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EXPERIMENTAL INVESTIGATIONS OF VIBRATIONS OF CYLINDER IN WATER AT A PLANE BOTTOM DUE TO RANDOM WAVES

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In the wave flume of the Institute of Hydroengineering vibrations of an elastically supported cylinder placed in the vicinity of a plane rigid bottom due to random water waves were investigated. The random waves were generated by feeding a time series generated on a computer into the control system of the wavemaker. The elevation of free surface above the cylinder and accelerations of the end points of the cylinder have been measured. Analysis of the measured surface elevations shows that the description by generalised Stokes' wave theory to the random case is justified. The starting point for the description of motion of the cylinder was the theory of ideal fluids supplied with the potential theory solutions. The addition of experimental terms to the differential equations of motion is discussed. The analysis in the frequency domain leads to the spectral densities based on simplified relations but without any empirical coefficients. The analysis in the time domain leads to a numerical solution that describes well the characteristic features of real behaviour. The experiments prove that the theoretical solution based on the theory of perfect fluids reproduces qualitatively the basic features of phenomenon and is a good starting point for formulations that may be used are reliable in engineering practice.

Key words: random water waves, wave-structure interaction, random vibrations

1. Introduction

The two-dimensional motion of a cylinder in an incompressible and inviscid fluid was studied by von Mueller (1929), Carpenter (1957), Yamamoto

(1974) and Wilde (1995). Within the theory of perfect fluids viscous effects are neglected. The separation of the boundary layer and shedding of vortices are considered for example by Sarpkaya (1976) and Sumer and Fredsoe (1988). Such effects are excluded from the present analysis. The experimental investigations of free vibrations by Wilde et al. (1995) showed that the influence of bottom proximity effect results in components with double eigenfrequencies that do not appear in solutions of a linear theory. The problem studied within the potential theory of perfect fluids shows that the interaction with the bottom is due to the nonlinear behaviour. This behaviour is pronounced when the gap is of the order of the cylinder radius and the velocities in the gap are large compared with velocities above the cylinder.

The problem of motion of pipelines in the proximity of sea bottom is important in offshore and coastal engineering. In a coastal region the bottom is soft and the waves are random. To have an insight into the behaviour, it is necessary to simplify the real problem. In the present study the bottom is plane and rigid. The influence of random water waves on vibrations of a cylinder are investigated.

The experiments were performed in the flume of the Institute of Hydroengineering. The experimental set-up is described. Free vibrations in air and water of the set-up were investigated to identify the mechanical system.

Narrow band random waves, with well defined stochastic properties, were generated in the flume by the feeding time series calculated by a computer program into the control system of the wavemaker. Transformation of the random wavemaker displacements into the random water waves is nonlinear. The extended Stokes' second order perturbation theory of water waves is used for analysis of the random water surface elevations. Approximate description of the transformation of of wavemaker motion into the water surface elevation fits the measured values. The properties of the fluid velocity fields are discussed.

The basic features of experimental results are presented. As an example, in Fig.1 the following, measured time series are plotted: of water surface elevation; horizontal and vertical components of acceleration as functions of number of steps for dominant periods $T = 1\,\mathrm{s}$ (a,b,c) and $T = 1.321\,\mathrm{s}$ (d,c,f) at gap width $e = 10\,\mathrm{mm}$ and randomness parameter $K = 2.5$. All the functions are random, caused by a sample function of the wavemaker random displacements with prescribed stochastic properties. For a given time period and randomness parameter the sample functions of water surface elevations, horizontal and vertical components of accelerations are somehow similar, but not the same. For example the shapes of wave groups are different. In the vertical accelerations the dominant frequency is twice the dominant frequency of the water wave.

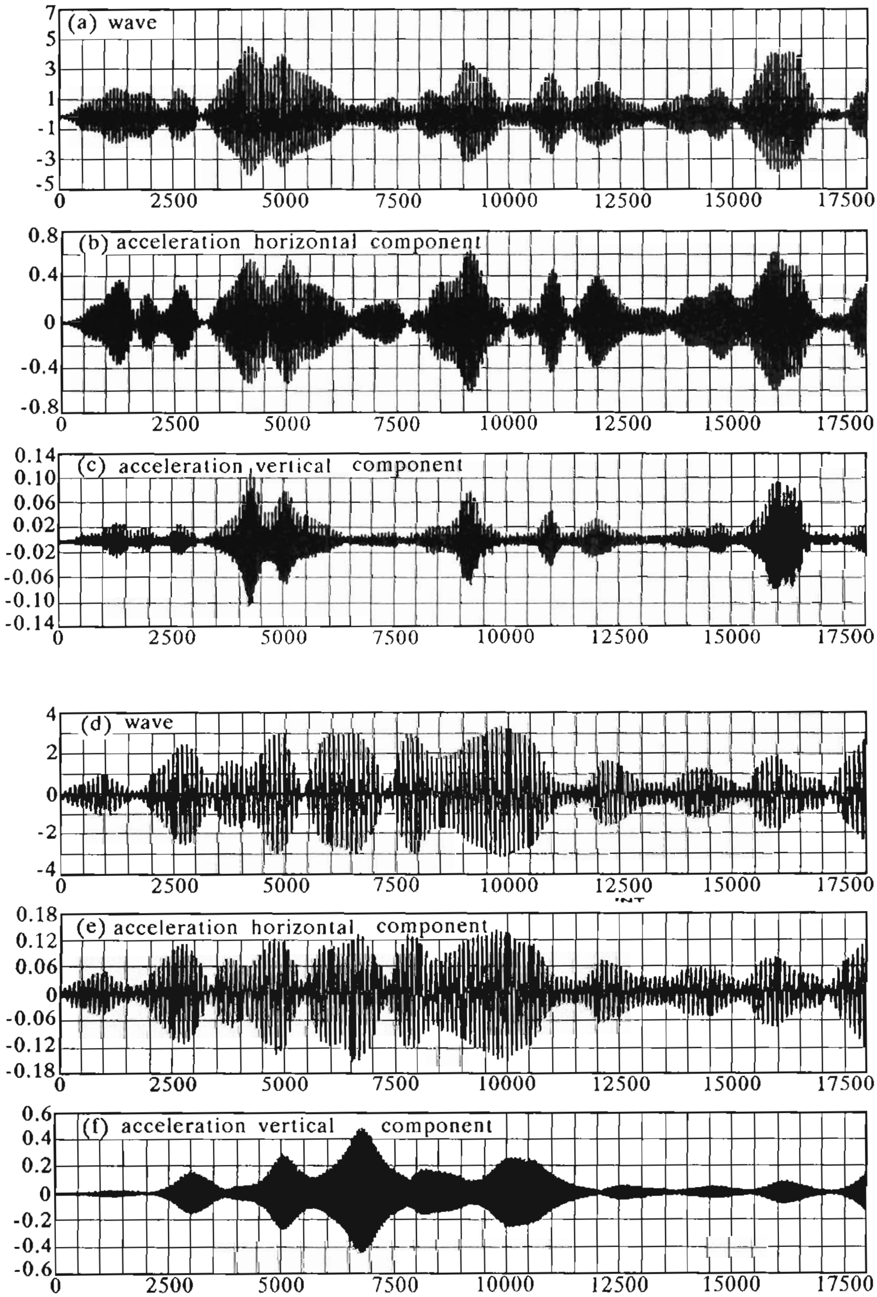

Fig. 1. Measured time series of surface elevation, horizontal and vertical components of acceleration versus the number of time steps for dominant periods $T = 1\,\mathrm{s}$ (a,b,c) and $T = 1.321\,\mathrm{s}$ (d,e,f), at $e = 10\,\mathrm{mm}$, $K = 2.5$

The properties of measured time series of accelerations are presented and a simplified mathematical model of transformation of the random water surface elevation into the accelerations of the cylinder is proposed.

