JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
2, **37**, 1999

FLOW PAST A SPHERE MOVING TOWARDS A WALL IN MICROPOLAR FLUID

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The paper presents the first "exact" solution to the problem of creeping flow past a sphere moving towards a wall in micropolar fluid. The analytical-numerical method is presented, that is a development of the boundary collocation technique previously used for solving many corresponding problems for a Newtonian fluid. The initial study of the method has been carried out and the results for a force acting on a sphere compared with their counterparts for a Newtonian fluid are presented. It is worth while to note that the drag force on a sphere depends on material constants of the micropolar fluid and the distance from the wall.

Key words: non-Newtonian fluid, collocation method, Stokes flow, micropolar fluid

1. Introduction

Since the 1970s the study of microhydrodynamics has been undertaken in large measure due to the numerical techniques development. Generally, these flows appear in suspension rheology, colloids, aerosols, microdevices. The presence of a wall changes the hydrodynamical parameters of a moving body and this phenomenon can be observed in many industrial and biological processes. Various numerical schemes have been constructed to solve these problems. For a thorough survey of the available results the Reader is reffered to Happel and Brenner (1985), Kim and Karilla (1991). Ganatos et al. (1980) proposed the method which enabled one calculate the low Reynolds number interaction for a sphere moving in a Newtonian fluid bounded by planar walls. The results can be applied to modelling diffusion of the plasmalemma vesicles across endothelial cells lining the artery wall (Weinbaum and Caro, 1976; Arminski et al., 1980) and diffusion of the molecules across the intercellular space between adjacent cells. Owing to the linearity of the Stokes equations and boundary conditions, the arbitrary motion of a sphere can be separated into the parallel and perpendicular ones, respectively, to the bonding walls.

The experimental results show, that when considering micro-scale fluid flows, several effects which are commonly excluded from the macro-scale fluid flow become increasingly important. This has introduced "anomalies" which the classical Newton theory of fluid is unable to explain. One of the effects not included in the classical Newton theory is the micro-rotation effect due to rotation of fluid molecules. This is of crucial importance when considering fluid flows in narrow channels (capillaries) and when the fluid under investigation reveals substructures. The biofluids fall into the category.

The micropolar fluid theory proposed by Eringen (1966) auguments the laws of classical continuum mechanics by incorporating the micro-rotation effects. In such fluids, the rigid particles included in a small volume element can rotate about the element centre represented by the micro-rotation vector. This local rotation of particles is an addition to the usual rigid body motion of the entire volume element. The laws of classical continuum mechanics are augumented with additional equations representing the conservation of microinertia moments and the balance of first stress moments for which consideration of the microstructure in a material accounts. Field equations are presented in terms of two independent kinematic vector fields, the velocity and microrotation vector and involve material constants. The stress tensor is not symmetric. Basing on the micropolar fluid theory the solutions have been obtained to many interesting physical problems in the field of lubricants, fluid with additives, blood, eletromagnetical suspensions, polymers and flows in microchannels. The up-to-date review can be found in Petrosyan (1984), Prohorenko and Migoun (1988), Łukaszewicz (1998).

The lack of solutions to the problems of micropolar fluid flow past a body in the presence of walls motivated the investigation. The micropolar model of fluid seems to be more realistic than the Newtonian one when we consider a small dimension of channel and properties of the biofluid. Results can be applied to biomechanics, the diffusion problem mentioned above and many other engineering issues.

In the paper, the Author concentrates on the creeping motion of a sphere towards a wall, treated as an initial study of a two walls problem. Our aim is to constuct an effective solving method and to answer the question: how do the material constants of micropolar fluid influence the force? The analytical-numerical method of solution is constructed and the force acting upon a sphere is calculated. It is based on the boundary collocation method, which is a particular case of the weighed residual method and is a very useful tool for solving the flow problems that can be formulated using linear equations. The biggest advantage of this method in comparison in the others ones (e.g. Finite Element Methods, Finite Difference Methods) is that for slightly more complicated regions it requires less work. The up-to-date bibliography and review of the results of the application of the boundary collocation method into mechanics can be found in Kołodziej (1987), Kim and Karilla (1991).

