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"STICK-SLIP" MOTION OF OPEN MANIPULATORS WITH FLEXIBLE DRIVES AND DRY FRICTION IN JOINTS

Andrzej Harlecki

Faculty of Machine Design
Technical University of Łódź Branch in Bielsko-Biała
e-mail: aharlecki@pb.bielsko.pl

An analysis of the phenomenon of "stick-slip" vibrations of arms of open manipulators with flexible drives and dry friction in joints is carried out in the paper. A method of physical and mathematical modelling of such systems for the purposes of dynamic analysis is presented. An example of dynamic analysis of a manipulator with revolute and prismatic joints has been carried out. Finally, some results of numerical simulation, illustrating the phenomenon of "stick-slip" motion, are presented.

Key words: dynamics, dry friction, open manipulator

1. Introduction

Modern industrial robots impose demanding requirement on the design process, especially in the view point of positioning accuracy of robot grippers. Flexibility of drive systems and friction in joints (kinematic pairs) exert a considerable influence on this accuracy. The clearances, occurring mainly in transmissions of drive systems, also determine the positioning accuracy.

The flexibility of drives, resulting mainly from the flexibility of transmissions used, is a main undesirable feature of mechanical devices. In the case of robot drives, it is mostly harmonic gears that are found since these gears are relatively light and small drive components revealing high transmission ratio capability (cf Abdelraheim and Seireg, 1995a). An example of physical modelling of harmonic drives was presented by Abdelraheim and Seireg (1995b).

Industrial practise proves that in many cases sufficient lubrication can not be ensured in joints of machines including robots, especially in prismatic (translational) joints. In this case small fluid friction which normally occurs may develop into significantly larger boundary friction, and in extreme cases there may even be dry friction. Large loads acting upon mating links and low relative velocities in joints, for example in the phases of starting-up or breaking of links, are conducive to the dry friction. This second effect is illustrated by the Stribeck curve known in tribological literature which presents a course of kinetic friction coefficient μ versus relative sliding velocity v in the joint considered (Fig.1).

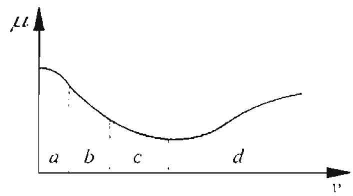

Fig. 1. Stribeck curve with regimes of: (a) dry friction, (b) boundary friction, (c) partial fluid friction, (d) full fluid friction

The dry friction, combined with the drive system flexibility (which is difficult to eliminate in practise), leads to a cumulation of undesirable effects, and finally the link vibrations called "stick-slip" vibrations occur. During these vibrations, very short phases of standstill (static friction phases) occur between the phases of relative motion of links (kinetic friction phases). It appears that so-called decreasing kinetic friction characteristics (Fig.2), usually present within the range of very small sliding velocities, are associated with the occurrence of this phenomenon (cf Harlecki and Wojciech, 1992).

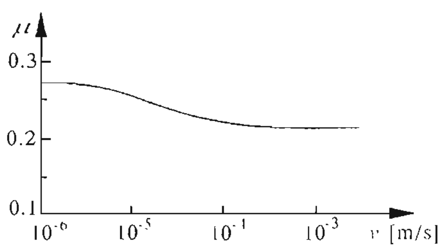

Fig. 2. Kinetic friction characteristic for pair of materials: steel-cast iron (Harlecki, 1995)

The problem of "stick-slip" vibrations is known in industry and has been dealt with by various authors. The phenomenon is particularly inconvenient in the case of machine tools. An extensive bibliography of investigations into this phenomenon in machine tool dynamics and attempts to eliminate it have been worked out. For example, the article by Soavi (1980), the often quoted paper by Bell and Burdekin (1969-70), and also the series of works by Kato and Matsubayashi (1970), Kato et al. (1970a,b) can be mentioned. Vibrations of the "stick-slip" type are met also in other devices. For example, Pfeifer (1992) considered the occurrence of this phenomenon in turbine blade dampers. Kolerus (1975) investigated the "stick-slip" problem in the case of engine wheels turning on rails. The "stick-slip" motion in micromechanisms was analysed by Suzuki and Itao (1995), and in telescopes by Hammerschlag (1986).

The problem of "stick-slip" perturbations in motion also arises in robotics where sticking behaviour is observed at final positioning. It is also difficult to eliminate this phenomenon in this case. In the publications devoted to the problem of "stick-slip" vibrations in robotics; e.g., Kubo et al.(1986), Ackermann and Müller (1990), the attempts to eliminate this problem by using suitable, often complicated, control methods have been described. Therefore, these articles concern the control problems.

However, the present paper concentrates on solving problems of dynamic analysis (forward dynamics) of robots with flexible drives and dry friction in joints. This is never a simple task in the case of mechanical devices such as robot manipulators with many degrees of freedom. Hence, the literature about this problem is rather scanty. The problems of forward dynamics in the case of robots with regard to dry friction in joints were solved among others by Gogoussis and Donath (1988,1990,1993), Wojciech (1995).

2. Characteristic of assumed physical models of manipulators

The method proposed can be used for dynamic analysis of any open spatial kinematic chain (manipulator) with n rigid links which are connected by revolute or prismatic joints (Fig.3).

In the case of rigid rotary or translational drive in a chosen joint i (where $1 \leqslant i \leqslant n$), the generalized coordinate q_i, describing motion of a link i, changes according to the assumed function of time t (Fig.4a and Fig.5a). This function can be treated as a kinematic input.

In the case of a flexible drive at this joint (Fig.4b and Fig.5b), the displacement q_i^* of the end of a non-dimensional spring, which models the drive flexibility, changes as an assumed function of time. On the other hand, the generalized coordinate q_i, as an unknown quantity, is calculated when the equations of motion are solved.

