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AXI-SYMMETRICAL CONTACT PROBLEM DURING BRAKING WITH FRICTIONAL HEATING AND WEAR

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The problem of transient contact with frictional heating and wear for two non-uniform sliding half-spaces is considered. One of the two half-spaces is assumed to be slightly curved to give a Hertzian initial pressure distribution; the other is a rigid non-conductor. On the assumption that the contact pressure distribution could be described by the Hertz formulas during whole the process of interaction, the problem is formulated in terms of one integral equation of the Volterra type an unknown radius of the contact area. A numerical solution of this equation is obtained using a piecewise-constant representation of unknown function. The influence of operating parameters on the contact temperature and the radius of the contact area is studied.

Key words: frictional heating, wear, contact pressure, temperature, braking system

1. Introduction

An approach to definition of the surface temperature of a disc brake based on the solution (cf Barber, 1980) of an axi-symmetrical contact problem for a half-space with frictional heat generation was proposed by Barber et al. (1985). Variations of the contact radius and temperature during uniform deceleration were investigated.

The effect of simultaneous frictional heating and wear on the temperature rise was studied by Yevtushenko and Chapovska (1995) on assumption that sliding was uniform. This paper is continuation of the investigation (cf Yevtushenko and Chapovska, 1995) undertaken the case of uniform reduction of speed from an initial value to zero.

2. Statement of the problem

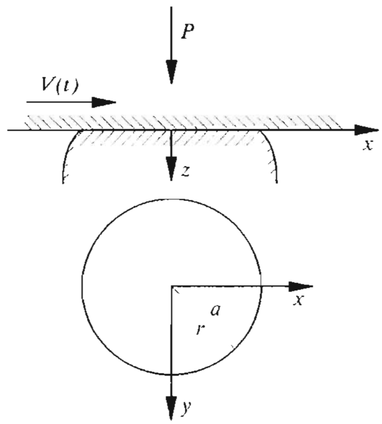

Fig. 1. Geometry of contact

Two semi-limited bodies are being compressed by a force P. One of the bodies slides on the surface of the other (Fig.1) at the time-dependent speed

$$V(t) = V_0\Big(1 - \frac{t}{t_s}\Big) \qquad t \le t_s \tag{2.1}$$

where

t – time

t_s – stopping time (duration of the stop)

V_0 – initial speed.

The sliding is accompanied by frictional heat generation over the contact interface having the form of a heat flux

$$q(r,t) = fV(t)p(r,t)H[a(t) - r]H(t_s - t) \tag{2.2}$$

directed into the elastic moving body. Here

f – coefficient of friction

p – contact pressure

r – radial coordinate

$a(t)$ – radius of the contact circle

$H(\cdot)$ – Heaviside step function.

It is assumed that:

1. The motionless body is a rigid thermoinsulator;

2. The convective heat transfer from the free surface of bodies is absent;

3. The relation between normal and tangential stresses in the contact area may be neglected. This does not mean that the tangential traction on the surface is neglected. Indeed, the work done against these tractions is the source of heat. However, the elastic displacement normal to the surface, caused by the tangential tractions, is much smaller than that produced by the normal tractions, and the coupling effect is negligible;

4. The wear takes place; we use the Archard law of wear with the coefficient of wear k_w (cf Goryazcheva and Dobychin, 1971);

5. The surface of the elastic body is slightly curved with the radius R_0 to give an initial Hertzian pressure distribution.

Due to the friction forces of sliding acting in the contact region the thermal distortion of the surface of elastic half-space appears, and the same time its wear takes place. The condition for the contact of bodies has the form

$$u_z^e(r,t) + u_z^t(r,t) + u_z^w(r,t) = \Delta(t) - \frac{r^2}{2R_0} \qquad r \le a(t) \qquad t \ge 0 \tag{2.3}$$

where

u_z^e – elastic displacements of the half-space boundary point in the direction of z axis

u_z^t – normal temperature displacements

u_z^w – vertical displacement of the elastic body as a rigid solid.

