic mathematics. On the other hand, an ethnomathematics perspective attempts to apply modeling to establish relations between local conceptual framework and the mathematical ideas, procedures, and practices developed by the members of distinct cultural groups. In this context, mathematics arises from emic rather than etic origins (Eglash et al., 2006). We must, therefore, search for alternative methodological approaches in order to record mathematical ideas, procedures, and practices that occur in different cultural contexts. One alternative approach to this methodology is called *ethnomodeling*, which is considered an application of ethnomathematics that adds a cultural perspective to the modeling process by studying mathematical phenomena within a culture, which are social constructions and culturally bound (Rosa & Orey, 2010a). Because it is a dynamic research program that is in permanent change and evolution, when justifying the need for a culturally bound view on mathematical modeling, our sources are rooted on the theoretical base of ethnomathematics (D'Ambrosio, 1990).

The rationale of this theoretical perspective is to discuss how ethnomodeling is the study of the mathematical ideas and procedures of local communities that uses dialectical relationships between emic and etic approaches on those mathematical practices. The contributions of this article to the mathematic education research literature is to bring to light a distinction about the types of processes that ethnomathematical and modeling research may include such as emic, etic, and dialectical approaches into the ethnomodeling research field.

This context allows us to state that the purpose of this article is to offer an alternative goal for research, which is the acquisition of both emic and etic knowledge for the implementation of ethnomodeling in classrooms. In order to achieve this goal, this article is structured in a way that guides the readers in sections that discuss the application of ethnomathematics along with the application of modeling techniques. This approach prepares readers to perceive the connection of cultural aspects of mathematics with its academic aspects.

Ethnomathematics

Ethnomathematics as a research paradigm is wider than traditional concepts of mathematics, ethnicity, or any current sense of multiculturalism. Ethnomathematics is described as the arts and techniques (*tics*) developed by members from diverse cultural and linguistic backgrounds (*ethno*) to explain, to understand, and to cope with their own social, cultural, environmental, political, and economic environments (*mathema*) (D'Ambrosio, 1990). *Ethno* refers to distinct groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring. Detailed studies of mathematical procedures and practices of distinct cultural groups most certainly allow us to further our understanding of the internal logic and mathematical ideas of diverse groups of people.

As depicted in Figure 1, we consider ethnomathematics as the intersection of cultural anthropology, mathematics, and mathematical modeling, which is used to help us understand and connect diverse mathematical ideas and practices found in our communities to traditional and academic mathematics (Rosa, 2000).

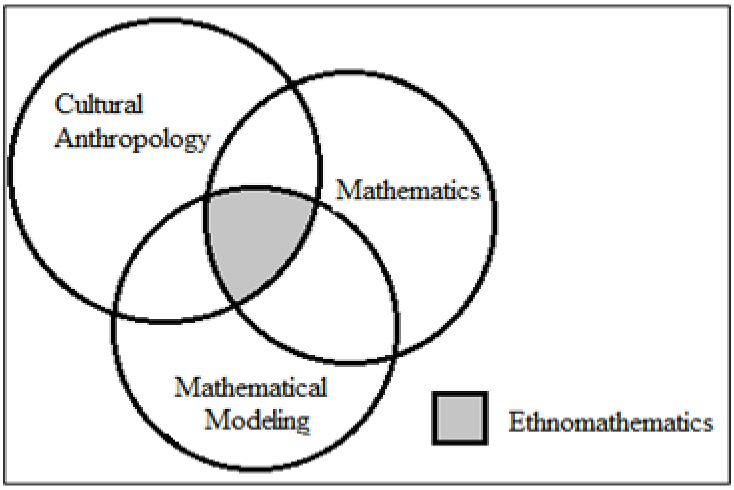

Figure 1. Ethnomathematics as an intersection of three research fields.
Source: Rosa (2000)

Ethnomathematics, as well, is a program that seeks to study how students have come to understand, comprehend, articulate, process, and ultimately use mathematical ideas, procedures, and practices that enable them to solve problems related to their daily activities (Rosa, 2000). This holistic context helps students to reflect, understand, and comprehend relations among all components of systems under study. In this regard, educators should be empowered to analyze the role of students' *ethnoknowledge* in the mathematics classroom (Borba, 1990), which is acquired by students in the process of learning mathematics in culturally relevant educational systems.

