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INTERACTION OF ELASTIC WAVES WITH A FLUID-SATURATED POROUS SOLID BOUNDARY

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The problem of energy reflection and transmission of an oblique incident plane harmonic wave at the surface of an elastic porous solid halfspace immersed in fluid is considered. The case of pure elastic behaviour of the system with incompressible material of the skeleton is analysed. Considerations are based on the two parametric theory of fluid-saturated porous solid in which the isotropic pore structure is described by the volume porosity and the parameter characterising tortuosity of pores. The influence of mechanical properties of both the physical constituents and angle of wave incidence on the energy reflection and transmission coefficients is analysed. Two general cases of the wave interaction are investigated: waves incident from the bulk fluid on the surface of porous halfspace, and waves (fast, slow and shear) incident from a fluid-saturated porous solid. Calculations have been made for the system composed of porous fused glass bead solid filled with water in one case and with ethyl alcohol in the other case and diagrams of the results have been shown.

Key words: saturated porous solids, wave reflection

1. Introduction

Analysis of the interaction of small amplitude waves propagating in a fluid with a surface of deformable porous solid halfspace immersed in that fluid is of great importance in modelling of a wide class of practical problems. In spite

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of geometrical simplicity of the system and the linear description such phenomena are very complicated. It is connected with complexity of the mechanical behaviour of saturated porous materials mainly due to the strong dynamic coupling between the deformable skeleton and moving pore fluid motion where the skeleton pore structure plays a very important role. Additional difficulties appear when the viscous interactions between fluid and porous skeleton are taken into account. In such a medium the three different kinds of waves may propagate (Biot, 1956; Plona, 1980); two compressional waves (fast and slow) and one shear wave, each of which falling on a discontinuity surface of porous medium, in general, induces again all these three waves.

Most of papers devoted to the theoretical study of wave interaction with surface of fluid-saturated deformable porous material, due to the complexity of this problem, concern the special cases: normal incidence of wave at the contact surface of two saturated porous media (Deresewicz and Rice, 1964; Geerstema and Smit, 1961), wave interaction with the free surface of saturated porous halfspace (Deresewicz, 1960; Deresewicz and Rice, 1962), or interaction of an oblique incident wave from fluid on a surface of saturated porous material (Albert, 1993; Santos et al., 1992). More detailed analysis of energy reflection and transmission coefficients of elastic waves at the boundary surface between bulk fluid and fluid-saturated porous solid is given by Wu et al. (1990) where the influence of angle of incidence and surface boundary conditions on these coefficients are studied.

The present paper aims at analysis of the problem of reflection and transmission of an oblique incident plane harmonic wave at a surface of porous solid halfspace immersed in fluid. The case of pure elastic behaviour of the isotropic medium is considered. Special attention is paid for the energy reflection and transmission coefficients to be obtained for the waves incident both from bulk fluid and fluid-saturated porous solid on the surface of porous skeleton, as well as to discussion of the dependence of these coefficients on the angle of wave incidence and on the dynamical properties of fluid.

Considerations are based on the assumption that the motion of barotropic bulk fluid is described by the Euler equation, while the motion of fluid-saturated porous solid is represented by the equations of poro-elasticity theory of saturated deformable porous media (Cieszko and Kubik, 1993, 1996a,b; Kubik, 1986, 1992). Within the framework of this theory the isotropic skeleton pore structure is described by two macroparameters: volume porosity and a parameter characterizing tortousity of pores. The additional assumption is made that the skeleton material is incompressible, which reduces the number of material constants of the medium to the three quantities: velocities of compressional and shear waves propagating in the dry skeleton, and velocity of the wave propagation in the bulk fluid (without skeleton). This provides a model in which dynamical properties of fluid-porous solid composition are represented by the properties of individual components i.e. the fluid and porous skeleton considered as single materials.

Calculations of the energy reflection and transmission coefficients as functions of the angle of wave incidence have been made for both water- and ethyl alcohol-saturated porous glass bead solids and the results obtained were presented graphically.

2. Dynamics of fluid-saturated porous solid

The description of mechanical behaviour of an elastic porous medium filled with fluid we base on the continuum immiscible mixture theory (Cieszko and Kubik, 1993, 1996a,b; Kubik, 1986, 1992), in which the geometrical pore structure is characterised by the two macroparameters (Kubik, 1992); i.e., volume porosity f_v that represents the fluid volume fraction and parameter κ which is the measure of inhomogeneity of the fluid micro-velocity in its relative flow to the porous solid being the ratio of the fluid kinetic energy expressed in terms of an average velocity to the average fluid kinetic energy. In general, both these parameters are space and time dependent field quantities. Within the framework of linear theory changes of κ do not influence the balance equations of the porous solid-fluid system and its initial value κ_0 is related to the Biot coupling parameter ρ_{12} (Biot, 1956), and the tortuosity parameter α_T used by Johnson (1986)

$$\frac{1}{\kappa_o} = 1 - \frac{\rho_{12}}{\overline{\rho}_o^f} = \alpha_T$$

where $\overline{\rho}_o^f$ is the fluid partial mass density in the reference configuration.

The equations of this theory concern both the physical components; fluid and porous solid, and the so-called virtual components of the porous medium. The first virtual component $(\cdot)^{(1)}$ is formed by the skeleton and the fluid associated with it and moving at the skeleton velocity $\boldsymbol{v}^{(1)}$ and the other $(\cdot)^{(2)}$ is the free fluid moving at the velocity $\boldsymbol{v}^{(2)}$. The partial densities $\rho^{(1)}$, $\rho^{(2)}$ and the partial Cauchy stresses $\mathbf{T}^{(1)}$, $\mathbf{T}^{(2)}$ of virtual constituents are related to the partial densities $\overline{\rho}^s$, $\overline{\rho}^f$ and the partial Cauchy stresses $\mathbf{T}^s$, $\mathbf{T}^f$

of the physical constituents by the following equations (Kubik, 1986)

$$\rho^{(1)} = \overline{\rho}^{s} + (1-\kappa)\overline{\rho}^{f} \qquad \rho^{(2)} = \kappa\overline{\rho}^{f}$$
$$\mathsf{T}^{(1)} = \mathsf{T}^{s} + (1-\kappa)\mathsf{T}^{f} \qquad \mathsf{T}^{(2)} = \kappa\mathsf{T}^{f}$$

where

$$\overline{\rho}^{f} = f_v\rho^{f} \qquad \overline{\rho}^{s} = (1-f_v)\rho^{s}$$

and ρ^f, ρ^s stand for the effective density of fluid and porous skeleton, respectively.

