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COUPLED FIELDS – MODELLING OF MATERIALS FOR MODERN TECHNOLOGIES

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A kind of review of the theory of Coupled (Combined) Fields is given in the paper. In introduction the foundation and development of phenomenological theories has been presented. In the next section the formulation and development of the theory of coupled fields are considered. In the present state it treats the mechanic, electromagnetic, thermal, diffusion and microstructural fields jointly. The range of possible applications of such an extended theory has been formulated. The main components of its foundation have been presented. Section 3 contains fundamental suggestions given to the construction of the constitutive equations conformal to modern thermodynamics of nonequilibrium processes. Foundations of the thermodynamic theory of constitutive equations have been considered and a sample of procedure of the constitutive equations construction has been given.

Key words: theories of continuous media, coupled fields, constitutive equations

1. Introduction

Theories of material continua, which are actually so fruitfully applied to solving many basic problems, erased in the 19th century. The development of partial differential equations contributed to forming of their foundations. Such prominent scientists as: Cauchy, Euler, d'Alambert, Jacobi, Bernoulli and others, who were involved into applications of mathematics, took part in the development of such theories of material continua as: elasticity, hydrodynamics, dissipative media, etc. In that time the macroscopic theory of heat i.e. thermodynamics was developed also. Starting from the practical science about heat engines, it was transformed into the ambitious, cognitive theory, dealing with very universal problems. In the mean time, the engineering sciences were developed successfully. They were: building structures, aerodynamics, machine building and others. Estimating, from the historical point of view, the importance of those macroscopic theories, modelling complicated physical processes in condensed matter, we can say that the results of their application exceeded any expectation.

The 19th century ended also with a great decisive success. Maxwell, recapitulating the experimental investigations, formulated the general theory of electromagnetism, from which the macroscopic electrodynamics of continuum media took its origin. It models processes in materials sensitive electrically and magnetically. Another serious problem, it was a proper description of physical phenomena on the molecular and atomic level, conformable with the experimental results. It caused the formulation of quantum mechanics-nonconventional and controversial theory. The old mechanical quantities, such as momentum, angular momentum and energy, have been changed by operators, and trajectories of material particles have been treated as stochastic objects. Thus, a gap appeared between the micro and macro phenomena in condensed matter descriptions. This gap is continuously filled up by applying asymptotic models, which generate connections between the micro and macro worlds.

2. Development and actual state of the theory of Coupled Fields

The macroscopic theories, applying models of material continua; i.e., Mechanics of Continuous Medium (MCM), Thermodynamics and Electrodynamics of Continuous Medium (ECM), for same time developed independently, overcoming internal drawbacks and sometimes even contradictions. As the sciences developed and found more frequent engineering applications, a tendency towards the common use of these principle sciences, was growing permanently. They were applied to descriptions of processes, in which mechanical, thermal, electromagnetic and diffusion phenomena occur jointly, interacting reciprocally. This way the theory of Coupled or Combined Fields was created. To its development essentially contributed Polish scientists Nowacki (1983) and Kaliski et al. (1987). It should be noted that the theory of couple fields has been developed permanently, overcoming internal imperfections, which occur mainly at the points of junction of multi-disciplinary sciences. After some time the range of the theory was extended also, by inclusion of the microstructural theories, in which, apart from classical combined fields, some additional, nonconventional ones (new degrees of freedom) were taken into account. They model, on a macroscopic level, some phenomena in organized material structures of ordered molecules and molecular complexes. Herein-after, we will call them "microbodies". They exert the non-negligible influences on the physical phenomena in such modified material continua. At each their point, except for the classical fields, there are defined some additional tensor fields and additional equations, following from the general laws of conservation type. The most popular such media are: micromorphic and micropolar material continua. The first of them models the situations, when the continuum of microbodies undergo the linear deformations only. Therefore, the field of microstructure is of affine tensors type. In the micropolar continua, the points (particles) of the medium undergo rigid rotations only. Therefore the microstructural field is the ortogonal tensors type. Such macroscopic theories based on a set of interacting material continua, can model physical phenomena in many kinds of materials; such as, ferroelectrics, ferromagnetics, liquid crystals, semiconductor, etc. Hence, the theories of many interactig material continua achieve a great success and are commonly accepted and applied in practice.

