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ON PARABOLIC REGULARIZATION OF HYPERBOLIC HEAT CONDUCTIVITY IN RIGID BODY[1]

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The higher order parabolic model of heat conduction is developed using the gradient theory with internal state variable. The model can be viewed as a parabolic regularization of the hyperbolic model of heat conduction developed in the earlier papers within the framework of modified Fourier laws.

1. Introduction

Classical Fourier law of heat conduction leads in general case to a non-linear system of the parabolic-type equation. In order to get the finite speed of propagation of thermal disturbances a modified Fourier law was presented by Kosiński (1989), in which the heat flux vector was proportional to the spatial gradient of an internal state variable, viewed as a semi-empirical temperature (cf Cimmelli and Kosiński (1991)) and related to the absolute temperature by an evolution equation. In this way the internal state variable represents a history of the absolute temperature. In that modification the resulting, final system of equations is hyperbolic. The equations are quasi-linear and hence they may involve some stability and uniqueness problems when numerical solutions of the wave-type is searched. In order to overcome these difficulties and give more detailed description of the complex phenomena of heat conduction more general model is required.

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Such a model has been just constructed within the framework of gradient generalization of the internal state variable theory (cf Kosiński and Wojno (1995)). In this theory the evolution equation for internal state variables and the energy balance equation lead to the final system of governing equations which is of the third order and parabolic type.

Although the main purpose of that paper was to develop a general set-up of the gradient theory, the first idea of parabolic regularization of the modified Fourier law of the semi-empirical temperature theory has been also mentioned. The aim of the present paper is just to present this regularization features using the simplest possible reasoning.

Thus in this paper the primitive, modified Fourier law of semi-empirical temperature has been perturbed by adding to the existing term proportional to $\nabla\beta$ a small term proportional to $\nabla\dot{\beta}$; or in other words to $\dot{\overline{\nabla\beta}}$. In fact this can be done by allowing the heat flux vector to depend on the gradient of absolute temperature, like in the classical Fourier law, and additionally on the gradient of internal state variable β. In this way the resulting heat conduction constitutive equation, due to the second prolongation of the evolution equation in β and the particular choice of the proportionality coefficients, can be regarded as a generalization of the Fourier law. This simple perturbation changes the type of the resulting field equations from hyperbolic to parabolic. It can be observed by appearance of the term with the third order mixed derivative.

We would like to emphasize that the parabolic regularization properties are features inherent of the gradient generalization of the internal state variable theory developed by Kosiński and Wojno (1995).

2. General framework

Kosiński (1989) introduced, for the first time, the gradient of a scalar internal state variable as a state variable in response functions of a thermoelastic material. In the course of examination of consequences following from the laws of thermodynamics he obtained a modified Fourier-type law and finite speeds of propagation of thermal and thermomechanical waves. That new model differs from the corresponding model, developed by Kosiński and Perzyna (1972) by the form of the evolution and constitutive equations. The model has been mostly applied to rigid heat conductors in 1D and 3D cases, and to thermoelastic solids in 1D case (cf Cimmelli and Kosiński (1991), (1992), (1993); Cimmelli et al. (1992); Frischmuth and Cimmelli (1995); Kosiński and Saxton (1991), (1993)).

Kosiński and Wojno (1995) solve some open problems of the new model with the internal state variable gradient. One of them is the question of the absence of the internal state variable from the constitutive function for free energy. Moreover, in that model the gradient of absolute temperature does not influence the response of the material. There were some reasons for constructing such a model, namely, to keep the generalization as simple as possible.

In the present paper we discuss the parabolic regularizaton existing in the approach presented by Kosiński and Wojno (1995).

If one considers the following scalar hyperbolic quasi-linear equation

$$u_t + g(u)_x = 0 \tag{2.1}$$

with a nonlinear function $g(u)$ of a scalar unkown function $u(t,x)$, then a global weak solution may not exist and not be unique, even for smooth initial data. Then one can try to find an admissible unique weak solution to Eq (2.1) as a limit of a sequence of approximate smooth solutions to Eq (2.1) generated by parabolic regularization. The Cauchy problem for a "diffusion processes" of the form

$$u_t + g(u)_x = \epsilon D(u)_{xx} \tag{2.2}$$

with a proper function $D(u)$ has for any nonvanishing ϵ a smooth and unique solution (cf Dafermos (1973); DiPerna (1983)). If a solution to Eq (2.1) is searched as a limit of sequnce of solutions to Eq (2.2), then we say that a parabolic regularization method has been applied to get the weak solution to the primitive hyperbolic equation Eq (2.1). Let us note that Eq (2.2) opens up the possibility of describing the structure of shock waves that is solutions to Eq (2.1) with strong discontinuity jumps.