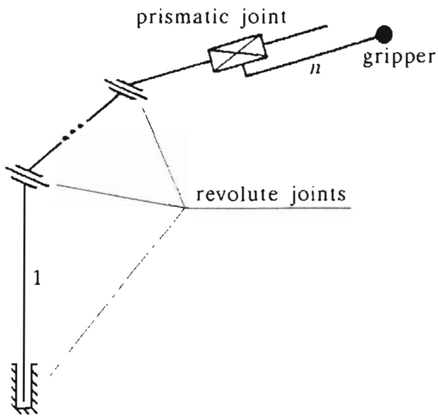

Fig. 3. Open spatial kinematic chain (manipulator)

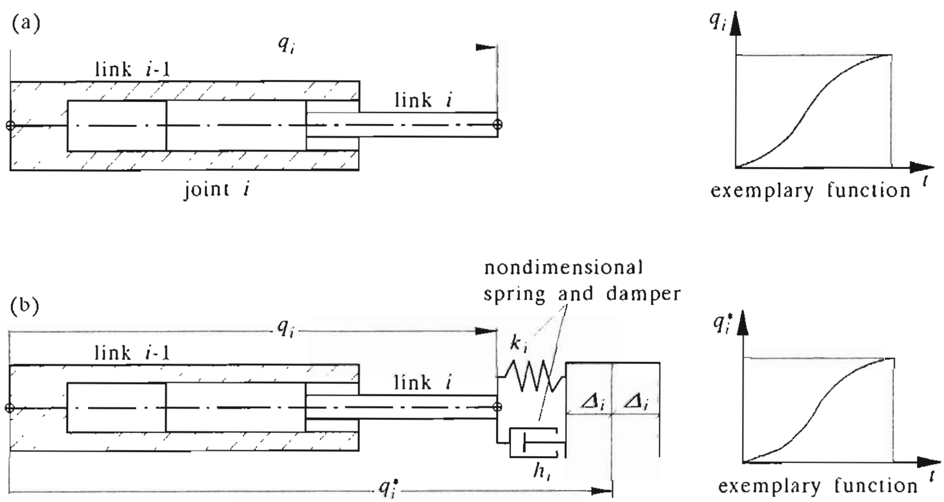

Fig. 4. Models of translational drives: (a) rigid, (b) flexible

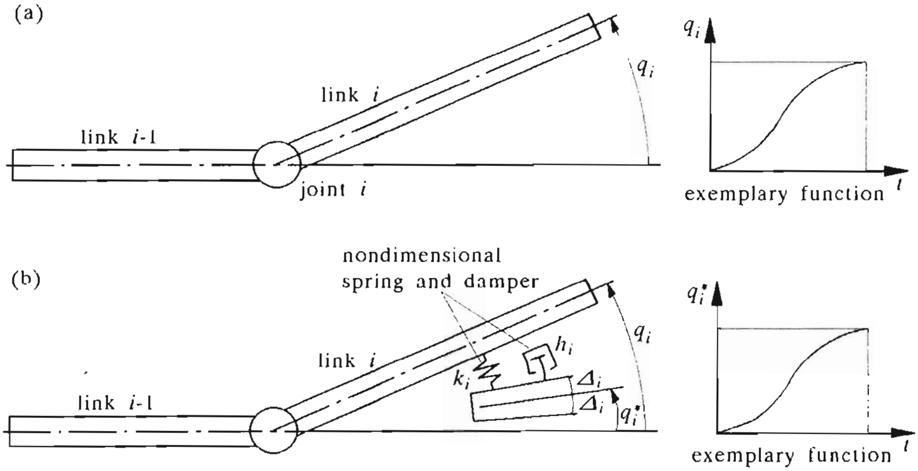

Fig. 5. Models of rotary drives: (a) rigid, (b) flexible

The models used allows us also to take into account a clearance occurring often in real drive systems (in figures denoted as Δ_i).

The driving force or moment of link i, connected with the previous $i-1$ link by a prismatic or revolute joint i, respectively, can be defined using the following formula

$$F_i^d = k_i\delta_i(q_i^* - q_i - \sigma_i\Delta_i) + h_i\delta_i(\dot{q}_i^* - \dot{q}_i) \tag{2.1}$$

where

$\dot{q}_i$	–	function q_i differentiated with respect to time
Δ_i	–	clearance in a drive i
k_i	–	stiffness coefficient of this drive
h_i	–	damping coefficient of this drive

and

$$\delta_i = \begin{cases} 1 & \text{when} \quad |q_i^* - q_i| \geqslant \Delta_i \\ 0 & \text{when} \quad |q_i^* - q_i| < \Delta_i \end{cases} \qquad \sigma_i = \operatorname{sgn}(q_i^* - q_i)$$

3. Mathematical model

The equations of motion of the manipulator can be derived on the basis of the Lagrange formalism using the Denavit-Hartenberg notation (cf Craig, 1989) for transformation of the coordinate systems.

If the vector $\boldsymbol{r}_i = [x_i, y_i, z_i, 1]^\top$, defining the position of chosen point in the coordinate system $x_i y_i z_i$ connected with the link i, is known, then the vector $\boldsymbol{r}_{i-1} = [x_{i-1}, y_{i-1}, z_{i-1}, 1]^\top$, defining the position of this point in the $x_{i-1} y_{i-1} z_{i-1}$ coordinate system connected with the link $i-1$, is defined as

$$\boldsymbol{r}_{i-1} = \mathbf{A}_i \boldsymbol{r}_i \qquad i = 1, ..., n \tag{3.1}$$

where $\mathbf{A}_i$ is the transformation matrix from $x_i y_i z_i$ to $x_{i-1} y_{i-1} z_{i-1}$ coordinate system.