Such a description includes the dynamical coupling between the pore fluid and porous skeleton (added mass effect) and the linear equations of this theory are equivalent (see Cieszko and Kubik, 1996; Kubik, 1992), to those of Biot (1956).

For porous materials with average and greater values of the volume porosity it is observed that small elastic deformations of a porous skeleton take place mainly at the cost of change in the pore volume. Therefore, we assume that the material of the skeleton is incompressible. It is a kinematical constraint imposed on the motion of the porous skeleton due to the constant effective mass density ρ^s of the skeleton during the deformation process, i.e.

$$\rho^{s} = \rho^{s}_{o}$$

where ρ^s_o is the effective mass density of the skeleton in the initial state of porous medium (reference configuration). This assumption provides a model in which all characteristic properties of the dynamics of fluid-porous solid composition are preserved and represented by the properties of individual components i.e. the fluid and porous skeleton treated as separate materials.

For the inviscid pore fluid and incompressible material of the skeleton the linear equations of this theory take following forms:

— continuity equations

$$\begin{aligned} -\frac{\partial f_v}{\partial t} + (1-f^o_v)\operatorname{div}\boldsymbol{v}^{(1)} &= 0 \\ f^o_v\frac{\partial\tilde{p}^f}{\partial t} + (1-\kappa_o f^o_v)\operatorname{div}\boldsymbol{v}^{(1)} + \kappa_o f^o_v\operatorname{div}\boldsymbol{v}^{(2)} &= 0 \end{aligned} \tag{2.1}$$

— equations of motion

$$\begin{aligned} \rho^{(1)}_o\frac{\partial\boldsymbol{v}^{(1)}}{\partial t} &= (1-f^o_v)\operatorname{div}(\mathsf{T}^{*s} + p^f\mathsf{I}) - (1-\kappa_o f^o_v)p^f_o\operatorname{grad}\tilde{p}^f \\ \rho^{(2)}_o\frac{\partial\boldsymbol{v}^{(2)}}{\partial t} &= -\kappa_o f^o_v p^f_o\operatorname{grad}\tilde{p}^f \end{aligned} \tag{2.2}$$

where

$$\tilde{\rho}^f = \frac{\rho^f - \rho_o^f}{\rho_o^f} \qquad\qquad \tilde{p}^f = \frac{p^f - p_o^f}{p_o^f}$$

and $\mathbf{T}^{*s}$ is the effective stress tensor in the porous skeleton related to the skeleton partial stress tensor $\mathbf{T}^s$ by

$$\mathbf{T}^s = \mathbf{T}^{*s}(1 - f_v)$$

This quantity, in the reference configuration, takes the value

$$\mathbf{T}^{*s} = -p_o^f \mathbf{I}$$

where p_o^f is the fluid pore pressure in this configuration. The quantities $\rho_o^{(1)}$, $\rho_o^{(2)}$ and ρ_o^s, ρ_o^f stand for the partial mass densities in the initial state of the medium.

Taking the incompressibility assumption of the skeleton material into account the constitutive relations for increments of the effective stresses are (cf Cieszko and Kubik, 1996a)

$$\begin{aligned} \Delta\mathbf{T}^{*s} + \Delta p^f \mathbf{I} &= 2\mu^* \mathbf{E} + \lambda^* \operatorname{tr}(\mathbf{E})\mathbf{I} \\ \Delta p^f &= a_o^2 \Delta\rho^f \end{aligned} \tag{2.3}$$

where

$$\Delta\mathbf{T}^{*s} = \mathbf{T}^{*s} - \mathbf{T}_o^{*s} = \mathbf{T}^{*s} + p_o^f \mathbf{I}$$

$$\Delta p^f = p^f - p_o^f \qquad\qquad \Delta\rho^f = \rho^f - \rho_o^f$$

and $\mathbf{E}$ is the infinitesimal strain tensor and a_o is the velocity of wave propagation in the bulk fluid.

The effective material constants μ^*, λ^* of the porous skeleton are related to the partial constants μ, λ (corresponding to Lame coefficients for nonporous medium), by

$$\lambda = (1 - f_v^o)\lambda^* \qquad\qquad \mu = (1 - f_v^o)\mu^*$$

The system of equations (2.1) ÷ (2.3) supplemented with the geometrical relations

$$\mathbf{E} = \frac{1}{2}\Big[\operatorname{grad}\boldsymbol{u}^{(1)} + (\operatorname{grad}\boldsymbol{u}^{(1)})^{\top}\Big] \tag{2.4}$$

reduces to the following two coupled dynamic equations representing the displacement fields of the skeleton and free fluid, respectively

$$
\begin{aligned}
\rho_o^{(1)}\frac{\partial^2 \boldsymbol{u}^{(1)}}{\partial t^2} &= \overline{\rho}_o^s V_{SH}^2\nabla^2\boldsymbol{u}^{(1)} + \Big[\overline{\rho}_o^s(V_{SL}^2 - V_{SH}^2) + \\
&+\rho_o^{(2)}c_o^2\Big(\frac{1-\kappa_o f_v^o}{\kappa_o f_v^o}\Big)^2\Big]\operatorname{grad}\operatorname{div}\boldsymbol{u}^{(1)} + \rho_o^{(2)}c_o^2\frac{1-\kappa_o f_v^o}{\kappa_o f_v^o}\operatorname{grad}\operatorname{div}\boldsymbol{u}^{(2)} \\
\rho_o^{(2)}\frac{\partial^2 \boldsymbol{u}^{(2)}}{\partial t^2} &= \rho_o^{(2)}c_o^2\frac{1-\kappa_o f_v^o}{\kappa_o f_v^o}\operatorname{grad}\operatorname{div}\boldsymbol{u}^{(1)} + \rho_o^{(2)}c_o^2\operatorname{grad}\operatorname{div}\boldsymbol{u}^{(2)}
\end{aligned}
\tag{2.5}
$$

where

$$
V_{SL} = \sqrt{\frac{2\mu+\lambda}{\overline{\rho}_o^s}} \qquad\qquad V_{SH} = \sqrt{\frac{\mu}{\overline{\rho}_o^s}}
$$

are the velocities of compressional and shear waves, respectively, in the dry skeleton, and

$$
c_o = a_o\sqrt{\kappa_o}
$$

is the velocity of wave propagation in an inviscid fluid filling rigid porous skeleton (Cieszko, 1992). The vectors $\boldsymbol{u}^{(1)}$ and $\boldsymbol{u}^{(2)}$ stand for the displacements of the skeleton and free fluid, respectively.

Eqs (2.5) describe wave propagation in the elastic porous skeleton, material of which is incompressible and saturated with a barotropic fluid. Form Eqs (2.5) it follows that the motion of porous skeleton and the fluid filling its pores is strongly coupled, and this fact is directly connected with the skeleton pore structure.

