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PROPAGATION OF PLANE WAVES IN STRATIFIED FLUID-SATURATED POROUS SOLIDS

STANISŁAW J. MATYSIAK

Institute of Hydrogeology and Engineering Geology, University of Warsaw

YURIĬ A. PYRYEV

Faculty of Mechanics and Mathematics, University of Lviv, Ukraine

Propagation of the harmonic plane waves in periodically stratified fluid-saturated porous solids is considered. Investigations are conducted on the basis of the linear homogenized theory of consolidation with microlocal parameters (cf Matysiak (1992)).

Key words: porous solid, stratified body, harmonic wave

1. Introduction

In view of their importance in numereous applications to geotechnical engineering, ground water hydrology, exploration geophysics, sejsmology, filtration and purification and sound absorption engineering, respectively, porous materials have received great attention in the literature on mechanics. There are many papers dealing with the development of basic equations governing behaviour of these materials as well as the mathematical analysis of the phenomena described by these equations (cf Biot (1941), (1956); Biot and Willis (1957); Crocket and Naghdi (1966); Derski (1979); Derski and Kowalski (1978); Luikow (1966); Paria (1966); Schneider and Bowen (1977); Zareckii (1967)).

This paper deals with the propagation of plane waves in a stratified fluid-saturated porous solids. The nonhomogeneous body under consideration in a natural (undeformed) configuration is composed of periodically repeated two elastic layers permeated by a network of interconnected pores filled with fluid. Perfect bonding between the layers is assumed. The inwestigations are conducted on the basis of the homogenized model with microlocal parameters (cf

Matysiak (1992)), in which the equations are given in terms of unknown macrodisplacements of the skeleton and fluid as well as certain extra unknowns called the kinematical microlocal parameters of skeleton and fluid. The homogenized model was derived by using the linear theory of consolidation given by Biot (1941), (1956), Biot and Willis (1957), Derski (1979). Following Biot (1957), Biot and Willis (1957) the homogenized model consists of two superposed continuous phases, each filling separately the whole space occupied by the aggregate.

The problems of wave propagation in a fluid-saturated porous medium composed of an elastic homogenous isotropic matrix permeated by a network of interconnected pores filled with liquid were considered by Biot (1956), (1962a,b), Dzięcielak (1980a,b), Kosmatka (1976), Mynett and Mei (1983), Mokryk and Pyryev (1985), Kończak (1992). The problems of shear waves in multilayered medium were considered by Kończak (1989). The homogenized model with microlocal parameters was applied by Jakubowska and Matysiak (1987), Bielski and Matysiak (1992) to some problems of wave propagation in the periodically layered elastic bodies.

The obtained solution can be used in solving some problems of geophysics (e.g., wave propagation in varved clays (cf Kaczyński and Matysiak (1993)), thin-layered limestone, sandstone shale).

2. Basic equations

Consider the nonhomogeneous body which in a natural (underformed) configuration is composed of periodically repeated two fluid-saturated porous elastic layers (see Fig.1). Let $0x_1x_2x_3$ be the Cartesian coordinate system such that the axis is x_2 normal to the layering. Let l_1, l_2 be the thicknesses of layers, and δ be the thickness of each repeated unit of the body, so $\delta = l_1 + l_2$. Let $\bar{\rho}$ denote the density of free fluid and $\rho^{(1)}$, $\rho^{(2)}$ be the densities of skeletons and non-free fluid of the subsequent layers. By $N^{(i)}$, $M^{(i)}$, $R^{(i)}$, $Q^{(i)}$, $i = 1,2$ we denote the porous media constants and by $b^{(i)}$, $i = 1,2$ the dissipation coefficients of the subsequent layers, respectively. Let t denote time, $\boldsymbol{u}(\boldsymbol{x},t) = [u_1(\boldsymbol{x},t), u_2(\boldsymbol{x},t), u_3(\boldsymbol{x},t)]$ be the displacement vector of skeleton and $\boldsymbol{U}(\boldsymbol{x},t) = [U_1(\boldsymbol{x},t), U_2(\boldsymbol{x},t), U_3(\boldsymbol{x},t)]$ be the displacement vector of fluid.