If the choice of the function $D(u)$ is based on mathematical arguments only, then one deals with the method called the artificial viscosity one. On the other hand if the choice of $D(u)$ results from physical arguments, in particular due to a proper choice of constitutive equation fully supported by physics, then we deal with a physical parabolic regularization.

The present paper tackles thermodynamic parabolic regularization of the hyperbolic heat conduction equations. To this aim one has to point out that the hyperbolic system of equations is of the second order, while its parabolic regularized form contains terms with the third order derivatives of the unkown function β.

2.1. Basic equations

Consider a rigid heat conductor $\mathcal{B}$ with the internal structure described by the influence of spatial gradient of the temperature on the response of a generic material point X of $\mathcal{B}$. According to the Kosiński and Wojno (1995) at any time t of any process restricted by the thermodynamic laws

$$\frac{\partial(\rho\epsilon)}{\partial t} + \mathrm{div}\boldsymbol{q} = \rho r$$
$$\frac{\partial(\rho\eta)}{\partial t} + \mathrm{div}\frac{\boldsymbol{q}}{\vartheta} \geq \frac{\rho r}{\vartheta} \tag{2.3}$$

where

- ρ – mass density
- $\boldsymbol{q}$ – heat flux vector
- ϵ – specific internal energy per units mass
- r – body heat supply
- η – specific entropy
- ϑ – absolute temperature.

the properties of such a conductor in terms of the free energy ψ, defined by

$$\psi = \epsilon - \eta\vartheta\ , \tag{2.4}$$

are given by the constitutive relations

$$\psi = \psi^*(\vartheta, \beta, \nabla\beta) \qquad \eta = -\partial_\vartheta\psi^*(\vartheta, \beta, \nabla\beta)$$
$$\boldsymbol{q} = \boldsymbol{q}^*(\vartheta, \nabla\vartheta, \beta, \nabla\beta) \tag{2.5}$$

in which β is the internal state variable and, at last, the symbol ∇ denotes the grad operator.

In what follows we think of $\nabla\beta$ as representing the history of $\nabla\vartheta$. Hence the internal parameter β must satisfy the initial evolution equation problem

$$\dot{\beta} = F(\vartheta, \nabla\vartheta, \beta, \nabla\beta)\ , \qquad \beta(t_0) = \beta_i\ , \tag{2.6}$$

in which t_0 denotes an initial instant and β_i is a correspondent initial distribution of β, assumed to be given for each X of $\mathcal{B}$. The dot over the variable β denotes its ordinary time derivative. Let us notice that at this stage the variable β can be either a scalar-valued or a vector-valued quantity.

To be compliant with the second law of thermodynamics $(2.3)_2$, the above equations were proved by Kosiński and Wojno (1995) to satisfy the orthogonality conditions

$$\partial_{\nabla\beta}\psi^* \cdot \partial_{\nabla\vartheta}F = 0 \qquad\qquad \partial_{\nabla\beta}\psi^* \cdot \partial_{\nabla\beta}F = 0 \, . \tag{2.7}$$

and also to fulfill the dissipation inequality in the form

$$-\rho\partial_{\nabla\beta}\psi^* \cdot \partial_\beta F\nabla\beta - \rho\partial_\beta\psi^* F - \rho\Big(\partial_{\nabla\beta}\psi^*\partial_\vartheta F + \frac{\boldsymbol{q}}{\rho\vartheta}\Big) \cdot \nabla\vartheta \geq 0 \tag{2.8}$$

When the orthogonality conditions (2.7) are satisfied nontrivially they generally lead to constrains between β, ϑ and their derivatives. In the case of isotropy these constrains are always of a differential type. However, in some anisotropic cases they can be of a function type or, as it was exemplified by Kosiński and Wojno (1995) for semi-linear constitutive equations, even do not yield any relationships between the temperature ϑ and internal parameter β.