Rapidly growing development of modern technologies of materials, implementing the actual achievements of macroscopic sciences, crates the urgent needs for construction of modern macroscopic models, which are able to describe effectively processes in materials of complicated internal structures. Such materials should satisfy many different, sometime contradictory requirements. For example:

- materials used for shields of space vehicles should be proof both against very high and very low temperatures,
- materials used for shields in nuclear power plants should be resistant to high temperatures and intensive thermonuclear radiations,
- in tokamaks they should be proof against high temperature and strong magnetic fields,
- the so called intelligent materials should be very sensitive to changes of some selected fields and parameters, and very resistant to changes of others,
- in chemical reactors materials should be sensitive to some kinds of reactions.

Here, the general question appears – is the theory of coupled fields, or interacting material continua, now able to design and investigate such materials? This is a question about the present state of their development and the readiness for fulfilling difficult and urgent tasks. Actually, such materials are designed by means of series of complicate experiments. To answer this question, the actual state of macroscopic theories of combined fields should be presented and discussed. Every macroscopic theory, as we know, rests on the following four pillars:

1. Theory of deformation and kinematics of deformable and microstructural continua
2. Macroscopic physical conservation laws and other macroscopic, fundamental relations (constraints)
3. Thermodynamic theory of constitutive relations (TTCR)
4. Dimensional analysis and the theory of similarity.

The components 1 and 2 of the theory are to be considered jointly. The main problem of the theory of combined fields appears at the confrontation of the mechanics of continuous media (MCM) and electrodynamics of continuous media (ECM). The equations of MCM remain invariant at the Galilean group of transformations and the equations of ECM at the Lorentz group. This difference can be eliminated by constructing of the relativistic theories of mechanics and thermodynamics. They will be also invariant at the Lorentz group of transformations in Minkowski's space. Fortunately, this complicated procedure can be avoided, if it is taken into account, that the Maxwell equations (ECM) are invariant also with respect to the Galilean group of transformations in asymptotic approximation, i.e., when velocities of material particles are much smaller than the light velocity $(V \ll c)$. Thus the uniform theory of coupled mechanic and electromagnetic phenomena exists. In this theory:

- A motion of the material medium modifies the electromagnetic fields
- Electromagnetic fields act on the material medium through the ponderomotoric force, which is a bilinear function of these fields.

Another basic problem of the microstructural continua, concerns the structure of the so called configuration space K. In MCM this space is a subset of the Euclidean space E^3. In microstructural spaces, like the space of thermal stresses or the continuous space of dislocations, K has a form of the Cartesian product of $U \subset E^3$ and a space of microstructure M. Hence

$$K = U \times M \tag{2.1}$$

where the space M should satisfy the following condition

$$p(K) = U \tag{2.2}$$

where p is a projection operator. The space M may be a non-Euclidean one with different kinds of geometrical connections, which can model different microstructural features. Properties of the media with microstructures of the type(2.1) may be reliably described in bundle spaces Δ with the following structure

$$\Delta = E(B, F, G, p) \tag{2.3}$$

where

E – bundle or fibre space
B – space of the base
F – microstructural fibre
G – structural group
p – projection operator.

3. Thermodynamic theory of constitutive equations

The main difficulty, which appears when constructing the phenomenological theories, lies in formulation of the proper constitutive equations (CEs), since they must model, in macroscopic approximation, basic physical properties of different kinds of material bodies. The behavior of very complicated structures of material bodies, should be described by some continuous fields, connected by the proper number of constitutive relations. It is not easy to postulate such relations for many reasons. First of all, the constitutive relations must be different for different material media, since they should model the behavior of condensed matter in different states of condensation (solid state, fluids, plasma, etc.). Secondly, the constitutive modelling should take into account such features as:

- sensitivity of the media to electromagnetic fields,

- microstructural structures of material bodies,if it influences their macroscopic properties. It concerns such basic features as: internal, microscopic symmetries, types of microbodies (crystals, polymers), level of non-regularity (polycrystals, amorphus bodies), thermal chaos (liquids, gases) and many other related features.