According to the results given by Matysiak (1992), where the governing equations of the homogenized model with microlocal parameters are derived,

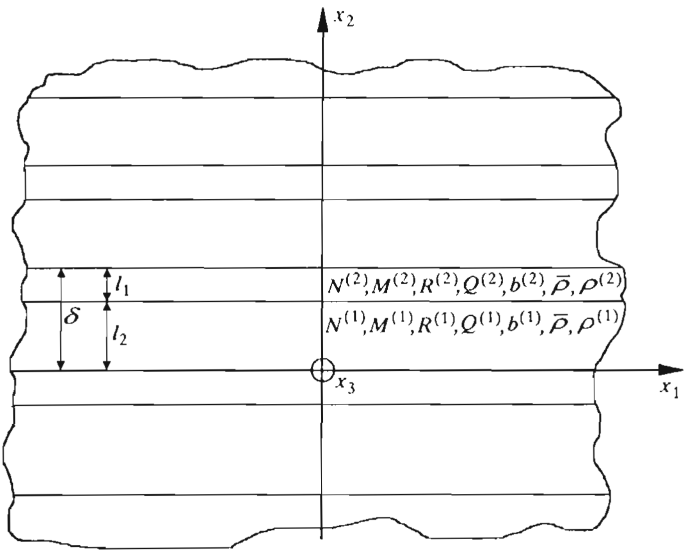

Fig. 1.

the displacement vectors are assumed as follows

$$
\begin{aligned}
u_i(\boldsymbol{x},t) &= w_i(\boldsymbol{x},t) + \underline{l(x_2)q_i(\boldsymbol{x},t)} \\
U_i(\boldsymbol{x},t) &= W_i(\boldsymbol{x},t) + \underline{l(x_2)Q_i(\boldsymbol{x},t)} \qquad i=1,2,3
\end{aligned}
\tag{2.1}
$$

where $l(\cdot):\ \mathbb{R}\rightarrow\mathbb{R}$ is a δ-periodic continuous function known a priori, called the shape function, given by

$$
\begin{aligned}
l(x_2) &= \begin{cases} x_2 - \frac{1}{2}l_1 & \text{for} \quad 0 \le x_2 \le l_1 \\ \frac{-\eta x_2}{1-\eta} - \frac{1}{2}l_1 + \frac{l_1}{1-\eta} & \text{for} \quad l_1 \le x_2 \le \delta \end{cases} \\
l(x_2) &= l(x_2+\delta)
\end{aligned}
\tag{2.2}
$$

where

$$
\eta = \frac{l_1}{\delta}
\tag{2.3}
$$

The functions $w_i(\cdot)$, $W_i(\cdot)$ are unknown and interpreted as the components of macrodisplacement vector of the skeleton and fluid, respectively. The unknown functions $q_i(\cdot)$ and $Q_i(\cdot)$ stand for the microlocal parameters of skeleton and fluid and correspond to the stratified material structure of the body.

Using the following notations

$$\begin{aligned}\widetilde{f} &\equiv \eta f_1 + (1-\eta) f_2 \\ [\![f]\!] &\equiv \eta(f_1 - f_2) \\ \widehat{f} &\equiv \eta f_1 + \frac{\eta^2}{1-\eta} f_2\end{aligned} \tag{2.4}$$

for an arbitrary δ-periodic function $f(\cdot)$ taking a constant value in a layer of the ith kind $(i = 1,2)$ and by substituting into Eqs (2.4) for f_i the values of the δ-periodic functions ρ, N, M, R, Q, b we obtain all the material modulae of the homogenized model with microlocal parameters (cf Matysiak (1992)).

Following the results given by Matysiak (1992), the governing equations of the homogenized model of periodically two-layered fluid-saturated bodies take the form (in the absence of body forces)[1]

$$\begin{aligned}&\widetilde{N} w_{i,jj} + (\widetilde{N} + \widetilde{M} + \widetilde{Q}) w_{j,ji} + (\widetilde{Q} + \widetilde{R}) W_{j,ji} + [\![N]\!] q_{i,2} + \Big([\![M]\!] + [\![Q]\!]\Big) q_{2,i} + \\ &+ [\![N]\!] q_{j,j}\delta_{i,2} + \Big([\![Q]\!] + [\![R]\!]\Big) Q_{2,i} = \widetilde{\rho} w_{i,tt} + \widetilde{b}(w_{i,t} - W_{i,t}) \\ &\widetilde{Q} w_{j,ji} + \widetilde{R} W_{j,ji} + [\![Q]\!] q_{2,i} + [\![R]\!] Q_{2,i} = \widetilde{\rho} W_{i,tt} - \widetilde{b}(w_{i,t} - W_{i,t})\end{aligned} \tag{2.5}$$