2.1. Waves in fluid-saturated porous medium

We use Eqs (2.5) of dynamics of fluid-saturated porous solid to derive the equations describing propagation of compressional and shear waves in such a medium.

Applying the divergence and rotation operators to Eqs (2.5) we obtain the equations for compressional waves that can be written in the matrix form

$$
\frac{\partial^2 \boldsymbol{\varepsilon}}{\partial t^2} = \nabla^2(\boldsymbol{\Psi\varepsilon}) \tag{2.6}
$$

and the equations for the shear waves

$$
\frac{\partial^2 \boldsymbol{\omega}^{(1)}}{\partial t^2} = V_V^2\nabla^2\boldsymbol{\omega}^{(1)} \qquad\qquad \frac{\partial^2 \boldsymbol{\omega}^{(2)}}{\partial t^2} = 0 \tag{2.7}
$$

where the vector $\boldsymbol{\varepsilon}$ is

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon^{(1)} \\ \varepsilon^{(2)} \end{bmatrix} \tag{2.8}$$

and

$$\varepsilon^{(k)} = \operatorname{div} \boldsymbol{u}^{(k)} \qquad \boldsymbol{\omega}^{(k)} = \operatorname{curl} \boldsymbol{u}^{(k)} \qquad k = 1,2$$

whereas $\boldsymbol{\Psi}$ is

$$\boldsymbol{\Psi} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix}$$

of the following elements

$$\begin{aligned} \Psi_{11} &= \frac{\overline{\rho}_o^s}{\rho_o^{(1)}} V_{SL}^2 + \Big(\frac{1-\kappa_o f_v^o}{\kappa_o f_v^o}\Big)^2 \frac{\rho_o^{(2)}}{\rho_o^{(1)}} c_o^2 & \qquad \Psi_{21} &= \frac{1-\kappa_o f_v^o}{\kappa_o f_v^o} c_o^2 \\ \Psi_{12} &= \frac{1-\kappa_o f_v^o}{\kappa_o f_v^o} \frac{\rho_o^{(2)}}{\rho_o^{(1)}} c_o^2 & \qquad \Psi_{22} &= c_o^2 \end{aligned} \tag{2.9}$$

The quantity V_V in Eq $(2.7)_1$, given by the relation

$$V_V = \sqrt{\frac{\mu}{\rho_o^{(1)}}} = V_{SH} \sqrt{\frac{\overline{\rho}_o^s}{\rho_o^{(1)}}} \tag{2.10}$$

is the velocity of shear wave propagation in porous skeleton filled with fluid. From Eqs (2.7) it is seen that in such a medium only one shear wave propagates.

To state the conditions for propagation of compressional waves we analyse Eq (2.6) for the dilatation vector $\boldsymbol{\varepsilon}$. Taking into account that $\partial^2/\partial t^2$ and ∇^2 are isotropic operators and do not change the direction of vector $\boldsymbol{\varepsilon}$, from Eq (2.6) we conclude that the vector $\boldsymbol{\varepsilon}$ can satisfy Eq (2.6) only in the case when it is collinear with the vector $\boldsymbol{\Psi\varepsilon}$. Thus, we obtain the algebraic equation for the eigenvalues of matrix $\boldsymbol{\Psi}$

$$\boldsymbol{\Psi\varepsilon} = V^2 \boldsymbol{\varepsilon} \tag{2.11}$$

where V^2 is a scalar coefficient that is the square of velocity of compressional wave propagating in porous medium filled with fluid.

The nonzero solution of Eq (2.11) requires the condition

$$\det(\boldsymbol{\Psi} - V^2 \mathsf{I}) = 0 \tag{2.12}$$

to be satisfied. This yields the equation for the velocity V in the form

$$V^4 + (\Psi_{11} + \Psi_{22})V^2 + \Psi_{11}\Psi_{22} - \Psi_{12}\Psi_{21} = 0$$

which has the two real solutions

$$V_I^2 = \frac{1}{2}(\Psi_{11} + \Psi_{22} + \sqrt{\Delta}) \qquad V_{II}^2 = \frac{1}{2}(\Psi_{11} + \Psi_{22} - \sqrt{\Delta}) \tag{2.13}$$

where

$$\Delta = (\Psi_{11} + \Psi_{22})^2 - 4(\Psi_{11}\Psi_{22} - \Psi_{12}\Psi_{21}) = (\Psi_{11} - \Psi_{22})^2 + 2\Psi_{12}\Psi_{21} > 0$$

The above results show that, in general, in a deformable porous solid filled with fluid two compressional waves propagate and the system of Eq (2.6) is purely hyperbolic.

The existence of two compressional waves in saturated porous medium was theoretically discovered by Biot (1956) and experimentally proved by Plona (1980). The wave propagating at the greater velocity defined by Eq $(2.13)_1$ is called the fast wave (wave of the first kind), and the wave propagating at the lower velocity defined by Eq $(2.13)_2$ is called the slow wave (wave of the second kind). These two velocities (eigenvalues of $\mathbf{\Psi}$) correspond the two vectors of dilatations $\boldsymbol{\varepsilon}_I$, $\boldsymbol{\varepsilon}_{II}$ (eigenvectors of $\mathbf{\Psi}$) given by the equation

$$\mathbf{\Psi}\boldsymbol{\varepsilon}_\alpha = V_\alpha^2 \boldsymbol{\varepsilon}_\alpha \qquad \alpha = I, II \tag{2.14}$$

The components of vectors $\boldsymbol{\varepsilon}_I$ and $\boldsymbol{\varepsilon}_{II}$ represent the values of dilatations of the skeleton and free fluid in the fast and the slow waves, respectively.

From Eq (2.14) we obtain

$$\varepsilon_\alpha^{(2)} = \Psi_\alpha \varepsilon_\alpha^{(1)} \qquad \alpha = I, II \tag{2.15}$$

where

$$\Psi_\alpha = \frac{V_\alpha^2 - \Psi_{11}}{\Psi_{12}}$$

Taking into account Eqs (2.14) and (2.15) we find out that the matrix equation (2.6) for the compressional waves can be replaced by the two equations

$$\frac{\partial^2 \varepsilon_\alpha^{(1)}}{\partial t^2} = V_\alpha^2 \nabla^2 \varepsilon_\alpha^{(1)} \qquad \alpha = I, II \tag{2.16}$$

referred to the dilatations of the porous skeleton in the fast and slow waves propagating in such a medium. Then dilatations of the free fluid in both waves are given by Eq (2.15).