The Lagrangian function and the nonlinear differential equations of two-dimensional motion of a cylinder in a deterministic variable in time velocity field are presented by P.Wilde (1997). The influence of nonlinear terms is discussed that leads to the introduction of simplifications. The final result is a set of two differential equations that has only quadratic terms in velocities and is suitable for the discussion of the random case.

A linear transformation of stochastic functions is standard and common in engineering applications to ocean engineering. In the case of nonlinear transformations, there is no simple theory ready for use, thus it is necessary to introduce simplifications into the analysis to explain the observed behaviour. The analysis in the frequency domain is presented. The formulae for spectral densities of coordinates of displacements are derived in the simplified description when the vertical coordinates of displacements of the cylinder are small compared with the horizontal ones. The agreement for basic qualitative features is good.

For a simple motion of cylinder, in a uniaxial flow, in engineering practice empirical terms are introduced. Our experiments showed that in view of the viscosity a term called the drag force should be added, to obtain a better agreement with the measured values. This term was introduced in a formal way by an extension of the Morison formula to the two-dimensional case.

A numerical method was used to obtain solutions in the time domain and to compare them with the measured values. In this analysis the sample functions of components of cylinder accelerations are constructed on the basis of the given sample function of wavemaker displacements. The measured and calculated time series are similar, but there are differences due the lack of a proper description of energy dissipation.

Sobierajski was responsible for the experiments. It was necessary to have a well tuned experimental set-up. The damping in air had to be very small so that the dissipation of energy in water was dominant. It was important that experiments with sample functions of a well defined random process could be repeated. The sampling frequency was 80 Hz and time series up to 25 000 cases were recorded.

M.Wilde was the author of the applied data processing methods within the control and acquisition system of the wavemaker. In Kalman filter analysis he used his original computer program to obtain decomposition into random components with dominant peak frequencies of the measured time series.

2. Experimental set-up

The experiments were carried out in a wave flume which had the test section 1.5 m high, 0.5 m broad, and, 30 m long. In the experiments the water depth was 0.60 m. The circular cylinder was a steel pipe with the radius $a = 0.0336$ m and the length $l = 0.480$ m. The cylinder surface was smooth. The cylinder was suspended on vertical and horizontal springs at a distance e, above the bottom, called the gap width. The vertical springs carried the deadweight of the cylinder, while the horizontal ones were prestressed in such a way that there was always tension during vibrations. The springs were attached to the cylinder at end points 1 and 2. The distance l_s, between these points, was equal to 0.368 m.

The spring constants; $k_x^{(1)}$, $k_x^{(2)}$, $k_y^{(1)}$, $k_y^{(2)}$, were determined experimentally by application of horizontal and vertical forces at end points 1 and 2, and measurements of the corresponding displacements. The resultant spring constants for uniform displacements were $k_x = 344.76\,\mathrm{Nm}^{-1}$ and $k_y = 585.90\,\mathrm{Nm}^{-1}$. The measurements showed that the springs were linear within the range of the displacements tested.

Four miniature low-impedance accelerometers (ENDEVCO Model 775 with the sensitivity of $500 \pm 5\%$ mV/g and the range ± 10 g) were used to measure accelerations of both ends of the cylinder in the x- and y-directions, respectively. The gauges were built into the cylinder. They were specially designed for very low frequencies, where very low residual noise is required. The amplitude linearity was $< 1\%$ of reading, and the frequency response was $\pm 5\%$ in the range $0.2 \div 1500$ Hz, according to technical specifications. The resonant frequency was 11 kHz.

The mechanical system had four degrees of freedom and could be described by four generalized coordinates: the displacements of the midpoints of the cylinder axis $x_0(t)$ and $y_0(t)$, and the rotations $\varphi_0(t)$ and $\psi_0(t)$, in the xz-plane and the yz-plane, respectively.

The identification of the system in air was performed. Free vibrations were induced by small initial displacements corresponding to horizontal, vertical and two rotational modes. Then the modes of vibrations were uncoupled and linear damping was assumed to fit the experimental data for the modes. The values of frequency and damping coefficients are shown in the first row of Table 1. The corresponding masses were obtained as $m_{cx} = 4.455$ kg and $m_{cy} = 4.432$ kg. For a perfect cylinder and massless springs and strings, the corresponding values in the horizontal and vertical planes should be equal. In the theoretical analysis the measured values were taken.

Table 1. Eigenfrequencies and coefficients of damping in air and still water

gap [mm]	horizontal free vibrations experiment	theory [Hz]	damping [1/s]	vertical free vibrations experiment	theory [Hz]	damping [1/s]
in air	1.40	1.400	0.02	1.83	1.830	0.05
70	1.17	1.182	0.13	1.54	1.544	0.16
20	1.16	1.156	0.20	1.51	1.509	0.17
10	1.13	1.134	0.18	1.48	1.480	0.17
5	1.11	1.111	0.17	1.45	1.450	0.17

The measurements of free vibrations of the cylinder in water were carried out for various values of the gap width $e = 70, 20, 10, 5\,\mathrm{mm}$. The vibrations were initiated either by horizontal or vertical displacements or rotations. Some mixed cases were considered. Data were decomposed into the components with dominant frequencies with the method of Kalman filters described by Wilde and Kozakiewicz (1993). The eigenfrequencies of the modes and the coefficients of linear damping are shown in the rows in Table 1. It should be stressed that linear damping is a crude approximation of the real behaviour in free vibrations. These values are used in the following analysis at the first step. Following the formulae given by Wilde (1995) and Wilde et al. (1995) the added masses of water were calculated for the cylinder initial positions. The calculated eigenfrequencies in water are shown in Table 1. They fit well the experimental values.

Random waves were generated in the wave flume with three values of the randomness parameter $K = 2.5, 1.0, 0.5$ and four dominant frequencies $f_f = 1.0, 1.321, 1.55. 1.9\,\mathrm{Hz}$. The details of the wave generation procedure are given in the next section.

The vibrations of the cylinder due to waves should be two-dimensional without rotations. In most cases this condition was satisfied. Significant rotations were introduced in resonant cases. The midpoint accelerations were calculated as the average values of corresponding accelerations at both ends of the cylinder.

3. Generation of random waves

There is no general theory of random water waves with finite amplitudes that leads to exact solutions. In experiments the case of waves with slowly changing amplitudes and phase shifts is considered, thus the waves correspond to narrow-band processes. To study the transformation of water surface elevations into the vibrations of the cylinder it is necessary to produce waves with known probabilistic properties. It was assumed that the displacements of the generator piston correspond to a stationary Gaussian process described by the equation

$$X_g(t) = A_g(t)\cos(\omega t) + D_g(t)\sin(\omega t) \tag{3.1}$$

where the processes $A_g(t)$ and $D_g(t)$ are three times differentiable and correspond to solutions of the following Ito stochastic differential equations

$$\begin{aligned} \Big(\frac{d}{dt}+\eta\Big)A_0(t) &= \alpha\frac{dB(t)}{dt} \\ \Big(\frac{d}{dt}+\eta\Big)A_1(t) &= \eta A_0(t) \\ &\dots \\ \Big(\frac{d}{dt}+\eta\Big)A_n(t) &= \eta A_{n-1}(t) \end{aligned} \tag{3.2}$$

where $n = 3$ and

ω	–	dominant angular frequency
η	–	parameter with dimension s^{-1}
$B(t)$	–	Brownian motion process
α	–	parameter
$A_g(t), D_g(t)$	–	independent processes governed by the same differential equations (3.2).