and

$$\begin{aligned}q_2 &= \frac{-2[\![N]\!]\widehat{R} w_{2,2} + \Big([\![Q]\!]\widehat{Q} - [\![M]\!]\widehat{R}\Big) w_{j,j} + \Big([\![R]\!]\widehat{Q} - [\![Q]\!]\widehat{R}\Big) W_{j,j}}{\widehat{M}\widehat{R} - \widehat{Q}^2} \\ Q_2 &= \frac{2[\![N]\!]\widehat{Q} w_{2,2} + \Big(\widehat{Q}[\![M]\!] - \widehat{M}[\![Q]\!]\Big) w_{j,j} + \Big(\widehat{Q}[\![Q]\!] - \widehat{M}[\![R]\!]\Big) W_{j,j}}{\widehat{M}\widehat{R} - \widehat{Q}^2} \\ q_1 &= -\frac{[\![N]\!]}{\widehat{N}}(w_{1,2} + w_{2,1}) \\ q_3 &= -\frac{[\![N]\!]}{\widehat{N}}(w_{2,3} + w_{3,2})\end{aligned} \tag{2.6}$$

Substituting the microlocal parameters given by Eqs (2.6) into Eqs (2.5) we obtain six linear partial differential equations with constant coefficients for macrodisplacements of skeleton and fluid and this system will be considered in the next sections.

[1]Summmation convention holds for the repeated indices and $f_{,i} \equiv \partial f/\partial x_i$, $f_{,t} \equiv \partial f/\partial t$, $\delta_{ij} = 1$ for $i = j$, $\delta_{ij} = 0$ for $i \neq j$

Following to results of paper (cf Matysiak (1992)), stresses and fluid pressures in the subsequent layers are expressed by the macrodisplacement vectors of skeleton and fluid as well as the microlocal parameters. It is emphasized, that since $|l(x_2)| < \delta$ for every $x_2 \in \mathbb{R}$, then for small δ the underlined terms in equations (2.1) are small and are neglected (see for an exact explanation in paper of Woźniak (1987)). However, $l_{,2}(\cdot)$ is not small and the terms involving $l_{,2}(\cdot)$ cannot be neglected. It leads to the following approximations

$$\begin{aligned} u_{i,\alpha} &\simeq w_{i,\alpha} & u_{i,2} &\simeq w_{i,2} + l_{,2}q_i \\ U_{i,\alpha} &\simeq W_{i,\alpha} & U_{i,2} &\simeq W_{i,2} + l_{,2}Q_i \\ u_{i,t} &\simeq w_{i,t} & U_{i,t} &\simeq W_{i,t} \end{aligned} \tag{2.7}$$

and $\alpha \in \{1,3\}$, $i \in \{1,2,3\}$.

3. Harmonic wave propagation

Consider now the problem of plane harmonic wave propagation in an arbitrary direction through an unbouned, periodically layered fluid-saturated porous body. To derive the dispersion equation, the homogenized model with microlocal parameters presented in Section 2 will be used.

We are looking for the solution of the equations of homogenized model in the form

$$\{\boldsymbol{w},\boldsymbol{W}\} = \{\boldsymbol{w}^{\pm},\boldsymbol{W}^{\pm}\}\exp[\mathrm{i}(kn_jx_j - \omega t)] \tag{3.1}$$

where

$\boldsymbol{w}^{\pm}, \boldsymbol{W}^{\pm}$	–	constant amplitudes of the macrodisplacements of skeleton and fluid
k	–	wave number
ω	–	circular frequency
n_j	–	components of the unit vector determining the direction of propagation, $j = 1,2,3$.

In Eq (3.1), which represents the harmonic waves, constants k and ω are complex quantities, so the lenght is $2\pi/\mathrm{Re}\,k$, the phase velocity is $\omega/\mathrm{Re}\,k$ and the period is $2\pi/\mathrm{Re}\,\omega$. Now we shall investigate the following problems of wave propagation.

3.1. Propagation of the waves normal to layering

Let the macrodisplacement vectors $\boldsymbol{w}$, $\boldsymbol{W}$ and the microlocal parameters