Finally, due to linearity of the considered equations, the general solution of Eq (2.6) can be written as a sum of the solutions of both Eqs (2.16). For dilatations of the skeleton and free fluid we have

$$\varepsilon^{(1)} = \varepsilon_I^{(1)} + \varepsilon_{II}^{(1)} \qquad \varepsilon^{(2)} = \Psi_I \varepsilon_I^{(1)} + \Psi_{II} \varepsilon_{II}^{(1)} \tag{2.17}$$

Eqs $(2.7)_1$ and (2.16) representing the special case of Eqs (2.5) form the set of three independent equations describing propagation of the shear, fast and slow waves in the saturated porous medium. This shows, that in the infinite porous medium each of these waves propagate independently of the other waves. However, in general, any mechanical excitation of such a medium, e.g. by a wave incident on the boundary of saturated porous medium, will produce all these three waves.

3. Interaction of acoustic waves propagating in fluid with the boundary of saturated porous halfspace

3.1. Formulation of the problem

We analyze the problem of reflection and transmission of a plane harmonic oblique wave incident both from the bulk fluid and the saturated porous solid on the Γ-surface being the boundary of porous halfspace (Fig.1). The considerations are limited to the case of purely elastic interactions. It is assumed that the fluid is barotropic, and the porous skeleton has the isotropic pore structure and the isotropic elastic properties, and its material is incompressible. Moreover, the displacement amplitudes of fluid and skeleton particles in the waves are assumed to be considerably shorter then the wave length. It enables one to use the linear description of the problem and to avoid the necessity for imposing the compatibility conditions on the movable boundary of deformable halfspace.

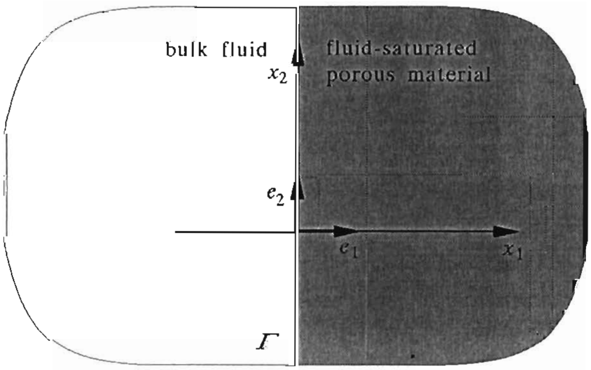

Fig. 1. Geometry of the problem

In such a case the wave propagation in a saturated porous halfspace

($x_1 > 0$) is described by the equations presented in Section 2 and the wave propagation in the bulk fluid ($x_1 < 0$) is given by the equation

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = a_o^2 \operatorname{grad} \operatorname{div} \boldsymbol{u} \tag{3.1}$$

where $\boldsymbol{u}$ is the displacement vector of the bulk fluid particles.

The acoustic fields in both regions ($x_1 > 0$, $x_1 < 0$) are coupled by means of the compatibility conditions of mechanical fields on their contact surface Γ ($x_1 = 0$). These conditions are (Cieszko and Kubik, 1993): continuity of the fluid mass flux; continuity of the effective fluid pressure and continuity of the normal component of the total stress vector. Also, due to the assumed fluid inviscidity, the skeleton tangential stresses disappear.

For the small disturbances of the medium the compatibility conditions written for displacement fields of components in both regions take the form

$$\begin{aligned}
&(\boldsymbol{u} - \boldsymbol{u}^{(1)})\Big|_\Gamma \cdot \boldsymbol{e}_1 = \kappa_o f_v^o (\boldsymbol{u}^{(2)} - \boldsymbol{u}^{(1)})\Big|_\Gamma \cdot \boldsymbol{e}_1 \\
&-f_v^o \operatorname{div} \boldsymbol{u}\Big|_\Gamma + (1 - \kappa_o f_v^o) \operatorname{div} \boldsymbol{u}^{(1)})\Big|_\Gamma + \kappa_o f_v^o \operatorname{div} \boldsymbol{u}^{(2)}\Big|_\Gamma = 0 \\
&\lambda \operatorname{div} \boldsymbol{u}^{(1)}\Big|_\Gamma + 2\mu \boldsymbol{e}_1 \cdot \operatorname{grad} (\boldsymbol{u}^{(1)} \cdot \boldsymbol{e}_1)\Big|_\Gamma = 0 \\
&\boldsymbol{e}_1 \cdot \operatorname{grad} (\boldsymbol{u}^{(1)} \cdot \boldsymbol{\tau}) + \boldsymbol{\tau} \cdot \operatorname{grad} (\boldsymbol{u}^{(1)} \cdot \boldsymbol{e}_1)\Big|_\Gamma = 0
\end{aligned} \tag{3.2}$$

where $\boldsymbol{\tau}$ is an arbitrary unit vector tangential to the Γ-surface.

Eqs $(2.7)_1$, (2.16) and (3.1) together with the compatibility conditions (3.2) fully describe the interaction between the acoustic fields in the halfspaces of bulk fluid and the saturated porous medium.

3.2. Solution to the problem

The acoustic fields induced by the wave incident from the bulk fluid on the boundary Γ of the porous medium is composed of the incident and reflected waves in the halfspace $x_1 < 0$ (Fig.2) and, according to the considerations presented in Section 2, is composed of three waves; two compressional ones (fast and slow) and one shear wave, in the halfspace $x_1 > 0$. We assume that the plane of wave incidence spanned over the normal versor of Γ-surface and the direction of incident wave agrees with the plane of Fig.2.

In order to solve the problem of wave interaction with the boundary of saturated porous solid we introduce the displacement potentials for all waves appearing in the system, i.e. ϕ_A, ϕ_D for the potentials of incident and reflected

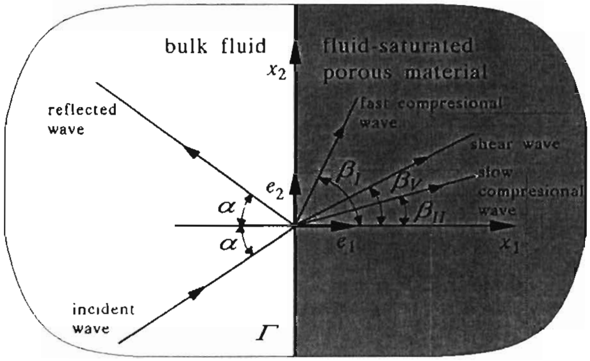

Fig. 2. Geometry of waves induced by a compressional wave incident from fluid

waves and ϕ_I, ϕ_{II}, $\boldsymbol{\psi}$ for the displacement potentials of porous skeleton in fast, slow and shear waves, respectively. Then the acoustic fields in the bulk fluid and the porous skeleton are given by

$$\boldsymbol{u} = \operatorname{grad}\phi_A + \operatorname{grad}\phi_D \qquad\qquad \boldsymbol{u}^{(1)} = \operatorname{grad}\phi_I + \operatorname{grad}\phi_{II} + \operatorname{rot}\boldsymbol{\psi} \tag{3.3}$$

for

$$\operatorname{div}\boldsymbol{\psi} = 0 \tag{3.4}$$

In the free fluid, by virtue of Eqs (2.15) and $(2.17)_2$ we have

$$\boldsymbol{u}^{(2)} = \Psi_I \operatorname{grad}\phi_I + \Psi_{II}\operatorname{grad}\phi_{II} \tag{3.5}$$

The potentials appearing in Eqs (3.3) ÷ (3.5) have to satisfy the suitable equations of wave propagation $(2.7)_1$, (2.16) and (3.1) written for the displacement potentials.