At the initial moment of time $t = 0$ the elastic displacements u_z^e and the contact pressure p, according to the assumption (5), are given by the Hertz formulas (cf Johnson, 1987)

$$u_z^e = \frac{3P(1-\nu)[2a^2(0) - r^2]}{16\mu a^3(0)} \qquad p = \frac{3P\sqrt{a^2(0) - r^2}}{2\pi a^3(0)} \qquad r \le a(0) \tag{2.4}$$

where

μ – shear modulus

ν – Poisson's ratio.

We approximate the normal displacements u_z^e due to frictional heating by the heat flux (2.2) and due to wear u_z^w at every moment of time $t > 0$ by a quadratic surface

$$u_z^t(r,t) + u_z^w(r,t) = C_0(t) + C_1(t)r^2 \qquad r \le a(t) \tag{2.5}$$

Taking into account the Eq (2.5), the boundary condition (2.3) has the form

$$u_z^e(r,t) = C_0^*(t) - C_1^*(t)r^2 \qquad r \leq a(t) \qquad t > 0 \tag{2.6}$$

where

$$C_0^*(t) = \Delta(t) - C_0(t) \qquad\qquad C_1^*(t) = C_1(t) + \frac{1}{2R_0}$$

$$C_k(0) = 0 \qquad k = 0,1$$

and, therefore, we may use the Hertz formulas (2.4) to find $u_z^e(r,t)$, $t > 0$ and $p(r,t)$, $t > 0$. The comparison of coefficients at r^2 in Eqs (2.4) and (2.6) yields

$$C_1^*(t) = \frac{3P(1-\nu)}{16\mu a^3(t)} \qquad t \geq 0 \tag{2.7}$$

Substituting for value of the contact radius $a(t)$ into Eqs (2.2), (2.4) from Eq (2.7), we obtain for $r \leq a(t)$, $t \geq 0$

$$p(r,t) = \frac{8\mu C_1^*(t)\sqrt{a^2(t) - r^2}}{\pi(1-\nu)} \tag{2.8}$$

$$q(r,t) = \frac{8fV(t)\mu C_1^*(t)\sqrt{a^2(t) - r^2}H(a-r)H(t)}{\pi(1-\nu)} \tag{2.9}$$

From Eqs (2.7) $\div$ (2.9) it follows that to define the contact pressure $p(r,t)$, frictional heat flux $q(r,t)$ and contact radius $a(t)$ it is enough to know a kind of the function $C_1^*(t)$.

We also note that the solution of the corresponding contact problem in the steady state of heat generation was obtained by Barber (1976). It is found that the contact radius is

$$a_0 = \frac{\pi K(1-\nu)}{1.566\alpha_t\mu f V_0(1+\nu)} \tag{2.10}$$

where

K – coefficient of thermal conductivity

α_t – coefficient of linear temperature expansion.

The parameter a_0, as it is evident from Eq (2.10), does not depend on the force P and is a limited value of a at $P \to \infty$. In a pure isothermal problem such a limit does not exist.

3. Reduction to the integral equation

We find the normal displacement $u_z^t(r,t)$, $r \geq 0$, $t > 0$ on the surface of elastic half-space heated by the heat flux $q(r,t)$ (2.2). To this end, we use a fundamental solution of the equation of thermal conductivity for an instantaneously acting point heat source of a constant power q (cf Carslaw and Jaeger, 1959) on the surface $z = 0$ of the elastic half-space

$$T(r,t) = \frac{q}{4\rho c\sqrt{(\pi kt)^3}} \exp\Big(-\frac{r^2}{4kt}\Big) \qquad r \geq 0 \qquad t > 0 \tag{3.1}$$

where
k – coefficient of thermal diffusivity
ρ – density
c – special capacity.

The vertical displacement corresponding to the temperature field (3.1) of the half-space surface is (cf Barber, 1972)

$$u_z^t(r,t) = -\frac{\alpha_t q(1+\nu)}{4\pi K t}\Phi\Big(1.5; 2; -\frac{r^2}{4kt}\Big) \qquad r \geq 0 \qquad t > 0 \tag{3.2}$$

where Φ is the degenerated hypergeometric function.