$\boldsymbol{q}$, $\boldsymbol{Q}$ be dependent on the variables (x_j, t) only. Then, eliminating the microlocal parameters from Eqs (2.5) (by using Eqs (2.6)) we obtain the following system of formulas for macrodisplacements w_j and W_j

$$\begin{aligned} L_{ij}(\partial x_i, \partial t)w_j(x_i,t) + M_{ij}(\partial x_i, \partial t)W_j(x_i,t) = 0 \\ N_{ij}(\partial x_i, \partial t)w_j(x_i,t) + R_{ij}(\partial x_i, \partial t)W_j(x_i,t) = 0 \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} &L_{ij}(\partial x_i, \partial t) \equiv l_{ij}\partial x_i^2 - \widetilde{\rho}\partial^2 t - \widetilde{b}\partial t \\ &M_{ij}(\partial x_i, \partial t) \equiv \delta_{ij}m_{ij}\partial x_i^2 + \widetilde{b}\partial t \\ &N_{ij}(\partial x_i, \partial t) \equiv \delta_{ij}n_{ij}\partial x_i^2 + \widetilde{b}\partial t \\ &R_{ij}(\partial x_i, \partial t) \equiv \delta_{ij}r_{ij}\partial x_i^2 - \widetilde{\rho}\partial^2 t - \widetilde{b}\partial t \\ &i,j = 1,2,3 \end{aligned} \tag{3.3}$$

and l_{ij}, m_{ij}, n_{ij}, r_{ij} are constans, which will be determined seperately for the considered wave types.

3.1.1. Longitudinal waves normal to the layering

For waves of this type the field variables are of the form

$$\{w_2, W_2\} = \{w_2^{\pm}, W_2^{\pm}\} \exp[\mathrm{i}(kx_2 - \omega t)] \tag{3.4}$$

while other field variables vanish identically. The above assumptions and Eqs (2.5) (after elimination of the microlocal parameters by using Eqs (2.6)) lead to Eqs (3.2) together with the notations given by Eqs (3.3), where $i = 2$, $j = 2$ and

$$\begin{aligned} l_{22} &= 2\widetilde{N} + \widetilde{M} + \widetilde{Q} + \Big(2[\![N]\!] + [\![M]\!] + [\![Q]\!]\Big)\gamma_1 + \Big([\![Q]\!] + [\![R]\!]\Big)\gamma_2 \\ m_{22} &= \widetilde{Q} + \widetilde{R} + \Big(2[\![N]\!] + [\![M]\!] + [\![Q]\!]\Big)\gamma_3 + \Big([\![Q]\!] + [\![R]\!]\Big)\gamma_4 \\ n_{22} &= \widetilde{Q} + [\![Q]\!]\gamma_1 + [\![R]\!]\gamma_2 \\ r_{22} &= \widetilde{R} + [\![Q]\!]\gamma_3 + [\![R]\!]\gamma_4 \\ \gamma_1 &= \Big(-2[\![N]\!]\widehat{R} + [\![Q]\!]\widehat{Q} - [\![M]\!]\widehat{R}\Big)\gamma^{-1} \\ \gamma_2 &= \Big(2[\![N]\!]\widehat{Q} + \widehat{Q}[\![M]\!] - \widehat{M}[\![Q]\!]\Big)\gamma^{-1} \\ \gamma_3 &= \Big([\![R]\!]\widehat{Q} - [\![Q]\!]\widehat{R}\Big)\gamma^{-1} \\ \gamma_4 &= \Big(\widehat{Q}[\![Q]\!] - \widehat{M}[\![R]\!]\Big)\gamma^{-1} \\ \gamma &= \widehat{M}\widehat{R} - \widehat{Q}^2 \end{aligned} \tag{3.5}$$

Substituting Eq (3.4) into Eqs (3.2) we obtain two algebraic equations in amplitudes $w_2^{\pm}$, $W_2^{\pm}$ in the form

$$\begin{aligned} L_{22}(\mathrm{i}k,-\mathrm{i}\omega)w_2^{\pm} + M_{22}(\mathrm{i}k,-\mathrm{i}\omega)W_2^{\pm} &= 0 \\ N_{22}(\mathrm{i}k,-\mathrm{i}\omega)w_2^{\pm} + R_{22}(\mathrm{i}k,-\mathrm{i}\omega)W_2^{\pm} &= 0 \end{aligned} \tag{3.6}$$

where L_{22}, M_{22}, N_{22}, R_{22} are determined in Eqs (3.3) by substituting constans $\mathrm{i}k$, $-\mathrm{i}\omega$ for differential operators ∂x_i and ∂t, respectively.