When the transformation matrices $\mathbf{A}_1, ..., \mathbf{A}_i$ are known, the position of this point in the inertial coordinate system $x_0 y_0 z_0$ can be expressed according to the formula

$$\boldsymbol{r}_0 = \mathbf{B}_i \boldsymbol{r}_i \tag{3.2}$$

where $\mathbf{B}_i = \mathbf{A}_1 \mathbf{A}_2 ... \mathbf{A}_i$ is the transformation matrix from the coordinate system $x_i y_i z_i$ to the inertial coordinate system.

Potential and kinetic energies of chain as well as the equations of the motion were formulated according to the procedure presented by Jurevič (1984).

Having differentiated with respect to time the matrices $\mathbf{B}_i$, the kinetic energy of the whole system can be calculated in the following way

$$T = \frac{1}{2} \sum_{i=1}^{n} \operatorname{tr} (\dot{\mathbf{B}}_i \mathbf{H}_i \dot{\mathbf{B}}_i^\top) \tag{3.3}$$

where $\mathbf{H}_i$ is inertial matrix of link i.

The potential energy can be written as follows

$$V = g\boldsymbol{a}\boldsymbol{b} \tag{3.4}$$

where

$$\boldsymbol{a} = [0, 1, 0, 0] \qquad\qquad \boldsymbol{b} = \sum_{i=1}^{n} m_i \mathbf{B}_i \boldsymbol{r}_{c_i}$$

and

- g – acceleration of gravity
- m_i – mass of the link i
- $\boldsymbol{r}_{c_i}$ – position vector of the centre of mass of the link i in the system $x_i y_i z_i$.

For further consideration the following is defined

$$\mathbf{B}_i^j = \frac{\partial \mathbf{B}_i}{\partial q_j} \qquad\qquad \mathbf{B}_i^{js} = \frac{\partial \mathbf{B}_i^j}{\partial q_s} \qquad\qquad i, j, s, = 1, ..., n \tag{3.5}$$

The above quantities can be given as

$$\mathbf{B}_i^j = \begin{cases} \mathbf{A}_1 ... \mathbf{A}_{j-1} \mathbf{D}_j \mathbf{A}_j \mathbf{A}_{j+1} ... \mathbf{A}_i & \text{when} \quad j \leqslant i \\ \mathbf{0} & \text{when} \quad j > i \end{cases}$$

$$\mathbf{B}_i^{js} = \begin{cases} \mathbf{A}_1 ... \mathbf{A}_{j-1} \mathbf{D}_j \mathbf{A}_j ... \mathbf{A}_{s-1} \mathbf{D}_s \mathbf{A}_s ... \mathbf{A}_i & \text{when} \quad j < s \leqslant i \\ \mathbf{A}_1 ... \mathbf{A}_{j-1} \mathbf{D}_j^2 \mathbf{A}_j ... \mathbf{A}_i & \text{when} \quad j = s \leqslant i \\ \mathbf{0} & \text{when} \quad j > i \text{ and } s > i \end{cases} \tag{3.6}$$

where $\mathbf{D}_j$ – matrices with constant coefficients.

The relations presented allow a complicated matrix differentiation to be replaced by simple multiplication. Thus

$$\dot{\mathbf{B}}_i = \sum_{i=1}^{n} \mathbf{B}_i^j \dot{q}_j \tag{3.7}$$

and so the kinetic energy (3.3) can be rewritten in the following way

$$T = \frac{1}{2} \sum_{i=1}^{n} \sum_{j,l=1}^{i} \operatorname{tr} \left(\mathbf{B}_i^j \mathbf{H}_i \mathbf{B}_i^{l\top} \right) \dot{q}_j \dot{q}_l \tag{3.8}$$

The method presented does not require the formulation of the equations of motion in generalized coordinates of joints with rigid drives, since these coordinates are known functions of time. The equations of motion are formulated therefore only in the coordinates of joints with flexible drives on condition that the phases of relative motion (kinetic friction phases) are present in these joints. If the phase of standstill (stiction phase) in a given joint occurs, the suitable generalized coordinate will become a known function of time. The equation referring to this coordinate is removed from the system of motion equations. As a result, the number of equations defining the motion of manipulator is variable.

The equation of motion for chosen joint s (where $1 \leqslant s \leqslant n$) with flexible drive, in which there is relative motion, takes the form

$$\sum_{i=1}^{n} D_{si} \ddot{q}_i + \sum_{j=1}^{n} \sum_{i=j}^{n} D_{sji} \dot{q}_j \dot{q}_i + D_s = F_s \tag{3.9}$$

where

$$D_{si} = \sum_{l=\max\{i,s\}}^{n} \operatorname{tr}\left(\mathbf{B}_l^i \mathbf{H}_l \mathbf{B}_l^{s\top}\right) \qquad D_s = g\boldsymbol{a}\boldsymbol{b}^*$$

$$D_{sji} = \delta_{ji} \sum_{l=\max\{i,j,s\}}^{n} \operatorname{tr}\left(\mathbf{B}_l^{ji} \mathbf{H}_l \mathbf{B}_l^{s\top}\right) \qquad \boldsymbol{b}^* = \sum_{i=s}^{n} m_i \mathbf{B}_i^s \boldsymbol{r}_{c_i}$$

$$\delta_{ji} = \begin{cases} 1 & \text{when} \quad j = i \\ 2 & \text{when} \quad j \neq i \end{cases}$$

Then, the current number of above equations is equal to the number of joints with flexible drives and the phase of relative motion.

The generalized force F_s is calculated as

$$F_s = F_s^d - F_s^f \tag{3.10}$$

The driving force (moment) F_s^d is defined by Eq (2.1).