The crucial element of the proposed analytical-numerical method consist in general solutions of the Stokes equations for a micropolar fluid in the spherical and cylindrical coordinates. They are derived in the paper and can be applied *to solution of another flow problems.*

The results show that:

(i) The material constants μ, κ, γ of micropolar fluid have a considerable influence on the force acting on the sphere.

(ii) Form of the boundary conditions to be satisfied by the microrotation vector on surfaces of the sphere affects the force.

(iii) Presented method can be extended to cover the flow past a sphere between parallel walls but it is beyond the scope of this paper. The Author intends to undertake the research in the near future.

2. Formulation of the problem

Let us consider a quasi-steady flow field of an incompressible micropolar fluid due to a translational axisymmetric motion of a sphere S_a of radius a towards the wall (Fig.1). The distance between the sphere and wall is denoted by d. In the polar coordinate system (r, θ, z) with the origin in the center of the moving sphere the wall surface is described as $z = -c$; $c = d + a$. The translational velocity of sphere S_a is $(0, 0, U)$.

The fluid is at rest at infinity. The Reynolds number is low.

Therefore, the equations of motion describing this flow are the Stokes equations for micropolar fluid and read

$$
\begin{aligned}
&(\mu + \kappa)\nabla^2 \boldsymbol{v} + \kappa \nabla \times \boldsymbol{\varpi} - \nabla p = \boldsymbol{0} \\
&(\alpha^m + \beta^m + \gamma)\nabla\nabla \boldsymbol{\varpi} - \gamma \nabla \times \nabla \times \boldsymbol{\varpi} + \kappa \nabla \times \boldsymbol{v} - 2\kappa \boldsymbol{v} = \boldsymbol{0}
\end{aligned}
\qquad (2.1)
$$

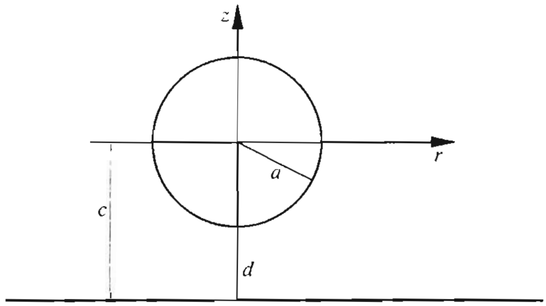

Fig. 1. Geometry of the flow

moreover, the mass conservation equation is satisfied: $\nabla \cdot \boldsymbol{v} = 0$, where $\boldsymbol{v}$ denotes the velocity of the fluid, $\boldsymbol{\varpi}$ is the microrotation vector. The positive constants μ, κ, α^m, β^m, γ characterise isotropic properties of the micropolar fluid.

The boundary conditions imposed on the moving sphere S_a are

$$\boldsymbol{v} = \boldsymbol{U} \qquad \boldsymbol{\varpi} = \alpha_1 \frac{1}{2}\zeta \tag{2.2}$$

and on the wall

$$\boldsymbol{v} = \boldsymbol{0} \qquad \boldsymbol{\varpi} = \alpha_2 \frac{1}{2}\zeta \tag{2.3}$$

Moreover at infinity

$$\Psi = 0 \tag{2.4}$$

$0 \leqslant \alpha_i \leqslant 1$, $i = 1, 2$.

Because of the axisymmetric geometry of the flow, the stream function $\Psi(r, z)$ can be used and the axial and radial velocity components expressed as

$$v_r = U\frac{1}{r}\frac{\partial \Psi}{\partial z} \qquad v_z = U\frac{-1}{r}\frac{\partial \Psi}{\partial r} \tag{2.5}$$

Moreover the microrotation vector has one component only.

