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BOUNDARY-ONLY FORMULATIONS OF THE BOUNDARY ELEMENT METHOD

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In the paper possible treatments of the domain integrals appearing in BEM are discussed. Major approaches are compared and critically evaluated with emphasis put on the dual and reciprocity reciprocity techniques. New trends in these methods involving new interpolation functions and decomposition are presented.

Key words: boundary element method, dual reciprocity method, multiple reciprocity method, boundary and domain integrals

1. Introduction

Complex engineering problems are efficiently solved nowadays only by using numerical methods. Finite differences (FDM), finite elements (FEM) or more recently boundary elements (BEM) are the principal methods used to determine both steady-state as well as transient fields. Each of the above techniques can be seen as a particular version of a more general approach i.e. the *Method of Weighted Residuals*. The differences between particular methods consist in the application of different weighting functions and in the number of integrations by parts performed over the domain. Many numerical tests carried out so far have proved that all the above mentioned techniques provide a remarkable and comparable degree of accuracy and are usually not very demanding in terms of computer time and memory. The versatility in representation of geometrically complicated regions allows one to model variety of engineering phenomena in objects of any shape with any kind of boundary conditions.

It should be pointed out, however, that in spite of many similarities, particular numerical techniques differ in the way they approach the problem, in the amount of required geometrical information, in the way they represent solution etc. From the user point of view it is important to notice that the FDM and FEM fall within the group of the so-called domain methods. This means that discretization of the whole domain is necessary when using these techniques. On the contrary, BEM expresses the solution to the problem in terms of an integral equation. In many cases this equation contains only the boundary integrals and only the boundary needs to be discretized. Solution at internal nodes (if needed) is obtained at the next stage making use of just the known boundary values. This stage is purely a postprocessing task.

As explained above the boundary element method *reduces* the dimensionality of the problem by one. The word reduces has been put into the inverted commas since the physical field retains its number of independent variables. The object which is discretized, i.e. the contour line (in 2D computations) or surface (in 3D computations), has one dimension less than the physical problem itself.

As already mentioned the Boundary Element Method consists of transforming the boundary value problem into an equivalent integral equation which is then solved numerically. The method was first used to solve the Laplace type partial differential equations which govern steady-state potential problems, including heat transfer problems. In these cases solution is expressed in terms of boundary integrals only, Brebbia et al. (1984), Brebbia and Dominguez (1989). The elegance of the formulation and simplicity of the computer implementation has stimulated the extension of the method onto more complicated engineering situations.

Problems with internal sources (or body forces), e.g. those described by the Poisson equation contain both domain and boundary integrals. The latter integral not only detracts from the elegance of the formulation but first of all affects numerical efficiency. This is why a substantial amount of research has been carried out in order to convert the domain integrals occurring in the BEM equations into boundary integrals. Several methods have been proposed so far and some of them will be discussed in this paper.

The aim of this paper is twofold. First, a general discussion on the treatment of domain integrals in BEM is given, including the fundamentals of the Dual Reciprocity Method (DRM) as well as the Multiple Reciprocity Method (MRM). These techniques attracted most attention for the last few years. Then recent advances in the DRM interpolations, computer implementation and substracturing are presented. This is then followed by main advantages and disadvantages of both techniques.

2. Boundary problem under consideration

The treatment of domain integrals will be studied reffering to the Poisson type equations which can describe, for example, a steady-state temperature field with heat sources acting inside a domain Ω with a boundary Γ

$$\nabla^2 u + \frac{1}{k} q_v = 0 \tag{2.1}$$

where

u – temperature

k – thermal conductivity

q_v – internal heat source (a known function).

It should be noted that this term can stand for real source existing inside the considered body, but also for a fictitious one when transient, nonlinear, convective etc effects are collected in the term q_v.

In order to obtain a unique solution Eq (2.1) one has to specify boundary conditions. However, because boundary conditions of any kind can be prescribed along the boundary Γ these equations will not be discussed hereby.

Application of the reciprocity theorem allows one to transform the boundary value problem (2.1) to the following integral equation (cf Brebbia et al., 1984; Brebbia and Dominguez, 1989)

$$kc_i u_i + \int\limits_{\Gamma} q^* u \, d\Gamma = \int\limits_{\Gamma} u^* q \, d\Gamma - \int\limits_{\Omega} u^* q_v \, d\Omega \tag{2.2}$$

where the fundamental solution u^* satisfies the following differential equation

$$\nabla^2 u^* = \delta_i \tag{2.3}$$

and has the form

$$u^* = \begin{cases} \dfrac{1}{2\pi} \ln(r) & \text{2D problems} \\[2ex] \dfrac{1}{4\pi} \dfrac{1}{r} & \text{3D problems} \end{cases} \tag{2.4}$$

In equation (2.4) δ_i is the Dirac function acting at the point i and r is the distance measured from that point.