How does the science overcome difficulties of constitutive modelling of different materials? In the past, one used to accumulate results of experiments and observations. By this way, the oldest constitutive equation – the Clapeyron equation for ideal gases, was established and further the Hook equation for elastic bodies, the equation for dissipative Newtonian fluid, etc. As the science was developed, the quasi-equlibrium thermodynamics in the Onsager approximation has been used. Due to existence, in this case, the thermodynamic potentials, there have been obtained many approximated constitutive equations, even for combined fields. However, they were, as usual, the linear ones and modelled only the most regular parts of processes.

Now, the encouraging results of the physics of solids and fluids are used, more frequently in constitutive modelling. The elaborated methods, applying the potentials of microscopic interactions between microbodies and the experimental results, enable statement and verification of some kinds of CEs.

On the turn of 1950s and 1970s the group of American scientists presented the modern version (using partly postulates) of MCM, including the thermodynamic theory of constitutive equations. The prominent members of this group are: Truesdell and Noll (1960), Noll (1974), Coleman and Owen (1974), and others. According to the included proposal the CEs should satisfy the following postulates:

1 – coexistence, 2 – determinizm, 3 – locality, 4 – material objectivity.

Moreover, they should satisfy the second law of thermodynamics. The above postulates were subject to severe criticism. They were criticized for too great superficiality and truisms. The postulate of the material objectivity with standed this criticism in the best condition, because it contains the essential requirements for the CEs. Namely, these equations should be invariant under any changes of non-inertial frame of references. It means, that the constitutive properties of material, should be invariant under any non-uniform motion of the observer, or a material should not posses any inertial features. Therefore, one can say, that the postulate of objectivity determines the notion of the constitutive material.

The presented postulates and the second law of thermodynamics form the frame or restrictions for the CEs. Hence only the CEs, which satisfy these restrictions are acceptable. It means, that satisfying them we can avoid only the improperly postulated CEs. Hence, they form only a set of necessary conditions. Construction of the CEs suitable for the material considered needs another methods to be applied. Among them, the experiments and promising results obtained in physic of condensed matter, are mostly used. The most important restrictions inposed on the form of the CEs follow from the postulate of objectivity and from the second law of thermodynamics.

Before formulation of these restrictions, we will present the general form of the CEs. The number of postulated constitutive relations results from the following difference:

number of material fields - number of equations describing the problem.

Mostly, the constitutive functions are unknown fields contained in the set of equations of the problem, while the remaining fields, which are to be established are the arguments (independent variables). However, such a choice is, arbitrary. The constitutive equations are generally postulated as tensor functions or functionals of tensor variables. The constitutive functionals, having a form of integrals in time, model the behavior of material with memory or equivalently with dissipations. The general form of constitutive equation is as follows

$$\mathbf{t} = \mathbf{t}(\chi, \mathbf{E}, \mathbf{H}, \mathbf{\Gamma}_i, T, ...) \tag{3.1}$$

where

$\mathbf{t}$ – tensor of stresses

χ – function of motion, a smooth homeomorphism in E^3

$\mathbf{E}, \mathbf{H}$ – electric and magnetic fields, respectively

$\mathbf{\Gamma}_i$ – fields of microstructure

T – temperature.

From the postulate of objectivity, it follows that the CEs should be invariant in regard to the group of rigid motions, i.e., translations and rotations. These motions have the form

$$\boldsymbol{x}' = \boldsymbol{c}(t) + \mathbf{Q}(t)\boldsymbol{x} \tag{3.2}$$

where $\boldsymbol{c}(t)$ is the translation vector of the reference frame, and $\mathbf{Q}(t)$ the ortogonal tensor of rotations. From translational invariance of the CEs it follows, that the tensor of stresses $\mathbf{t}$ can not depend on the function of motion χ but only on its gradient $\nabla\chi$. From the rotational invariance the relation follows

$$\mathbf{Q}\mathbf{t}\mathbf{Q}^{\top} = \mathbf{t}(\mathbf{Q}\nabla\chi\mathbf{Q}^{\top}, \mathbf{Q}\mathbf{E}, \mathbf{Q}\mathbf{H}, \mathbf{Q}\mathbf{\Gamma}_i, ...) \tag{3.3}$$

satisfied for any ortogonal matrix $\mathbf{Q}$. Moreover, the tensor of stresses should satisfy Eq (3.3) with regard to the group of material symmetry G_m.