For the harmonic waves of frequency f $(\omega = 2\pi f)$ we obtain

$$\begin{aligned}
&\phi_A = \operatorname{Re}\Big(\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{-2\pi\mathrm{i}\boldsymbol{k}^o\cdot\boldsymbol{x}}\Big) && \phi_D = \operatorname{Re}\Big(D\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{-2\pi\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\Big)\\
&\phi_\alpha = \operatorname{Re}\Big(R_\alpha\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{-2\pi\mathrm{i}\boldsymbol{k}^\alpha\cdot\boldsymbol{x}}\Big) && \alpha = I, II\\
&\boldsymbol{\psi} = \operatorname{Re}\Big(R_V\boldsymbol{m}\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{-2\pi\mathrm{i}\boldsymbol{k}^V\cdot\boldsymbol{x}}\Big)
\end{aligned} \tag{3.6}$$

where D, R_α, R_V are the amplitudes of particular waves. The amplitude of incident wave, without any loss of generality, can be assumed to be equal to unity.

The wave vectors $\boldsymbol{k}^o$, $\boldsymbol{k}$, $\boldsymbol{k}^\alpha$, $\boldsymbol{k}^V$ are related to the wave numbers

$$k = \frac{f}{a_o} \qquad k^\alpha = \frac{f}{V_\alpha} \qquad k^V = \frac{f}{V_V} \tag{3.7}$$

and the direction versors $\boldsymbol{\mu}_o$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}_\alpha$, $\boldsymbol{\nu}_V$ of waves propagation by

$$\boldsymbol{k}^o = k\boldsymbol{\mu}_o \qquad \boldsymbol{k} = k\boldsymbol{\mu} \qquad \boldsymbol{k}^\alpha = k^\alpha \boldsymbol{\nu}_\alpha \qquad \boldsymbol{k}^V = k^V \boldsymbol{\nu}_V \tag{3.8}$$

for $\alpha = I, II$.

The unit vector $\boldsymbol{m}$ in Eq $(3.6)_4$, due to the condition (3.4), is normal to the direction vector $\boldsymbol{\nu}_V$ of the shear wave propagation, and the vector product $\boldsymbol{m} \times \boldsymbol{\nu}_V$ defines the direction of displacement of the skeleton particles in the shear wave.

Substitution of Eq (3.6) into Eqs (3.3), (3.5) and next into Eqs (3.2) leads to the following set of equations for waves amplitudes

$$\begin{aligned}
&a_I K^I R_I + a_{II} K^{II} R_{II} + (1-\kappa_o) f_v^o R_V \sin\alpha - D\cos\alpha = -\cos\alpha \\
&\frac{a_I}{\overline{V}_I^2} R_I + \frac{a_{II}}{\overline{V}_{II}^2} R_{II} - f_v^o D = f_v^o \\
&\frac{b_I}{\overline{V}_I^2} R_I + \frac{b_{II}}{\overline{V}_{II}^2} R_{II} - 2K^V R_V \sin\alpha = 0 \\
&2K^I R_I \sin\alpha + 2K^{II} R_{II}\sin\alpha - \Big(\frac{1}{\overline{V}_V^2} - 2\sin^2\alpha\Big) R_V = 0
\end{aligned} \tag{3.9}$$

where

$$a_X = 1 + \kappa_o f_v^o (\Psi_X - 1) \qquad b_X = \Big(\frac{\overline{V}_{SL}}{\overline{V}_{SH}}\Big)^2 - 2\overline{V}_X \sin^2\alpha \qquad X = I, II$$

and

$$K^X = -\frac{1}{\overline{V}_X}\sqrt{1 - \overline{V}_X^2 \sin^2\alpha} \qquad X = I, II, V \qquad \overline{(\cdot)} = \frac{(\cdot)}{a_o}$$

Additionally, we obtain that the directions of all waves induced by the wave incident on the Γ-surface lay in the plane of wave incidence and the direction angles of the fast, slow and shear waves in that plane are given by Snell's low

$$\frac{\sin\alpha}{a_o} = \frac{\sin\beta_I}{V_I} = \frac{\sin\beta_{II}}{V_{II}} = \frac{\sin\beta_V}{V_V} \tag{3.10}$$

Also, the displacement vectors of the skeleton particles in the shear wave lay in the plane of wave incidence. Therefore

$$\boldsymbol{m} = \boldsymbol{e}_3$$

The solution of Eq (3.9) determines the amplitude reflection and transmission coefficients for the wave incident from the bulk fluid. In order to calculate these coefficients for energy we have to derive expressions for the average energy carried by particular waves propagating in the considered system.

Generally, the flux of mechanical energy in any process in a fluid-saturated porous medium is given by the Poynting vector that for description used in this paper takes form

$$\boldsymbol{P} = -\mathbf{T}^{(1)}\boldsymbol{v}^{(1)} - \mathbf{T}^{(2)}\boldsymbol{v}^{(2)} \tag{3.11}$$

Similarly, the Poynting vector for the bulk fluid is

$$\boldsymbol{P}^f = -\mathbf{T}\boldsymbol{v} = p^f\boldsymbol{v} \tag{3.12}$$

Representing these two vectors in the quadratic form of displacements and velocities and applying the displacement potentials (3.6) the Poynting vectors (3.11) and (3.12) can be specified for the waves propagating in the saturated porous medium and the bulk fluid, respectively. Applying the time averaging over the wave period we obtain

$$\begin{aligned} \tilde{\boldsymbol{P}}_X &= I_X \boldsymbol{\nu}_X \qquad &\text{for} \quad X = I, II, V \\ \tilde{\boldsymbol{P}}^f_Y &= I_Y \boldsymbol{\mu}_Y \qquad &\text{for} \quad Y = A, D \end{aligned} \tag{3.13}$$

where

$$\begin{aligned} I_X &= \frac{\omega^4}{2V_X^3}|R_X|^2\Big(2\mu + \lambda + \frac{\rho_o^f a_o^2}{f_v^o}[1 + \kappa_o f_v^o(\Psi_X - 1)]\Big)^2 \qquad X = I, II \\ I_V &= \mu\frac{\omega^4}{2V_V^3}|R_V|^2 \qquad I_A = \rho_o^f\frac{\omega^4}{2a_o} \qquad I_D = \rho_o^f\frac{\omega^4}{2a_o}|D|^2 \end{aligned} \tag{3.14}$$

are the intensities of the fast, slow, shear, incident and reflected waves, respectively. Taking Eq (3.13) into account the energy reflection and transmission coefficients are

$$\gamma_D = \frac{I_D}{I_A}|D|^2 \qquad \gamma_Y = \frac{I_Y}{I_A}\frac{\cos\beta_Y}{\cos\alpha} \qquad Y = I, II, V \tag{3.15}$$