The method of calculation of forces or moments of kinetic friction F_s^f, which are functions of so-called joint forces f_s and moments n_s, will be presented below for prismatic and revolute joints, respectively. Joint forces and moments can be determined using the approach based on the recursive Newton-Euler formulation (cf Craig, 1989; Luh et al., 1980) which consists in realisation of two loops. The first one of them enables velocities and accelerations and subsequently the inertial forces and torques to be calculated, from the link 1 to the last link n. The second loop allows the joint forces and moments to be determined, from the last joint n to the joint 1.

3.1. Prismatic joints

It is assumed that the slider is part of link s, and the slideway is part of link $s-1$ (Fig.6a). It is assumed, moreover, that the slider can contact with slideway, as shown in Fig.6b, only at certain points, namely at the corners.

Knowing the components f'_{x_s}, f'_{y_s} of joint forces f'_s and the components n'_{x_s}, n'_{y_s} of joint moments n'_s, transformed to the centre of slider, it is relatively easy to calculate normal reaction forces acting at the slider corners. The whole normal load acting upon the slider is the sum of the components

$$\begin{aligned} N_s = {} & |N_{fy_s} + N_{nx_s} + N_{nz_s}^y| + |N_{fy_s} + N_{nx_s} - N_{nz_s}^y| + \\ & + |N_{fy_s} - N_{nx_s} + N_{nz_s}^y| + |N_{fy_s} - N_{nx_s} - N_{nz_s}^y| + |N_{fx_s} - N_{ny_s} + N_{nz_s}^x| + \\ & + |N_{fx_s} - N_{ny_s} - N_{nz_s}^x| + |N_{fx_s} + N_{ny_s} + N_{nz_s}^x| + |N_{fx_s} + N_{ny_s} - N_{nz_s}^x| \end{aligned} \tag{3.11}$$

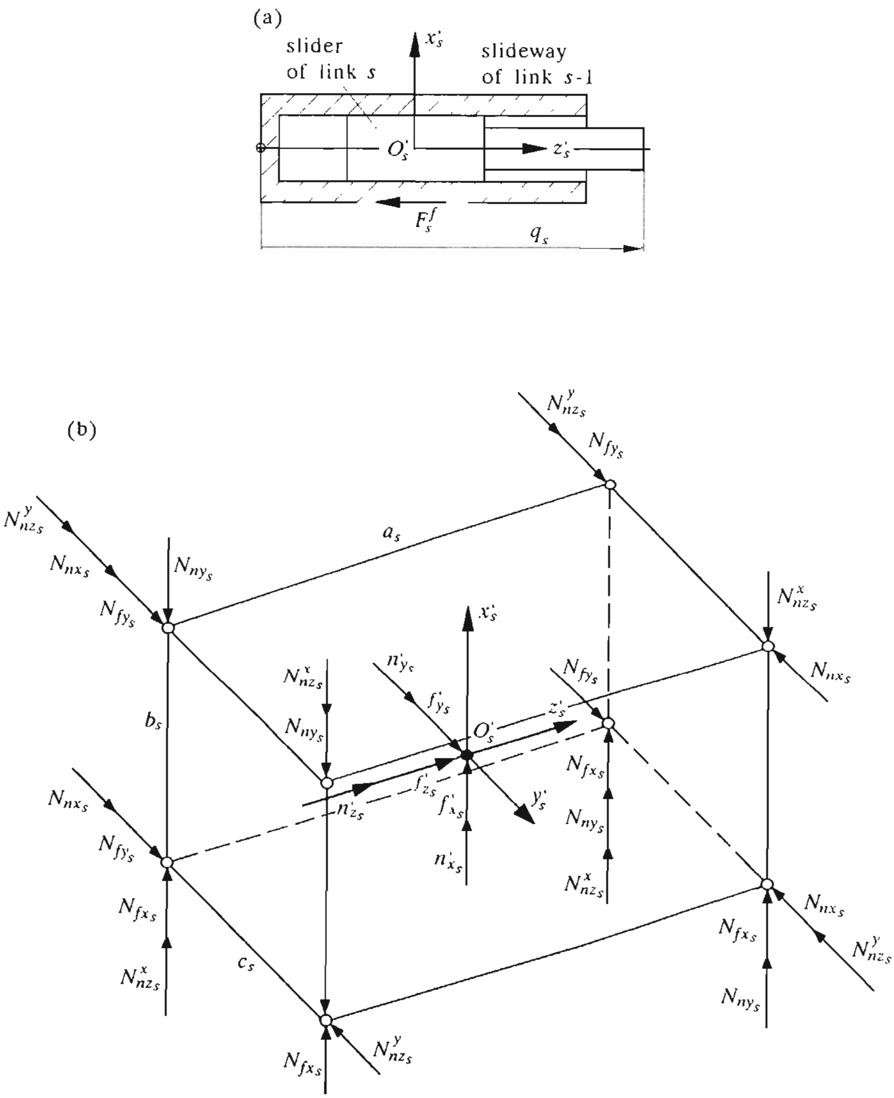

Fig. 6. (a) Prismatic joint, (b) normal reaction forces acting upon slider

where

$$N_{fx_s} = \frac{1}{4} f'_{x_s} \qquad N_{fy_s} = \frac{1}{4} f'_{y_s} \qquad N_{nx_s} = \frac{1}{2} \frac{n'_{x_s}}{a_s}$$
$$N_{ny_s} = \frac{1}{2} \frac{n'_{y_s}}{a_s} \qquad N^x_{nz_s} = \frac{1}{2} n'_{z_s} \frac{c_s}{b_s^2 + c_s^2} \qquad N^y_{nz_s} = \frac{1}{2} n'_{z_s} \frac{b_s}{b_s^2 + c_s^2}$$

Now, kinetic friction forces can be defined using the Coulomb formula

$$F^f_s = \mu_s N_s \operatorname{sgn} \dot{q}_s \tag{3.12}$$

where μ_s is the kinetic friction coefficient in the joint considered.