The heat flux analog q^* is defined as

$$q^* = -k \frac{\partial u^*}{\partial n} \tag{2.5}$$

The domain integral in Eq (2.2)

$$D = \int_{\Omega} u^* q_v \, d\Omega \tag{2.6}$$

causes that discretization of this equation can not be restricted to the boundary Γ only. In early BEM works (cf Brebbia, 1978; Brebbia and Walker, 1979) integrals of this type have been calculated by domain discretization. If the sources term q_v is a known function of space only, the domain integral does not introduce any new unknowns. Subdivision of the domain into cells is however cumbersome and time consuming, and particularly in three dimensions this is a difficult task even when automatic mesh generators are available. Moreover, integration over the whole domain has to be performed as many times as the total number of nodes. This affects noticeably efficiency of the method and causes BEM to lose its main advantage which is the boundary-only formulation of the problem.

Gipson (1987) in his paper devoted to solutions of the Poisson equation using BEM proposed to employ the Monte Carlo method to perform numerical integrations over the domain. This idea is simply to implement but it usually requires a large number of random integrals to achieve a good accuracy.

In many practical situations the domain integral appearing in Eq (2.6) can be transformed into its equivalent boundary form, which is a subject of the next section.

3. Transformation of the BEM domain integrals to the boundary

Transformation methods discussed in this section are in general restricted to the domain integral appearing in Eq (2.6). For notation convenience the source term q_v is replaced by a generalized body force called b and this integral is written in the following form

$$D = \int_{\Omega} u^* b \, d\Omega \tag{3.1}$$

The transformation methods which have been proposed so far fall into one of the following main groups:

- methods related to particular solutions
- methods related to the so-called Galerkin technique.

The particular solution, noted here as $\hat{u}$, obviously satisfies Eq (2.1) and the body forces can be expressed in terms of a particular Laplacian solution (cf Azevedo and Brebbia, 1988; Cruse et al., 1989)

$$b = -k\nabla^2\hat{u} \tag{3.2}$$

Substitution of Eq (3.2) into Eq (3.1) makes the integration by parts possible and yields

$$D = -k\int_{\Omega} u^*\nabla^2\hat{u}\, d\Omega = -k\int_{\Omega}\hat{u}\nabla^2 u^*\, d\Omega + \int_{\Gamma}(u^*\hat{q} - q^*\hat{u})\, d\Gamma \tag{3.3}$$

Taking into account the definition of fundamental solution and particularly the property of Dirac function, one arrives at the following boundary formulation

$$D = -kc_i\hat{u}_i + \int_{\Gamma}(u^*\hat{q} - q^*\hat{u})\, d\Gamma \tag{3.4}$$

When the particular solution is not known (and this is the majority of practical situations) Nardini and Brebbia proposed in the beginning of eighties Nardini and Brebbia (1982), (1985) that heat source term is generally approximated at $N + L$ points (N boundary nodes and L internal nodes) using the following interpolation

$$b = \frac{1}{k}q_v = \sum_{j=1}^{N+L} f_j\alpha_j \tag{3.5}$$

where f_j are arbitrary approximating functions and α_j are unknown coefficients. In early works, e.g. by Nardini, Partridge, usually $f = 1 + r$ was used. This function is recognized to have the so-called local support, i.e. the influence of points located closer to the collocation point is stronger than that of distant points. This feature is very important at the stage of determining the coefficients α_j. Simply, the interpolation functions without this property can lead to a divergent algorithm.

The technique is nowadays widely known as the *dual reciprocity method.* It uses interpolation functions f_j that allow for easy analytical solutions for the following equations

$$\nabla^2\hat{u}_j = f_j \tag{3.6}$$

More details of the DRM are given in many references, e.g. by Partridge et al. (1992), Partridge and Brebbia (1989). It has already been successfully applied to solving many engineering problems, Partridge et al. (1990) and

(1992), including nonlinear cases, e.g. Wrobel and Brebbia (1987). As already mentioned the DRM can be considered as an extension of the particular solution technique.

Instead of applying interpolation (3.5) Tang (1988) proposed to expand the body force term b into its Fourier series. In general, to obtain the Fourier coefficients one needs to integrate over the whole domain. In order to perform this integration analytically Tang expanded the body force term within an *overdimensioned* but simple shaped auxiliary region Ω' covering the domain Ω. For 2D problems the usual choice regarding domain Ω' is a rectangle. whereas for the 3D case it might be a parallelepiped or sphere. For such simple shapes the Fourier base functions are also fairly simple. As a result it is relatively easy to find particular solutions to the Fourier base functions. Taking into account the orthogonality of base functions and following the DRM transformation procedure the author arrives at the series of boundary integrals.