Eqs (3.6) yield the amplitude relation

$$\frac{w_2^{\pm}}{W_2^{\pm}} = \frac{M_{22}(\mathrm{i}k,-\mathrm{i}\omega)}{L_{22}(\mathrm{i}k,-\mathrm{i}\omega)} = -\frac{m_{22}k^2 + \mathrm{i}\widetilde{b}\omega}{l_{22}k^2 - \widetilde{\rho}\omega^2 - \mathrm{i}\widetilde{b}\omega} \tag{3.7}$$

and the frequency equation

$$k^4 - k^2(a_{22}\omega^2 + \mathrm{i}a_{21}\omega) + a_{04}\omega^4 + \mathrm{i}a_{03}\omega^3 = 0 \tag{3.8}$$

where

$$\begin{aligned} a_{22} &= \frac{\widetilde{\rho}r_{22} + \bar{\rho}l_{22}}{l_{22}r_{22} - m_{22}n_{22}} \\ a_{21} &= \widetilde{b}\,\frac{l_{22} + m_{22} + n_{22} + r_{22}}{l_{22}r_{22} - m_{22}n_{22}} \\ a_{04} &= \frac{\bar{\rho}\widetilde{\rho}}{l_{22}r_{22} - m_{22}n_{22}} \\ a_{03} &= \widetilde{b}\,\frac{\widetilde{\rho} + \bar{\rho}}{l_{22}r_{22} - m_{22}n_{22}} \end{aligned} \tag{3.9}$$

Assuming that ω is a real constant Eq (3.8) has then the roots $\pm k_1$, $\pm k_2$, where

$$\begin{aligned} k_1(\omega) &= \sqrt{\frac{\omega}{2}}\sqrt{a_{22}\omega + \mathrm{i}a_{21} + \sqrt{\Omega(\omega)}} \\ k_2(\omega) &= \sqrt{\frac{\omega}{2}}\sqrt{a_{22}\omega + \mathrm{i}a_{21} - \sqrt{\Omega(\omega)}} \end{aligned} \tag{3.10}$$

and

$$\begin{aligned} &\omega_{\pm} \equiv \mathrm{i}\omega_0^I \pm \omega_0^R \qquad && \omega_0^R = 2\frac{\sqrt{p_2}}{p_1} \qquad \omega_0^I = -\frac{p_3}{p_2} \\ &p_1 = a_{22}^2 - 4a_{04} && p_2 = a_{22}a_{21}a_{03} - a_{21}^2a_{04} - a_{03}^2 \\ &p_3 = a_{22}a_{21} - 2a_{03} \end{aligned} \tag{3.11}$$

Denoting by

$$V_n(\omega) = \frac{\omega}{\text{Re } k_n(\omega)} \qquad \eta_n(\omega) = \text{Im } k_n(\omega) \qquad n = 1,2 \tag{3.12}$$

where $V_n(\omega)$ is the phase velocity and $\eta_n(\omega)$ is the attenuation coeficient. We have

$$k_n(\omega) = \frac{\omega}{V_n(\omega)} + \mathrm{i}\eta_n(\omega) \qquad n = 1,2 \tag{3.13}$$

The solution to the considered problem of longitudinal waves propagating in the direction normal to the layering can be written in the form

$$\begin{aligned} w_2(x_2,t) &= w_2^+ \exp\Big[-\eta_1 x_2 - \mathrm{i}\omega\Big(t - \frac{x_2}{V_1}\Big)\Big] + w_2^- \exp\Big[\eta_1 x_2 - \mathrm{i}\omega\Big(t + \frac{x_2}{V_1}\Big)\Big] + \\ &- \frac{M_{22}(\mathrm{i}k_2, -\mathrm{i}\omega)}{L_{22}(\mathrm{i}k_2, -\mathrm{i}\omega)V_2}\Big\{W_2^+ \exp\Big[-\eta_2 x_2 - \mathrm{i}\omega\Big(t - \frac{x_2}{V_2}\Big)\Big] + \\ &+ W_2^- \exp\Big[\eta_2 x_2 - \mathrm{i}\omega\Big(t + \frac{x_2}{V_2}\Big)\Big]\Big\} \\ W_2(x_2,t) &= -\frac{N_{22}(\mathrm{i}k_1, -\mathrm{i}\omega)}{R_{22}(\mathrm{i}k_1, -\mathrm{i}\omega)}\Big\{w_2^+ \exp\Big[-\eta_1 x_2 - \mathrm{i}\omega\Big(t - \frac{x_2}{V_1}\Big)\Big] + \\ &+ w_2^- \exp\Big[\eta_1 x_2 - \mathrm{i}\omega\Big(t + \frac{x_2}{V_1}\Big)\Big]\Big\} + \\ &+ W_2^+ \exp\Big[-\eta_2 x_2 - \mathrm{i}\omega\Big(t - \frac{x_2}{V_2}\Big)\Big] + W_2^- \exp\Big[\eta_2 x_2 - \mathrm{i}\omega\Big(t + \frac{x_2}{V_2}\Big)\Big] \end{aligned} \tag{3.14}$$

The solution given by Eqs (3.14) contains two types of harmonic waves: modified elastic waves and modified acoustics waves corresponding to the exponents $t \pm x_2/V_1$ and $t \pm x_2/V_2$, respectively. The superscripts $(\cdot)^+$ and $(\cdot)^-$ are correspond to the wave moving in the positive and negative directions, respectively.