Ethnomodeling

Ethnomodeling is the study of mathematical ideas and procedures elaborated by members of distinct cultural groups. It involves the mathematical practices developed, used, and presented in diverse situations in the daily life of the members of these groups (Rosa & Orey, 2010a). This context is holistic and allows those engaged in this process to study mathematics as a system taken from their own reality in which there is an equal effort to create an understanding of all components of these systems as well as the interrelationship among them (D'Ambrosio, 1993; Bassanezi, 2002; Rosa & Orey, 2003).

Researchers and investigators such as Ascher (2002), Eglash (1999), Gerdes (1991), Orey (2000), Urton (1997), and Rosa and Orey (2009) "have revealed [in their research] sophisticated mathematical ideas and practices that include geometric principles in craft work, architectural concepts, and practices in the activities and artifacts of many indigenous, local, and vernacular cultures" (Eglash et al., 2006, p. 347). These concepts are related to numeric relations found in measuring, calculation, games, divination, navigation, astronomy, modeling, and a wide variety of other mathematical procedures and cultural artifacts (Eglash et al., 2006). In this context, ethnomodeling may be considered as the intersection area of cultural anthropology, ethnomathematics, and mathematical modeling, which can be used "as a tool towards pedagogical action of an ethnomathematics program, students have been shown to learn how to find and work with authentic situations and real-life problems" (Rosa & Orey, 2010a, p. 60). Figure 2 shows ethnomodeling as an intersection of three research fields.

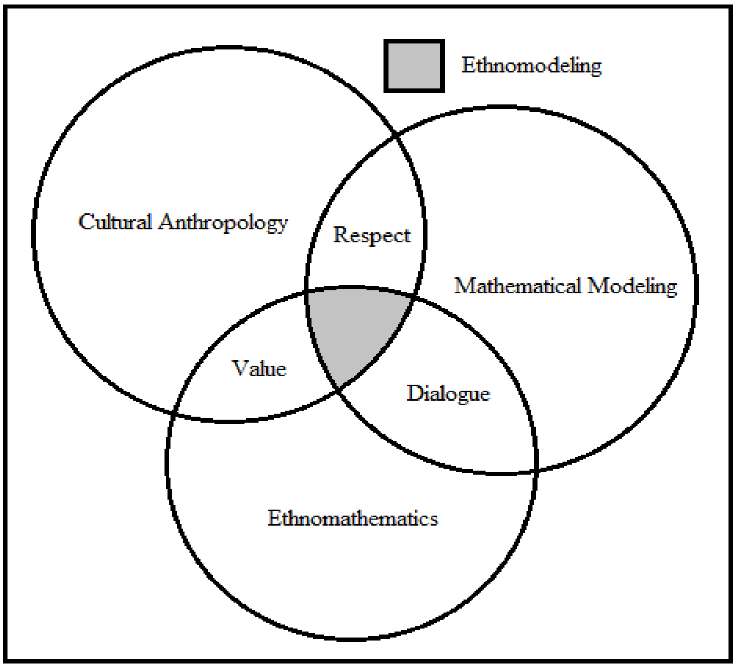

Figure 2. Ethnomodeling as an intersection of three research fields.
Source: Rosa & Orey (2010a)

Researchers such as Eglash and colleagues (2006) and Rosa and Orey (2006) use the term *translation* to describe the process of modeling local cultural systems (*emic*), which may have a Western academic mathematical representation (*etic*). In other words, ethnomathematics makes use of modeling by attempting to use it to establish a relationship between the local conceptual framework (emic) and the mathematics embedded in relation to local designs. On the other hand, often indigenous designs are merely analyzed from a Western view (etic) such as the applications of the symmetry classifications from crystallography to indigenous textile patterns (Eglash et al., 2006). In some cases, "the translation to Western mathematics is direct and simple such as counting systems and calendars" (Eglash et al., 2006, p. 347). However, there are cases in which mathematical ideas and concepts are "embedded in a process such as iteration in bead work, and in Eulerian paths in sand drawings" (Eglash et al., 2006, p. 348). In our point of view, the act of translation applied in this process is best referred to as ethnomodeling in which "mathematics knowledge can be seen as arising from emic rather than etic origins" (Eglash et al., 2006, p. 349).