The process described by Eqs (3.1) and (3.2) was studied by Wilde and Kozakiewicz (1993). The stochastic process is Gaussian, ergodic and asymptotically stationary. The mean value is zero and the formulae for correlation function and spectral density are given. The process is well defined in the mathematical way in continuum. To obtain a time series the differential equations were transformed into stochastic difference equations for points in time $k\Delta t$, where k is an integer and Δt is the time step. Details are given in the above mentioned book. The resulting random time series is well defined too. This time series was fed into the control system of the wavemaker.

In the case η approaches zero the sample functions correspond to trigonometric functions with a random amplitude and phase shift. In the case η goes to infinity the sample functions become a Gaussian white noise. Let us relate the value of this parameter to the dominant period of the process T by

$$\eta = \frac{1}{KT} \tag{3.3}$$

where K is a dimensionless randomness parameter. In our case we used the values $K = 2.5,\ 1.0,\ 0.5$ corresponding to narrow-band processes ($K \to 0$ corresponds to a white noise and $K \to \infty$ corresponds to a harmonic function). The experiments showed that the calculated time series and the measured displacements of the piston were very close.

The water surface elevations due to wave motion were measured over the position of the cylinder at rest represented by the coordinates $x_p = 0$ and y_p. The problem of transformation of the process of piston into the surface elevation was discussed by Androsiuk (1994), Androsiuk and Szmidt (1994). In the present analysis the measured surface elevation will be the starting point. The measurements showed that there were some peaks in the wave spectral density, one corresponded to the dominant frequency of the piston motion control process and the other frequencies to its multiple values. This phenomenon indicates that the transformation of the piston motion process to the measured water elevation process is nonlinear. In the Kalman filter decomposition for the components mathematical models corresponding to twice differentiable functions ($n = 2$ in Eqs (3.2)) were used. The parameters of the mathematical models were chosen in such a way that the remainder (the difference between the measured time series and the sum of estimated components) had no peaks and corresponded almost to a white noise.

The second order Stokes wave theory was used in the analysis. Stokes applied the perturbation technique to the equations of finite amplitude wave theory. The considered method is valid for

$$\frac{H}{L} \ll 1 \qquad\qquad \frac{H}{h} \ll (kh)^2 \quad \text{for} \quad kh < 1$$

where

- H – wave height
- L – wave length
- h – depth of water
- k – wave number, $k = 2\pi/L$.

The waves generated in the experiments fulfilled these conditions.

For the second order Stokes wave theory the water surface elevation profile is given by the relation

$$\eta(t) = \frac{H}{2}\cos(kx - \omega t + \beta) + \Big(\frac{H}{2}\Big)^2 \frac{\gamma}{h} \cos[2(kx - \omega t + \beta)] \tag{3.4}$$

where

$$\gamma = \frac{kh}{2}\coth(kh)\Big[1 + \frac{3}{2\sinh^2(kh)}\Big]$$

and β is a constant phase shift and the wave number and angular frequency are related by the dispersion equation

$$\frac{\omega^2 h}{g} = kh\tanh(kh) \tag{3.5}$$

where g is the acceleration of the gravitational field.

The potential function Φ is

$$\begin{aligned}
\Phi &= \Phi_1 + \Phi_2\\
\Phi_1 &= \frac{\omega H}{2k}\frac{\cosh(ky)}{\sinh(kh)}\sin(kx - \omega t + \beta)\\
\Phi_2 &= \frac{3\omega H^2}{32}\frac{\cosh(2ky)}{\sinh^4(kh)}\sin[2(kx - \omega t + \beta)]
\end{aligned} \tag{3.6}$$

where the first function corresponds to the first component and the second – to the second one with the double frequency of the first component. It is seen that the second order Stokes wave theory is nonlinear, the surface elevation (3.4) and the potential function (3.6) are expressed as sums of two terms with distinct frequencies and H and H^2, respectively. When the wave height H is small the second term may be omitted and the solution corresponds to a linear sine wave. In such a case the wave frequency corresponds to the frequency of piston motion. For large wave heights one has to take more terms in the Stokes perturbation approximation.

The velocity vector field is equal to the potential gradient. It should be mentioned that the velocities are given in Euler description. The Stokes theory is obtained by a perturbation method and is an approximate formulation. In standard text books, for example by Chakrabarti (1987), the partial derivative with respect to time is taken for the accelerations of the fluid particles. Such an approach is consistent with the assumptions accepted.

The theory of the regular wave was extended for a narrow band random case by Wilde and Romańczyk (1989). It was assumed

$$\begin{aligned}
&\frac{H}{2} \to \frac{H(t)}{2} && \beta \to \beta(t)\\
&A(t) = \frac{H(t)}{2}\cos[\beta(t)] && D(t) = \frac{H(t)}{2}\sin[\beta(t)]
\end{aligned} \tag{3.7}$$

and

$$\dot{A}(t) \ll \omega A(t) \qquad \dot{D}(t) \ll \omega D(t) \tag{3.8}$$

Substituting Eqs (3.7) into the expression for the water surface elevation (3.4) yields

$$\begin{aligned} \eta(x,t) &= A(t)\cos(kx-\omega t) - D(t)\sin(kx-\omega t) + \\ &+ \frac{\gamma}{h}\Big\{A^*(t)\cos[2(kx-\omega t)] + D^*(t)\sin[2(kx-\omega t)]\end{aligned} \tag{3.9}$$

where

$$A^*(t) = A^2(t) - D^2(t) \qquad D^*(t) = 2A(t)D(t)$$

The potential function (3.6) goes over to

$$\begin{aligned} \Phi_1 &= \frac{\omega}{k}\frac{\cosh(ky)}{\sinh(kh)}\Big[A(t)\sin(kx-\omega t) + D(t)\cos(kx-\omega t)\Big] \\ \Phi_2 &= \frac{3\omega}{8}\frac{\cosh(2ky)}{\sinh^4(kh)}\Big\{A^*(t)\sin[2(kx-\omega t)] + D^*(t)\cos[2(kx-\omega t)]\Big\}\end{aligned} \tag{3.10}$$

This potential function satisfies the Laplace equation for all values of the time parameter t. The kinematic and dynamic boundary conditions on the free water surface are satisfied when the time derivatives of $A(t)$ and $D(t)$ are neglected according to the assumption expressed by Eqs (3.8), thus the relations are true only for $x = 0$ and its small neighbourhood. When the functions $A(t)$ and $D(t)$ are known the functions $A^*(t)$ and $D^*(t)$ may be calculated from the relations given in Eq (3.9). For a sample function of piston motion the corresponding functions $A(t)$ and $D(t)$ may be calculated for any fixed distance from the wavemaker using the linear theory of waves. It should be stressed that this is an approximate description and its validity has to be checked by experiments.

The values of the functions $A(t)$, $D(t)$, $A^*(t)$ and $D^*(t)$ were calculated from comparison of the theoretical formulation (3.9) for $x = 0$

$$\eta(t) = A(t)\cos(\omega t) + D(t)\sin(\omega t) + \frac{\gamma}{h}\Big[A^*(t)\cos(2\omega t) + D^*(t)\sin(2\omega t)\Big] \tag{3.11}$$

with the measured data. Eq (3.11) has two components with dominant angular frequencies ω and 2ω. The measured time series were decomposed into two processes with two dominant frequencies. The Stokes wave theory leads to a relation given in Eqs (3.9) between the functions $A(t)$, $D(t)$ and $A^*(t)$, $D^*(t)$. The comparison of the estimated values of $A^*(t)$, $D^*(t)$ with those calculated according to the formulae in Eqs (3.9), based on estimates of $A(t)$, $D(t)$, showed good agreement. In some experiments more components were detected, but the first two were dominant and thus the application of second order Stokes' approximation was justified. For the experiments the second term gives a contribution that is of the order of 10%.