Restrictions following from the second law of thermodynamics are very strong, regarding that the thermodynamics of nonequilibrium processes is continuously developing. We are to present an example of its formulation, being the most proper for combined fields, worked out in the framework of thermodynamics of nonequilibrium processes by Müller (1967), Maugin and Eringen (1977), Tiersten (1971), and others, and developed in Poland by Maruszewski (1986). The second law of thermodynamics for nonequilibrium processes has been formulated in the form of the following Clausius-Duhem inequality (C-D)

$$\frac{d}{dt}\int\limits_V \rho s\, dv + \int\limits_{\partial V} \varPhi_i n_i\, d(\partial V) - \int\limits_V \frac{\rho r}{T}\, dv \geq 0 \tag{3.4}$$

where

s – density of entropy

$\varPhi_i$ – flux of entropy

$\rho r/T$ – entropy production caused by the external heat fluxes.

The proposed formulation differs from the classical one, suitable for quasi-equilibrium processes, and has the following form:

— the flux of entropy

$$\varPhi_i = \frac{q_i}{T} + k_i \qquad \text{or} \qquad \varPhi_i = \frac{q_i - q_i^{\alpha\alpha}}{\Theta} \tag{3.5}$$

where Θ is the so called generalized temperature, for which the constitutive relation is formulated,

— $q_i^{\alpha\alpha}$ is the flux of energy caused by chemical reactions

$$q_i^{\alpha\alpha} = \mu^\alpha j_i^\alpha \tag{3.6}$$

where

μ^α – chemical potential

j_i^α – elementary flux.

The local form of the C-D inequality is as follows

$$\rho\frac{ds}{dt} + (\varPhi_i + k_i)_{,i} - \frac{\rho r}{\Theta} \geq 0 \tag{3.7}$$

The main step in the postulation of general form of the CEs is the choice of a set of arguments (vector of state). For the arguments, there may stay not only fields but also their time and space derivatives or their fluxes. The postulate of coexistence states only that the list of chosen arguments is universal and should be the same for all CEs. However, it can differ for different kinds of materials. We should notice here, that this step involves some kind of subjectivism, for one can establish an arbitrary large set of arguments and after that eliminate them, using general physical constraints. But this procedure can violate the universality of the established set of arguments and contradict the postulate of coexistence.

Following Maruszewski (1986), the constitutive equations for magneto-thermo-diffusion, in conducting magnetic medium, are presented. The following set of arguments is postulated

$$\Pi = (\varepsilon_{ij}, E_i, H_i, c, c_{,i}, T_{,i}, \dot{T}, q_i, j_i) \tag{3.8}$$

where

ε_{ij} – tensor of small deformations
E_i, H_i – electric and magnetic fields, taking a motion of the medium into consideration
c – concentration of diffusing medium
T – temperature
q_i, j_i – fluxes of heat and diffusing medium.

It is one of possible sets of arguments. Wanting to know, what are the results of the above choice of arguments, the conservation law of energy is used. Its local form is following

$$\rho\frac{dU}{dt} + \frac{\partial U_z}{\partial t} = \rho r + f_i v_i + t_{ij} v_{j,i} - q_{j,j} - (\mathbf{EH})_{j,j} \tag{3.9}$$

where

U – density of internal energy
U_z – density of energy of external actions
f_i – density of external forces
t_{ij} – tensor of full stresses.