In this context, the emphasis of ethnomodeling takes into consideration the essential processes found in the construction and development of mathematical knowledge, which includes often curious and unique aspects of collection, creativity, and invention. It is impossible to imprison mathematical concepts in registers of univocal designation of reality because there are distinct systems that provide an unambiguous representation of reality as well as universal explanations (Craig, 1998). This means that mathematics cannot be conceived as a universal language because its principles, concepts, and foundations are not the same everywhere (Rosa, 2010). In this regard, "the choice among equivalent systems of representation can only be founded on considerations of simplicity, for no other consideration can adjudicate between equivalent systems that univocally designate reality" (Craig, 1998, p. 540). The processes of production of mathematical ideas, procedures, and practices operate in the register of the interpretative singularities regarding the possibilities for a symbolic construction of knowledge in different cultural groups (Rosa & Orey, 2006).

Mathematical Phenomena, Modeling, and Ethnomodeling

Throughout history, researchers and investigators have made extensive use of mathematical modeling procedures ranging from statistical methods for the elucidation of patterns in behavior to the mathematical representations of the logic processes of indigenous and local conceptual systems. Mathematical modeling has been considered by some to be a pedagogical tool and by others as a way to understand anthropological and archaeological research (Read, 2002). Yet others have decried the use of mathematical, and in particular statistical and quantitative, modeling as fundamentally in opposition to a humanistic approach to understanding human behavior and knowledge that takes into account contingency and historical embeddedness and, in turn, decries universality. However, we argue that traditional mathematical modeling does not fully take into account the implications of cultural aspects of human social systems.

The cultural component in this process is critical; it "emphasize[s] the unity of culture, viewing culture as a coherent whole, a bundle of [mathematical] practices and values" (Pollak & Watkins, 1993, p. 490) that are incompatible with the rationality of the elaboration of traditional mathematical modeling process. In the context of mathematical forms of knowledge, however, what is meant by the cultural component varies widely. It ranges from viewing mathematical practices as procedures learned, acquired, and transmitted to members of cultural groups across generations to mathematical practices viewed as made up of abstract symbols with an internal logic (Rosa, 2000). If the former is considered, then it is the process by which knowledge transmission takes place from one person to another, which is central to elucidating the role of culture in the development of mathematical knowledge (D'Ambrosio, 1993). If the latter is considered, then culture plays a far reaching and constructive role with respect to mathematical practices that cannot be induced simply through observation of these practices (Eglash et al., 2006).

In this regard, mathematical knowledge developed by members of a specific cultural group consists of abstract symbol systems whose form is the consequence of a unique internal logic. These symbols are then learned through instances of usage within this cultural context. These members also learn about what is derived from those instances, which helps them to form a cognitively based understanding of the internal logic of their mathematical symbolic systems. The cognitive aspects needed in this procedure are primarily decision -making processes by which the members of cultural groups either accept or reject an ethnomodel as part of their own repertoire of mathematical knowledge. We believe that the conjunction of these two scenarios appears to be adequate enough to encompass the full range of cultural mathematical phenomena.

In effect, there are two ways in which we recognize, represent, and make sense of a mathematical phenomenon that impinges upon our sensory system. First, there is a level of cognition that we share, to varying degrees, with the members of our own and other cultural groups. This level would include cognitive models that we may elaborate on at a non-conscious level, which serves to provide an internal organization of external mathematical phenomena in order to provide the basis upon which a mathematical practice takes place. Second, there is a culturally constructed representation of external mathematical phenomena that also provides its internal organization. However, this representation arises through the formulation of abstract and conceptual structures that provides forms and organizations for external mathematical phenomena. In other words, cultural constructs provide representations for systems taken from reality.

The implications for mathematical modeling are that models of cultural constructs are considered as symbolic systems organized by the internal logic of members of cultural groups. We agree with Eglash and colleagues (2006) and Rosa and Orey (2010b) who argued that models built without a first-hand sense for the world being modeled should be viewed with suspicion. Researchers and investigators, if not blinded by their prior theory and ideology, should come out with an informed sense of distinctions that make a difference from the point of view of the mathematical knowledge of the work being modeled. In doing so, they should, in the end, be able to tell outsiders (etic) what matters to insiders (emic).

Ethnomodeling privileges the organization and presentation of mathematical ideas and procedures developed by the members of distinct cultural groups in order to facilitate its communication and transmission across generations. These members construct ethnomodels of mathematical practices found in sociocultural systems (Rosa & Orey, 2010b), which link cultural heritage with the development of mathematical practice. It is our understanding that this approach may help the organization of the pedagogical action that takes place in classrooms through the use of the emic and etic aspects of this mathematical knowledge.