Other models of prismatic joints with dry friction were presented also by Luh et al. (1980), Szwedowicz (1991), Sharan et al. (1993).

3.2. Revolute joints

It is assumed that the pin is part of link s, and the sleeve is part of link $s-1$. Knowing the components f_{x_s}, f_{y_s}, of joint forces f_s and the components n_{x_s}, n_{y_s}, of joint moments n_s acting at the pin (Fig.7), the kinetic friction moment can be calculated as the following sum

$$F^f_s = F^f_{A_s} + F^f_{B_s} + F^f_{C_s} \tag{3.13}$$

The components of kinetic friction moment are defined as follows

$$\begin{aligned} F^f_{A_s} &= \mu_{A_s} R_{A_s} \frac{d_{A_s}}{2} \operatorname{sgn} \dot{q}_s \\ F^f_{B_s} &= \mu_{B_s} R_{B_s} \frac{d_{B_s}}{2} \operatorname{sgn} \dot{q}_s \\ F^f_{C_s} &= \frac{2}{3} \mu_{C_s} |f_{z_s}| \frac{d_{C_s}}{2} \operatorname{sgn} \dot{q}_s \end{aligned} \tag{3.14}$$

The normal reaction forces take the form

$$\begin{aligned} R_{A_s} &= \frac{1}{l_s} \sqrt{(l_{B_s} f_{x_s} - n_{y_s})^2 + (l_{B_s} f_{y_s} + n_{x_s})^2} \\ R_{B_s} &= \frac{1}{l_s} \sqrt{(l_{A_s} f_{x_s} + n_{y_s})^2 + (l_{A_s} f_{y_s} - n_{x_s})^2} \end{aligned} \tag{3.15}$$

The method of solving the equations of motion requires, at every step of integration, the application of a special iterative procedure based on the Newmark algorithm. This procedure was presented in detail by Wojciech (1995).

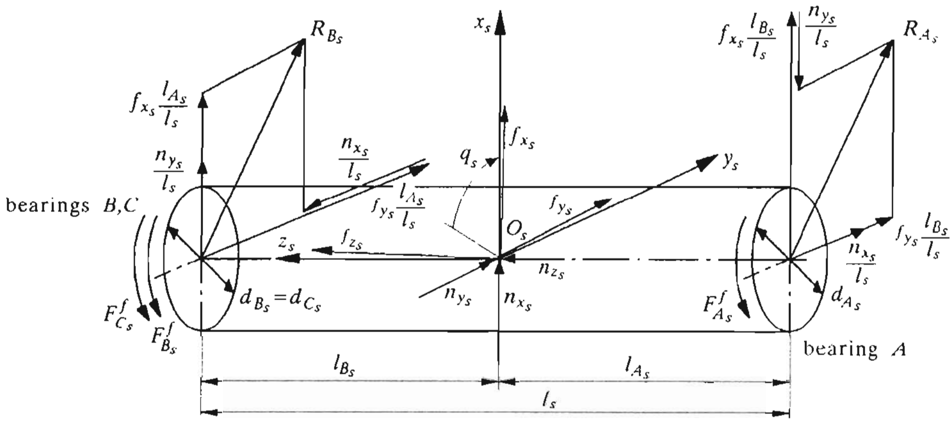

Fig. 7. Normal reaction forces acting at radial A, B and thrust C bearings of pin of link s

The friction in the joints causes a violent decrease in velocity, even to zero. The signal indicating possible occurrence of stiction phase in the given joint s is the change of sign of relative velocity $\dot{q}_s$ in this joint. However, the motion stops only when the following condition is fulfilled at the same time

$$|F_s^d - F_s^*| \leqslant \overline{F}_s^f \tag{3.16}$$

Knowing the component f'_{z_s} of joint force f'_s in the prismatic joint s (Fig.6b) and the component n_{z_s} of joint moment n_s in the revolute joint s (Fig.7), the force F_s^* can be defined as

$$F_s^* = \begin{cases} f'_{z_s} & \text{for the prismatic joint} \\ n_{z_s} & \text{for the revolute joint} \end{cases} \tag{3.17}$$

The static friction force or moment $\overline{F}_s^f$ can be defined by using Eqs (3.12) or (3.14) and (3.15), respectively, substituting for the values of kinetic friction coefficients μ_s or μ_{A_s}, μ_{B_s}, μ_{C_s} the values of static friction coefficients.

Transmission from the stiction phase to the kinetic friction phase in the joint considered will follow when the following condition is fulfilled

$$|F_s^d - F_s^*| > \overline{F}_s^f \tag{3.18}$$

Other models of revolute joints with dry friction were considered also by Wu et al. (1986), Szwedowicz (1991), Sharan and Dhanaraj (1993), Frączek (1993), Kłosowicz (1990).

4. Results of calculations

The analysed model of four-links spatial manipulator is shown in Fig.8. Motion of links is defined by angle q_1, q_2, q_4 and translational q_3 generalized coordinates. The following types of drives are used in the model: in the revolute joint 1 – flexible drive, in the revolute joint 2 – rigid drive, in the prismatic joint 3 – flexible drive, in the revolute joint 4 – rigid drive.