After introduction of the density of free energy F, defined by the relation

$$F = U - \Theta s - B_i M_i \tag{3.10}$$

and making the most of the energy conservation law (3.9), we obtain a very complicated form of the C-D inequality, containing more than 20 terms, which have a structure of products of the postulated constitutive variables and their derivatives and, moreover, the bilinear forms, containing different combinations of derivatives of the free energy density F, the generalized temperature Θ, the fluxes q_i, j_i, and others. Since many terms in the C-D inequality are multiplied by the constitutive arguments or their derivatives which do not appear in the set of postulated ones, therefore they should be put equal to zero. Hence, the constitutive equations are obtained and moreover, some set of equations having forms of the constitutive constraints. As an example, we present the two terms, taken from the C-D inequality:

1. First term

$$\Big[-\rho\Big(\frac{\partial F}{\partial \varepsilon_{ij}}+s\frac{\partial \Theta}{\partial \varepsilon_{ij}}\Big)+t_{ij}-B_kM_k\delta_{ij}\Big]\dot{\varepsilon}_{ij} \tag{3.11}$$

Since $\dot{\varepsilon}_{ij}$ does not exist among the constitutive arguments, then the term in square brackets should be equal to zero. Hence, we obtain the constitutive equation for the full stresses

$$t_{ij}=\rho\Big(\frac{\partial F}{\partial \varepsilon_{ij}}+s\frac{\partial \Theta}{\partial \varepsilon_{ij}}\Big)+M_kB_k\delta_{ij} \tag{3.12}$$

The full stresses, i.e. mechanic and electromagnetic are expressed through derivatives of the two thermodynamic potentials F and Θ. Therefore we have the CEs for stresses in nondissipative and quasi-equilibrium processes, i.e. when such potentials exist. They are invalid for dissipative processes far from the thermodynamic equilibrium, i.e. for very rapid and intensive ones. For moderately intensive processes, it can be assumed that the local thermodynamic equilibrium exists and such potentials too.

2. Second term

$$\rho\Big(\frac{\partial F}{\partial \dot{T}}+s\frac{\partial \Theta}{\partial \dot{T}}\Big)\ddot{T} \tag{3.13}$$

Since $\ddot{T}$ does not exist among constitutive variables, therefore

$$\frac{\partial F}{\partial \dot{T}}+s\frac{\partial \Theta}{\partial \dot{T}}=0 \tag{3.14}$$

This is one of the simplest constitutive constraints between potentials F and Θ, see Maruszewski (1986) for details.

The presented results, concerning construction of the constitutive equations need some discussion. On the one hand, we have the set of postulated constitutive variables and on the other some number of relations, following from the second law of thermodynamics in quasi-equlibrium approximation. This situation can be exploited for making the set of prescribed constitutive arguments more "realistic" varying even for different equations. Such a procedure remains in accordance with the postulate of coexistence. This way it is applicable. The analyse of constitutive constraints can even lead to some contradictions, explanation of which may improve and justify the CEs. A good example of such a procedure is presented by Maruszewski (1986). It concerns mainly verification of the structure of potentials F and Θ.

Expanding these potentials into Taylor series, in the state of thermodynamic equilibrium, the set of asymptotic CEs for nondissipative, moderately intensive processes is obtained. For example, the CE for the full stresses, in linear approximation, have the following form

$$t_{ij} = c_{ijkl}\varepsilon_{kl} - h_{ijk}B_k - \lambda^1_{ij}c - \lambda^2_{ij}T \tag{3.15}$$

where $-c_{ijkl}$, h_{ijk}, λ^1_{ij}, λ^2_{ij} – tensors of material constants. Their structure depends on material symmetries of medium. In the considered case there are 16 material tensors,vectors and scalars.

After elimination of nondissipative terms, the C-D inequality takes the following form

$$\Big(\Theta\frac{\partial k_i}{\partial t} - \frac{q_i}{\Theta}\frac{\partial\Theta}{\partial T}\Big)T,_i - \rho\Big(\frac{\partial F}{\partial T} + s\frac{\partial\Theta}{\partial T}\Big)\dot{T} + j_iE_i - \mu_i j_i - \Pi_i J_i \geq 0 \tag{3.16}$$

where

$$\Pi_i = \rho\Big(\frac{\partial F}{\partial j_i} + s\frac{\partial\Theta}{\partial j_i}\Big) \tag{3.17}$$

The above terms model the entropy production or the degradation of energy. They can be treated as generalized dissipations and therefore the sum of terms in the above inequality should be positive definite. If we confine our considerations to the quasi-equlibrium processes, then the CEs will be obtained in a similar way as above. However, in this case, the tensors of material constants, besides the requirements following from the material symmetries, should satisfy a set of conditions in a form of inequalities, which follows from inequality (3.16).