On the basis of the solution (3.1), (3.2) temperature and the vertical displacements of the boundary $z = 0$ of the half-space heated by the flux $q(r,t)$ Eq (2.2) can be rewritten as follows

$$T(r,t) = \frac{1}{4\rho c\sqrt[3]{\pi k}} \int\limits_0^t \int\limits_0^{a(\tau)} \int\limits_0^{2\pi} sq(s,\tau)\exp(-X^2)\frac{d\theta ds d\tau}{\sqrt{(t-\tau)^3}} \tag{3.3}$$

$$u_z^t(r,t) = -\frac{\alpha_t(1+\nu)}{4\pi K} \int\limits_0^t \int\limits_0^{a(\tau)} \int\limits_0^{2\pi} sq(s,\tau)\Phi(1.5; 2; -X^2)\frac{d\theta ds d\tau}{t-\tau} \tag{3.4}$$

where

$$X^2 = \frac{r^2 - 2rs\cos(\theta) + s^2}{4k(t-\tau)} \qquad r \leq a(t) \qquad t > 0$$

We use the wear law in the form (cf Goryazcheva and Dobychin, 1988)

$$u_z^w(r,t) = k_w \int\limits_0^t p(r,\tau)\, d\tau \qquad r \leq a(t) \qquad t > 0 \tag{3.5}$$

Such a relationship is valid in rigid regimes of wear by abrasive particles and in some cases of the fatigue wear.

In view of Eq (2.10) the normal displacements $u_z^t(r,t)$ and $u_z^w(r,t)$, see Eqs (3.4) and (3.5), for the heat flux $q(r,t)$ Eq (2.9) take the form

$$u_z^t(r,t) = -\frac{16k}{1.556\pi a_0}\int\limits_0^t\int\limits_0^{A(\tau)}\int\limits_0^{2\pi}\Big(1-\frac{\tau}{t_s}\Big)C_1^*(\tau)2\sqrt{k(t-\tau)(A^2-S^2)}\cdot$$

$$\cdot\Phi\Big(1.5;2;-(R^2-2RS\cos\theta+S^2)\Big)S\;d\theta dS d\tau \tag{3.6}$$

$$u_z^w(r,t) = -\frac{8V_0\mu k_w}{\pi(1-\nu)}\int\limits_0^t\Big(1-\frac{\tau}{t_s}\Big)C_1^*(\tau)\sqrt{a^2(\tau)-r^2}\;d\tau \tag{3.7}$$

$$A^2 = \frac{a^2}{4k(t-\tau)} \qquad R^2 = \frac{r^2}{4k(t-\tau)} \qquad S^2 = \frac{s^2}{4k(t-\tau)} \tag{3.8}$$

We denote

$$\Phi_1(R,S) \equiv \int\limits_0^{2\pi}\Phi\Big(1.5;2;-(R^2-2RS\cos(\theta)+S^2)\Big)\;d\theta \tag{3.9}$$

In the vicinity of zero the degenerated hypergeometric function permits one to perform the following decomposition

$$\Phi(1.5;2;-R^2) = \sum_{i=0}^{\infty}\frac{(2i+1)!!(-R^2)^i}{(2i)!!(i+1)!} \tag{3.10}$$

Substituting the series (3.10) in the right-hand side of Eq (3.9) and calculating the integral (cf Gradshtein and Ryzhik, 1971)

$$\int\limits_0^{2\pi}\Big(1-2\frac{R}{S}\cos\theta+\frac{R^2}{S^2}\Big)^i\;d\theta = 2\pi\sum_{j=0}^{i}(C_j^i)^2\Big(\frac{R}{S}\Big)^{2j} \qquad 0\le\frac{R}{S}<1 \tag{3.11}$$

where C_j^i is the binomial coefficient, we find

$$\Phi_1(R,S) = 2\pi\sum_{i=0}^{\infty}\frac{(2i+1)!!(-R^2)^i}{(2i)!!(i+1)!}\sum_{j=0}^{i}(C_j^i)^2\Big(\frac{R}{S}\Big)^{2j} \tag{3.12}$$

By virtue of Eqs (3.9), (3.12) from Eq (3.6) it follows

$$
\begin{aligned}
u_z^t(r,t) &= -\frac{16k}{1.556\pi a_0} \int\limits_0^t \int\limits_0^{A(\tau)} \Big(1-\frac{\tau}{t_s}\Big) C_1^*(\tau) \sqrt{k(t-\tau)(A^2-S^2)} \cdot \\
&\cdot \Phi_1(R,S) S\, dS d\tau \qquad R \leq A(t) \qquad t > 0
\end{aligned}
\tag{3.13}
$$