The Emic and Etic Constructs of Ethnomodeling

In the ethnomodeling approach, the emic constructs are the accounts, descriptions, and analyses expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the members of the cultural group under study (Lett, 1996). This means that an emic construct is in accordance with the perceptions and understandings deemed appropriate by the insider's culture. The validation of the emic knowledge comes with a matter of consensus, which is the consensus of local people who must agree that these constructs match the shared perceptions that portray the characteristic of their culture (Lett, 1996).

In other words, the emic approach tries to investigate mathematical phenomena and their interrelationships and structures through the eyes of the people native to a particular cultural group. It is paramount to note that particular research techniques used in acquiring emic mathematical knowledge has nothing to do with the nature of that knowledge. In this regard, the "emic mathematical knowledge may be obtained either through elicitation or observation because it is possible that objective observers may infer local perceptions" (Lett, 1996, p. 382) about mathematical ideas, procedures, and practices developed through history.

It is necessary to state that etic constructs are considered accounts, descriptions, and analyses of mathematical ideas, concepts, procedures, and practices expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers, researchers, and investigators (Lett, 1996). An etic construct is precise, logical, comprehensive, replicable, and observer-researcher independent. In so doing, the validation of the etic knowledge thus becomes a matter of logical and empirical analysis, in particular, the logical analysis of whether the construct meets the standards of comprehensiveness and logical consistency, and then the empirical analysis of whether or not the mathematical concept has been replicated (Lett, 1996).

It is important to emphasize that particular research techniques used in the acquisition of etic mathematical knowledge has no bearing on the nature of that knowledge. The etic knowledge may be obtained at times through elicitation as well as observation, because it is entirely possible that native informants possess scientifically valid knowledge (Lett, 1996). In this sense, we agree with D'Ambrosio (1990) who states that researchers and investigators have to acknowledge and recognize that members of distinct cultural groups possess scientifically mathematically valid knowledge.

The Dialectical Approach in Ethnomodeling Research

The emic perspective is concerned about differences that make mathematical practices unique from an insider's point of view. We argue that emic ethnomodels are grounded in the mathematical ideas, procedures, and practices that matter to the insiders' view of the mathematical world being modeled. On the other hand, many ethnomodels are etic in the sense that they are built on an outsider's view about the mathematical world being modeled. In this context, etic ethnomodels represent how the modeler thinks the world works through systems taken from reality while emic ethnomodels represent how people who live in such worlds think these systems work in their own reality.

We also would like to point out that the emic perspective always plays an important role in ethnomodeling research, yet the etic perspective should also be taken into consideration in this process. In this perspective, the emic ethnomodels sharpen the question of what an agent-based model should include to serve practical goals in modeling research. Thus, mathematical ideas and procedures are etic if they can be compared across cultures using common definitions and metrics. On the other hand, the focus of the analysis of these aspects are emic if mathematical ideas, concepts, procedures, and practices are unique to a subset of cultures that are rooted on the diverse ways in which etic activities are carried out in a specific cultural setting.

Currently, the debate between emic and etic is one of the most intriguing questions in ethnomathematics and mathematical modeling research since researchers continue to deal with questions such as:

1. Are there mathematical patterns that are identifiable and/or similar across cultures?
2. Is it better to focus on these patterns particularly arising from the culture under investigation?

While emic and etic are often thought of as creating a conflicting dichotomy, they were originally conceptualized as complementary viewpoints (Pike, 1967). According to this context, rather than posing a dilemma, the use of both approaches deepens our understanding of important issues in scientific research and investigations (Berry, 1999). A suggestion for dealing with this dilemma is to use a combined emic-etic approach, rather than simply applying emic or etic dimensions to study and examine mathematical procedures and practices employed by the members of distinct cultural groups. A combined emic-etic approach requires researchers to first attain emic knowledge developed by the members of the cultural groups under study. This may allow researchers to put aside their cultural biases so that they may be able to become familiar with the cultural differences that are relevant to the members of these groups (Berry, 1990).

Usually, in ethnomodeling research, an emic analysis focuses on a single culture and employs descriptive and qualitative methods to study a mathematical idea, concept, procedure, or practice of interest. Its focus is on the study within a cultural context in which the researcher tries to examine relative internal characteristics or logic of the cultural system. In this perspective, meaning is gained relative to the context and therefore not easily, or if at all, transferable to other contextual settings. For example, it is not intended to compare the observed mathematical patterns in one setting with mathematical patterns in other settings.