The velocities at a point $(0,y)$ are

$$\begin{aligned} u_x(0,y,t) &= \omega\frac{\cosh(ky)}{\sinh(kh)}\Big[A(t)\cos(\omega t)+D(t)\sin(\omega t)\Big]+\\ &+\frac{3\omega k}{4}\frac{\cosh(2ky)}{\sinh^4(kh)}\Big[A^*(t)\cos(2\omega t)+D^*(t)\sin(2\omega t)\Big]\\ u_y(0,y,t) &= \omega\frac{\sinh(ky)}{\sinh(kh)}\Big[-A(t)\sin(\omega t)+D(t)\cos(\omega t)\Big]+\\ &+\frac{3\omega k}{4}\frac{\sinh(2ky)}{\sinh^4(kh)}\Big[-A^*(t)\sin(2\omega t)+D^*(t)\cos(2\omega t)\Big] \end{aligned} \tag{3.12}$$

It should be noted that we measured the elevations at the point $x = 0$ and thus we have no information about the velocities in the other profiles. As an approximation we have to take the partial time derivative to get the acceleration. It is not possible to calculate the material time derivative without additional assumptions or analysis. It should be mentioned, that the Stokes approximate solution is a good one for surface elevations and the experimental verification of velocities is not perfectly reliable.

4. Results of experiments

Large parts of the measured time series for the dominant wave periods $T = 1\,\mathrm{s}$ and $T = 1.321\,\mathrm{s}$, randomness parameter $K = 2.5$ and gap width $e = 10\,\mathrm{mm}$ are depicted in Fig.1. The horizontal axis corresponds to the number of time steps. The vertical axes correspond to the surface elevation, horizontal and vertical components of acceleration, respectively. The values are plotted in different scales.

These plots show an overall relationship between the values of these physical random functions. In this paragraph the statistical properties in the frequency domain are discussed as a basis for the construction of a simplified mathematical description.

The input to the mechanical system are velocities of the fluid which are described by the narrow band stationary processes with two peaks at ω and

2ω in the spectral densities. Therefore it is reasonable that in the output – the measured accelerations – one may expect peaks in the spectral densities at multiples of ω and eigenfrequencies of the system. In filtering the twice differentiable mathematical model was used and such values of the randomness parameter K were fixed that the rest had no significant peaks and corresponded almost to a white noise.

In experiments it was important that the cylinder did not hit the bottom. Thus, for smaller gaps random waves with smaller standard deviations were generated and consequently the number of components in water surface elevations decreased with the gap width. All the measured time series where decomposed into random processes with dominant peaks corresponding to peak frequencies. The standard deviations of components were calculated. The results for periods $T = 1$ s and $T = 1.321$ s, random parameters $K = 2.5$ and $K = 0.5$ and gap widths $e = 70$ mm and $e = 10$ mm are shown in Table 2. In general, when the randomness parameter K decreased the standard deviations had to be diminished to avoid hitting.

Table 2. Standard deviations for the filtered components of surface elevations

T [s]	K	gap [mm]	σ_f [cm]	σ_{2f} [cm]	σ_{3f} [cm]
1.000	2.5	70	2.2746	0.3509	0.1189
1.000	0.5	70	1.8523	0.3180	0.0
1.321	2.5	70	2.3721	0.1791	0.0344
1.321	0.5	70	1.8978	0.2198	0.0
1.000	2.5	10	1.2625	0.1947	0.0
1.000	0.5	10	1.0337	0.1015	0.0
1.321	2.5	10	1.2164	0.0476	0.0
1.321	0.5	10	1.0504	0.0	0.0

The frequency of free vibrations in the horizontal plane mode is equal to some 1.15 Hz and thus it is close to the frequency corresponding to the wave period of 1 s. In such a case the decomposition is crude. The frequency of free vibrations in the vertical plane mode equals some $f_{fy} = 1.5$ Hz. For $T = 1.321$ s the frequency $f = 0.757$ Hz and thus $2f = 1.51$ Hz is very close to the eigenfrequency. It is not possible to distinguish between these two modes. The system is close to resonance for the first wave period and in resonance for the double frequency corresponding to the second wave period.

Some results of analysis of the measured time series of accelerations are presented in Table 3. From the analysis of all results several conclusions were drawn. For the large gap $(e = 70\,\text{mm})$ the influence of the bottom is not big and thus in engineering applications a model of motion of a cylinder in an infinite space of fluid may be used. The standard deviations of free vibrations are small in general, and in many cases can not be detected in measured data. They are large when the frequency of free vibrations is close to the dominant frequency of forced vibrations. They increase when randomness increases, as measured by the parameter K. The vertical components are in general small when compared with the corresponding horizontal ones.

Table 3. Standard deviations for the filtered components of accelerations

T [s]	K	gap		σ_{fx} [m/s^2]	σ_{fy} [m/s^2]	σ_f [m/s^2]	σ_{2f} [m/s^2]
1.000	2.5	70	horizontal	0.1595		0.1951	0.0098
1.000	0.5	70	horizontal	0.3171		0.0959	0.0
1.321	2.5	70	horizontal	0.0020		0.0849	0.0125
1.321	0.5	70	horizontal	0.0662		0.0848	0.0
1.000	2.5	70	vertical		0.0125	0.0214	0.0130
1.000	0.5	70	vertical		0.0275	0.0210	0.0076
1.321	2.5	70	vertical		0.0	0.0020	0.0990
1.321	0.5	70	vertical		0.0	0.0014	0.0469
1.000	2.5	10	horizontal	0.0566		0.1614	0.0027
1.000	0.5	10	horizontal	0.2212		0.0793	0.0
1.321	2.5	10	horizontal	0.0		0.0516	0.0034
1.321	0.5	10	horizontal	0.0430		0.0535	0.0
1.000	2.5	10	vertical		0.0	0.0066	0.0166
1.000	0.5	10	vertical		0.0008	0.0065	0.0164
1.321	2.5	10	vertical		0.0	0.0020	0.0990
1.321	0.5	10	vertical		0.0	0.0014	0.0469

When the wave frequency or its double enters a resonance zone the standard deviations grow. For $T = 1.321$ s there is resonance in vertical vibrations, and thus there is only one significant peak with the double of the dominant wave frequency.

5. Lagrangian, the differential equations

In the discussed case, let us modify the Lagrangian proposed by Wilde (1997) to include the masses of cylinder m_{cx} and m_{cy} obtained from the experimental results. First, let us consider an approximation in which we assume a velocity field homogeneous in space with the value at the point $(0,0)$ as a function of time. Thus according to Eqs (3.12), in the neighbourhood of the origin of the fixed coordinate system (where the cylinder moves) the velocity field is approximated by

$$u_x(x,y,t) = u_x(0,0,t) \qquad u_y(x,y,t) = 0 \tag{5.1}$$

In the considered case we take the Lagrangian in the form

$$\begin{aligned} L &= \frac{1}{2}\rho V_C[1 + C_m(v)]\Big[(u_x - \dot{x}_0)^2 + \dot{y}_0^2\Big] + \\ &+ \frac{1}{2}(m_{cx} - \rho V_C)\dot{x}_0^2 + \frac{1}{2}(m_{cy} - \rho V_C)\dot{y}_0^2 - \frac{1}{2}k_x x_0^2 - \frac{1}{2}k_y(y_0 - y_p) \end{aligned} \tag{5.2}$$

where $V_C = \pi a^2$ is the volume of the cylinder unit length. The corresponding Lagrange's differential equations are

$$\begin{aligned} &[m_{cx} + \rho V_C C_m(v)]\ddot{x}_0 - \rho V_C[1 + C_m(v)]\dot{u}_x + \rho\frac{V_C}{a}C_m'(v)(\dot{x}_0 - u_x)\dot{y}_0 + k_x x_0 = 0 \\ &[m_{cy} + \rho V_C C_m(v)]\ddot{y}_0 + \rho\frac{V_C}{2a}C_m'(v)\Big[-(\dot{x}_0 - u_x)^2 + \dot{y}_0^2\Big] + k_y(y_0 - y_p) = 0 \end{aligned} \tag{5.3}$$