The phase velocity $V_n(\omega)$ presented in Eqs (3.12) and (3.10) is subjected to dispersion. The attenuation coeficient η_n and the phase velocity V_n are functions of the reduced frequency ω.

Remark. The behaviour of the wave number range at the extremes of ω may be predicted from Eqs (3.10) and (3.11). It leads to the following relations

$$\begin{aligned} k_1(\omega) &\sim \frac{\omega}{c_+} + \mathrm{i}\eta_+ + O(\omega^{-1}) \qquad \omega \to \infty \\ k_2(\omega) &\sim \frac{\omega}{c_-} + \mathrm{i}\eta_- + O(\omega^{-1}) \qquad \omega \to \infty \end{aligned} \tag{3.15}$$

and

$$
\begin{aligned}
k_1(\omega) &\sim \mathrm{i}\sqrt{-\mathrm{i}\omega}\sqrt{a_{21}}\Big(1 + o(\omega)\Big) && \omega \to 0 \\
k_2(\omega) &\sim -\frac{\omega}{c_0} + \frac{1}{4}\mathrm{i}\omega^2\mu_0^2 + o(\omega^3) && \omega \to 0
\end{aligned}
\tag{3.16}
$$

where

$$
\begin{aligned}
c_\pm &= \sqrt{\frac{2}{a_{22} \pm \sqrt{p_1}}} && \eta_\pm = \pm\frac{1}{2\eta c_\pm}\frac{c_0^2 - c_\pm^2}{c_-^2 - c_+^2} \\
\mu_0^2 &= \frac{2\eta_0}{c_0^3}(c_0^2 - c_+^2)(c_-^2 - c_+^2) && \eta_0 = \frac{a_{04}}{a_{03}} \qquad c_0 = \sqrt{\frac{a_{21}}{a_{03}}}
\end{aligned}
\tag{3.17}
$$

It can be shown that

$$
c_- > c_0 > c_+ > 0 \qquad\qquad \eta > 0 \tag{3.18}
$$

3.1.2. Transverse waves normal to the layering

For the waves of this type the field variables are of the form

$$
\begin{aligned}
\{w_j, W_j\} &= \{w_j^\pm, W_j^\pm\}\exp[\mathrm{i}(kx_2 - \omega t)] \qquad j = 1,3 \\
w_2 &= W_2 = 0
\end{aligned}
\tag{3.19}
$$

Eliminating the microlocal parameters from Eqs (2.5) by using Eqs (2.6) we obtain the system of formulae for macrodisplacements in the form given by Eqs (3.2) and (3.3) where

$$
\begin{aligned}
l_{2j} &= \widetilde{N} - \frac{[\![N]\!]^2}{\widehat{N}} && M_{2j} = \widetilde{b}\partial t \\
N_{2j} &= \widetilde{b}\partial t && R_{2j} = -\widetilde{\rho}\partial^2 t - \widetilde{b}\partial t \qquad j = 1,3
\end{aligned}
\tag{3.20}
$$

Substituting Eqs (3.19) into Eqs (3.2) we arrive at the following frequency equation

$$
-k^2(\bar{\rho}\omega + \mathrm{i}\widetilde{b}) + b_{04}\omega^3 + \mathrm{i}b_{03}\omega^2 = 0 \tag{3.21}
$$

where

$$
b_{04} = \frac{\bar{\rho}\widetilde{\rho}}{l_{2j}} \qquad\qquad b_{03} = \frac{\widetilde{b}(\widetilde{\rho} + \bar{\rho})}{l_{2j}} \tag{3.22}
$$

The roots of Eq (3.21) are k_3 and $-k_3$, provided that

$$k_3 = \omega\sqrt{\frac{b_{04}\omega + \mathrm{i}b_{03}}{\bar{\rho}\omega + \mathrm{i}\tilde{b}}} \tag{3.23}$$