This means that the primary goal of an emic approach is a descriptive idiographic orientation of mathematical phenomena because it puts emphasis on the uniqueness of each mathematical idea, procedure, or practice developed by the members of cultural groups. Thus, if researchers and educators wish to highlight meanings of these generalizations in local or emic ways, then they need to refer to more precisely specified mathematical events. In contrast, an etic analysis would be comparative, examining many distinct mathematical cultural practices by using standardized methods (Lett, 1996). This means that the etic approach tries to identify lawful relationships and causal explanations valid across different cultures. Thus, if researchers and educators wish to make statements about universal or etic aspects of mathematical knowledge, these statements need to be phrased in abstract ways.

On the other hand, an etics approach may be a way of examining the emics of members of cultural groups because it may be useful for looking deeply at, discovering, and elucidating emic systems that were developed by members of different cultural groups (Pike, 1954). In so doing, while traditional concepts of emic and etic are important points of view for understanding and comprehending cultural influences on mathematical modeling, we would like to propose a distinctive view on ethnomathematics and modeling research, which is referred as a *dialectical approach* (Martin & Nakayama, 2007). In this approach, the etic perspective claims that the mathematical knowledge of any given cultural group will have no necessary priority over its competing emic meaning. According to this point of view, it is necessary to depend "on acts of 'translation' between emic and etic perspectives" (Eglash et al., 2006, p. 347). In other words, the cultural specificity may be better understood with the background of communality and the universality of theories and methods and vice versa.

In this context, these claims must be verified with methods independent of the subjectivity of the observer and researcher in order to achieve a scientific character. Therefore, it becomes important to analyze the insights that have been acquired through subjective and culturally contextualized methods. The rationale behind the emic-etic dilemma is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur.

The Wine Barrel: The Dialectical Ethnomodel

The ethnomodel that offers an ethnomodeling methodology example was elaborated by a group of Brazilian students who studied wine production (Bassanezi, 2002). The motivation of their study was to find the volume of wine barrels by applying the techniques learned by ancestors of the wine producers who came to Southern Brazil as Italian immigrants in the early twentieth century. Since then the production of wine has become an essential agricultural activity for the economy of that Brazilian region.

The ethnomodeling process. In order to conduct their research, initially, students visited wineries to conduct interviews with the wine producers. Subsequently, they collected data that were supplemented by the literature review on the chosen theme. The ethnological and historical research of the construction of wine barrels theme was the first stage of the ethnomodeling process. In the ethnological study, students identified characteristics of this particular group so that they were able to understand some of the cultural elements that shape their mathematical thinking (Bassanezi, 2002). In this context, students found out that, in addition to producing wine, wine producers construct their own wooden wine barrels by using geometric schemes inherited from their ancestors in Italy.

During their research, students also found out that to construct these barrels with pre-established volume, it is necessary for the wine producers to cut wooden staves to fit perfectly. This process drew the attention of the students who were interested in exploring the mathematical ideas the wine producers were using in their geometric schemes. For example, figure 3 shows a geometric scheme made by the wine producers in the construction of wine barrels.

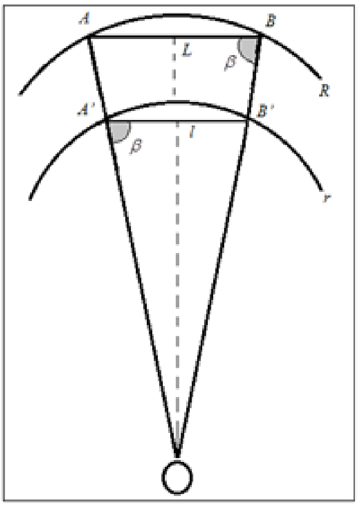

Figure 3. Geometric scheme made by wine producers in the construction of wine barrels.
Source: Bassanezi (2002, p. 47)

In the scheme in figure 3, *L* is the maximum width of the stave, ℓ is the width to be determined and β is the fitting angle between the staves, which depend on the initial width of the stave *L* and the volume required for the wine barrel. In figure 4, the *R* is the radius of the base of the larger circle that represents the base of the barrel, *r* is the radius of the smaller circle that represents the barrel cover and *H* is the height of the barrel.

The wine producers construct barrels shaped like a truncated cone by interlocking wooden staves whose dimensions are 2.5 cm in length and width ranging from 5cm to 10cm (Bassanezi, 2002).