The obtained differential equations are nonlinear and coupled due to the interaction with the bottom. When the distance from the bottom to the cylinder centre is great then $C_m(v) \to 1$, $C_m'(v) \to 0$ and Eqs (5.3) reduce to

$$\begin{aligned} &[m_{cx} + \rho V_C]\ddot{x}_0 + k_x x_0 = 2\rho V_C \dot{u}_x \\ &[m_{cy} + \rho V_C]\ddot{y}_0 + k_y(y_0 - y_p) = 0 \end{aligned} \tag{5.4}$$

Such a case almost occurs in the experiments for the gaps of $e = 70\,\text{mm}$ in width. It should be however stressed that in this case we should disregard the interaction of the cylinder with the bottom and express the external velocity field as the first term in a Taylor series around the cylinder position at rest $x = 0$, $y = y_p$. This would lead us to Eqs (5.4) supplemented in the second equation on the right-hand side by a term due to the vertical component of acceleration.

From the discussion of statistical analysis results of the measured time series it follows that the nonlinear term in the first equation of (5.3) has only a very slight influence and that the important term that affects the horizontal displacements is the term due to the horizontal component of acceleration. In the last term in the second expression the value of $\dot{y}_0^2(t)$ is very small compared with the other terms and may be neglected. In the case the displacements of cylinder are infinitesimal, the dimensionless parameter v is very close to $v = y_p/a$. When the terms corresponding to linear damping are added the following simplified differential equations result

$$\ddot{x}_0(t) + 2\kappa_x\dot{x}_0(t) + 4\pi^2 f_x^2 x_0(t) = F_1\dot{u}_x(t)$$

$$\ddot{y}_0(t) + 2\kappa_y\dot{y}_0(t) + 4\pi^2 f_y^2[y_0(t) - y_p] = F_2\dot{u}_y(t) + F_3\Big[(\dot{x}_0 - u_x)^2 + u_y^2\Big] \tag{5.5}$$

$$F_1 = \frac{\rho V_C[1 + C_m(v_p)]}{m_{cx} + \rho V_C C_m(v_p)} \qquad F_2 = \frac{\rho V_C[1 + C_m(v_p)]}{m_{cy} + \rho V_C C_m(v_p)}$$

$$F_3 = \frac{\rho V_C C_m'(v_p)}{2a[m_{cy} + \rho V_C C_m(v_p)]}$$

where the damping coefficients are estimated from the tests for free vibrations. In the equations the vertical components of acceleration and velocity are zero. The cylinder is close to the bottom but at a finite distance. We may look at an expansion around the centre of the cylinder at rest and generalise the problem as was proposed by Wilde (1997). For the random case we limit our discussion to the simplified case described by Eqs (5.5).

The differential equations have constant coefficients. The first equation is linear and there is no problem to find a solution for any function of time on the right side and any initial conditions. When the horizontal component of the displacement of the cylinder centre is known the right-hand side of the second equation is a known function of time. There is no problem to find a solution.

6. Simplified spectral densities of cylinder displacements

Let us calculate the spectral densities based on the simplified differential equations of the problem (5.5) and take the first terms into account only in water surface elevations and velocity fields.

Wilde and Kozakiewicz (1993) defined two processes $X(t)$ and $Y(t)$

$$\begin{aligned} X(t) &= A(t)\cos(\omega t) + D(t)\sin(\omega t) \\ Y(t) &= -A(t)\sin(\omega t) + D(t)\cos(\omega t) \end{aligned} \tag{6.1}$$

and a process in complex variables

$$Z(t) = X(t) + \mathrm{i}Y(t) \tag{6.2}$$

It is easy to verify that

$$\begin{aligned} Z^2 &= \widetilde{X}(t) + \mathrm{i}\widetilde{Y}(t) \\ \widetilde{X}(t) &= [A^2(t) - D^2(t)]\cos(2\omega t) + 2A(t)D(t)\sin(\omega t) \\ \widetilde{Y}(t) &= -[A^2(t) - D^2(t)]\sin(2\omega t) + 2A(t)D(t)\cos(\omega t) \end{aligned} \tag{6.3}$$

The envelope $W(t)$ of the processes (6.1) is

$$W(t) = \sqrt{X^2(t) + Y^2(t)} = \sqrt{A^2(t) + D^2(t)} \tag{6.4}$$

From Eqs (6.3) and (6.4) it follows

$$X^2(t) = \frac{1}{2}[W^2(t) + \widetilde{X}(t)] \qquad Y^2(t) = \frac{1}{2}[W^2(t) - \widetilde{Y}(t)] \tag{6.5}$$

In these notation the measured surface elevations given by Eq (3.11) may be written as

$$\eta(t) = X(t) \tag{6.6}$$

and the velocities are

$$\begin{aligned} u_x(0, y_p, t) &= \omega \coth(ky_p)X(t) \\ u_y(0, y_p, t) &= \omega \tanh(ky)Y(t) \end{aligned} \tag{6.7}$$

For the narrow-band processes derivatives of the slowly varying functions $A(t)$ and $D(t)$ may be neglected compared with the derivatives of trigonometric functions (assumption expressed by Eqs (3.8)). Thus it follows

$$\begin{aligned} \dot{u}_x(0, y_p, t) &= \omega^2 \coth(ky_p)Y(t) \\ \dot{u}_y(0, y_p, t) &= -\omega \tanh(ky)X(t) \end{aligned} \tag{6.8}$$

Now we express the stochastic processes of velocities and their derivatives in terms of random functions $X(t)$ and $Y(t)$ with known stochastic properties. The spectral densities of both functions are equal to the spectral density of measured time series of elevations of the free water surface. Thus the spectral density of x_0, as given by the first differential equation in (5.5) is

$$S_{x_0x_0} = \omega^4 \coth^2(ky_p)\frac{S_{\eta\eta}(\lambda)}{(\omega_x^2 - \lambda^2)^2 + 4\kappa_x^2\lambda^2} \tag{6.9}$$

One peak is due to the peak in spectral densities and a second one may be expected when λ is equal to the corresponding eigenfrequency.

To obtain the spectral density of vertical displacements one has to calculate the spectral density of the right-hand side of the second equation (5.5). In the right-hand side a random function has to be squared. Eqs (6.5) are used to express the squares of random functions. It should be noted that the correlation and spectral properties of all random functions that appear on the right-hand side are known. A computer program was written to calculate the spectral density $S_{R_2R_2}(\lambda)$ of the right-hand side of the second differential equation in (5.5). The left-hand side is a linear differential equation with constant coefficients and the stationary solution may be calculated in a standard way. Finally, the spectral density $S_{y_0y_0}(\lambda)$ of vertical displacements is

$$S_{y_0y_0} = \frac{S_{R_2R_2}(\lambda)}{(\omega_y^2 - \lambda^2)^2 + \kappa_y^2\lambda^2} \tag{6.10}$$

In the spectral density of the right-hand side one may expect a peak at the wave frequency, a peak for its double value due to the square and a peak at the eigenfrequency of horizontal vibrations. The fourth peak may appear when λ equals the angular eigenfrequency that corresponds to the case of resonance in vertical vibrations.