The solution to the considered problem of transverse waves propagating in the direction normal to the layering takes the form

$$\begin{aligned}
w_j(x_2,t) &= w_j^{+}\exp\Big[-\eta_3 x_2 - \mathrm{i}\omega\Big(t - \frac{x_2}{V_3}\Big)\Big] + w_j^{-}\exp\Big[\eta_3 x_2 - \mathrm{i}\omega\Big(t + \frac{x_2}{V_3}\Big)\Big] \\
W_j(x_2,t) &= \frac{\mathrm{i}\tilde{b}}{\bar{\rho}\omega + \mathrm{i}\tilde{b}}\Big\{W_j^{+}\exp\Big[-\eta_3 x_2 - \mathrm{i}\omega\Big(t - \frac{x_2}{V_3}\Big)\Big] + \\
&+ W_j^{-}\exp\Big[\eta_3 x_2 - \mathrm{i}\omega\Big(t + \frac{x_2}{V_3}\Big)\Big]\Big\}
\end{aligned} \tag{3.24}$$

where $j = 1,3$ and

$$V_3 \equiv V_3(\omega) = \frac{\omega}{\mathrm{Re}\; k_3(\omega)} \qquad\qquad \eta_3 \equiv \eta_3(\omega) = \mathrm{Im}\; k_3(\omega) \tag{3.25}$$

Remark. From Eq (3.23) it follows that

$$\begin{aligned}
k_3(\omega) &\sim \frac{\omega}{c} + \mathrm{i}\eta_0 + O(\omega^{-1}) && \omega \to \infty \\
k_3(\omega) &\sim \frac{\omega}{c_0} + \frac{1}{4}\mathrm{i}\omega^2\mu_1^2 + o(\omega^3) && \omega \to 0
\end{aligned} \tag{3.26}$$

where

$$c = \sqrt{\frac{l_{2j}}{\bar{\rho} + \tilde{\rho}}} \qquad\qquad \mu_1 = \frac{2\bar{\rho}^2}{\tilde{b}}\sqrt{l_{2j}(\bar{\rho} + \tilde{\rho})} \tag{3.27}$$

3.2. Propagation of waves in the direction of layering

We assume now that the macrodisplacement vectors $\boldsymbol{w}$, $\boldsymbol{W}$ as well as the microlocal parameters $\boldsymbol{q}$, $\boldsymbol{Q}$ depend on the variables (x_j, t), $j = 1$ or $j = 3$. Eqs (2.5) and (2.6) reduce to the form given by Eqs (3.2) and (3.3) provided that $i = 1,3$, $j = 1,2,3$. So, we have the following cases.

3.2.1. Longitudinal waves in the direction of layering

For the waves of this type we have

$$\begin{aligned} &\{w_j, W_j\} = \{w_j^{\pm}, W_j^{\pm}\} \exp[\mathrm{i}(kx_j - \omega t)] \qquad j = 1 \quad \text{or} \quad j = 3 \\ &\{w_2, W_2\} \equiv= \{0, 0\} \end{aligned} \tag{3.28}$$

The wave equations are obtained from Eqs (3.2) and (3.3), where $i = j = 1$ or $i = j = 3$ and

$$\begin{aligned} l_{jj} &= 2\tilde{N} + \widetilde{M} + \tilde{Q} + \Big(\llbracket M \rrbracket + \llbracket Q \rrbracket\Big)\beta_1 + \Big(\llbracket Q \rrbracket + \llbracket R \rrbracket\Big)\beta_2 \\ m_{jj} &= \tilde{Q} + \tilde{R} + \Big(\llbracket M \rrbracket + \llbracket Q \rrbracket\Big)\gamma_3 + \Big(\llbracket Q \rrbracket + \llbracket R \rrbracket\Big)\gamma_4 \\ n_{jj} &= \tilde{Q} + \llbracket Q \rrbracket\beta_1 + \llbracket R \rrbracket\beta_2 \\ r_{jj} &= r_{22} \\ \beta_1 &= \Big(\llbracket Q \rrbracket\hat{Q} - \llbracket M \rrbracket\hat{R}\Big)\gamma^{-1} \\ \beta_2 &= \Big(\hat{Q}\llbracket M \rrbracket - \widehat{M}\llbracket Q \rrbracket\Big)\gamma^{-1} \end{aligned} \tag{3.29}$$