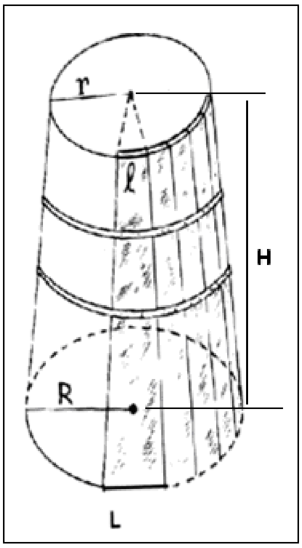

Figure 4. Wine barrel shaped like a truncated cone
Source: Bassanezi (2002, p. 48)

In order to determine the volume of the wine barrel, wine producers approximated its volume by applying a procedure called *averaging cone* (Bassanezi, 2002), which is used in the construction of wooden wine barrels. It is important to state here that the volume determined by the averaging cone formula is an approximation of the volume obtained by applying the academic formula that provides the volume of the truncated cone.

The averaging cone formula is given by:

$$V \cong \pi \bullet r_m^{\ 2} \bullet H$$

They also apply the *averaging radius* procedure, which is given by formula II:

$$r_m = \frac{r + R}{2}$$

By replacing formula II into formula I, formula III is given by:

$$V \cong \pi \bullet \left(\frac{r+R}{2}\right)^2 \bullet H$$

Figure 4 also shows that the fitting angle β between the two wooden staves is obtained by considering that:

- R is the radius of the base of the wine barrel.
- L is the width of the wooden stave of the wine barrel in its base.
- All juxtaposed wooden staves form a circumference at the base of the wine barrel.

In this process, it is possible to observe that the scheme used in figure 3 is an orthogonal projection of one of the wine barrel wooden staves as shown in figure 5.

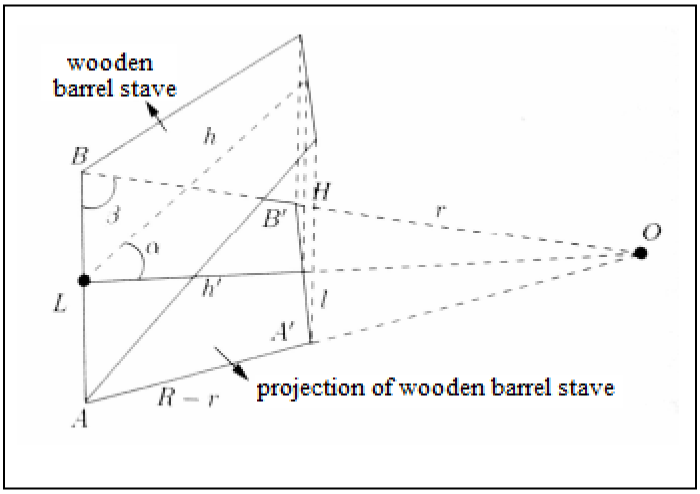

Figure 5. Orthogonal projection of a wine barrel wooden slate
Source: Bassanezi (2002, p. 49)

According to the etic approach by developing the mathematical model used in academic mathematics, the volume of the truncated cone is given by the formula:

$$V = \frac{1}{3} \bullet \pi \bullet H \bullet \left(R^2 + rR + r^2\right)$$

In the emic approach by developing the ethnomodel used by the wine producers the volume of the wine barrel is given by the formula:

$$V \cong \pi \bullet \left(\frac{r+R}{2}\right)^2 \bullet H$$

The construction of the wine barrel process is an excellent example that typifies the connection between ethnomathematics and mathematical modeling

(D'Ambrosio, 2002) through ethnomodeling. Thus, this method presents an approximated calculation of the area of the volume of the wine barrel as employed by the members of this specific cultural group.

Some Considerations about the Ethnomodeling Wine Barrel Construction

An emic observation of this mathematical practice sought to understand it for constructing wine barrels from the perspective of local dynamics and relationships as influenced within the culture of the wine producers themselves. On the other hand, an etic perspective uses some aspects of academic mathematics to translate this mathematical practice in order to amplify the understanding of those from a different cultural background by explaining this practice from the point of view of the outsider.

In this context, the emic viewpoint clarifies intrinsic cultural distinctions while the etic perspective seeks objectivity as an outside observer across cultures. This is the dialectical approach, which concerns the stability of relationships between these two different cultural approaches. In our point of view, both perspectives are essential to understand human behaviors (Pike, 1996), especially, social and cultural behaviors that help to shape mathematical ideas, procedures, and practices developed by the members of distinct cultural groups.