Thus, on the example of the magneto-thermo-diffusion in conductor, the procedure of obtaining of the CEs for quasi-equilibrium processes, in linear approximation, has been ilustrated. They are proper for modelling phenomena in interacting material continua or coupled fields, slow enough and not very intensive. By the way, the following issues should be decided:

- settling the set of constitutive variables,
- verification of this set, applying the obtained constitutive constraints,
- establishing the tensors of material constants and experimental methods for their determination,

- determination of the inequality relations for constants in dissipative materials.

To obtain the theory, being able to model arbitrary, nonequlibrium processes in the media with interacting material continua (coupled fields), such principal problems must be properly solved:

1. Taking into account arbitrary deformations and introducing two descriptions, i.e., material and spacial.
2. Definition of geometry and kinematics for the microstructural media.
3. Construction of nonlinear approximations for the CEs, applying the tensor functions of tensor variables and the theory of tensor invariants.
4. Development of the theory of the thermodynamic processes far from thermodynamic equilibrium.

The actual state of above four problems is following:

Ad. 1. The problem of great deformations for combined fields can be treated as being worked out satisfactory enough. The main results one can find in the publications of Maugin and Eringen (1977), Tiersten (1971), and Hoffmann (1989) and many others.

Ad. 2. The microstructural media were investigated by Eringen (1966), Toupin (1964), Mindlin (1964), and in Poland by Rymarz (1991), (1993), Nowacki (1981) and Maruszewski (1988b), (1993). For many years the thorough investigations of liquid crystals have been continued. They contain some kind of singular microstructure, which gives contradictory results and causes many misunderstandings. Sławianowski presented a new proposal, based on application of the theory of moving repers in modelling the phenomena of interacting material fields. However, generally, the problems of modelling of microstructures in material continua should be treated as still open which need further investigations.

Ad. 3. Constructions of the nonlinear CEs, in general form and polynomial approximations, were investigated by Spencer (1971), Rivlin (1955), Wang (1970), Boehler (1987), and many others. The general level of development of the theory of tensor functions depending on tensor arguments allows one to construct the CEs for coupled fields. Constructions of the CEs for liquid crystals are presented in the papers of Eringen (1979), Rymarz (1991) and for magnetics by Maruszewski (1988a).

Ad. 4. Extension of the thermodynamics for the processes far from the thermodynamic equilibrium, developed by Glansdorff and Prigogine (1970), on the processes in combined fields should be treated as an open problem. This extension is of the greatest importance, since such an extension allows for the new types of instabilities mainly these of the self-organization and deterministic chaos type.

4. Conclusion

Thus we can confirm, that a possibility of constructing the constitutive equations for interacting material continua, creates the ability to design new materials, satisfying the extremal requirements of modern technologies. Hence, it is the problem of highest priority for the theory of combined fields. However, as it follows from the presented revue, it remains still a great amount of work to do.

References

1. BOEHLER J.P., 1987, *Application of Tensor Functions in Solid Mechanics*, Springer Verlag, Wien
2. COLEMAN B.D., OWEN D.R., 1974, A Mathematical Foundation for Thermodynamics, *Arch. Rat. Mech. Anal.*, **54**
3. ERINGEN C.A., 1966, Linear Theory of Micropolar Elasticity, *J. Math. Mech.*, **15**
4. ERINGEN C.A., 1979, Continuum Theory of Nematic Liquid Crystals Subjected to Electromagnetic Fields, *J. Math. Phys.*, **20**
5. GLANSDORFF P., PRIGOGINE I., 1970, *Thermodynamic Theory of Structure, Stability and Fluctuations*, WILEY-Interscience
6. HOFFMANN T., 1989, *Dynamika ciał odkształcalnych z efektami elektromagnetycznymi*, Wyd. Politechniki Poznańskiej
7. KALISKI J., NOWACKI W., RYMARZ C., WŁODARCZYK E., 1987, Review of S. Kaliski Scientific Achievements in the Domain of Coupled Fields, *Int. J. Engin. Sci.*, **25**, 8
8. MARUSZEWSKI B., 1986, *Termodynamiczne podstawy magnetotermodyfuzji i elektrotermodyfuzji w ośrodkach ciągłych*, Wyd. Politechniki Poznańskiej
9. MARUSZEWSKI B., 1988a, Evolution Equations of Thermodiffusion in Paramgnets, *Int. J. Engen. Sci.*, **20**
10. MARUSZEWSKI B., 1988b, On a Disloction Core Tensor, *Phys. Stat. Sol.*, **168**