To obtain the spectral densities of second derivatives it is sufficient to multiply the spectral densities (6.9) and (6.10) by λ^4. The spectral densities (for the gap width $e = 10\,\mathrm{mm}$ and parameters of the dynamic system given in Table 1, and surface wave parameters: dominant period $T = 1\,\mathrm{s}$, standard deviation $1.23\,\mathrm{cm}$, $K = 2.5$) are depicted in Fig.2. To compare with the Fourier coefficients the square roots of the spectral densities are plotted as functions of frequencies. Two peaks are in the spectral densities of the horizontal accelerations and four in the vertical ones. They correspond to the wave frequency and its double value and to the eigenfrequencies of dynamic system. The solutions are very sensitive to the values of damping parameters and the widths of the spectral densities of the input. The corresponding spectral densities for the dominant period $T = 1.321\,\mathrm{s}$ are shown in Fig.3. In this case, in the vertical accelerations, the eigenfrequency is very close to the double of dominant frequency of the wave and due to resonance the amplitude of this component is large. Comparison with the amplitudes of Fourier series coefficients depicted in Fig.5 and Fig.6 shows that the simplified theoretical solution gives a good insight into the character of spectral densities. The calculated spectral densities are based on a simplified theoretical formulation only and no experimental coefficients are included.

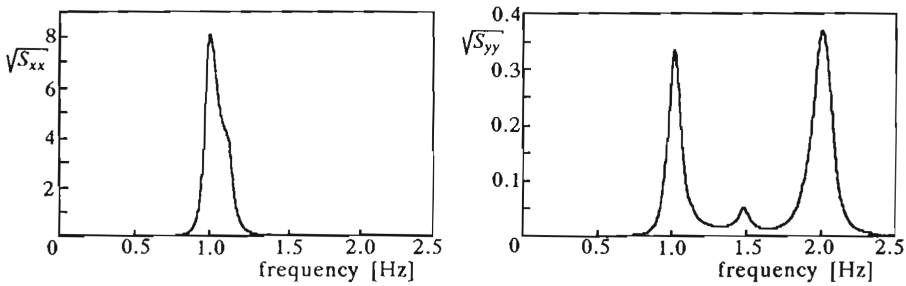

Fig. 2. Square roots of spectral densities of horizontal and vertical accelerations of the cylinder centre in $\mathrm{cm/s^{\frac{3}{2}}}$ for the dominant period $T = 1\,\mathrm{s}$

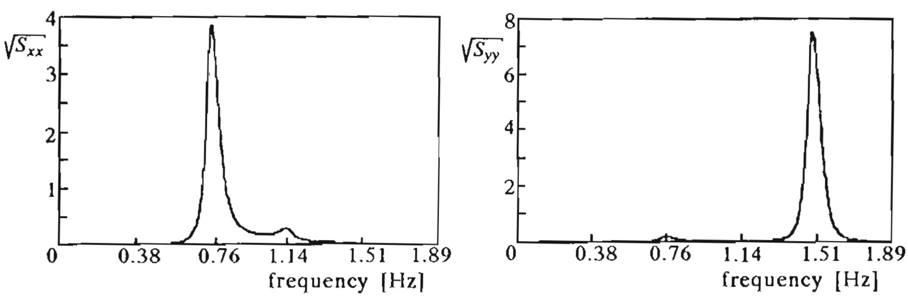

Fig. 3. Square roots of spectral densities of horizontal and vertical accelerations of the cylinder centre in $\mathrm{cm/s^{\frac{3}{2}}}$ for the dominant period $T = 1.321\,\mathrm{s}$

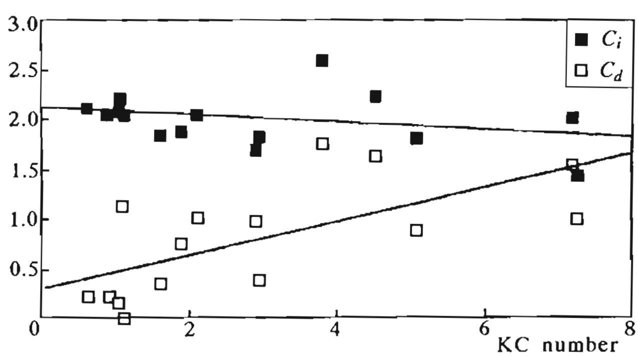

Fig. 4. C_i and C_d coefficients versus the Keulegan-Carpenter numbers for harmonic water waves

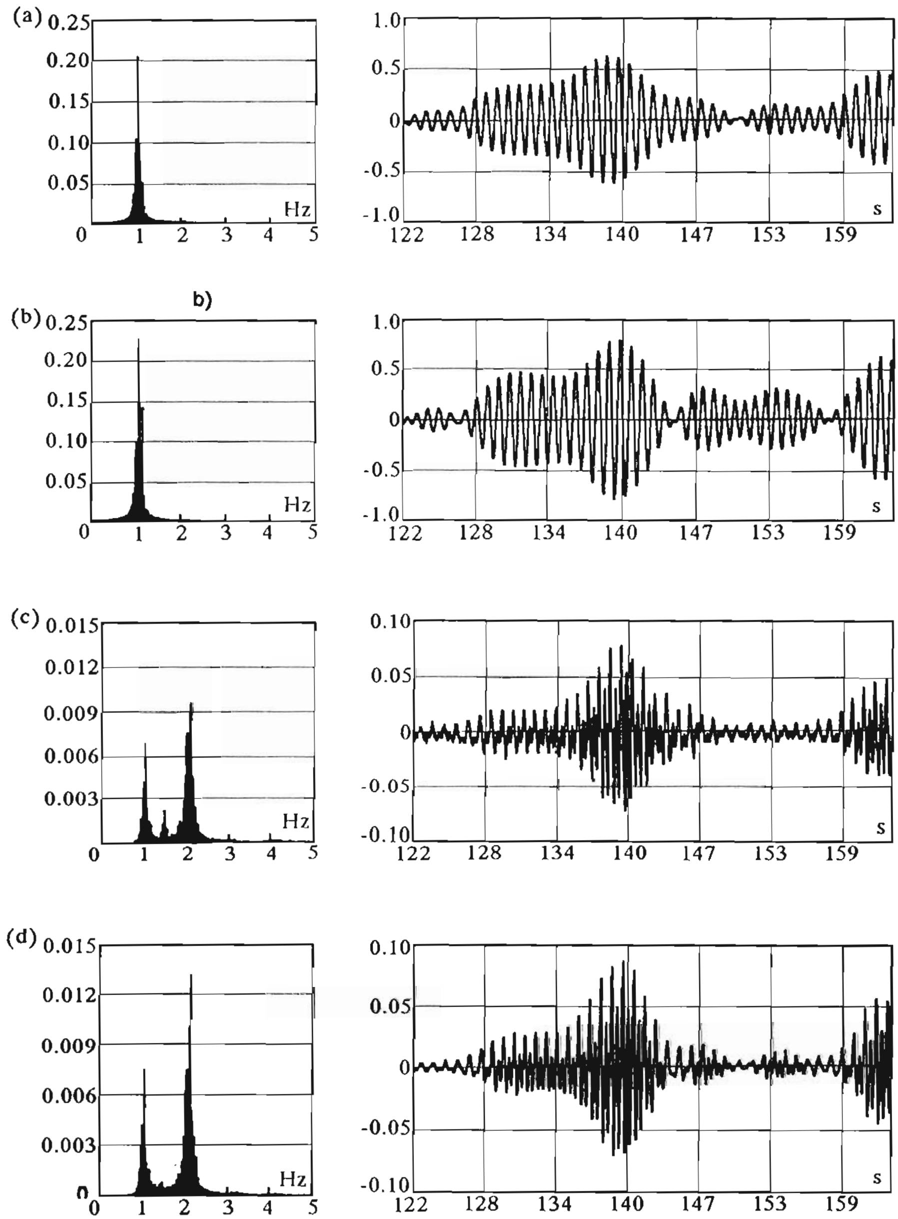

Fig. 5. Comparison of horizontal accelerations measured (a), calculated (b) and vertical accelerations measured (c), calculated (d) of the cylinder centre in m/s^2 for $T = 1\,s$, $e = 10\,mm$, $K = 2.5$