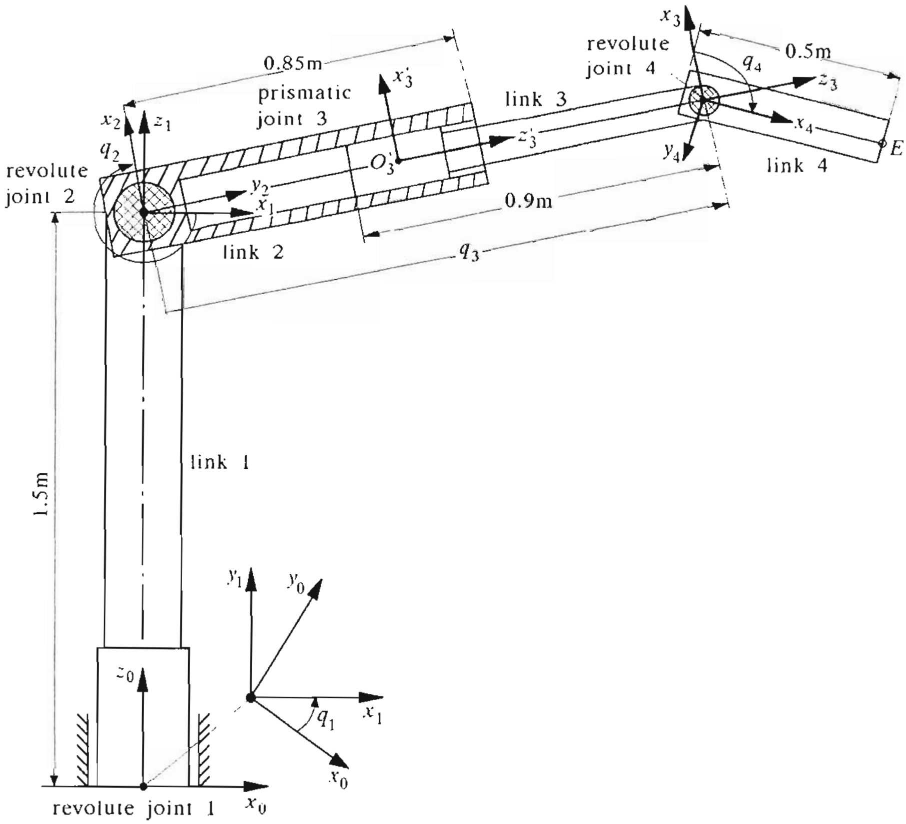

Fig. 8. Model of the analysed four-links manipulator

The main dimensions of the manipulator are shown in Fig.8. Some other dimensions and physical data of the system are specified below:

– diameters of pins (sleeves) of revolute joint 1: $d_{A_1} = d_{B_1} = d_{C_1} = 0.1\,\mathrm{m}$

– dimensions of slider in prismatic joint 3: $a_3 = 0.2\,\mathrm{m}$, $b_3 = c_3 = 0.07\,\mathrm{m}$

- distances of radial bearings A and B in joint 1: $l_1 = 0.12\,\mathrm{m}$, $l_{A_1} = l_{B_1} = 0.06\,\mathrm{m}$
- stiffness coefficients of drives 1,3: $k_1 = k_3 = 10^6\,\mathrm{Nm}^{-1}$
- damping coefficients of drives 1,3: $h_1 = h_3 = 0$
- kinetic friction coefficients in radial bearings A, B of joint 1: $\mu_{A_1} = \mu_{B_1} = 0.2$
- kinetic friction coefficient in thrust bearing C of joint 1: $\mu_{C_1} = 0.2$
- kinetic friction coefficient in prismatic joint 3: $\mu_3 = 0.2$.

The values of static friction coefficients were assumed to be equal to the values of kinetic friction coefficients.

The worked out physical and mathematical models of the manipulator considered allows one to carry out its dynamic analysis. The results of numerical simulation of the manipulator motion enable definition of the expected influence of different parameters, characterising the system properties, on its dynamic behaviour. The data characterising the physical system were assumed in general on the basis of literature and one should treat them as approximate values. In order to study more distinctly the influence of some system parameters on stability of motion, the values of some data were assumed to be rather improbable, for example zero values of damping coefficients in almost all examples of calculations. The author realizes that accurate complex dynamic analysis of real manipulators would require identification of their physical parameters. Large rigidity characterising the drive systems of arms made it necessary to emply a very small step of integration in calculation. The numerical experiments showed that in some cases the assumption of an integration step equal to at least $0.2 \cdot 10^{-5}\,\mathrm{s}$ ensures sufficient calculation accuracy. This fact is illustrated by the plots of real velocity $\dot{q}_3$ of link 3 as a function of time t obtained for different integration steps (Fig.10). The plots of input functions used, defining the required displacements $\dot{q}_1$, $\dot{q}_3$ of links 1 and 3 with flexible drives, are presented in Fig.9.

Analysing the curves shown in Fig.10, we can see that the velocity of link 3 assumes the zero value on many occasions (particularly in the case a) because of large friction, and so parasitic, transitory standstill phases appear. Occurrence of these phases is signalled by the thick sections in the course of the so-called coefficient of motion c_m. Such motion can be treated as a typical example of the "stick-slip" vibrations. It appears that even a small change in the value of an integration step leads to a quite different form of the "stick-slip" phenomenon. As the numerical calculations have proved, the assumption of a step smaller than $0.2 \cdot 10^{-5}\,\mathrm{s}$ seems to be unnecessary because it does not