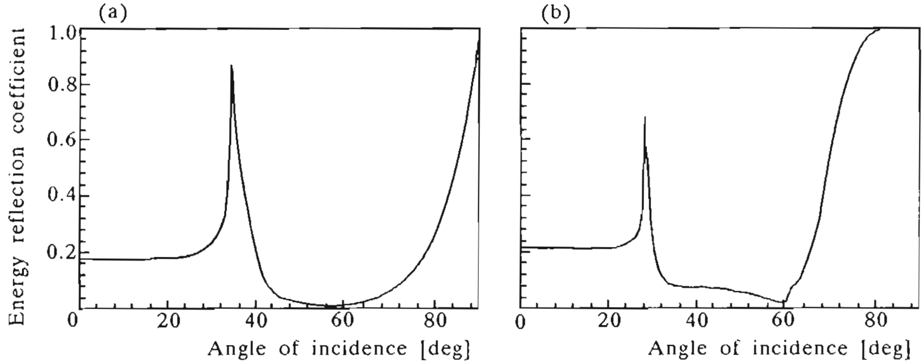

Fig. 3. Energy reflection coefficient of a compressional wave: (a) – incident from water on the interface between water and water-saturated porous fused glass bead solid, (b) – incident from ethyl alcohol on the interface between ethyl alcohol and ethyl alcohol-saturated porous fused glass bead solid

They satisfy the equation

$$\gamma_I + \gamma_{II} + \gamma_V + \gamma_D = 1$$

resulting from the wave energy conservation.

Numerical results for the energy reflection γ_D and transmission coefficients γ_Y, $(Y = I, II, V)$ for the wave incident from a fluid on the surface of saturated porous solid are shown in Fig.3 and Fig.4. The calculations have been made for the porous material made of fused glass bead solid of the physical parameters assumed after Wu et al. (1990), i.e. $\rho_o^s = 2500\,\mathrm{kg/m^3}$, $V_L = 2250\,\mathrm{m/s}$, $V_{SH} = 1350\,\mathrm{m/s}$, $f_v^o = 0.38$, $\kappa_o = 1/1.8$. The diagrams shown in Fig.3a and Fig.4a refer to the porous medium saturated with water $(\rho_o^f = 1000\,\mathrm{kg/m^3}$, $a_o^w = 1500\,\mathrm{m/s})$ while those in Fig.3b and Fig.4b concern the porous material saturated with ethyl alcohol $(\rho_o^f = 790\,\mathrm{kg/m^3}$, $a_o^w = 1130\,\mathrm{m/s})$. For the water-saturated porous medium velocities of the fast and slow compressional waves and the shear wave are: $V_I^w = 2660\,\mathrm{m/s}$, $V_{II}^w = 900\,\mathrm{m/s}$, $V_V^w = 1280\,\mathrm{m/s}$, and these velocities in the porous medium filled with alcohol take the values: $V_I^a = 2400\,\mathrm{m/s}$, $V_{II}^a = 750\,\mathrm{m/s}$, $V_V^a = 1300\,\mathrm{m/s}$. Since $V_I^w > a_o^w$ and $V_V^w < a_o^w$, from Snell's law (3.10) it results that there is only one critical angle $\alpha_k^I = \arcsin(a_o^w/V_I^w) = 34°$ at which the normal component of the fast wave in the water-saturated porous medium disappears. On the other hand, in the porous solid filled with alcohol both the velocities V_I^a and V_V^a are greater then the velocity a_o^a in the bulk alcohol, therefore there are the two critical angles: $\alpha_k^I = \arcsin(a_o^a/V_I^a) = 28°$ and $\alpha_k^V = \arcsin(a_o^a/V_V^a) = 60°$ at which the normal components of the fast and shear waves disappear. Fig.3

and Fig.4 prove that for angles of incidence close to their critical values the energy reflection and transmission coefficients change rapidly.

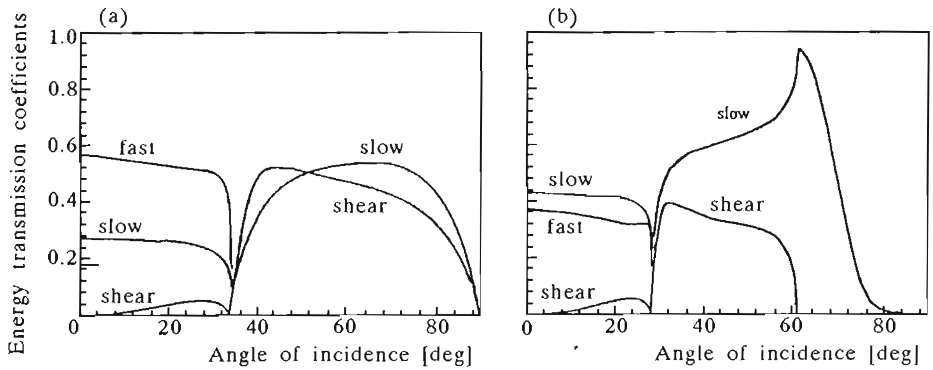

Fig. 4. Energy transmission coefficients of a compressional wave: (a) – incident from water on the interface between water and water-saturated porous fused glass bead solid, (b) – incident from ethyl alcohol on the interface between ethyl alcohol and ethyl alcohol-saturated porous fused glass bead solid

Comparing graphs in Fig.3 and in Fig.4 it is seen that the change of dynamical properties of the fluid in the considered system significantly influences the distribution of energy transmitted through the Γ-surface. For subcritical angles of incidence ($\alpha < 28°$) the replacement of water with ethyl alcohol does almost not influence the energy carried by the reflected and shear waves. This replacement, however, increases considerably the amount of energy carried by the slow wave what occurs at expanse of the energy for the fast wave. Similarly, for intercritical angles of incidence ($34° < \alpha < 60°$) the energies of reflected waves in both cases are of the same, very small amount. The energy carried by the slow wave in ethyl alcohol-saturated porous material is greater then that in a porous material filled with water and increases with the increase in the angle of incidence but this time it takes place at expanse of energy of the shear wave.

