JOURNAL OF THEORETICAL AND APPLIED MECHANICS
51, 1, pp. 3-13, Warsaw 2013

LEAST SQUARES METHOD MODIFICATION APPLIED TO THE NASGRO EQUATION

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The paper has been intended to present some modification of Least Squares Method (LSM) as used for describing of experimentally gained fatigue crack propagation data by means of the NASGRO equation. In particular, the specific nature of the NASGRO equation and consequent difficulties with theoretical description of test data have been shown. An algorithm has been presented of how to find coefficients of the NASGRO equation for the modified LSM criterion. Computations have been performed for the aluminum alloy 2024 taken from the rotor blades of Mi-8 helicopter.

Key words: fatigue crack propagation, NASGRO equation, method of least squares (LSM)

1. Introduction

The analysis of fatigue crack propagation is the most important factor in the study of stability and lifespan of structural components, but it may require time and expense to investigate it experimentally. Computer simulation is especially useful for studying the crack propagation problem in cases when it is difficult to obtain detailed results by direct experimentation. Hence, in order to be efficient, fatigue crack propagation software should estimate the remaining life of any construction or structural component both experimentally and by simulation. The critical size of the crack can be calculated using material constants which have been derived experimentally and from the constant amplitude crack propagation curve, crack size-life data and curve using crack propagation software. Many publications in the field of fracture mechanics prove significant development in the numerical analysis of test data from fatigue crack propagation tests.

The maximum likelihood method and the second moment approximation is a simple stochastic crack growth analysis method, and the crack growth rate is considered to be random variable. A deterministic differential equation is used for the crack growth rate, while it is assumed that parameters in this equation are random variables. The analytical methods are implemented into engineering use and to estimate the statistics of the crack growth behavior (Xing and Hong, 1999).

However, due to the number and complexity of mechanisms involved in this problem, there are probably as many equations as there are researchers in the field. Though many models have been developed, none of them enjoys universal acceptance. In more detail, each model can only account for one or several phenomenological factors. Moreover, the applicability of each model varies from case to case, there is no general agreement among the researchers to select any fatigue crack growth model in relation to the concept of fatigue crack behaviour (Farahmand, 2006; Kujawski, 2001; Mohanty *et al.*, 2009; Murthy *et al.*, 2004).

Mathematical models proposed e.g. by Paris, Forman, and further modifications thereof describe crack propagation with account taken of such factors as: material properties, geometry of a test specimen/structural component, acting loads and the sequence of these loads (Paris and Erdogan, 1963; Forman *et al.*, 1967; Willenborg *et al.*, 1971; Wheeler, 1972; Kłysz, 2001). Application of the NASGRO equation, derived by Forman and Newman from NASA, de Koning from NLR and Henriksen from ESA, of the general form (Paris and Erdogan, 1963; Forman *et al.*, 1967)

$$\frac{da}{dN} = C\Big(\frac{1-f}{1-R}\Delta K\Big)^n \frac{\Big(1-\frac{\Delta K_{th}}{\Delta K}\Big)^p}{\Big(1-\frac{K_{max}}{K_c}\Big)^q} \tag{1.1}$$

has significantly extended possibilities of describing the crack growth rate tested according to the standard (Wheeler, 1972). The coefficients stand for (Paris and Erdogan, 1963; Forman *et al.*, 1967; [11], [12])

a	–	crack length [mm]
N	–	number of load cycles
C, n, p, q	–	empirical coefficients
R	–	stress ratio
ΔK	–	the stress-intensity-factor (SIF) range that depends on the size of the specimen, applied loads, crack length, $\Delta K = K_{max} - K_{min}$
ΔK_{th}	–	the SIF threshold, i.e. minimum value of ΔK, from which the crack starts to propagate

$$\Delta K_{th} = \Delta K_1 \sqrt{\frac{a}{a+a_0}} \frac{\Big(\frac{1-R}{1-f}\Big)^{1+RC_{th}}}{(1-A_0)^{(1-R)C_{th}}} \tag{1.2}$$

or

$$\Delta K_{th} = \Delta K_0 \sqrt{\frac{a}{a+a_0}} \Big(\frac{1-f}{(1-A_0)(1-R)}\Big)^{-(1+C_{th}R)} \tag{1.3}$$

where: a_0 – structural crack length that depends on the material grain size [mm], ΔK_0 – threshold SIF at $R \to 0$, ΔK_1 – threshold SIF at $R \to 1$, C_{th} – curve control coefficient for different values of R; equals 0 for negative R, equals 1 for $R \geqslant 0$, for some materials it can be found in the NASGRO database