Replacing the velocity with the derivatives of stream function in the Stokes equations (2.1), we obtain

$$\begin{aligned} &-(\mu+\kappa)L_1^2(\Psi) + \kappa L_1(r\varpi) = 0 \\ &-\gamma L_1(r\varpi) + \kappa L_1(\Psi) - 2\kappa r\varpi = 0 \end{aligned} \tag{2.6}$$

L_1 is the generalised axisymmetric Stokes operator $L_1 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$.

The above equations are fourth-order partial differential equations, linear for Ψ and ϖ. After elimination from Eqs (2.6) the microrotation vector ϖ we arrive at

$$L_1^2(L_1 - \lambda^2)\Psi = 0 \tag{2.7}$$

with the microrotation being given by

$$\varpi = \frac{1}{2r}\Big(L_1\Psi + \frac{\gamma(\mu+\kappa)}{\kappa^2}L_1^2\Psi\Big) \tag{2.8}$$

and constant λ^2 defined as

$$\lambda^2 = \frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)} \tag{2.9}$$

To solve Eq (2.7) which is equivalent to the Stokes equations of micropolar fluid, the boundary conditions (2.2) $\div$ (2.4) have to be reformulated. On the moving sphere S_a they are

$$\Psi = \frac{1}{2}r^2 \qquad \frac{\partial\Psi}{\partial z} = 0 \qquad \varpi = \alpha_1\frac{1}{2}\zeta \tag{2.10}$$

and on the wall

$$\Psi = \frac{\partial\Psi}{\partial z} = 0 \qquad \varpi = \alpha_2\frac{1}{2}\zeta \tag{2.11}$$

Moreover, at infinity

$$\Psi = 0 \tag{2.12}$$

The stream function Ψ that satisfies the partial differential equation (2.7) with the corresponding boundary conditions (2.10) $\div$ (2.12) and the microrotation vector defined by Eq (2.8) represent the flow field for the problem under consideration.

3. Solution procedure

The alghoritm for the flow field determination can be constructed as follows:

Stage (i) – We seek the solution of Eq (2.7) as a sum

$$\Psi = \Psi_u + \Psi_w \tag{3.1}$$

The part Ψ_u represents the general solution of the Stokes equation in the spherical coordinates. It can be expanded into an infinite series. Ψ_w represents the general solution of the Stokes equation in the cylindrical coordinates regular in the flow field and is given by an integral. Using Eqs (2.5) we can write the formulae for velocity and microrotation vectors.

Stage (ii) – In order to obtain a unique solution the unknown constants from the general solutions have to be determined. First, we impose the boundary conditions on the wall Eqs (2.3).

As a result we get equations which can be easily inverted. This fact allows us to express the unknown constants which appeared in the integrand into series.

Having done it, the axial v_r, and radial v_z velocities of the fluid flow and the microrotation vector $\boldsymbol{\varpi}$ can be rewritten in terms of an infinite series in which still the unknown constant appears.

Next, we truncate the infinite series and impose the boundary conditions on the sphere at a finite number of discrete points. The collocation technique is applied.

Stage (iii) – Then we solve the derived linear set of the equations using a numerical method. At this stage the solution is known.

To apply the algorithm we should first find the general solutions of the Stokes equation for the micropolar fluid in the cylindrical and spherical coordinates. The stream function Ψ is split up into two parts

$$\Psi = \Psi^1 + \Psi^2 \tag{3.2}$$

where Ψ^2 and Ψ^1 satisfy the second order differential equations resulting from Eq (2.7)

$$L_1^2\Psi^1 = 0 \qquad\qquad (L_1 - \lambda^2)\Psi^2 = 0 \tag{3.3}$$

In spite of that Ψ^1 is the well known general solution of the classical case (cf Happel and Brenner, 1985) our problem reduces to solving the second order differential equation $(3.3)_2$.