The existence of critical angle for the shear wave in the alcohol-saturated porous medium causes that for angles of incidence greater than this critical angle the whole energy of incident wave is transmitted by the reflected and slow wave only and with the increase in the angle of incidence the energy of slow wave rapidly decreases. For angles $\alpha > 80°$ the incident wave is almost completely reflected from the surface of alcohol-saturated porous solid.

The results obtained for the energy reflection and transmission coefficients for the wave incident from water on the surface of porous fused glass bead solid are in good agreement with those obtained by Wu et al. (1990). The same concerns the energy reflection and transmission coefficients for fast and slow compressional waves and the shear wave in water-saturated porous material, not presented here. Therefore, hereinafter, we restrict our considerations to the analysis of such coefficients for alcohol-saturated porous fused glass bead solid only.

4. Interaction of acoustic waves incident from the saturated porous halfspace

In the case when the fast, slow or shear wave propagating in porous fused glass bead solid saturated with alcohol is incident on its contact surface with the bulk alcohol three reflected and one transmitted waves will be generated in each case. The procedure of solution to these problems is analogical to that presented above for the wave incident from fluid. Changes in the acoustical fields in both halfspaces of the system should be only taken into account. Here instead of the solution procedure details we will present the results only.

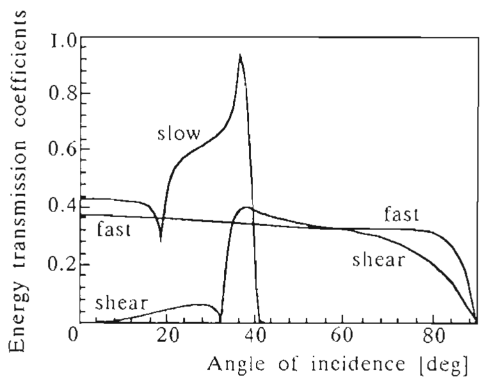

Fig. 5. Energy transmission coefficients of fast, slow and shear waves incident from ethyl alcohol-saturated porous fused glass bead solid on its contact surface with bulk ethyl alcohol

Fig.5 shows the energy transmission coefficients for fast, slow and shear waves incident from porous fused glass bead solid saturated with alcohol on the surface Γ for different values of the angle of wave incidence. Comparing the transmission coefficients in Fig.5 with those in Fig.4b it is seen that the curves for particular waves have the same character. It is easy to find, that for the angles related by Snell's law (3.10) the suitable curves in both figures coincide. It means that the energy transmission coefficients for wave incident from bulk fluid on the Γ-surface and characterizing the energies transmitted by the fast, slow and shear waves, are equal to the transmission coefficients for the waves propagating in the opposite direction. Physical explanation of this statement does need any further theoretical investigations.

As a consequence of the noticed equalities we find that the transmission coefficients γ_X^L $(X = I, II, V)$ of the impulse waves through a layer of porous material immersed in fluid are the squares of transmission coefficients for the wave incident from bulk fluid on the surface of porous material presented in Fig.4b.

$$\gamma_X^L = (\gamma_X)^2 \qquad X = I, II, V \tag{4.1}$$

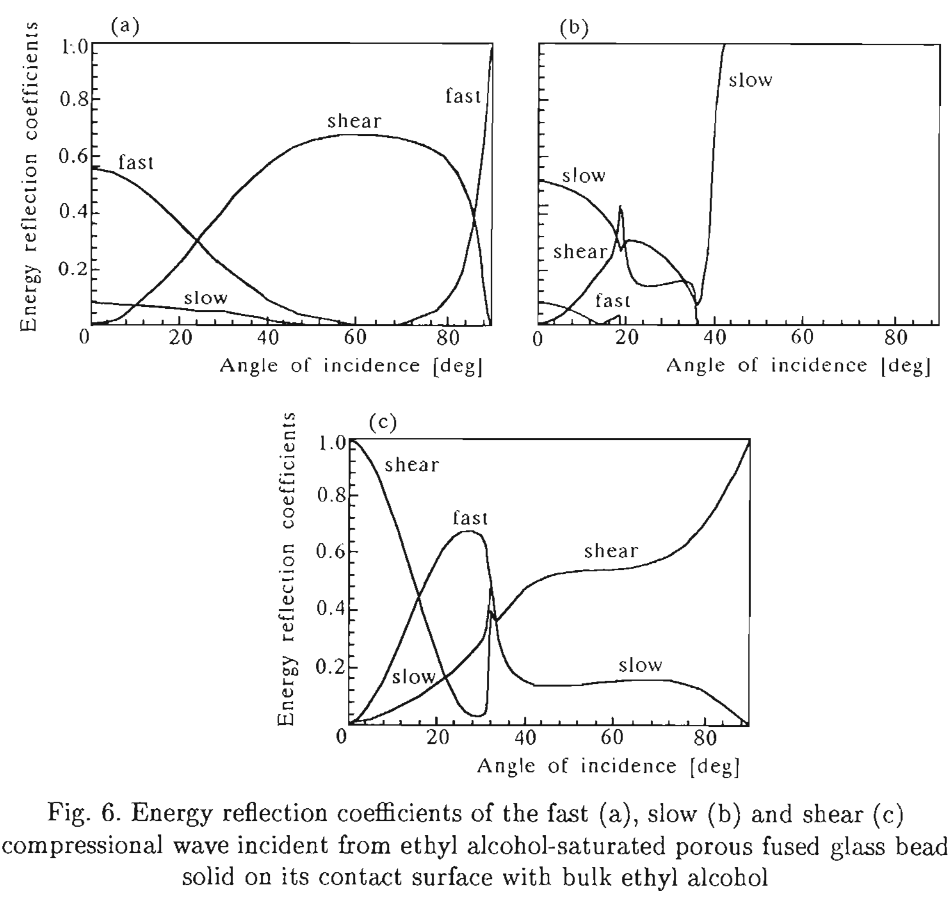

Fig. 6. Energy reflection coefficients of the fast (a), slow (b) and shear (c) compressional wave incident from ethyl alcohol-saturated porous fused glass bead solid on its contact surface with bulk ethyl alcohol

Fig.6 shows the energy reflection coefficients for a fast, slow and shear waves, respectively, incident from a porous material saturated with alcohol on the Γ-surface.