K_{max}	–	SIF for the maximum loading force in the cycle
K_c	–	critical value of SIF
f	–	Newman's function describing closing of the crack

$$f = \begin{cases} \max(R, A_0 + A_1R + A_2R^2 + A_3R^3) & \text{for} \quad R \geqslant 0 \\ A_0 + A_1R & \text{for} \quad -2 \leqslant R < 0 \end{cases} \tag{1.4}$$

where A_0, A_1, A_2, A_3 coefficients are equal

$$\begin{aligned} A_0 &= (0.825 - 0.34\alpha + 0.05\alpha^2)\sqrt[\alpha]{\cos\Big(\frac{\pi}{2}\frac{S_{max}}{\sigma_0}\Big)} & \qquad A_1 &= (0.415 - 0.071\alpha)\frac{S_{max}}{\sigma_0} \\ A_2 &= 1 - A_0 - A_1 - A_3 & A_3 &= 2A_0 + A_1 - 1 \end{aligned} \tag{1.5}$$

$\alpha, S_{max}/\sigma_0$ – Newman's empirical coefficients.

Determination of the above coefficients for the equation that correctly approximates test data is difficult and causes some singularities when the Least Squares Method (LSM) is used; it can become feasible when the below proposed modification is introduced.

2. Data analysis

Analysis was based on fatigue tests conducted with the Round Compact Tension (RCT) specimen (Fig. 1) made of duralumin alloy. Analyzed specimens were of thickness $B = 2\,\text{mm}$, width $W = 27.5\,\text{mm}$, initial crack length $a = 7\,\text{mm}$. Tests were conducted in accredited AFIT Laboratory for Materials Strength Testing, according to appropriate standard [2], under laboratory conditions, with the loading frequency 15 Hz, for three values of stress ratio $R = 0.1$; 0.5; 0.8. The crack length was measured with the COD *clip gauge* using the compliance method. The crack propagation rate was determined using the polynomial method. The test data come from the examination of the 2024 aluminum alloy taken from the Mi-8 helicopter rotor blades [4].

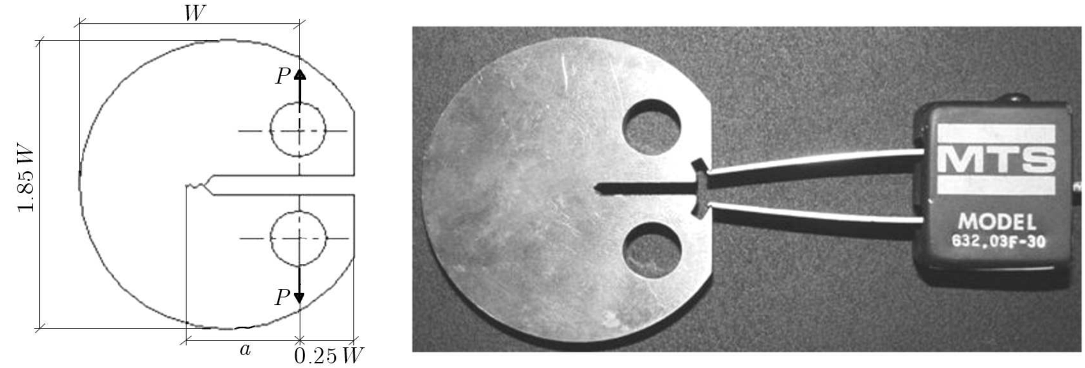

Fig. 1. RCT specimen used in fatigue crack propagation tests

The formula that describes the stress intensity factor for the RCT specimen is as follows

$$K_I = \frac{P}{B\sqrt{W}} Y \tag{2.1}$$

where: P – applied force, Y – the shape function of the specimen, for the RCT specimen ([2], Fuchs and Stephens, 1980)

$$Y = \frac{2 + \frac{a}{W}}{\sqrt{1 - \frac{a}{W}}} \Big[0.886 + 4.64\frac{a}{W} - 13.32\Big(\frac{a}{W}\Big)^2 + 14.72\Big(\frac{a}{W}\Big)^3 - 5.56\Big(\frac{a}{W}\Big)^4\Big] \tag{2.2}$$

where a/W is a nondimensional crack length.

The compliance function to compute the crack length in the RCT specimen has the form

$$\frac{a}{W} = 1 - 4.459u + 2.066u^2 - 13.041u^3 + 167.627u^4 - 481.4u^5 \tag{2.3}$$

where: u – compliance described by the following formula

$$u = \frac{1}{1 + \sqrt{\frac{EB\,COD}{F}}}$$

and E – Young's modulus, COD – Crack Opening Displacement.