The method was originally proposed to deal with potential and elasticity problems. However, it has been successfully extended by Itagaki and Brebbia (1988) to solve neutron diffusion problems.

The second group of methods is related to the so-called *Galerkin technique*, Brebbia and Dominguez (1989), Brebbia et al. (1984), Cruse (1977), Telles (1986). In its original form it permits of conversion of domain integrals to the boundary for a limited selection of the body force terms, i.e. those obeying the Laplace equation

$$\nabla^2 b = 0 \tag{3.7}$$

The transformation to the boundary is basically accomplished through integration by parts and making use of the fundamental solution v^* for a biharmonic equation, i.e.

$$\nabla^4 v^* = \delta_i \tag{3.8}$$

It is easy to prove that the function v^* reads

$$u^* = \begin{cases} \dfrac{r^2}{8\pi}(\ln r + 1) & \text{2D problems} \\ \dfrac{r}{8\pi} & \text{3D problems} \end{cases} \tag{3.9}$$

as well as satisfies the condition

$$\nabla^2 v^* = u^* \tag{3.10}$$

Hence, by virtue of Eqs (3.10) and (3.7) the reciprocity theorem yields

$$D = \int\limits_{\Omega} v^* b \, d\Gamma = \int\limits_{\Gamma} \left(b \frac{\partial u^*}{\partial n} - \frac{\partial b}{\partial n} v^* \right) d\Gamma \tag{3.11}$$

The multiple reciprocity method (MRM), developed by Nowak in the late 1980s generalizes these concepts (cf Brebbia and Nowak, 1989; Nowak, 1988, 1989, 1992; Nowak and Brebbia, 1989, 1992). The method introduces a set of the so-called *higher order fundamental solutions*

$$\nabla^2 u^*_{j+1} = u^*_j \qquad q^*_j = -k \frac{\partial u^*_j}{\partial n} \qquad j = 0, 1, 2, ... \tag{3.12}$$

as well as a sequence of the source function Laplacians

$$\nabla^2 b_j = b_{j+1} \qquad w_j = -k \frac{\partial b_j}{\partial n} \qquad j = 0, 1, 2, ... \tag{3.13}$$

and operates on the domain integral in a recurrent manner. As a result, the technique can lead in the limit to the exact boundary-only formulation of the problem

$$D = \frac{1}{k} \sum_{j=0}^{\infty} \int\limits_{\Gamma} (q^*_{j+1} b_j - u^*_{j+1} w_j) \, d\Gamma \tag{3.14}$$

where $b_0 = b$ and $u^*_0 = u^*$.

The multiple reciprocity method was proposed by Nowak (1988) to cope with thermal processes and has since then been gradually extended to solve other engineering problems (cf Nowak and Brebbia, 1989a,c; Nowak, 1989; Brebbia and Nowak, 1992). It has already been applied to solving the Helmholtz equation, calculating eigenvalues; analysing fluid flow problems; investigation of elasticity and thermoelasticity, as well as solving critical safety and vibration problems. Full discussion on the MRM can be found in Nowak and Neves (1994) while solutions of nonlinear problems are reported by Nowak (1995).

4. New trends in the treatment of BEM domain integrals

Recent works on the domain integrals treatment show that many international efforts have been made generally in the following two directions:

- Extension of the DRM and MRM into new applications. It is interesting to notice that fluid dynamics groups are particularly very active in the DRM research. Fluid flow problems are attacked by the DRM approaches using both fundamental solutions, i.e. to the Navier-Stokes equation as well as to the Laplace one (cf Popov and Power, 1966; Szczygiel and Nowak, 1992)
- Progress in the accuracy and robustness of the DRM which was possible mainly because of the more solid mathematical bases formulated recently for this technique. The following two improvements have to be pointed out in that context
  - Decomposition (zoning) which reduces considerably numerical errors (cf Mingo and Power, 1966). The reason why results obtained using zoning are much more accurate is still not fully clear. Usually, reduction of the size of patches is pointed out. In another words, formulation is more similar to that of FEM. It is however important to stress that *boundary nature* of this solution is still preserved. Another reason could be an additional constrain involving fluxes, which in the case of zoning are forced to be continuous across each interface in this approach
  - New DRM interpolation functions which allow now for better representation of the behaviour of source term inside the domain. This issue is discussed in the next subsection.