For the fast compressional wave (Fig.6a) there is no critical angles of incidence since the fast wave velocity V_I^a is the greatest velocity appearing in the system. Therefore, for each angle of incidence, the fast wave induces all three reflected waves and the energy reflection coefficients form smooth curves. It is seen that the fast wave induces a very weak slow reflected wave.

For the case of slow compressional wave (Fig.6b) there are three critical angles since the slow wave velocity V_{II}^a is the smallest velocity appearing in the system. The first critical angle is connected with deccy of the fast reflected wave, second – with deccy of the shear wave, and third – with deccy of the transmitted wave. For values greater than the third critical angle total energy of the slow wave reflects from the Γ-surface and further is carried by the slow wave only. It is seen in Fig.6b that slow wave induces very weak reflected fast wave.

For the energy reflection coefficients of shear wave, shown in Fig.6c, the only one critical angle exists, i.e., for the reflected fast compressional wave.

5. Final remarks

In the paper the problem of energy reflection and transmission of an oblique incident plane harmonic wave at the contact surface between the fluid-saturated porous solid and the bulk fluid has been considered. Discussion has been limited to the pure elastic behavior of the system. The assumption of incompressibility of the skeleton material does provide a model in which all characteristic properties of dynamics of fluid-porous solid composition are preserved and represented by the properties of fluid and porous skeleton treated as single materials.

It is shown that the obtained values of energy reflection and transmission coefficients for waves interacting with surface of porous glass bead solid halfspace immersed in water are in good agreement with the results of Wu et al. (1990) based on Biot's theory. Comparison of these results with calculations made for porous glass bead solid saturated with ethyl alcohol proved that the dynamical properties of fluid filling the porous skeleton influence substantially the energy of waves induced by the wave interacting with the surface of porous material. Particularly, the energy transmitted by the slow wave induced in alcohol-saturated porous glass bead solid halfspace is much more greater than that induced in water-saturated porous glass bead solid halfspace. This result is important for experimental investigations of slow waves in saturated porous materials which due to strong attenuation can be hardly observed.

Moreover, it was shown that the transmission coefficients of energy transmitted by fast, slow and shear waves through the layer of porous material are equal to the squares of such transmission coefficients through the surface of porous material.

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References

1. ALBERT G.A., 1993, A Comparison Between Wave Propagation in Water-Saturated and Air-Saturated Porous Materials, *J. Appl. Phys.*, **73**, 28-36
2. BIOT M.A., 1956, Theory of Propagation of Elastic Waves in Fluid-Saturated Porous Solid, *JASA*, **28**, 168-178
3. CIESZKO M., 1992, Influence of a Rigid Skeleton Pore Structure on Wave Propagation in a Fluid-Filled Porous Medium, *TiPM*, **9**, 61-71
4. CIESZKO M., KUBIK J., 1993, On the Compatibility Conditions in the Fluid-Saturated Porous Solid Contact Problems, *Arch. Mech.*, **45**, 1, 77-91
5. CIESZKO M., KUBIK J., 1996, Constitutive Relations and Internal Equilibrium Condition for Fluid-Saturated Porous Solid. Linear Description, *Arch. Mech.*, **48**, 5, 911-923
6. CIESZKO M., KUBIK J., 1996, Constitutive Description of Fluid-Saturated Porous Media with Immiscible Elastic Constituents, In *Problems of Environmental and Damage Mechanics*, edit. W.Kosiński, R.de Boer, D.Gross, Proceeding of SolMec'96 Conference, 39-52
7. DERESIEWICZ H., 1960, The Effect of Boundaries on Wave Propagation in Liquid-Filled Porous Plane. I. Reflection of Plane Waves at a Free Plane Boundary (No-Dissipative Case), *Bull. Seismol. Soc. Am.*, **50**, 599-607
8. DERESIEWICZ H., RICE J.T., 1962, The Effect of Boundaries on Wave Propagation in a Liquid-Filled Porous Solid. III. Reflection of Plane Waves at a Free Plane Boundary (General Case), *Bull. Seismol. Soc. Am.*, **52**, 595-625
9. DERESIEWICZ H., RICE J.T., 1964, The Effect of Boundaries on Wave Propagation in a Liquid-Filled Porous Solid. V. Transmission Acros a Plane Interface, *Bull. Seismol. Soc. Am.*, **51**, 409-416

10. GEERSTMA J., SMIT D.G., 1961, Some Aspects of Elastic wave Propagation in Fluid-Saturated Porous Solids, *Geophysics*, **26**, 169-181

11. JOHNSON D.L., 1986, Recent Developments in the Acoustic Properties of Porous Media, in: *Frontiers in Physical Acoustics*, XCIII Corso, Soc. Italiana di Fisica Bologna

12. KUBIK J., 1986, On Internal Coupling in Dynamic Equations of Fluid-Saturated Porous Solid, *Int. J. Engng Sci.*, **24**, 981-989

13. KUBIK J., 1992, Pore Structure in Dynamic Behavior of Saturated Materials, *Transport in Porous Media*, **9**, 15-24

14. PLONA T.J., 1980, Observation of a Second Bulk Compressional Wave in a Porous Medium at Ultrasonic Frequencies, *Appl. Phys. Lett.*, **36**, 259-261

15. SANTOS J.E., CORBERO J.M., RAVAZZDI C.L., HENSLEY J.L., 1992, Reflection and Transmission Coefficients in Fluid-Saturated Porous Media, *JASA*, **91**, 1911-1923

16. WU K., XUE Q., ADLER L., 1990, Reflection and Transmission of Elastic Waves from a Fluid-Saturated Porous Solid Boundary, *JASA*, **87**, 6, 2349-2358

Oddziaływanie fal sprężystych z brzegiem nasyconego płynem ośrodka porowatego

Streszczenie

W pracy rozważono zagadnienie odbicia i przenikania fal harmonicznych ukośnie padających na powierzchnię porowatej półprzestrzeni zanurzonej w płynie. Przeanalizowano czysto sprężyste zachowanie się układu o nieściśliwym materiale szkieletu. Do rozważań wykorzystano dwuparametrową teorię deformowalnego ośrodka porowatego nasyconego płynem, w której struktura porów szkieletu charakteryzowana jest przez porowatość objętościową i krętość porów. Przeanalizowano wpływ mechanicznych własności składników fizycznych oraz kąt padania fali na energetyczne współczynniki odbicia i przenikania fal. Rozważono dwa przypadki: gdy fala pada na powierzchnię porowatej półprzestrzeni od strony samego płynu oraz przypadek, gdy fale (szybka, wolna i poprzeczna) padają od strony nasyconego płynem porowatego ośrodka. Obliczenia przeprowadzono dla ośrodka porowatego ze spiekanych kulek szklanych wypełnionych wodą w jednym przypadku oraz alkoholem etylowym w drugim przypadku.

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