Results of fatigue crack growth rate tests for 9 specimens (3 specimens for each value of R) are presented in the figures below and approximated with NASGRO equation – in different configurations, separately for each specimen or divided into groups with different procedures used for defining coefficients of the above mentioned equation.

The degree of fit of the theoretical description to test data using the NASGRO equation can be almost perfect if a single da/dN-ΔK propagation curve is considered. In Fig. 2, sample curves for 3 selected specimens are shown, each of which was tested at different R. It can be concluded that the defined NASGRO equation coefficients are valid. The coefficients were defined using the Least Square Method in which the criterion is to achieve the minimum sum of squared deviations between approximated and approximating values of the form

$$S = \sum_{i=1}^{n} (\overline{y}_i - y_i)^2 \tag{2.4}$$

where $\overline{y}_i$ are approximations of the test data y_i (i.e. values of da/dN_i).

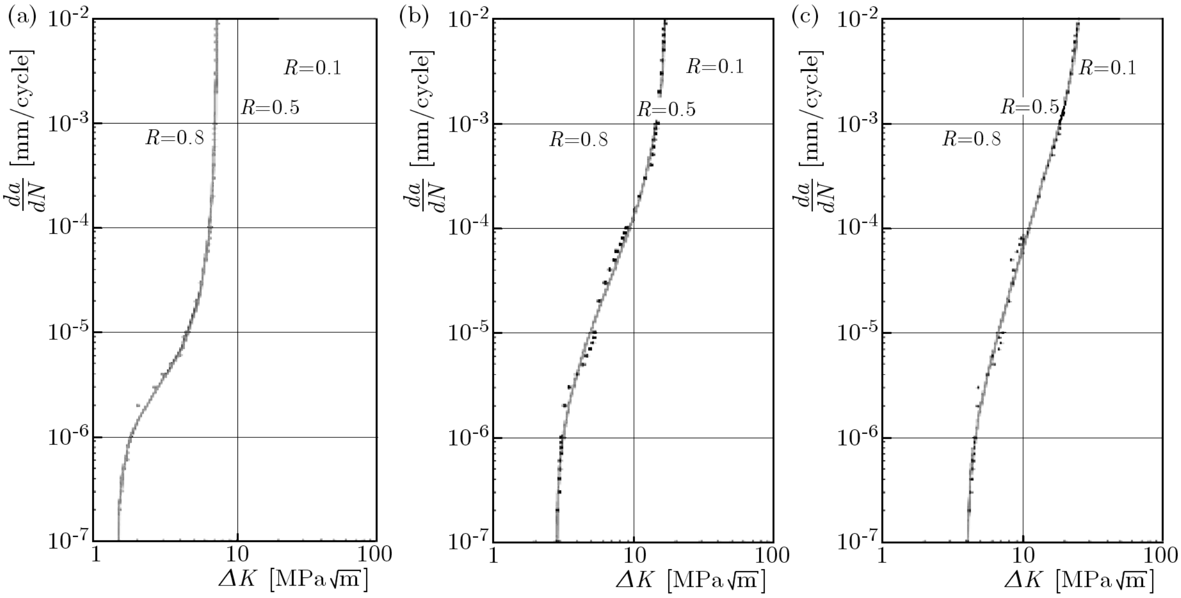

Fig. 2. Approximation of the test data for a single da/dN-ΔK curve with the NASGRO equation, for: (a) $R = 0.8$; (b) $R = 0.5$ and (c) $R = 0.1$

However, if one applies these coefficients to describe the courses of two remaining data sets corresponding to specimens tested at different values of R, then this description is not valid – Fig. 3 (curves plotted against test data points of all 9 tested specimens – description thereof under such conditions would, of course, be analogous – for readability reasons they are not shown on the graphs). It can be seen that for these cases the description of test data for specimens tested at different values of R is not valid.

It can also be seen in Figs. 3b and 3c that the approximation in the range of high values of ΔK and da/dN (critical range) is better than that for smaller values (threshold range).