In the cylindrical coordinates Eq $(3.3)_2$ can be rewritten as

$$\Big(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \lambda^2\Big)\Psi^2 = 0 \tag{3.4}$$

It can be solved by the variables separation as follows

$$\Psi^2 = R(r)Z(z) \tag{3.5}$$

The two second order equations result

$$\begin{aligned} &\frac{d^2R(r)}{dr^2} - \frac{1}{R(r)}\frac{dR(r)}{dr} = \alpha^2 R(r) \\ &\frac{d^2Z(z)}{dz^2} - (\alpha^2+\lambda^2)Z(z) = 0 \end{aligned} \tag{3.6}$$

The first of these equations has the solution

$$R(r) = ArJ_1(\alpha r) + BrY_1(\alpha r) \tag{3.7}$$

whereas the solution of the latter is

$$Z(z) = G_1(\alpha)\mathrm{e}^{-\delta z} + G_2(\alpha)\mathrm{e}^{\delta z} \tag{3.8}$$

where $\delta = \sqrt{\lambda^2+\alpha^2}$.

Symbol J_1 denotes the Bessel function, Y_1 is the Weber function.

The condition for Ψ at infinity (2.12) demands that $B=0$ and $G_2=0$.

Combining (3.7) and (3.8), yields

$$\Psi^2 = \int\limits_0^\infty G_1(\alpha)\mathrm{e}^{-\delta z}ArJ_1(\alpha r)\,d\alpha \tag{3.9}$$

Hence, we are able to write the general solution of the Stokes equations for the micropolar fluid in the cylindrical coordinates, which are regular everywhere in the flow field. It is given by the Fourier-Bessel integral

$$\Psi_w = \int\limits_0^\infty \Big[B(\alpha)\mathrm{e}^{-\alpha z} + D(\alpha)\alpha z\mathrm{e}^{-\alpha z} + G_1(\alpha)\mathrm{e}^{-\delta z}\big)rJ_1(\alpha r)\,d\alpha \tag{3.10}$$

Here $B(\alpha)$, $D(\alpha)$, $G(\alpha)$ are unknown functions of α.

In the spherical coordinates, Eq $(3.3)_2$ assumes the form

$$\Big(\frac{\partial^2}{\partial\varrho^2} - \frac{1-\zeta^2}{\varrho^2}\frac{\partial^2}{\partial\zeta^2} - \lambda^2\Big)\Psi^2 = 0 \tag{3.11}$$

with $\zeta = \cos\theta$.

We look for the solution of the form

$$\Psi^2 = R(\varrho)Z(\zeta) \tag{3.12}$$

Substituting Eq (3.12) into Eq (3.11) yields

$$
\begin{aligned}
&-\frac{1-\zeta^2}{Z}\frac{d^2 Z}{d\zeta^2} - n(n-1)Z = 0 \\
&\frac{\varrho^2}{R}\frac{d^2 R}{d\varrho^2} - n(n-1)R - \varrho\lambda^2 R = 0
\end{aligned}
\tag{3.13}
$$

The first equation of Eqs (3.13) is the Gegenbauer equation (cf Happel and Brenner, 1985), solutions of which are the Gegenbauer functions of degree $(-1/2)$

$$
Z(\zeta) = I_n(\zeta) + H_n(\zeta) \tag{3.14}
$$

of the first $I_n(\zeta)$, and the second kind $H_n(\zeta)$, of order n.

The second equation of Eqs (3.13) (cf Kamke, 1976) is the Bessel modified equation of the solution

$$
R(\varrho) = \sqrt{(\varrho)}Z_{n-1/2}(i\lambda\varrho) \tag{3.15}
$$

where $Z_{n-1/2}$ denotes the sum of the Bessel functions

$$
Z_{n-1/2}(\mathrm{i}z) = C_1 I_{n-1/2}(z) + C_2 K_{n-1/2}(z) \tag{3.16}
$$

Note, that the solution derived by Ramkissoon and Majumdar (1976) may be obtained by putting into Eq (3.16) $n = 2$.