It is easy to notice that the conditions (2.7) can be fulfilled trivially in four ways, the most interesting of which is obtained when $\partial_{\nabla\vartheta}F = \mathbf{0}$ and $\partial_{\nabla\beta}F = \mathbf{0}$. Under these conditions the constitutive relations (2.5) simplify to

$$\begin{aligned} &\psi = \psi^*(\vartheta, \beta, \nabla\beta) \qquad\qquad \eta = -\partial_\vartheta\psi^*(\vartheta, \beta, \nabla\beta) \\ &\boldsymbol{q} = \boldsymbol{q}^*(\vartheta, \nabla\vartheta, \beta, \nabla\beta) \, , \end{aligned} \tag{2.9}$$

and Eq (2.8) becomes

$$-\rho\partial_{\nabla\beta}\psi^* \cdot \partial_\beta F\nabla\beta - \rho\partial_\beta\psi^* F - \rho\Big(\partial_{\nabla\beta}\psi^*\partial_\vartheta F + \frac{\boldsymbol{q}}{\rho\vartheta}\Big) \cdot \nabla\vartheta \geq 0 \, , \tag{2.10}$$

while the evolution equation (2.6) admits the simpler form

$$\dot{\beta} = F(\vartheta, \beta) \tag{2.11}$$

Thus under the above specified conditions we arrive at the generalized theory of conductivity in which the free energy function ψ may be unconventionally enriched by the dependence also on $\nabla\beta$, leaving at the same time the function F in the evolution equation in the classical form of the ordinary differential equation, comparatively easy to integrate.

2.2. Regularization properties

Let us think of the conductor as being isotropic and obeying the orthogonality conditions (2.7). Next concentrate attention to the case in which the constitutive equations (2.9) admit the form

$$\psi = \psi^*(\vartheta,\beta,|\nabla\beta|) \qquad \eta = -\partial_\vartheta\psi^*(\vartheta,\beta,|\nabla\beta|)$$
$$\boldsymbol{q} = \zeta a\partial_\vartheta F(\vartheta,\beta)\nabla\vartheta + [q(\vartheta,\beta,|\nabla\beta|) + \zeta a\partial_\beta F(\vartheta,\beta)]\nabla\beta \tag{2.12}$$

where the function F is given by Eq (2.11) and $a,\zeta \geq 0$ stand for a dimensional coefficient and a nondimensional parameter, respectively. With this constitutive equations (2.8) becomes

$$\begin{aligned}
&-\rho\partial_{\nabla\beta}\psi^*(\vartheta,\beta,|\nabla\beta|)\cdot\partial_\beta F(\vartheta,\beta)\nabla\beta - \rho\partial_\beta\psi^*(\vartheta,\beta,|\nabla\beta|)F(\vartheta,\beta) + \\
&-\vartheta^{-1}\Big[q(\vartheta,\beta,|\nabla\beta|) - \zeta a\partial_\beta F(\vartheta,\beta)\Big]\nabla\vartheta\cdot\nabla\beta + \\
&-\rho\Big[\partial_{\nabla\beta}\psi^*(\vartheta,\beta,|\nabla\beta|)\partial_\vartheta F(\vartheta,\beta) + \zeta a(\rho\vartheta)^{-1}\partial_\vartheta F(\vartheta,\beta)\nabla\vartheta\Big]\cdot\nabla\vartheta \geq 0
\end{aligned} \tag{2.13}$$

Although the above equation for heat flux vector does not describe the most general properties of the isotropic heat conductor its simple form is just sufficient to show the natural regularization properties of the formulated theory which can be used to obtain numerical solutions for singularly loaded heat conductors with semi-empirical temperature (cf Cimmelli et al. (1992)).

For this purpose, let us consider the system $(2.3)_1$, (2.11) of the energy balance and the evolution equations. First, from the initial two prolongations of Eq (2.11) we obtain the expressions

$$\begin{aligned}
\dot\vartheta &= \tau^*(\vartheta,\beta)(\ddot\beta - \partial_\beta F\dot\beta) \\
\nabla\vartheta &= \tau^*(\vartheta,\beta)(\nabla\dot\beta - \partial_\beta F\nabla\beta)
\end{aligned} \tag{2.14}$$

for the first derivatives of ϑ. Next by use of the constitutive equations (2.12) and Eqs (2.14) we can give to the energy equation $(2.3)_1$ the form