However, if one tries to describe all 9 curves, for the same LSM criterion, then the effect would be as shown in Fig. 4. Graphs 4a correspond to the case when constant values of K_c and ΔK_{th} parameters are assumed for all values of R. Graphs 4b correspond to the theoretical case with different values of K_c and ΔK_{th} parameters assumed, following the test results for individual values of R. Graphs 4c for the assumed different values of K_c and ΔK_{th} parameters are, according to the test results, for individual specimens/curves. The presented approximations have been obtained while complying with the LSM criterion, i.e. the minimum of the sum S (in case 4a, a scaled measure of the sum S is equal to 161.5; in case 4b, to 167.8 – this could indicate in favour of the first case). However, it can be seen that in both cases the description is not valid – only for the highest values of da/dN the approximating curves show good correlation with the test data.

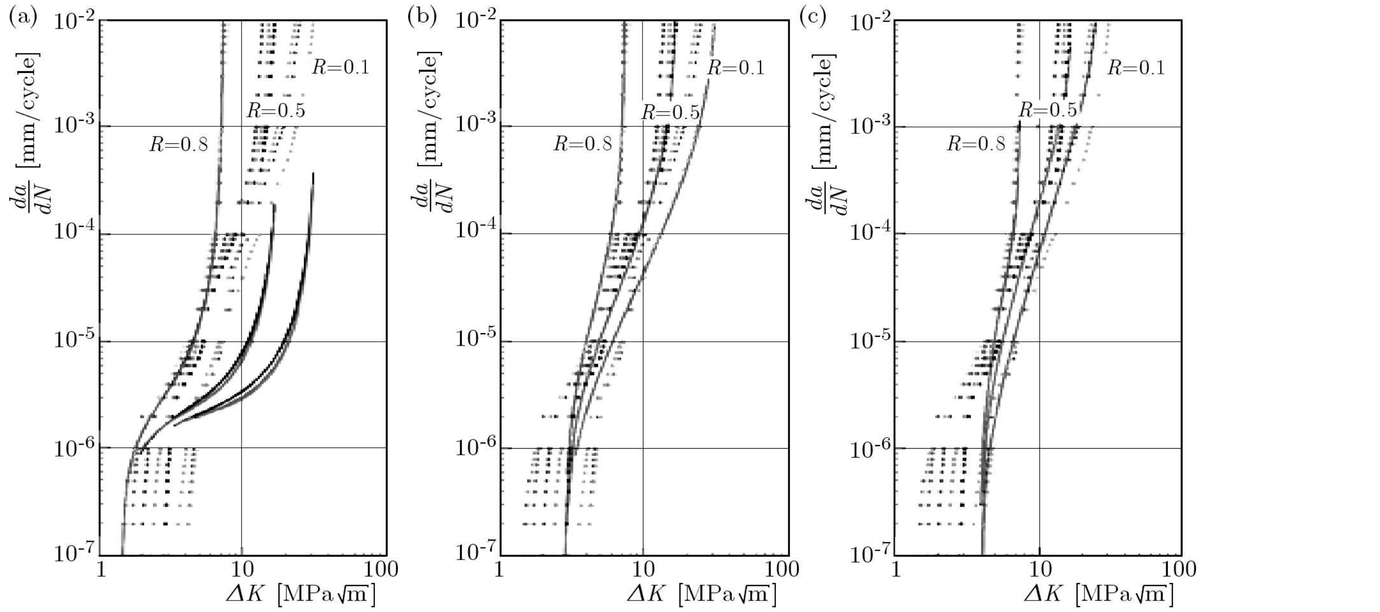

Fig. 3. Approximation of the test data for two different da/dN-ΔK curves with different values of R, based on the same coefficients as used to plot graphs in Fig. 2