We define the function

$$
\Phi_2(R,A) = \int\limits_0^A \sqrt{A^2-S^2}\,\Phi_1(R,S) S\, dS \tag{3.14}
$$

Taking into acount the form of the function $\Phi_1(R,S)$ given by (3.12) and the value of the integral (cf Gradshtein and Ryzhik, 1971)

$$
\int\limits_0^A \sqrt{A^2-S^2}\, S^{2i-2j+1}\, dS = \frac{(2i-2j)!!}{(2i-2j+3)!!} A^{2i-2j+3} \tag{3.15}
$$

we have

$$
\Phi_2(R,A) = -2\pi A \sum_{i=0}^{\infty} \frac{(2i+1)!!(-A^2)^{i+1}}{(2i)!!(i+1)!} \sum_{j=0}^{i} (C_j^i)^2 \frac{(2i-2j)!!}{(2i-2j+3)!!} \Big(\frac{R}{A}\Big)^{2j} \tag{3.16}
$$

Including Eqs (3.14), (3.16), from Eq (3.13) we obtain

$$
\begin{aligned}
u_z^t(r,t) &= \frac{16k}{1.566 a_0} \sum_{i=0}^{\infty} \frac{(-1)^{i+1}(2i+1)!!}{(2i)!!(i+1)!} \sum_{j=0}^{i} (C_j^i)^2 \frac{(2i-2j)!!}{(2i-2j+3)!!} \cdot \\
&\cdot \int\limits_0^t \Big(1-\frac{\tau}{t_s}\Big) C_i^*(\tau) \Big(\frac{r}{a}\Big)^{2j} A^{2i+2} a\, d\tau
\end{aligned}
\tag{3.17}
$$

Having changed the order of summation in the right-hand side of Eq (3.17) and taking into account the first of two terms with respect to j, we find

$$
\begin{aligned}
u_z^t(r,t) &= \frac{16k}{1.566 a_0} \sum_{j=0}^{1} \sum_{i=j}^{\infty} \frac{(-1)^{i+1}(2i+1)!!}{(2i)!!(i+1)!} (C_j^i)^2 \frac{(2i-2j)!!}{(2i-2j+3)!!} \cdot \\
&\cdot \int\limits_0^t \Big(1-\frac{\tau}{t_s}\Big) C_i^*(\tau) \Big(\frac{r}{a}\Big)^{2j} A^{2i+2} a\, d\tau
\end{aligned}
\tag{3.18}
$$

The normal surface displacement of the half-space surface owing to the wear (3.5) is given in the form

$$u_z^w(r,t) = \frac{8V_0\mu k_w}{\pi(1-\nu)} \int\limits_0^t \Big(1-\frac{\tau}{t_s}\Big) C_1^*(\tau) a(\tau) \Big(1-\frac{r^2}{2a^2}+\cdots\Big)\, d\tau \tag{3.19}$$

Comparing the coefficients at r^2 in Eqs (2.5), (3.18), (3.19), we obtain

$$\begin{aligned} C_1^*(t) &= \frac{8k}{1.566a_0}\sum_{i=1}^{\infty}\frac{i}{(i+1)!}\int\limits_0^t \frac{(-A^2)^{i+1}C_1^*(\tau)}{a(\tau)}\, d\tau + \\ &- \frac{4V(t)\mu k_w}{\pi(1-\nu)}\int\limits_0^t \Big(1-\frac{\tau}{t_s}\Big)\frac{C_1^*(\tau)}{a(\tau)}\, d\tau + C_1^*(0) \\ C_1^*(0) &= \frac{1}{2R_0} \end{aligned} \tag{3.20}$$