Finally, in order for this process to be successful as well as mathematics to be valued as a discipline whose contents can be considered as human creations, it is necessary to understand and modify the environment we live. In this regard, we can use ethnomodeling in order to link theory into practice by the inclusion of the dialectical approach into the mathematics curriculum.

The Dialectical Approach into a Mathematical Curriculum

Classrooms cannot be isolated from their communities because they form part of a well-defined cultural practice. Classrooms form learning environments that facilitate pedagogical practices, which are developed by using an ethnomodeling approach (Rosa, 2010). When students come to school, they bring with them values, norms, and concepts that they have acquired in their home-community cultural environments. Some of these concepts are mathematical in nature (Bishop, 1994). However, mathematical concepts of the school curriculum are presented in a way that may not be directly related to the cultural backgrounds of the students.

It has been hypothesized that low attainment in mathematics could be due to a lack of cultural consonance in mathematics curriculum. In this regard, the pedagogical elements necessary to develop a mathematics curriculum are found in the school community itself (Bakalevu, 1998). Thus, the field of ethnomodeling presents tangible possibilities for educational initiatives that introduce mathematical ideas via rich problems that engage students in doing mathematics and will aid them in developing the mathematical reasoning and problem-solving abilities used by expert problem solvers (Rosa, 2010).

According to this context, mathematical knowledge of the members of distinct cultural groups combined with Western-mathematical knowledge systems may result in a dialectical approach to mathematics education. In this regard, *academic mathematics* (etic) is efficient and appropriate to solve many problems and there is no reason to replace it. However, *local mathematical practices* (emic) are good in solving other kinds of problems. The combination of these two domains of ethnomathematics offers greater possibilities to understand and explain different problem-solving situations (Rosa & Orey, 2010a).

In this context, an emic analysis of a mathematical phenomenon is based on local structural or functional elements of a particular cultural group while an etic analysis is based on predetermined general concepts external to that cultural group (Lovelace, 1984). The emic perspective provides internal conceptions and perceptions of mathematical ideas and concepts while the etic perspective provides the framework for determining the effects of those beliefs on the development of the mathematical knowledge. In this perspective, the acquisition of mathematical knowledge is based on the applications of current mathematics curriculum, which may be assessed based on multiple instructional methodologies across various cultures.

In this regard, it could indeed be that one of the reasons for failure of students' achievement in many educational systems is that curriculum developers by using a *one size fits all* program have ignored unique emic perspectives regarding the recognition of distinct cultural backgrounds within the schools. A dialectal approach supports the recognition of the existence of other mathematical knowledge systems, which are found in many schools and urban centers.

In other words, an ethnomodeling curriculum provides an ideological basis for learning with and from the diverse cultural and linguistic elements presented by members of distinct cultural groups (Rosa & Orey, 2010a). In this kind of curriculum it would be crucial to understand that etic constructs are mathematical ideas, procedures, and practices that are assumed to apply in all cultural groups while emic constructs only apply to members of specific cultural groups. This means that there is concern for cultural bias occurring if educators and researchers assume that an emic construct is actually etic (Eglash et al., 2006), which results in an imposed etic perspective in which a culture-specific idea is wrongly imposed on the members of another cultural group.

This kind of curriculum is intended to make school mathematics more relevant and meaningful to students because it is based on students' knowledge, which allows teachers to have more freedom and creativity to choose academic mathematical topics to be covered in the lessons. It is through dialogue that teachers can apply mathematical themes that help students to elaborate the mathematics curriculum. In this regard, teachers engage students in the critical analysis of the dominant culture as well as the analysis of their own culture through an ethnomodeling perspective. It also means that it is necessary to investigate the conceptions, traditions, and mathematical practices of a particular cultural group with the intention of incorporating these concepts into the mathematics curriculum (Rosa, 2010).

A classroom environment in which teachers are applying an ethnomodeling curriculum would be full of examples that draw on the students' own experiences and makes use of experiences common in their cultural environments. These examples would be vehicles for communicating mathematical ideas, which themselves would remain relatively unchanged. In other words, ethnomathematics aims to draw from the students' cultural experiences and practices of the individual learners, the communities, and the society at large (Rosa, 2010). This means that ethnomathematics uses these cultural experiences as vehicles to not only make mathematics learning more meaningful but also, and more importantly, to provide students with the insights of mathematical knowledge as embedded in their social and cultural environments (Rosa & Orey, 2008).