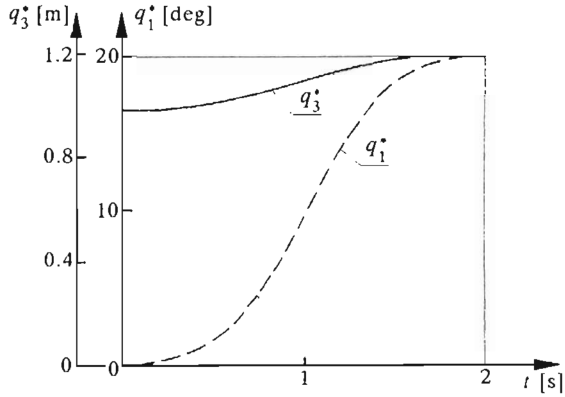

Fig. 9. Courses of the input functions

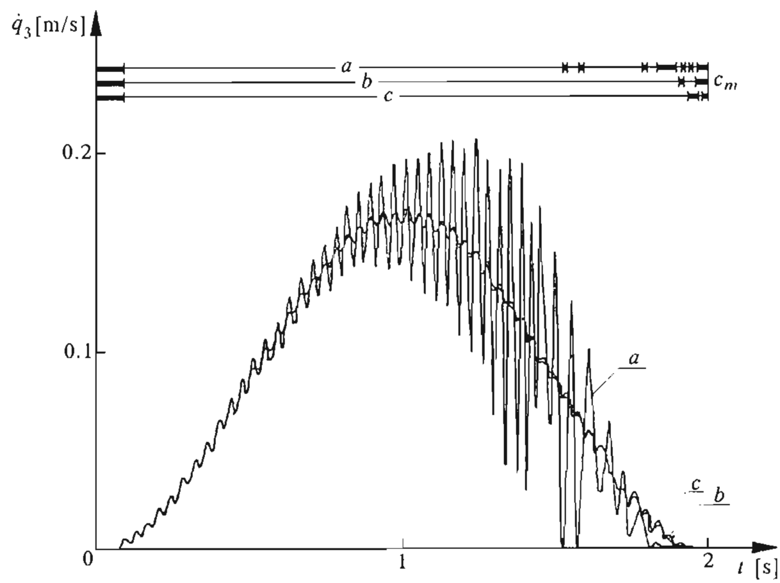

Fig. 10. Influence of the values of integration step on accuracy of calculations: (a) $0.5 \cdot 10^{-4}$ s, (b) $0.2 \cdot 10^{-4}$ s, (c) $0.2 \cdot 10^{-5}$ s

affect the accuracy of results. The computations were carried out by using a personal computer, and so the times of numerical simulations were long (even to several hours) due to small integration steps. For this reason, in general times of manipulator motion no longer than 2 s were considered.

Below, the results of some selected numerical calculations are presented.

Fig.11 illustrates an influence of different values of kinetic fricton coefficients in joint 3 on the positioning accuracy ε of the manipulator end E defined as follows

$$\varepsilon = \sqrt{(x_E - x_E^*)^2 + (y_E - y_E^*)^2 + (z_E - z_E^*)^2} \tag{4.1}$$

where

x_E, y_E, z_E – coordinates of the point E in the case when all drives of manipulator are considered to be rigid

x_E^*, y_E^*, z_E^* – coordinates of this point in the case when the drives of links 1 and 3 are modelled as flexible.

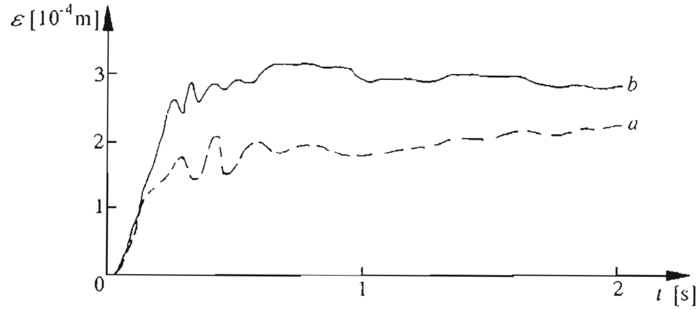

Fig. 11. Influence of the kinetic friction coefficients on positioning accuracy: (a) $\mu_3 = 0.1$, (b) $\mu_3 = 0.2$

It can be seen that larger friction influences negatively the accuracy of motion of of manipulator links.

Fig.12 presents the plots of difference ε_3, where $\varepsilon_3 = q_3^* - q_3$, between the required and real displacements of link 3 determined when both constant (case a) and decreasing (case b) kinetic friction characteristics were taken into account in joints 1 and 3 with flexible drives. The graphs illustrate the bad effect of decreasing friction characteristics on stability of motion. This bad effect is demonstrated by the increase of vibrations. The thick sections in the course of the coefficient of motion c_m, numerous especially in the case of decreasing characteristic, denote appearance of stiction phases.