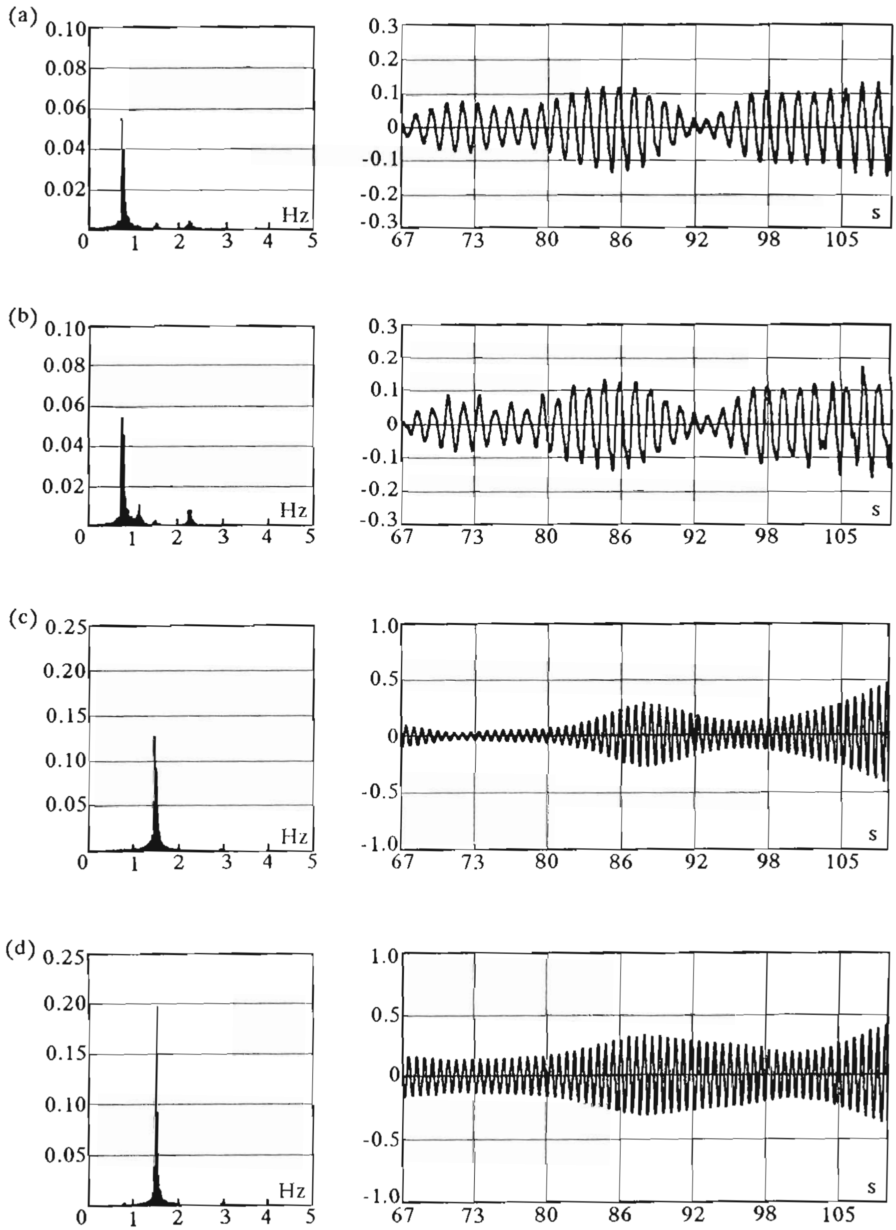

Fig. 6. Comparison of horizontal accelerations measured (a), calculated (b) and vertical accelerations measured (c), calculated (d) of the cylinder centre in m/s^2 for $T = 1.321\,\mathrm{s}$, $e = 10\,\mathrm{mm}$, $K = 2.5$

7. The influence of Morison's type terms

The problem of hydrodynamic forces on piles is very important in offshore engineering. A lot of experiments were performed on fixed cylinders in special set-ups for homogeneous in space and harmonic in time velocity fields. The resultant force in line with the velocity is expressed by the Morison et al. (1950)

$$R_i = C_i \rho V_C \dot{u}_x + C_d \frac{\rho V_C}{\pi a} |u_x| u_x \tag{7.1}$$

where the first term on the right-hand side is called the inertia force and the second one, the drag force. The values of coefficients C_i and C_d depend upon the Keulegan-Carpenter number KC and the Reynolds number Re defined by

$$\mathrm{KC} = \frac{u_{max} T}{2a} \qquad \mathrm{Re} = \frac{u_{max} 2a}{\nu} \tag{7.2}$$

where T is the period and ν is the coefficient of kinematic viscosity. Sarpkaya has published graphs that are used in engineering practice and reproduced in many textbooks, e.g. Chakrabarti (1987). It should be noted that the values of coefficients depend upon the period of harmonic velocity. For narrow band stochastic process the coefficients corresponding to the dominant frequencies may be used as first approximation. Experiments reveal that a lift force acts perpendicularly to the fluid velocity direction.

In engineering applications, a one dimensional relative motion of a cylinder in the x direction is represented by the differential equation

$$(m_{cx} + \rho V_C C_m)\ddot{x}_0 + 2\kappa_{xa}\dot{x}_0 + \omega_x^2 x_0 = C_i \rho V_C \dot{u}_x + C_d \frac{\rho V_C}{\pi a} |u_x - \dot{x}_0|(u_x - \dot{x}_0) \tag{7.3}$$

In the simplified case the first differential equation (5.5) has a similar form. Thus we may look at an extension. In the right-hand side we replace a theoretical coefficient by an experimental one

$$1 + C_m(v) \rightarrow C_i \tag{7.4}$$

and add a drag term. Such changes take care of the viscosity of the fluid neglected in our description. We left the coefficient of added mass C_m unchanged because it fits well the measured free vibrations in water.

In our experiments the cases of cylinder motion in regular waves were also investigated. We limit our discussion to the cases of measurements for deterministic harmonic waves. Therefore, the first component was filtered from the measured surface elevation and the corresponding velocity was calculated.

The component in the horizontal acceleration was found in a similar way. For regular waves these components are linear combinations of sine and cosine functions. Substitution led to two algebraic equations with unknown values of C_i and C_d.

The values of Keulegan-Carpenter numbers are small and indicate that all the experiments correspond to the cases where the inertia term is dominant. The Reynolds numbers are small for the considered cases and the differences are insignificant for all experiments. The coefficients plotted versus the Keulegan-Carpenter numbers are depicted in Fig.4. The scatter is large but not surprising. It should be noted that in many cases for the harmonic wave the measured accelerations were not regular. This is due to the detachment of vortices that introduces a random behaviour. In this analysis we considered the influence of bottom proximity only in the added mass coefficient. The average value of C_i is 2 and declines with the increase of the KC number. The C_d values are small and increase. For these KC values, in standard engineering practice, the values $C_i = 1.6 \div 2.1$ and $C_d = 0.6 \div 0.8$ are taken.