$$\begin{aligned}
&\rho c_v(\vartheta,\beta,|\nabla\beta|)\tau^*(\vartheta,\beta)\ddot\beta + \rho\Big[\partial_\beta\epsilon^*(\vartheta,\beta,|\nabla\beta|) - c_v(\vartheta,\beta,|\nabla\beta|)\tau^*(\vartheta,\beta)\times \\
&\times\partial_\beta F(\vartheta,\beta)\Big]\dot\beta + \rho\partial_{\nabla\beta}\epsilon^*(\vartheta,\beta,|\nabla\beta|)\cdot\nabla\dot\beta + \zeta a\Delta\dot\beta + q(\vartheta,\beta,|\nabla\beta|)\Delta\beta + \\
&+\Big[\partial_\vartheta q(\vartheta,\beta,|\nabla\beta|)\tau^*(\vartheta,\beta)\partial_\beta F(\vartheta,\beta) + \partial_\beta q(\vartheta,\beta,|\nabla\beta|)\Big]\nabla\beta\cdot\nabla\beta + \\
&\quad +\partial_\vartheta q(\vartheta,\beta,|\nabla\beta|)\tau^*(\vartheta,\beta)\nabla\beta\cdot\nabla\dot\beta + \partial_{\nabla\beta}q(\vartheta,\beta,|\nabla\beta|)\otimes\nabla\beta\cdot\nabla\nabla\beta = \rho r
\end{aligned} \tag{2.15}$$

where

$$\begin{aligned}
\tau^*(\vartheta,\beta) &:= \Big[\partial_\vartheta F(\vartheta,\beta)\Big]^{-1} \\
c_v(\vartheta,\beta,|\nabla\beta|) &:= \partial_\vartheta\epsilon^*(\vartheta,\beta,|\nabla\beta|)
\end{aligned}$$

denotes the specific heat. Eqs (2.15) and (2.11) make now the system of two partial differential equations (PDEs) in the dependent variables ϑ and β.

Let us assume that $\partial_\vartheta F(\vartheta, \beta) \neq 0$. If so, the evolution equation (2.11) can be solved with respect to ϑ. The solution makes it possible to express Eq (2.15) as the PDE with respect to the parameter β only. At last this PDE can be presented as a first order quasi-linear system of four PDEs in terms of β, $v = \dot{\beta}$, $\boldsymbol{p} = \nabla\beta$, $\boldsymbol{z} = \nabla v$. It was shown by Kosiński and Wojno (1995) that because of the presence of the term $\zeta a \Delta\dot{\beta}$ in Eq (2.15), the system obtained this way is a parabolic one. Thus for any $t > 0$ its solution will depend continuously on the parameter ζ, we can therefore write

$$\vartheta = \vartheta^*(x, t; \zeta) \qquad \beta = \beta^*(x, t; \zeta) \tag{2.16}$$

Now let us think of ζ as being small and consider the limit case of the above problem as $\zeta \to 0$. Denoting the limits Eqs (2.16) by

$$\vartheta_0 = \vartheta^*(x, t; 0) \qquad \beta_0 = \beta^*(x, t; 0) \tag{2.17}$$

from Eqs (2.12) and (2.13) we have the limit constitutive equations

$$\begin{aligned} &\psi_0 = \psi^*(\vartheta_0, \beta_0, |\nabla\beta_0|) \qquad \eta_0 = -\partial_\vartheta \psi^*(\vartheta_0, \beta_0, |\nabla\beta_0|) \\ &\boldsymbol{q}_0 = q(\vartheta_0, \beta_0, |\nabla\beta_0|)\nabla\beta_0 \ , \end{aligned} \tag{2.18}$$

constrained by the inequality

$$\begin{aligned} &-\rho\partial_{\nabla\beta}\psi^*(\vartheta_0, \beta_0, |\nabla\beta_0|) \cdot \partial_\beta F(\vartheta_0, \beta_0)\nabla\beta_0 - \rho\partial_\beta\psi^*(\vartheta_0, \beta_0, |\nabla\beta_0|) \times \\ &\times F(\vartheta_0, \beta_0) - \rho\Big[\partial_{\nabla\beta}\psi^*(\vartheta_0, \beta_0, |\nabla\beta_0|)\partial_\vartheta F(\vartheta_0, \beta_0) + \\ &+(\rho\vartheta_0)^{-1} q(\vartheta_0, \beta_0, |\nabla\beta_0|)\nabla\beta_0\Big]\nabla\vartheta_0 \geq 0 \end{aligned} \tag{2.19}$$