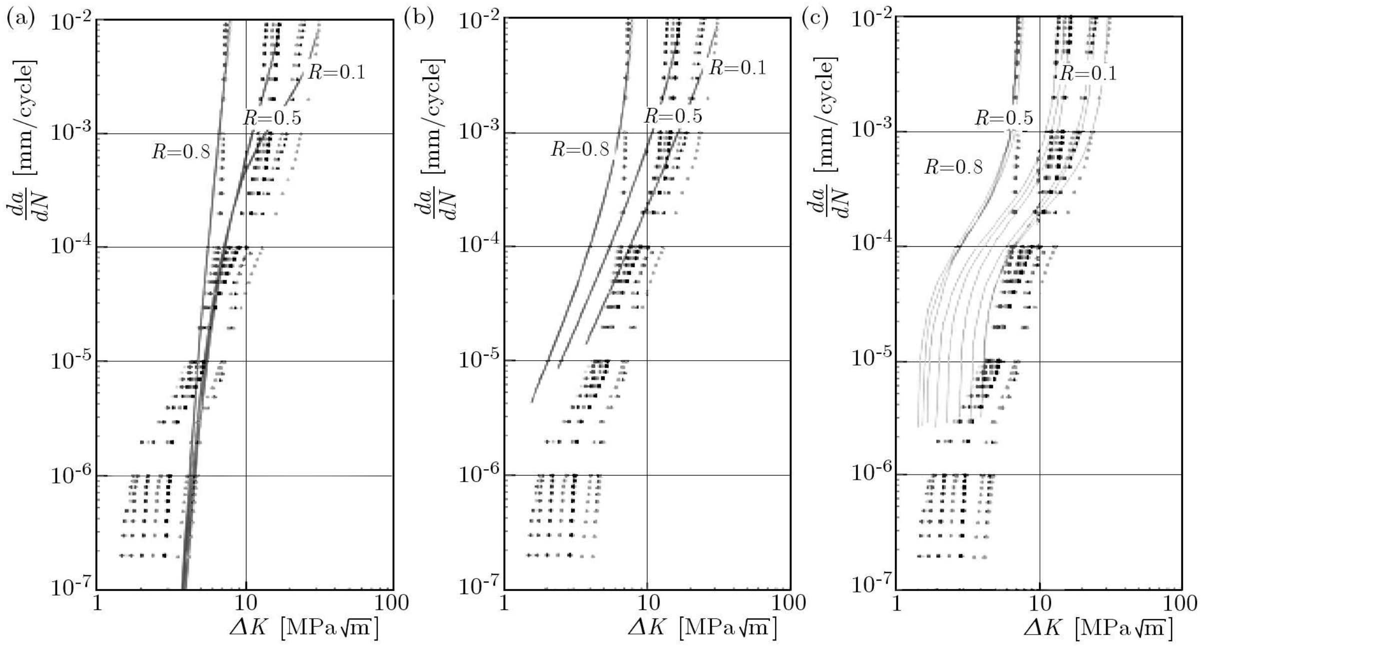

Fig. 4. Approximations of the test data for 9 da/dN-ΔK curves using the NASGRO equation, for cases: (a) $K_c = \text{const}$, $\Delta K_{th} = \text{const}$; (b) theoretically assumed $K_c = f(R)$, $\Delta K_{th} = f(R)$; (c) theoretically assumed K_c and ΔK_{th} fitted separately for each curve

This is a result of chosen LSM criterion (2.4) that has the following properties, which sometimes may prove their disadvantages:

- value of the sum S increases regarding the order of magnitude, as values of the approximated numbers grow – if the values of test data are of the order e.g. 10, 1000, 1000000, then with the scatter of 10% the summed differences are of the order 1, 100, 100000 – and therefore, consequently:
  - the same, e.g. 2-, 5-times change in the test results in different as to the order of magnitude values of the summed differences;

– the dynamics of changes in the total value of S depends on the size of values of differences; as a quadratic function it is characterized by a linear function of the derivative, which also means that in the case of differences close to zero (e.g. 10^{-5}, 10^{-8}, etc.) this dynamics is much smaller than for differences of higher values, which affects the "flexibility" of the performed approximation;
– if the test data represent a wide range of values (e.g. from 1 to 100000 or from 10^{-8} to 10^{-2}), the values approximated near the lower range give much smaller contribution to the sum S than the values approximated near the upper range – which means that, for example, tens or hundreds of test points which differ by 100% around the value of 1 are much less important in this approximation than one or a few points of 1% difference around the value of 100000;

- hence, the approximation is always "asymmetric" since it gives a better approximation for higher values of the test data neglecting at the same time differences around lower values – an example of such an approximation is shown in Fig. 1. It shows a good fit of the theoretical description of the 9 curves around large values of da/dN (above 10^{-4} mm/cycle) with a total misfit around the smallest values (below 10^{-5} mm/cycle).

If the test data fit in a wide range of values, e.g. 5 orders of magnitude as in this case from 10^{-2} to 10^{-7} mm/cycle, then the sum of squared deviation S will be much more sensitive to differences between the highest test values and the value of the approximating function, while differences even by two or three orders of magnitude for lower values do not significantly contribute to the value of this sum.

Hence, the misfit of the approximating function for low values of da/dN, practically for values lower by only one order of magnitude than the maximum values of da/dN, i.e. below 10^{-3} mm/cycle. Within this range the theoretical description is rather random and has rather no effect on the value of the sum S,which indicates that this criterion is rather useless for this type of analysis.