Substituting Eqs (3.28) into Eqs (3.2) together with Eqs (3.3) and (3.29) we arrive at the frequency equation

$$k^4 - k^2(c_{22}\omega^2 + \mathrm{i}\omega c_{22}) + c_{04}\omega^4 + \mathrm{i}c_{03}\omega^3 = 0 \tag{3.30}$$

where

$$\begin{aligned} c_{22} &= \frac{l_{jj}\bar{\rho} + r_{jj}\tilde{\rho}}{l_{jj}r_{jj} - m_{jj}n_{jj}} \\ c_{21} &= \tilde{b}\,\frac{l_{jj} + r_{jj} + m_{jj} + n_{jj}}{l_{jj}r_{jj} - m_{jj}n_{jj}} \\ c_{04} &= \frac{\bar{\rho}\tilde{\rho}}{l_{jj}r_{jj} - m_{jj}n_{jj}} \\ c_{03} &= \tilde{b}\,\frac{\tilde{\rho} + \bar{\rho}}{l_{jj}r_{jj} - m_{jj}n_{jj}} \qquad j = 1 \quad \text{or} \quad j = 3 \end{aligned} \tag{3.31}$$

The roots of Eq (3.30) are given by Eqs (3.10), (3.11), (3.15), (3.16) and (3.17) provided that the constants a_{22}, a_{21}, a_{04}, a_{03} are replaced by c_{22}, c_{21}, c_{04}, c_{03} given in Eqs (3.31) and (3.29).

3.2.2. Transverse waves in the direction of layering

For the shear horizontal waves in the direction of layering we have

$$\begin{aligned} &\{w_j, W_j\} = \{w_j^{\pm}, W_j^{\pm}\} \exp[\mathrm{i}(kx_n - \omega t)] \qquad j \neq n \quad j,n \in \{1,3\} \\ &\{w_2, W_2\} \equiv= \{0,0\} \end{aligned} \tag{3.32}$$

Substituting Eqs (3.32) into Eqs (2.5) and (2.6) we obtain the system of equations in the form given by Eqs (3.2) and (3.3), where

$$\begin{aligned} l_{jn} &= \widetilde{N} & L_{ij} &= l_{jn}\partial x_j^2 - \widetilde{\rho}\partial^2 t - \widetilde{b}\partial t \\ M_{jn} &= \widetilde{\rho}\partial t & N_{jn} &= \widetilde{b}\partial t \\ R_{ij} &= -\widetilde{\rho}\partial^2 t - \widetilde{b}\partial t & & j \neq n \quad j,n \in \{1,3\} \end{aligned} \tag{3.33}$$

By analogy to Case 3.1.2 the frequency equation takes the form

$$-k^2(\bar{\rho}\omega + \mathrm{i}\widetilde{b}) + d_{04}\omega^3 + \mathrm{i}d_{03}\omega^2 = 0 \tag{3.34}$$

where

$$d_{04} = \frac{\bar{\rho}\widetilde{\rho}}{\widetilde{N}} \qquad\qquad d_{03} = \frac{\widetilde{b}(\widetilde{\rho} + \bar{\rho})}{\widetilde{N}} \tag{3.35}$$

The roots of Eq (3.35) are $\pm k_3$, where k_3 is given in Eq (3.23) provided that the constants b_{04}, b_{03} are replaced by the constants d_{04}, d_{03}, respectively.

4. Final remarks

The present paper is devoted to the problems of plane harmonic waves propagating in the periodically stratifield fluid-saturated porous infinite solids. The exact solutions of the considered problems within the framework of the homogenized model with microlocal parameters have been obtained. The dispersion relations for harmonic waves propagating parallel and normal to the direction of layering have been presented. The wave number ranges at the extremes of the circular frequency has been discussed.

Assuming that the skeleton is homogeneous so

$$\Big(\rho^{(1)}, N^{(1)}, M^{(1)}, R^{(1)}, Q^{(1)}, b^{(1)}\Big) = \Big(\rho^{(2)}, N^{(2)}, M^{(2)}, R^{(2)}, Q^{(2)}, b^{(2)}\Big) \tag{4.1}$$

then Eqs (2.6) yield

$$q_1 = q_2 = q_3 = Q_2 \equiv 0 \tag{4.2}$$

Substituting Eq (4.2) into Eqs (2.5) and using Eq (4.1) we arrive at the equations of the theory of consolidation for the case of homogeneous elastic skeleton (cf Derski (1979); Derski and Kowalski (1978)). Assumptions (4.1) together with the obtained solutions given by Eqs (3.14), (3.24) as well as the frequency equations for all considered cases lead to the case of homogeneous skeleton.

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Propagacja fal płaskich w nasyconych cieczą porowatych ośrodkach warstwowych

Streszczenie

W pracy rozpatrzono zagadnienie propagacji płaskich fal harmonicznych w nasyconych cieczą porowatych ośrodkach warstwowych. Analiza została przeprowadzona wykorzystując liniową homogenizowaną teorię konsolidacji z parametrami mikrolokalnymi (Matysiak (1992)).

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