4.1. Thin plate splines

Early works on the dual reciprocity have shown that although a variety of functions can in principle be used as a basic approximation function, good results were usually obtained with simple expansions, the most popular of which is $f_j = 1 + r_j$, where r_j is the distance between represented fixed collocation points, $\boldsymbol{y}_j$, and a field point $\boldsymbol{x}$, at which the sought field is approximated. In the DRM literature given, the choice is based on experience gained numerical tests rather than on formal mathematical analysis.

Recently, mathematicians have pointed out, from cf Goldberg and Chen (1994), that according to the radial bases functions in interpolation theory, the best possible 2D approximation for b in Eq (3.5) is given the following formula

$$b = \sum_{j=1}^{N+L} (r_j^2 \log r_j)\alpha_j + ax + by + c \tag{4.1}$$

Such function is known as the so-called thin plate spline. It is important to add that the above best interpolant still possesses the local support, but it additionally reveals another important feature, i.e. its curvature between collocation points is minimum. This is why the function (4.1) guarantees the best possible interpolation of source term b.

It is also interesting to note that the first term on the right-hand side of Eq (4.1) (involving $\log r$) is very much the same as the fundamental solution of the first order utilized in the multiple reciprocity method. This suggests deep links between the DRM and MRM which need to be investigated.

The interpolation function (4.1) contains three additional coefficients comparing with Eq (3.5). Thus, three more equations to Eq (4.1) are required. They are as follows (cf Goldberg and Chen, 1994)

$$\sum_{j=1}^{N+L} \alpha_j = \sum_{j=1}^{N+L} \alpha_j x_j = \sum_{j=1}^{N+L} \alpha_j y_j = 0 \tag{4.2}$$

The set of Eqs (4.1) and (4.2) allows one to determine the components of vector $\boldsymbol{b}$ for each boundary node and each internal point. Remaining part of the algorithm follows the standard path.

5. Conclusions

Recent developments of the treatment of domain integrals in BEM offer two alternative techniques, namely the Dual Reciprocity and the Multiple Reciprocity Methods which both utilize the reciprocity theorem although the philosophy behind each method is different. The MRM leads to the exact boundary-only formulation whereas the DRM is based on approximation of the body force term using interpolating functions.

It should however be stressed that this approximation can require the substantial number of internal points as the approximation based on boundary nodes only may not guarantee a sufficient accuracy. The numerical tests carried out so far indicate that first internal points included affect solution considerably. The influence of the following points is usually weaker. However, up-to-now, a criterion of how many internal points is required has not been proposed.

Also location of the internal poles is very important and generally requires some knowledge about the sought field behaviour. This is why many sets of interpolating functions have been tried including the Fourier expansions.

New DRM interpolation functions improve significantly the accuracy of it. The price one have to pay for this improvement is more difficult computer implementation of the technique. It has been noticed that combining these new interpolation functions with a zoning produced very encouraging results.

The DRM formulation involves the matrix F based on interpolation functions. If the body force in Eq (3.5) is a known function, then the vector $\boldsymbol{\alpha}$ may be obtained explicitly using the Gauss elimination. If $\boldsymbol{b}$ is an unknown function, e.g. depends on solution $b = b(x, y, u)$ then $\boldsymbol{\alpha} = \mathsf{F}^{-1}\boldsymbol{b}$ is used and the matrix F must be inverted. In the case when the inversion of F is necessary this operation together with the multiplication of matrices for large number degrees of freedom can be time consuming.

It is worth noticing that numerical implementation of both methods is very simple and straightforward. In any case a right hand side vector requires to be calculated by multiplication of appropriate matrices.

Both techniques were found to give accurate results. It should, however, be remembered that the DRM usually requires certain number of internal points, which are not used in the MRM. Providing the discretization is appropriate to represent properly the behaviour of the solution and the number of terms in the series (3.14) guarantee its convergence, the MRM gives more accurate results. The DRM produces a less accurate solution but with less computational effort. It is important to point out that for many cases where the functions sought are relatively smooth the DRM gives good results very cheaply. Thus, the choice between DRM or MRM depends strongly on the boundary problem.

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W pełni brzegowe sformułowania w metodzie elementów brzegowych

Streszczenie

W artykule omówiono sposoby obliczania całek po całym obszarze występujących w metodzie elementów brzegowych. Podstawowe algorytmy obliczeniowe zostały krytycznie porównane ze szczególnym uwzględnieniem metod wykorzystujących zasady wzajemności: podwójną oraz wielokrotną. Zaprezentowano również trendy w tych technikach obliczeniowych, a w szczególności nowe funkcje interpolujące oraz metodę dekompozycji obszaru.

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