Going on to dimensionless variables and parameters by the substitution into Eq (3.20)

$$\begin{aligned} t^* &= \frac{4kt}{a_0^2} & \tau^* &= \frac{4k\tau}{a_0^2} & b(t^*) &= \frac{a(t^*)}{a_0} \\ B^2 &= \frac{b^2(t^*)}{t^*-\tau^*} & t_s^* &= \frac{4kt_s}{a_0^2} & k_w^* &= \frac{V_0\mu k_w}{\pi(1-\nu)k} \end{aligned} \tag{3.21}$$

and taking into account that

$$\sum_{i=1}^{\infty}\frac{(-B^2)^{i+1}}{(i+1)!} = 1-\Psi(B) \tag{3.22}$$

where

$$\Psi(B) = (1+B^2)\exp(-B^2)$$

we come to the integral equation of Volterra type for the function $b(t^*)$

$$\frac{1}{b^3(t^*)} = 1.277\int\limits_0^{t^*}\Big(1-\frac{\tau^*}{t_s^*}\Big)\frac{1-0.783k_w^*-\Psi(B)}{b^4(\tau^*)}\, d\tau^* + \frac{1}{b^3(0)} \qquad t^* \geq 0 \tag{3.23}$$

4. Numerical algorithm

We build a numerical procedure of the solution of the nonlinear integral (3.23). To this end we divide the integration interval $[0, t^*]$ into l equal parts of the length Δt using the points $t_k = k\Delta t$, $k = 0, 1, 2, ..., l$, $0 \le t_k \le t_s = s\Delta t$. For small times $(t \to 0)$ we have $B \to \infty$, $\Psi(B) \to 0$, Eq (3.23) may be rewritten as

$$\frac{1}{b^3(t^*)} = 1.277 \int\limits_0^{t^*-\Delta t} \Big(1 - \frac{\tau^*}{t_s^*}\Big)[D - \Psi(B)]\frac{d\tau^*}{b^4(\tau^*)} + +1.277D \int\limits_{t^*-\Delta t}^{t^*} \Big(1 - \frac{\tau^*}{t_s^*}\Big)\frac{d\tau^*}{b^4(\tau^*)} + \frac{1}{b^3(0)} \qquad 0 \le t^* \le t_s^* \tag{4.1}$$

where $D = 1 - 0.783k_w^*$.

At the first time step $t^* = t_1 = \Delta t$ and from Eq (4.1) it follows that

$$\frac{1}{b^3(t_1)} = 1.277D \int\limits_0^{t_1} \frac{d\tau^*}{b^4(\tau^*)} + \frac{1}{b^3(0)} \tag{4.2}$$

The solution of the Eq (4.2) is

$$b_1 \equiv b(t_1) = b(0) - 0.425Dt_1\Big(1 - \frac{t_1}{2t_s}\Big) \tag{4.3}$$

where

$$b_0 \equiv b(0) = \frac{a(0)}{a_0} \tag{4.4}$$

At the kth time step $t^* = t_k = k\Delta t$, $b_k = b(t_k)$ and by the quadrature formula of the trapezoidal rule it follows from Eq (4.1)

$$\frac{1}{b_k^3} = 0.638\Delta t\Big\{2\sum_{i=0}^{k-1}{}'' \frac{h_i}{b_i^4}[D - \Psi(B_i)] + D\Big(\frac{h_{k-1}}{b_{k-1}^4} + \frac{h_k}{b_k^4}\Big)\Big\} + \frac{1}{b_0^3} \tag{4.5}$$

where

$$h_i = 1 - \frac{t_i}{t_s} \qquad i = 0, 1, ...k$$
$$B_0^2 = \frac{b_0^2}{k\Delta t} \qquad B_i^2 = \frac{b_i^2}{(k-i)\Delta t} \qquad i = 1, 2, ..., k-1 \tag{4.6}$$

(here and hereinafter the summation sign $\sum''$ means that the first and the last term of the series must be multiplied by 1/2). Having multiplied the both parts of Eq (4.5) by $b_k^4 \neq 0$ we have

$$b_k^4 - d_{1k}b_k + d_{0k} = 0 \qquad k = 1,2,...,s \tag{4.7}$$

where

$$d_{1k} = \frac{1}{B_k} \qquad d_{0k} = 0.638D\Delta t\frac{h_k}{B_k}$$

$$B_k = 0.638\Delta t\left\{2\sum_{i=0}^{k-1}{}'' \frac{h_i}{b_i^4}[D - \Psi(B_i)] + D\frac{h_i}{b_{k-1}^4}\right\} + \frac{h_{k-1}}{b_0^3}$$

The largest positive real root of the order polynomial fourth (4.7) will give a required value of the dimensionless radius of the contact area $b(t_k)$ at the kth time step.