Cimmelli et al. (1992) showed that the above inequality yielded for the heat flux vector $\boldsymbol{q}_0$ the equation of the Fourier-type in $\nabla\beta_0$

$$\boldsymbol{q}_0 = -\alpha^*(\vartheta_0, \beta_0, |\nabla\beta_0|)\nabla\beta_0 \ , \tag{2.20}$$

with the coefficient $\alpha^* = -q(\vartheta_0, \beta_0, |\nabla\beta_0|)$ defined by the thermodynamic potential

$$\alpha^*(\vartheta_0, \beta_0, |\nabla\beta_0|) := \rho\vartheta_0|\nabla\beta_0|^{-1}\partial_\vartheta F(\vartheta_0, \beta_0)\partial_{|\nabla\beta_0|}\psi^*(\vartheta_0, \beta_0, |\nabla\beta_0|) \tag{2.21}$$

In Cimmelli et al. (1992) no dependence of ψ on β was present, however, this does not influence the final result (2.20). It was also shown that Eq $(2.18)_1$ and the limit form of the evolution equation (2.11)

$$\dot{\beta}_0 = F(\vartheta_0, \beta_0) \tag{2.22}$$

must still satisfy the inequality

$$
\begin{aligned}
&-(\vartheta_0\partial_\vartheta F(\vartheta_0,\beta_0))^{-1}\partial_\beta F(\vartheta_0,\beta_0)\alpha^*(\vartheta_0,\beta_0,|\nabla\beta_0|)|\nabla\beta_0|^2 + \\
&-\rho\partial_\beta\psi^*(\vartheta_0,\beta_0,|\nabla\beta_0|)F(\vartheta_0,\beta_0) \geq 0
\end{aligned}
\tag{2.23}
$$

The limit form of the energy equation (2.15) can be written as

$$
\begin{aligned}
&\rho c_v(\vartheta_0,\beta_0,|\nabla\beta_0|)\tau^*(\vartheta_0,\beta_0)\ddot{\beta}_0 + \rho\Big[\partial_\beta\epsilon^*(\vartheta_0,\beta_0,|\nabla\beta_0|) + \\
&-c_v(\vartheta_0,\beta_0,|\nabla\beta_0|)\tau^*(\vartheta_0,\beta_0)\partial_\beta F(\vartheta_0,\beta_0)\Big]\dot{\beta}_0 + \\
&+\rho\partial_{\nabla\beta}\epsilon^*(\vartheta_0,\beta_0,|\nabla\beta_0|)\cdot\nabla\dot{\beta}_0 + \Big[\partial_\vartheta q(\vartheta_0,\beta_0,|\nabla\beta_0|)\times \\
&\times\tau^*(\vartheta_0,\beta_0)\partial_\beta F(\vartheta_0,\beta_0) + \partial_\beta q(\vartheta_0,\beta_0,|\nabla\beta_0|)\Big]\nabla\beta_0\cdot\nabla\beta_0 + \\
&+\partial_\vartheta q(\vartheta_0,\beta_0,|\nabla\beta_0|)\tau^*(\vartheta_0,\beta_0)\nabla\beta_0\cdot\nabla\dot{\beta}_0 + \partial_{\nabla\beta}q(\vartheta_0,\beta_0,|\nabla\beta_0|)\otimes \\
&\otimes\nabla\beta_0\cdot\nabla\nabla\beta_0 = \rho r
\end{aligned}
\tag{2.24}
$$

Similarly to Eq (2.15) this equation can now be expressed as a system of three PDEs in terms of β_0, $v_0 = \dot{\beta}_0$, $\boldsymbol{p}_0 = \nabla\beta_0$, which was proved by Kosiński and Wojno (1995) to be no longer the parabolic but a hyperbolic one, with the speed of generally nonsymmetric propagation given by the relation