Hence, it seems reasonable to use one of the following criterion modifications, which will allow one to remove the above stated problems:

- changing the form of the criterion, or
- using logarithmic values of da/dN

$$S = \sum_{i=1}^{n} (\overline{\log y_i} - \log y_i)^2 \qquad \text{or} \qquad S = \sum_{i=1}^{n} (\log \overline{y}_i - \log y_i)^2 \tag{2.5}$$

In the present study, the first variant has been examined due to the fact that it is more general since it does not limit itself only to positive values of predicted y_i, which is a requirement in the second variant. In the case of crack propagation test data, all the da/dN values are positive; therefore the second variant could also be used.

3. Modification of the optimisation criterion

The following modified LSM criterion proposed in Kłysz *et al.* (2010) is suggested to test data described by means of the NASGRO equation, i.e. for the sum of squared deviations of the form

$$S = \sum_{i=1}^{n} \Big(\frac{\overline{y}_i - y_i}{y_i}\Big)^2 = \sum_{i=1}^{n} \Big(\frac{\overline{y}_i}{y_i} - 1\Big)^2 \tag{3.1}$$

instead of standard (2.4).

The fraction in brackets in formula (3.1), as a relative error, is a measure of deviation independent of the order of magnitude of the compared values (approximated and approximating ones), so that the contribution of all the test data is equally "strong" to the total error S, which should have good influence on the approximation within the whole range.

In order to carry out the approximation of the test data, it is necessary to calculate coefficients of the approximating equation used to determine $\overline{y}_i$. Equation (1.1), after applying logarithms, takes the form

$$\log\frac{da}{dN} = \log C + n\log\Big(\frac{1-f}{1-R}\Delta K\Big) + p\log\Big(1-\frac{\Delta K_{th}}{\Delta K}\Big) - q\log\Big(1-\frac{K_{max}}{K_c}\Big) \tag{3.2}$$

and can be presented in the following general way

$$\overline{y} = b_0 + b_1 f_1 + b_2 f_2 + b_3 f_3 \tag{3.3}$$

Coefficients b_i $(i = 0,1,2,3)$ are directly connected with C, n, p and q $(b_0 = \log C$, $b_1 = n$, $b_2 = p$, $b_3 = -q)$, whereas functions f_i are dependent on ΔK and R and include all the remaining coefficients of the NASGRO equation. Coefficients b_i of the approximating equation are calculated from the minimum condition of equation (3.1), i.e.

$$\frac{\partial S}{\partial b_k} = \frac{\partial \sum_{i=1}^{n}\Big(\frac{b_0+b_1f_{1,i}+b_2f_{2,i}+b_3f_{3,i}-y_i}{y_i}\Big)^2}{\partial b_k} = 0 \qquad k = 0,1,2,3 \tag{3.4}$$

This leads to the following system of equations

$$\begin{aligned}
\frac{\partial S}{\partial b_0} &= 2\sum_{i=1}^{n}\Big[\Big(\frac{b_0+b_1f_{1,i}+b_2f_{2,i}+b_3f_{3,i}-y_i}{y_i}\Big)\frac{1}{y_i}\Big] = 0\\
\frac{\partial S}{\partial b_1} &= 2\sum_{i=1}^{n}\Big[\Big(\frac{b_0+b_1f_{1,i}+b_2f_{2,i}+b_3f_{3,i}-y_i}{y_i}\Big)\frac{f_1}{y_i}\Big] = 0\\
\frac{\partial S}{\partial b_2} &= 2\sum_{i=1}^{n}\Big[\Big(\frac{b_0+b_1f_{1,i}+b_2f_{2,i}+b_3f_{3,i}-y_i}{y_i}\Big)\frac{f_2}{y_i}\Big] = 0\\
\frac{\partial S}{\partial b_3} &= 2\sum_{i=1}^{n}\Big[\Big(\frac{b_0+b_1f_{1,i}+b_2f_{2,i}+b_3f_{3,i}-y_i}{y_i}\Big)\frac{f_3}{y_i}\Big] = 0
\end{aligned} \tag{3.5}$$