It may be shown that in the absence of wear ($k_w^* = 0$, $D = 1$) the existence of such a root is ensured by satysfying the condition

$$\Delta t < \frac{1.35h_k}{\sqrt[3]{B_k}} \tag{4.8}$$

From Eqs (4.8) and (4.6) it follows that the time step duration depends essentially on the stopping time t_s. If $k_w^* \neq 0$, then it is difficult to obtain an estimation of the type (4.8) and the choice of Δt is made by means of selection.

5. Temperature

Owing to the symmetry of the problem the maximum temperature is reached at the centre of the circular heating region $r \leq a(t)$. Taking Eqs (2.1), (2.2), (2.10) and (3.21) into account, from Eq (3.3) at $r = 0$ we have

$$T(t^*) = \frac{4T_{\max}}{\pi\sqrt{\pi}}\int_0^{t^*}\left(1 - \frac{\tau^*}{t_s^*}\right)\Phi_3(B)\frac{d\tau^*}{b^3(\tau^*)} \qquad 0 \leq t^* \leq t_s^* \tag{5.1}$$

Here

$$T_{\max} = \frac{3fV_0P}{8a_0K} = \frac{0.587\alpha_t f^2V_0^2P\mu(1+\nu)}{\pi K^2(1-\nu)} \tag{5.2}$$

is the maximum temperature at the centre of the circular contact region $r \leq a_0$ in the steady state of heat generation in long braking $(t_s \to \infty)$ (cf Barber, 1980). From Eq (5.2) it follows that for a given power of the heat source fV_0P, the maximum temperature may be reduced if we use materials of low shear modulus μ. Such are composite materials of brake shoes pads which are more effective in exploitation than cast iron shoes of large shear modulus (cf Barber et al., 1985).

The function $\Phi_3(B)$ in Eq (5.1) is

$$\Phi_3(B) = \frac{1}{2}B[1 - F_1(B)] \tag{5.3}$$

where

$$F_1(B) = B^{-1}\exp(-B^2)\int\limits_0^B \exp(\xi^2)\, d\xi$$

is the Dousson integral (cf Abramovitsand Stegun, 1979).

The function $F_1(B)$ is computed using its expansion into a polynomial series (cf Barber and Martin-Moran, 1982)

$$F_1(B) = \begin{cases} \sum\limits_{i=0}^{\infty} \dfrac{(-2B^2)^i}{(2i+1)!!} & B < 3 \\[2ex] \dfrac{(2i-1)!!}{(2B^2)^{i+1}} & B > 3 \end{cases} \tag{5.4}$$

6. Numerical analysis

Input parameters of the problem are the following three dimensionless values: stopping time t_s^* (3.21), coefficient of wear k_w^* (3.21) and initial radius of the contact circle b_0 (4.4).

The dimensionless contact area radius b (3.21) variation versus the ratio t^*/t_s^* at the fixed values $b_0 = 10$, $k_w^* = 0$ (continuous curves) and $k_w^* = 0.2$ (dashed curves) for two values t_s^* are shown in Fig.2a. The corresponding distribution of the dimensionless temperature $T^* = T(t^*)/T_{max}$ is shown in Fig.2b. The value of the coefficient of wear $k_w^* = 0.2$ is appropriate for ceramics/metal interfaces at high sliding speeds (cf Richardson and Finger, 1983). From Fig.2 one can see that the maximum temperature in the contact region is reached at the instant when the contact area radius is minimum. Wear reduces to increasing of the contact area and to decreasing of the contact