$$
\begin{aligned}
&\rho c_v(\vartheta_0,\beta_0,|\nabla\beta_0|)\tau^*(\vartheta_0,\beta_0)\lambda^2 - \rho\partial_{\nabla\beta}\epsilon^*(\vartheta_0,\beta_0,|\nabla\beta_0|)\cdot\boldsymbol{n}\lambda + \\
&+\partial_\vartheta\alpha^*(\vartheta_0,\beta_0,|\nabla\beta_0|)\tau^*(\vartheta_0,\beta_0)\nabla\beta_0\cdot\boldsymbol{n}\lambda - \partial_{\nabla\beta}\alpha^*(\vartheta_0,\beta_0,|\nabla\beta_0|)\otimes \\
&\otimes\nabla\beta_0\cdot\boldsymbol{n}\otimes\boldsymbol{n} - \alpha^*(\vartheta_0,\beta_0,|\nabla\beta_0|)\boldsymbol{n}\cdot\boldsymbol{n} = 0
\end{aligned}
\tag{2.25}
$$

where $\boldsymbol{n}$ is the unit normal to the wave front.

The above limiting process reveals natural regularization properties of the theory introduced in Section 1 for isotropic heat conductors. This properties can be particularly useful for finding numerical singular solutions to the quasi-linear systems, which come from the thermodynamics of heat conductor with semi-empirical temperature.

To make this comment reliable let us assume that we must find numerically the singular solution to the problem given by the equations $(2.3)_1$, (2.11) and $(2.12)_{1,2}$ together with the heat flux vector

$$
\boldsymbol{q}_0 = -\alpha^*(\vartheta,\beta,|\nabla\beta|)\nabla\beta\,, \tag{2.26}
$$

where the coefficient α^* is expressed by the thermodynamic potential

$$
\alpha^*(\vartheta,\beta,|\nabla\beta|) := \rho\vartheta|\nabla\beta|^{-1}\partial_\vartheta F(\vartheta,\beta)\partial_{|\nabla\beta|}\psi^*(\vartheta,\beta,|\nabla\beta|) \tag{2.27}
$$

which yields the hyperbolic set of PDEs. If according to Eq $(2.12)_3$ we replace Eq (2.26) by the heat flux vector

$$\boldsymbol{q} = \zeta a \partial_\vartheta F(\vartheta,\beta)\nabla\vartheta - \Big[\alpha^*(\vartheta,\beta,|\nabla\beta|) - \zeta a \partial_\beta F(\vartheta,\beta)\Big]\nabla\beta \ , \tag{2.28}$$

then the resulting problem, which in the light of Eq (2.13) must be constrained now by the inequality

$$\begin{aligned} &-\Big[\vartheta\partial_\vartheta F(\vartheta,\beta)\Big]^{-1}\partial_\beta F(\vartheta,\beta)\alpha^*(\vartheta,\beta,|\nabla\beta|)|\nabla\beta|^2 + \\ &-\rho\partial_\beta\psi^*(\vartheta,\beta,|\nabla\beta|)F(\vartheta,\beta) + \\ &+\zeta a(\vartheta)^{-1}\Big[\partial_\vartheta F(\vartheta,\beta)\nabla\vartheta - \partial_\beta F(\vartheta,\beta)\nabla\beta\Big]\cdot\nabla\vartheta \geq 0 \ , \end{aligned} \tag{2.29}$$

can be reduced to the parabolic set of PDEs. If only the parameter ζ is sufficiently small then its numerical solution smoothly approximates the singular solution giving thus regularization of the problem under investigation.

It is worth to notice, that owing to the prolongation $(2.14)_2$ the heat flux vector can be written in the simpler form

$$\boldsymbol{q} = \zeta a \dot{\overline{\nabla\beta}} - \alpha^*(\vartheta,\beta,|\nabla\beta|)\nabla\beta \tag{2.30}$$

This form reveals that as a matter of fact the heat flux vector (2.26) has been perturbed by adding to one small "viscosity" term $\zeta a\dot{\overline{\nabla\beta}}$ or equivalently the term $\zeta a\nabla\dot{\beta}$.

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O parabolicznej regularyzacji hiperbolicznego przewodnictwa ciepła w ciele sztywnym

Streszczenie

W pracy pokazano, jak paraboliczny model wyższego rzędu dla przewodnictwa ciepla, wprowadzajacy gradientową teorię wewnętrznych zmiennych stanu, może być wykorzystany do naturalnej parabolicznej regularyzacji hiperbolicznego modelu przewodnictwa, sformułowanego we wcześniejszych pracach w ramach zmodyfikowanego prawa Fouriera.

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