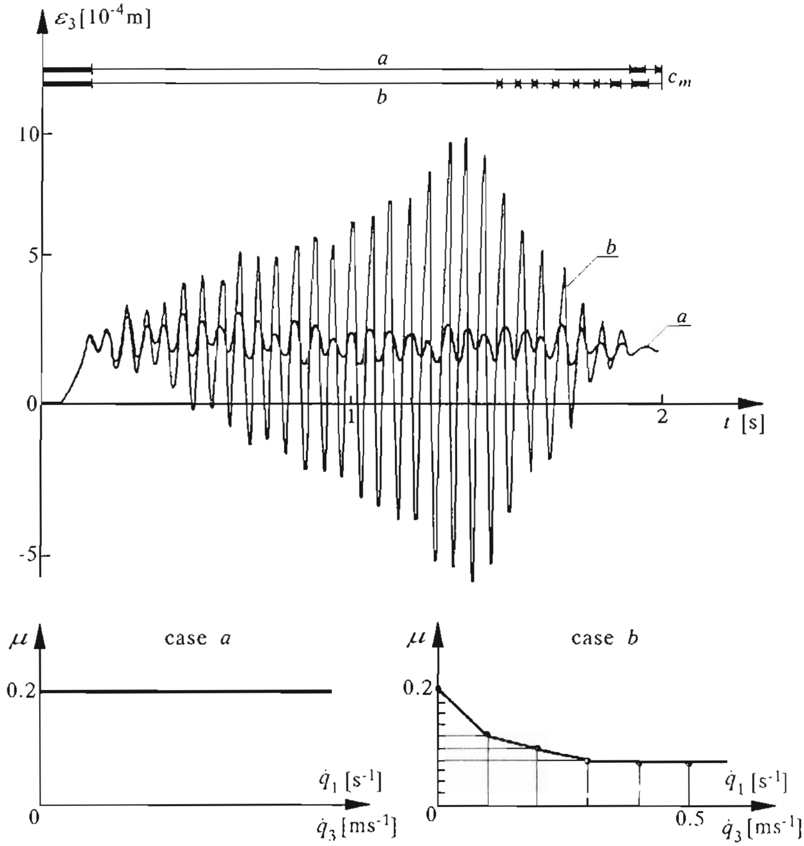

Fig. 12. Influence of the kinetic friction characteristics on accuracy of displacement of link 3

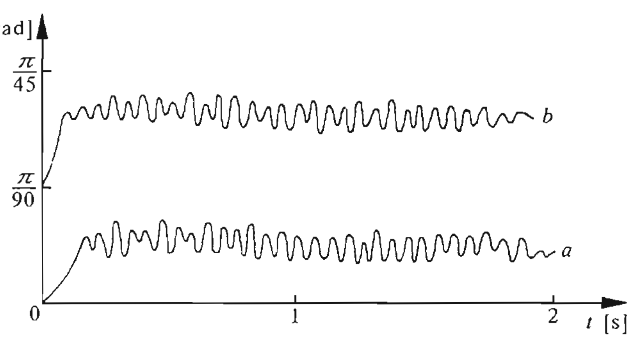

Fig. 13. Accuracy of displacement of link 1 for various clearances in drives: (a) $\Delta_1 = \Delta_3 = 0$, (b) $\Delta_1 = \pi/90$, $\Delta_3 = 0.002\,\mathrm{m}$

Next characteristic property of the drive system is the clearance resulting mainly from backlash in drive gears. The influence of clearances Δ_1, Δ_3 in the drive systems of link 1 and 3 on the accuracy of movement of link 1 is presented in Fig.13. The two cases are illustrated: without and with clearances, respectively, introduced into drives. In the second case, the difference ε_1, where $\varepsilon_1 = q_1^* - q_1$, between the required and real displacements of link 1 is larger and more or less constant in the time of numerical simulation.

The method described allows also for determination of the influence of different dimensions, such as for example dimensions of joints, on the stability of motion. For example, the Fig.14 illustrates the influence of length a_3 of slider in joint 3 on the positioning accuracy ε of the manipulator end E.

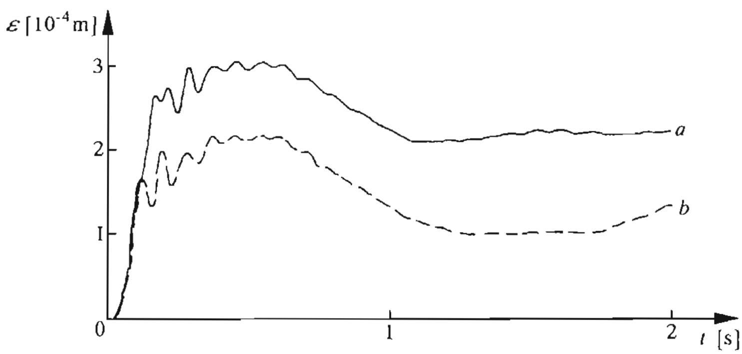

Fig. 14. Positioning accuracy for different lengths of slider in joint 3: (a) $a_3 = 0.2\,\mathrm{m}$, (b) $a_3 = 0.4\,\mathrm{m}$

The possibility of damping occurrence in the drive systems of links 1 and 3 was taken into account in the last case. Using the suggestions put foreward by Soavi (1980), the values of damping coefficients were assumed: $h_1 = 10^3\,\mathrm{Ns/rad}$ and $h_3 = 10^3\,\mathrm{Ns/m}$. The increase in the slider length a_3 causes a decrease in values of the normal reaction forces and as a result of smaller values of friction forces and larger positioning accuracy.

5. Conclusions

It can be seen from the above plots that the method elaborated allows for determination of the influence of different parameters characterising properties of manipulator drives and joints on the stability of system motion. Selected cases of possible large instability of the system in the form of "stick-slip" perturbations were presented. In general, it can be stated that the results of this work are useful in defining expected positioning errors when investigating manipulator behaviour for variable physical and constructional parameters of system.

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Ruch typu "stick-slip" otwartych manipulatorów z podatnymi napędami i suchym tarciem w parach kinematycznych

Streszczenie

Tematem artykułu jest analiza zjawiska drgań typu "stick-slip" ramion otwartych manipulatorów. Zaproponowano pewną metodę opracowania fizycznych i matematycznych modeli manipulatorów w postaci otwartych łańcuchów kinematycznych dla potrzeb analizy dynamicznej, uwzględniającej podatność napędów i tarcie suche w parach kinematycznych. Przeprowadzono przykładową analizę dynamiczną wybranego manipulatora z postępową i obrotowymi parami kinematycznymi. W podsumowaniu zaprezentowano wybrane wyniki obliczeń numerycznych, ilustrujące wpływ suchego tarcia w parach kinematycznych na zjawisko drgań typu "stick-slip", a w konsekwencji stabilność ruchu członów manipulatora.

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