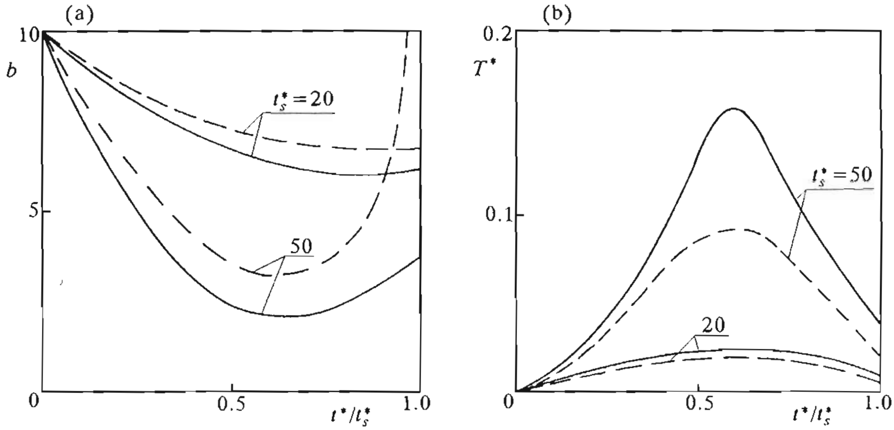

Fig. 2. Dimensionless contact radius b (a) and dimensionless temperature T^* (b) charts versus the t^*/t_s^* without ($k_w^* = 0$, continuous curves) and with ($k_w^* = 0.2$, dashed curves) wear

temperature. At small ($t_s^* \leq 20$) values of the stopping time the influence of wear on the contact radius b and on the temperature T^* is negligible. Another picture is observed in long ($t_s^* \geq 50$) regimes of braking. In this case at $t^* \simeq 0.8t_s^*$, $k_w^* \neq 0$ a sharp increase of the contact region takes place (Fig.2a). So, the contact circle radius at $t^* = t_s^* = 50$ in the absence of wear ($k_w^* = 0$) is only 30% of the corresponding value at $k_w^* = 0.2$. The maximum contact temperature is reached approximately after the half stopping time ($t^* \simeq 0.5t_s^*$).

The stopping time depends on the friction force and can be determined from the equation

$$\frac{d(MV)}{dt} = -fP \tag{6.1}$$

where M is mass of the system referred to the radius of the braking surface.

Since the reduction of mass M due to wear is small and taking Eq (2.1) into account, from Eq (6.1) we obtain

$$fP = \frac{MV_0}{t_s} \tag{6.2}$$

Further, using Eqs (3.21), from Eq (6.2) we find

$$t_s = \frac{a_0^2 t_s^*}{4k} = \frac{MV_0}{fP} \tag{6.3}$$

Thus, the maximum steady temperature T_{max} (5.2) takes the form

$$T_{max} = \frac{3MV_0^2}{2a_0^3ct_s^*} \tag{6.4}$$

where $MV_0^2/2$ is the initial kinetic energy of the system.

We note

$$T_1 = \frac{3MV_0^2}{2a_0^3c} \qquad T_1^* = \frac{T}{T_{\max}t_s^*} = \frac{T}{T_1} \tag{6.5}$$

In the physical sense a_0^3c is the heat capacity of a cube of given material with a side length a_0. Therefore, T_1 is the temperature which is equal to the ratio of initial kinetic energy of the system to the amount of heat in one third of this cube.

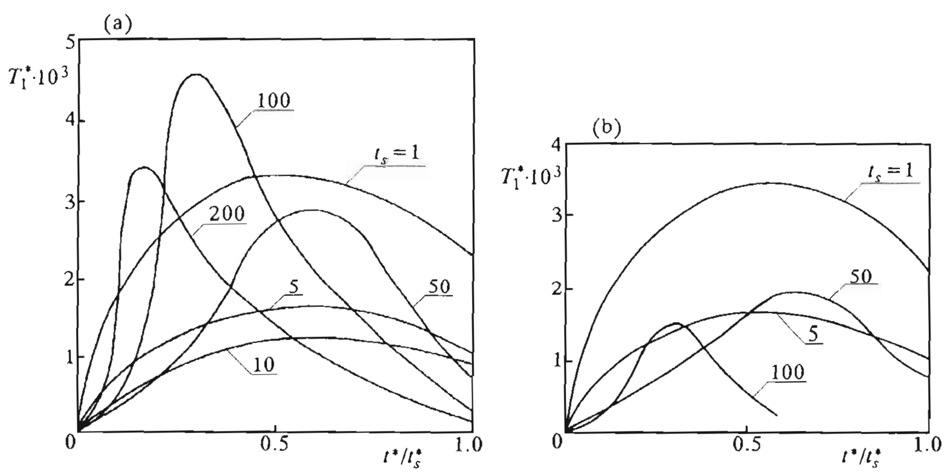

Fig. 3. Dimensionless temperature T_1^* charts versus the t^*/t_s^* at $b_0 = 10$ in absence of wear (a) and at $b_0 = 10$ and $k_w^* = 0.2$ (b)