There is no, justified by experiments, generalisation of the drag term to the two- dimensional case when the cylinder is in motion. It is reasonable to take in Eq (7.1) relative velocity and to assume that the direction of the drag force coincides with the relative velocity vector. Thus, to introduce the inertia and drag terms it is enough to replace the inertia terms in Eqs (5.5) according to the following formulae

$$
\begin{aligned}
\frac{\rho V_C[1 + C_m(v_p)]}{m_{cx} + \rho V_C C_m(v_p)} \dot{u}_x \quad &\Rightarrow \quad \frac{\rho V_C C_i}{m_{cx} + \rho V_C C_m(v_p)} \dot{u}_x + \\
&\quad + \frac{\rho V_C C_d W_d}{\pi a[m_{cx} + \rho V_C C_m(v_p)]} (u_x - \dot{x}_0) \\
\frac{\rho V_C[1 + C_m(v_p)]}{m_{cy} + \rho V_C C_m(v_p)} \dot{u}_y \quad &\Rightarrow \quad \frac{\rho V_C C_i}{m_{cy} + \rho V_C C_m(v_p)} \dot{u}_y + \\
&\quad + \frac{\rho V_C C_d W_d}{\pi a[m_{cy} + \rho V_C C_m(v_p)]} (u_y - \dot{y}_0)
\end{aligned}
\tag{7.5}
$$

where

$$
W_d = \sqrt{(u_x - \dot{x}_0)^2 + (u_y - \dot{y}_0)^2}
$$

It should be stressed that such approach is aimed at formulating a simplified engineering description. To study the problem from the point of view of mechanics it is necessary to find an exact description of the behaviour especially the terms that represent energy dissipation.

8. Numerical solution, comparison with experiments

In the numerical solution the starting point was the Lagrangian in the general form

$$
\begin{aligned}
L &= \frac{1}{2}\rho V_C[1 + C_m(v)]\Big[(u_x - \dot{x}_0)^2 + (u_y - \dot{y}_0)^2\Big] + \\
&+ \frac{1}{2}(m_{cx} - \rho V_C)\dot{x}_0^2 + \frac{1}{2}(m_{cy} - \rho V_C)\dot{y}_0^2 - \frac{1}{2}k_x x_0^2 - \frac{1}{2}k_y(y_0 - y_p)
\end{aligned}
\tag{8.1}
$$

and Hamilton's principle of least action

$$
I = \int L(x_0, y_0, \dot{x}_0, \dot{y}_0, t)\, dt \tag{8.2}
$$

Analytic integration was replaced by the simplest numerical procedure, the derivatives by finite differences and the values by the averages over the assumed time step. Finally the action integral was reduced to an algebraic equation in the unknown values $x_0(r\Delta t)$ and $y_0(r\Delta t)$. The necessary condition for the extreme value, vanishing of the partial derivatives with respect to $x_0(s\Delta t)$ and $y_0(s\Delta t)$ was used, that led to two sets of algebraic equations. The equations were changed to include the terms corresponding to the Morison formula. The equations were solved by Newton's method. In the comparisons the values of linear damping coefficients in air for the mechanical system were taken. The values of the coefficients corresponding to the Morison formula were assumed equal to $C_i = 2.1$ and $C_d = 0.6$. Thus the damping of the system is nonlinear mainly due to the interaction with the fluid as given by the drag term.

Let us compare the measured accelerations with the calculated ones. In the Fig.5 the amplitudes of the Fourier coefficients and graphs of corresponding functions are shown for the case $T = 1\,\mathrm{s}$, $K = 2.5$, $e = 10\,\mathrm{mm}$. The first graph corresponds to the measured horizontal accelerations and the second to the calculated ones, while the third to measured vertical accelerations and the fourth to calculated ones. There are differences but the character is reproduced in the calculated accelerations. In Fig.6 the corresponding graphs are shown for the case $T = 1.32\,\mathrm{s}$ with the same values other of parameters. It is possible to look for the values of coefficients C_i and C_d that give a better fit for the considered interval but one can not expect to obtain a very good fit for the whole measured time series.

9. Results and conclusions

The measured surface elevations of the generated random waves were well described by an extension of the Stokes wave theory to the case of waves with slowly varying amplitudes and phase shifts.

In the considered cases, as the first approximation, an extension of the classic wave theory to the case of waves with slowly varying amplitudes and phase shifts may be used to calculate the velocities at the cylinder centre. It must be emphasized that the velocities were not measured due to the lack of precise gauges in our laboratory. More research, especially in the field of measurements and analysis of the velocity fields, is needed. The authors believe the discrepancies are also due to rough estimates of the velocity fields in water waves in flumes.

The measured vertical accelerations revealed interaction of the cylinder with the plane and rigid bottom in the form of a peak at the double of dominant water wave frequency. The influence of the interaction on the horizontal accelerations was so weak that it could not be detected in our tests.

The calculated spectral densities of accelerations based on simplified theoretical solutions with no empirical coefficients give good insight into the analysis of behaviour in the frequency domain.

In the time domain analysis, the theoretical description was supplemented by experimental terms corresponding to the Morison formula extended to the two dimensional case in a formal way. The inertia coefficient modifies the amplitudes and the drag coefficient describes the damping of the mechanical system due to the interaction with the fluid. The value of the second coefficient controls the behaviour in resonance. The comparison with measured values shows that there are differences but the overall character is reproduced.

The comparison with theoretical simplified solutions shows that is very important to have reliable estimates of velocities of the undisturbed fluid field in the vicinity of the cylinder centre and a good description of dissipation of energy.

The experimental analysis shows that the theoretical analysis based on the theory of ideal fluids and potential theory solutions gives a good starting point for the analysis of the proximity of the bottom on the cylinder vibrations.

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Badania doświadczalne drgań cylindra w wodzie w pobliżu dna wywołanych powierzchniową falą losową

Streszczenie

W kanale falowym Instytutu Budownictwa Wodnego PAN zbadano, wywołane losowymi falami wodnymi, drgania sprężyście podpartego cylindra, umieszczonego w pobliżu płaskiego i sztywnego dna. Powierzchniowe fale losowe były wywołane przez ruch płyty generatora fal określony przez obliczony na komputerze ciąg czasowy realizacji i zadany do systemu sterowania falorobu. Mierzono wzniesienia powierzchni swobodnej wody ponad cylindrem oraz przyspieszenia końców cylindra. Analiza pomierzonych wzniesień pokazuje, że uzasadniony jest opis w ramach teorii Stokes'a uogólniony na przypadek fal losowych o wąskim widmie (wolnozmienne amplitudy i przesunięcia fazowe). Punktem wyjścia dla opisu teoretycznego ruchu cylindra była teoria nieściśliwej cieczy doskonałej oraz ruchu potencjalnego. Podano propozycje uzupełnień równań zagadnienia o człony uwzględniające w sposób przybliżony efekty lepkości cieczy. Przy przyjęciu założeń upraszczających, analiza w dziedzinie częstotliwości prowadzi do wyrażeń na gęstości widmowe przemieszczeń i przyspieszeń cylindra. Analiza w dziedzinie czasu prowadzi do obliczenia numerycznego realizacji, które odzwierciedlają podstawowe cechy pomierzonych ciągów czasowych przyspieszeń cylindra. Badania doświadczalne potwierdzają, że rozwiązania teoretyczne bazujące na teorii potencjalnej cieczy doskonałej odzwierciedlają podstawowe cechy zjawisk obserwowanych w eksperymencie i są dobrym punktem wyjścia dla formułowania dla praktyki zawodowej metod inżynierskich.

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