While it seems that only the members of a specific cultural group living within the culture can provide an emic perspective of the mathematical knowledge that is generated by their own culture, when teachers and local members of a community come together in research/study groups they can find creative ways in which to use elements of their own culture, knowledge, and language in the elaboration of curricular activities and pedagogical practices. They also can create zones of safety in which resistance to conventional practices can be expressed and innovative approaches to schooling can be investigated and practiced. The work of these teacher groups may have theoretical implications for community-based teacher preparation. Factors influencing development of these groups and their ability to affect change need to be discussed along with the challenges of transferring their cultural creations to the wider institutions of schooling. According to this context, teachers and members of the community decide what mathematics content needs to be taught to students. In our opinion, this is how teachers may reconcile between the many emics that students from distinct cultural backgrounds bring to the classrooms.

An ethnomodeling curriculum that combines key elements of local and academic knowledge in a dialectical approach helps students to manage knowledge and information systems taken from their own cultural background and creatively apply this knowledge to other situations. This means that ethnomodeling can be considered part of a critical mathematics education because it provides a learning process in which teachers encourage the use of multiple sources of knowledge from different cultural contexts. In this approach, acquired knowledge is centered, located, oriented, and grounded on the cultural backgrounds of the students, which could be applied and translated appropriately by them and thus equip them to be fully productive locally and globally. According to Rosa and Orey (2010b), ethnomodeling is a pedagogical approach valuable for reaching this goal.

The nature of the previous mathematical knowledge of the students lends themselves to the principle of sequencing in curriculum development. By giving educators the freedom to start with previous mathematical knowledge and experience of their students, we can move from the familiar to the unfamiliar and from the concrete to the abstract in the process of promoting the acquisition of mathematical knowledge (Rosa & Orey, 2006) and should include moves from emic to etic perspectives and vice versa.

In this dialectical context, an ethnomodeling curriculum provides the underlying philosophy for knowledge generation and exchange within and between distinct systems of mathematical knowledge. This concept of an ethnomodeling curriculum approach ensures a more balanced integration of the affective domain of educational objectives that are essential to the recognition and utilization of the students' previous knowledge.

Final Considerations

Today, numerous diverse indigenous and local mathematical knowledge systems and traditions are at risk of becoming extinct because of the rapidly changing natural and cultural environments and a fast pacing economic, social, environmental, political, and cultural changes occurring on a global scale. Many local mathematical practices disappear because of intrusion and imposition of foreign etic knowledge and technologies or from the development concepts that promise short-term gains or solutions to problems faced by the members of a specific cultural group without considering their emic knowledge to solve these problems. Not unlike the loss of global tropical rainforests, the tragedy of the impending disappearance of indigenous and local knowledge is most obvious when a diversity of skills, technologies, and cultural artifacts, problem solving strategies and techniques, and expertise are lost to all of us before being understood and/or achieved and archived.

Defined in that manner, the usefulness of the emic and etic distinction is evident. Like all human beings, researchers, educators, and teachers have been enculturated to some particular cultural worldview. They, therefore, need a means of distinguishing between the answers they derive as enculturated members and the answers they derive as observers. Defining emics and etics in epistemological terms provides a reliable means of making that distinction. In this perspective, culture is a lens, shaping reality; it can be considered a blueprint, specifying a plan of action. At the same time, a culture is unique to a specific group of people. By utilizing the research provided by both emic and etic approaches through ethnomodeling, we gain a more complete understanding of the cultural groups of interest. In this article, the ethnomodeling process was investigated from an ethnomathematical perspective as the cultivation of grape vines and the production of wine barrels are strongly linked to the history and culture of people in the Southern region of Brazil.

We have offered an alternative goal for research, which is the acquisition of both *emic* and *etic* knowledge for the implementation of an ethnomodeling research. Emic knowledge is essential for an intuitive and empathic understanding of mathematical ideas of a culture and it is essential for conducting effective ethnographic fieldwork. Furthermore, emic knowledge is a valuable source of inspiration for etic hypotheses. Etic knowledge, on the other hand, is essential for cross-cultural comparison, the essential components of ethnology, because such comparison necessarily demands standard units and categories. We also offered here a third approach on ethnomodeling research, which is the dialectical approach that makes use of both emic and etic knowledge and understandings through the processes of dialogue.

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