It is a system of 4 linear equations with 4 unknowns b_i, which after transformation takes the form

$$\begin{aligned}
&n\sum_{i=1}^{n}\frac{1}{y_i} - b_0\sum_{i=1}^{n}\frac{1}{y_i^2} - b_1\sum_{i=1}^{n}\frac{f_{1,i}}{y_i^2} - b_2\sum_{i=1}^{n}\frac{f_{2,i}}{y_i^2} - b_3\sum_{i=1}^{n}\frac{f_{3,i}}{y_i^2} = 0\\
&n\sum_{i=1}^{n}\frac{f_{1,i}}{y_i} - b_0\sum_{i=1}^{n}\frac{f_{1,i}}{y_i^2} - b_1\sum_{i=1}^{n}\frac{f_{1,i}^2}{y_i^2} - b_2\sum_{i=1}^{n}\frac{f_{1,i}f_{2,i}}{y_i^2} - b_3\sum_{i=1}^{n}\frac{f_{1,i}f_{3,i}}{y_i^2} = 0\\
&n\sum_{i=1}^{n}\frac{f_{2,i}}{y_i} - b_0\sum_{i=1}^{n}\frac{f_{2,i}}{y_i^2} - b_1\sum_{i=1}^{n}\frac{f_{1,i}f_{2,i}}{y_i^2} - b_2\sum_{i=1}^{n}\frac{f_{2,i}^2}{y_i^2} - b_3\sum_{i=1}^{n}\frac{f_{2,i}f_{3,i}}{y_i^2} = 0\\
&n\sum_{i=1}^{n}\frac{f_{3,i}}{y_i} - b_0\sum_{i=1}^{n}\frac{f_{3,i}}{y_i^2} - b_1\sum_{i=1}^{n}\frac{f_{1,i}f_{3,i}}{y_i^2} - b_2\sum_{i=1}^{n}\frac{f_{2,i}f_{3,i}}{y_i^2} - b_3\sum_{i=1}^{n}\frac{f_{3,i}^2}{y_i^2} = 0
\end{aligned} \tag{3.6}$$

and is easily solved by subtracting in the following steps:

— eliminating b_0

$$
\begin{gathered}
\frac{n\sum\limits_{i=1}^{n}\frac{1}{y_i}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{n\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}-b_1\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}^2}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}\right)-b_2\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}f_{2,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}\right) \\
-b_3\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}\right)=0 \\
\frac{n\sum\limits_{i=1}^{n}\frac{1}{y_i}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{n\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}-b_1\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}f_{2,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}\right)-b_2\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{2,i}^2}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}\right) \\
-b_3\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{2,i}f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}\right)=0 \\
\frac{n\sum\limits_{i=1}^{n}\frac{1}{y_i}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{n\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}-b_1\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{1,i}f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}\right)-b_2\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{2,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{2,i}f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}\right) \\
-b_3\left(\frac{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{1}{y_i^2}}-\frac{\sum\limits_{i=1}^{n}\frac{f_{3,i}^2}{y_i^2}}{\sum\limits_{i=1}^{n}\frac{f_{3,i}}{y_i^2}}\right)=0
\end{gathered}
\tag{3.7}
$$

what gives 3 equations of the general form

$$
B_k-b_1B_{1,k}-b_2B_{2,k}-b_3B_{3,k}=0 \qquad k=1,2,3 \tag{3.8}
$$

— eliminating b_1

$$
\begin{aligned}
&\frac{B_1}{B_{1,1}}-\frac{B_2}{B_{2,1}}-b_2\Big(\frac{B_{1,2}}{B_{1,1}}-\frac{B_{2,2}}{B_{2,1}}\Big)-b_3\Big(\frac{B_{1,3}}{B_{1,1}}-\frac{B_{2,3}}{B_{2,1}}\Big)=0 \\
&\frac{B_1}{B_{1,1}}-\frac{B_3}{B_{3,1}}-b_2\Big(\frac{B_{1,2}}{B_{1,1}}-\frac{B_{3,2}}{B_{3,1}}\Big)-b_3\Big(\frac{B_{1,3}}{B_{1,1}}-\frac{B_{3,3}}{B_{3,1}}\Big)=0
\end{aligned}
\tag{3.9}
$$

what gives 2 equations of the general form

$$
C_k-b_2C_{2,k}-b_3C_{3,k}=0 \qquad k=2,3 \tag{3.10}
$$

— eliminating b_2

$$
\frac{C_2}{C_{2,2}}-\frac{C_3}{C_{2,3}}-b_3\Big(\frac{C_{3,2}}{C_{2,2}}-\frac{C_{3,3}}{C_{2,3}}\Big)=0 \tag{3.11}
$$