11. MARUSZEWSKI B., 1993, On An Anisotropy-Grain Tensor, *Phys. Stat. Solid.*, **178**
12. MAUGIN G.A., 1988, *Continuum Mechanics of Electromagnetic Solids*, Elsevier, North Holland, Amsterdam
13. MAUGIN G.A., ERINGEN A.C., 1977, On the Equations of the Electrodynamics of Deformable Bodies of Finite Extent, *Journ. Mech.*, **16**
14. MINDLIN R.D., 1964, Microstructure in Linear Elasticity, *Arch. Rat. Mech. Anal.*, **16**
15. MÜLLER I., 1967, On the Entropy Inequality, *Arch. Rat. Mech. Anal.*, **26**
16. NOLL W., 1974, *A Mathematical Theory of the Mechanical Behavior of Continuous Medium, The Foundation of Mechanics and Thermodynamics*, Springer Verlag, Berlin
17. NOWACKI W., 1981, *Teoria niesymetrycznej sprężystości*, PWN, Warszawa
18. NOWACKI W., 1983, *Efekty elektromagnetyczne w ciałach stałych odkształcalnych*, PWN, Warszawa
19. RIVLIN R.S., 1955, Further Remarks on the Stress-Deformation Relations for Isotopic Materials, *J. Rat. Mech. An.*, 4
20. RYMARZ C., 1991, Ferroelectric Liquid Crystals as Micropolar Medium in Bundle Space, *J. Tech. Phys.*, 1, **32**
21 RYMARZ C., 1993, *Mechanika ośrodków ciągłych, rdz. 11, Nieklasyczne modele kontinuum materialnego*, PWN, Warszawa
22. SPENCER A.J.M., 1971, *Continuum Physics*, **1**, Part III, *Theory of Invariants*, Academic Press
23. TIERSTEN H.F., 1971, On the Nonlinear Equations of Thermoelectroelasticity, *Int. J. Engin. Sci.*, **9**
24. TRUESDELL C., NOLL W., 1960, The Nonlinear Fields Theories of Mechanics, *Handbuch der Physik*, III/3, Springier Verlag, Berlin
25. TOUPIN R.A., 1964, Theories of Elasticity with Couple-Stresses, *Arch. Rat. Mech. Anal.*, **17**
26. WANG C.C., 1970, A New Representation Theorem for Isotropic Functions, Part I and II, *Arch. Rat. Mech. Anal.*, **36**

Pola połączone – opis materiałów dla nowoczesnych technologii

Praca stanowi rodzaj przeglądu teorii pól połączonych. We wstępie przedstawiono podstawy i rozwój teorii fenomenologicznych ośrodków ciągłych. W następnym punkcie rozważono powstanie i rozwój teorii pól połączonych. W aktualnym stanie traktuje ona łącznie pola: mechaniczne, elektromagnetyczne, termiczne, dyduzji i mikrostrukturalne. Przedstawiono zakres możliwych zastosowań tak rozszerzonej teorii. Podano podstawowe składowe (filary) jej budowy. Punkt 3 zawiera podstawowe rozważania dotyczące konstrukcji równań konstytutywnych zgodnych ze współczesną termodynamiką procesów nierównowagowych. Przeanalizowano podstawy termodynamicznej teorii równań konstytutywnych oraz podano przykład procedury konstrukcji równań konstytutywnych.