Fig.3a shows the dependence of T_1^* on t^*/t_s^* at the absence of wear ($k_w^* = 0$) for the fixed values t^*. The behaviour of T_1^* will be different in relative to the values of stopping time. Two intervals of its change may be choosed:

- Rapid regime ($t_s^* < 50$). High contact temperatures do not develop and with increasing of t_s^* the temperature falls. In this regime the maximum temperature T_1^* is reached nearly in the half of the stopping time

- Long regime $(t_s^* > 50)$. The braking period is so large that the surface temperature is sufficient for hot spots to arise. The temperature will reach the maximum value T_{max} in the steady state and, therefore, from Eq (6.5) we have

$$T_1^* = \frac{1}{t_s^*} \tag{6.6}$$

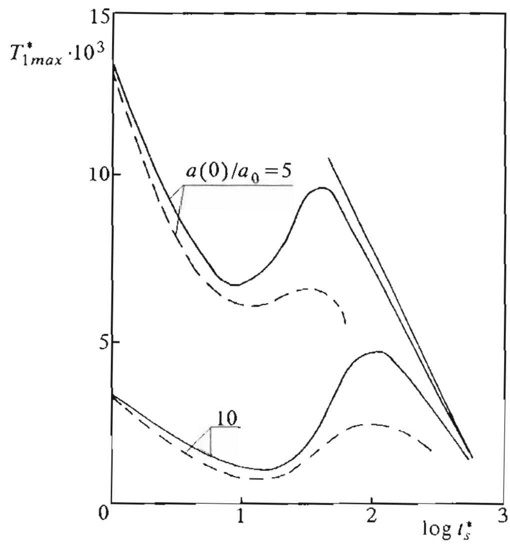

Fig. 4. Maximum temperature T_1^* on braking time t_s^* without $(k_w^* = 0$, continuous curves) and with $(k_w^* = 0.2$, dashed curves) wear

The influence of wear on the contact temperature is shown in Fig.3b. In this case the maximum contact temperature is reached during the rapid regime of braking. The maximum temperature T_1^* variations versus the stopping time t_s^* at $b_0 = 5$; 10 and $k_w^* = 0$ (continuous curves) and $k_w^* = 0.2$ (dashed curves) are shown in a semi-logarithmic scale in Fig.4. We see that there is the stopping time t_s^* at which the temperature reaches the minimum value. At slow braking $(t_s^* > 100)$, the temperature reached the steady state value and T_1^* asymptotically tends to the value (6.6).

7. Conclusions

The following main conclusions can be drawn from the obtained solution:

- The introduction of wear into the analysis of the thermoelastic contact of two sliding half-spaces with uniform retardation resalts in the maximum temperature and increasing of the contact radius
- The maximum contact temperature and the minimum contact radius are reached near the half of the stopping time $(t \approx 0.5t_s)$
- Significant increasing of the contact area due to wear takes place at the end of braking $(t \geq 0.8t_s)$
- In a slow braking regime at absence of wear the increase of contact temperature is sufficient for developing of the hot spots
- When the wear takes place, the greatest value of the temperature T_1, which is independent of the initial kinetic energy, is reached during rapid braking.

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Osiowosymetryczne kontaktowe zagadnienie podczas hamowania z uwzględnieniem generacji ciepła

Streszczenie

W pracy rozpatrzono zagadnienie kontaktowe z uwzględnieniem generacji ciepła dla dwóch ślizgających się półprzestrzeni. O jednej z półprzestrzeni założono, że jest gładko zakrzywiona co powoduje Hertzowski początkowy rozkład ciśnienia. Druga z półprzestrzeni jest izolatorem. Wykorzystując założenie, że ciśnienie kontaktowe może być założone w postaci rozkładu otrzymanego przez Hertza podczas trwania oddziaływania, zagadnienie zostało zredukowane do równania całkowego typu Voltera na nieznany promień obszaru kontaktu. Rozwiązanie numeryczne zostało przedstawione wykorzystując kawałkami stałą reprezentację nieznanej funkcji. Zbadano wpływ parametrów charakteryzujących ciała na temperaturę w obszarze kontaktu oraz promień obszaru kontaktu.

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