hence

$$
b_3=\frac{\frac{C_2}{C_{2,2}}-\frac{C_3}{C_{2,3}}}{\frac{C_{3,2}}{C_{2,2}}-\frac{C_{3,3}}{C_{2,3}}}=\frac{\dfrac{\frac{B_1}{B_{1,1}}-\frac{B_2}{B_{2,1}}}{\frac{B_{1,2}}{B_{1,1}}-\frac{B_{2,2}}{B_{2,1}}}-\dfrac{\frac{B_1}{B_{1,1}}-\frac{B_3}{B_{3,1}}}{\frac{B_{1,2}}{B_{1,1}}-\frac{B_{3,2}}{B_{3,1}}}}{\dfrac{\frac{B_{1,3}}{B_{1,1}}-\frac{B_{2,3}}{B_{2,1}}}{\frac{B_{1,2}}{B_{1,1}}-\frac{B_{2,2}}{B_{2,1}}}-\dfrac{\frac{B_{1,3}}{B_{1,1}}-\frac{B_{3,3}}{B_{3,1}}}{\frac{B_{1,2}}{B_{1,1}}-\frac{B_{3,2}}{B_{3,1}}}} \tag{3.12}
$$

Hence, the coefficient b_2 can be calculated from one of formulae (3.9); secondly, the coefficient b_1 from one of equations (3.7), and finally, the coefficient b_0 from one of equations (3.6).

The result of approximation with the coefficients calculated by means of the modified LSM criterion is shown in Fig. 5, and values of these coefficients are presented in Table 1.

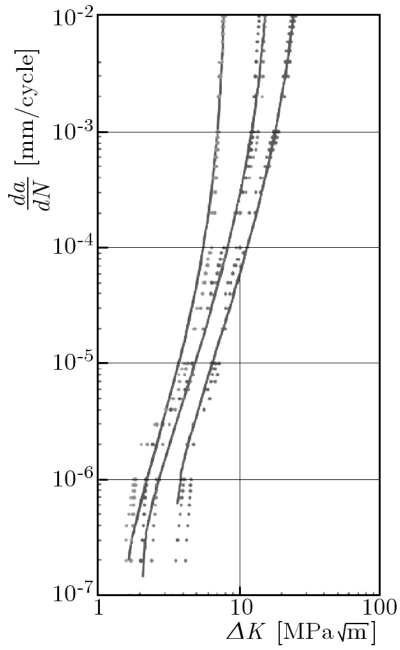

Fig. 5. Approximation of da/dN-ΔK test data based on modified LSM criterion (3.1)

Table 1. The NASGRO equation coefficients for curves from Fig. 5 and correlation coefficients for the obtained approximation

α	S_{max}/σ_0	a_0	ΔK_{th}	K_c	C	n	p	q	Correlation factor
2.0	0.3	0.0381	1.60 2.08 3.61	41.17 35.22 33.16	$7.172 \cdot 10^{-8}$	3.0089	0.2452	2.0012	0.8517

Coefficients ΔK_{th} and K_c have been assumed constant for individual tests for a given value of R – estimated also by the criterion of the minimum sum S.

Significant improvement in the theoretical description of the test data da/dN-ΔK is clearly visible within the whole range.

4. Conclusion

Using the Least Square Method in its classical form (2.4) to determine coefficients, e.g. for the NASGRO equation describing fatigue crack propagation curves da/dN-ΔK, is ineffective since this approximation criterion does not prove correct when values of the approximated curves (da/dN) are within the range of few orders of magnitude. The test data are not evenly distributed along the curve (ranges: threshold, stable growth, critical) and several curves grouped in sets for different values of R are being approximated at the same time.

In this paper, modification (3.1) of the approximation criterion has been proposed for the theoretical description of test curves da/dN-ΔK, which results in an improvement of the approximation effects. The presented methodology shows the following advantages:

- it is effective while approximating individual, several, as well as large numbers of sets/curves of test data,
- it is more accurate, smaller scatters of the test data are observed within the same groups (for single test),
- it can be used for other similar analyses involving test data regression since it contains a universal procedure not associated only with da/dN-ΔK crack propagation curves.

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Modyfikacja metody najmniejszych kwadratów w zastosowaniu do równania NASGRO

Streszczenie

W pracy zaprezentowano modyfikacje Metody Najmniejszych Kwadratów (MNK) w zastosowaniu do opisu przy pomocy równania NASGRO danych doświadczalnych propagacji pęknięć zmęczeniowych. W szczególności wskazano na szczególną postać równania NASGRO i wynikające z tego trudności w teoretycznym opisie wyników badań doświadczalnych. Przedstawiono algorytm do wyznaczania współczynników tego równania przy zmodyfikowanym kryterium MNK. Obliczenia przeprowadzono dla stopu aluminium 2024 pobranego z łopat wirnika śmigłowca Mi-8.

Manuscript received September 29, 2011; accepted for print November 23, 2011