Upon collecting the results we find as possible solutions of Eq (3.11)

$$
\Psi^2 = \sum_{n=2}^{\infty} A_n I_{n-1/2}(\varrho\lambda) I_n(\zeta) \tag{3.17}
$$

In spite of the fact that Ψ^1 is the well known general solution of the classical case (cf Happel and Brenner, 1985) in accord with (3.2), a complete solution regular in the flow field for the stream function in the spherical coordinates reads

$$
\Psi_u = \sum_{n=2}^{\infty} [B_n\varrho^{-n+1} + D_n\varrho^{-n+3} + A_n I_{n-1/2}(\varrho\lambda)] I_n(\zeta) \tag{3.18}
$$

A_n, B_n and D_n are unknown constants which will be determined from the equations resulting from the non-slip boundary conditions on the surface of the sphere.

Before, that we write the formula for the microrotation, Eq (2.8), some simplifications should be made. From Eqs (3.3) it follows (cf Ramkissoon and Majumdar, 1976) that

$$
L_1\Psi = L_1\Psi^1 + \lambda^2\Psi^2 \qquad \dots \qquad L_1^2\Psi = \lambda^2 L_1^2\Psi^2 = \lambda^4\Psi^2 \tag{3.19}
$$

Substituting Eqs (3.2) and (3.19) into Eq expression (2.4), yields

$$\varpi = \frac{1}{2r}(L_1\Psi^1 + \delta\Psi^2) \tag{3.20}$$

where

$$\delta = \lambda^2\Big(1 + \frac{\gamma(\mu+\kappa)}{\kappa^2}\lambda^2\Big) \tag{3.21}$$

From the above it follows that the microrotation vector ϖ can be written as

$$\begin{aligned}\varpi &= \sum_{n=2}^{\infty}\Big[B_n\varrho^{-n+1} + \delta A_n I_{n-1/2}(\varrho\lambda)\Big]I_n(\zeta) + \\ &+ \int_0^{\infty}\Big[B(\alpha)\mathrm{e}^{-\alpha z}(\alpha z) + \delta G(\alpha)\mathrm{e}^{-\delta z}\Big]J_1(\alpha r)\,d\alpha\end{aligned} \tag{3.22}$$

Eq (3.2) is written now in mixed coordinates: the cylindrical coordinate system and the spherical coordinate system. In order to differentiate Eq (3.2) and impose the no-slip conditions along the wall we have to employ the relation between the spherical and cylindrical coordinates

$$\varrho = \sqrt{r^2+z^2} \qquad\qquad \theta = \cos^{-1}\frac{z}{\sqrt{r^2+z^2}} \tag{3.23}$$

We denote by u_r, u_z, w_r, w_z the velocity components derived from Eqs (2.5) after substituting for the stream functions Ψ_u (3.18) and Ψ_w (3.10), respectively.

After differentiation one obtains the formulae for the axial and radial components of velocity v_r and v_z

$$\begin{aligned}v_r &= \sum_{n=2}^{\infty}(B_n\mathcal{B}_{rn} + D_n\mathcal{D}_{rn} + A_n\mathcal{A}_{rn}) + \int_0^{\infty}\mathcal{E}(\alpha,z)\alpha J_1(\alpha r)\,d\alpha = u_r + w_r \\ v_z &= \sum_{n=2}^{\infty}(B_n\mathcal{B}_{zn} + D_n\mathcal{D}_{zn} + A_n\mathcal{A}_{zn}) + \int_0^{\infty}\mathcal{F}(\alpha,z)\alpha J_0(\alpha r)\,d\alpha = u_z + w_z - U\end{aligned} \tag{3.24}$$

and for microrotation

$$\varpi = \sum_{n=2}^{\infty} B_n\mathcal{B}_{on} + A_n\mathcal{A}_{on} + \int_0^{\infty}\mathcal{G}^0(\alpha,z)\alpha J_1(\alpha r)\,d\alpha \tag{3.25}$$

The function $\mathcal{A}_{rn}$, $\mathcal{B}_{rn}$, ..., $\mathcal{D}_{zn}$, ..., $\mathcal{G}$ are listed in Appendix A1.

Thus, the boundary conditions $(2.2)_1$ which should be satisfied by the velocity v on the wall yield

$$u_r = -w_r \qquad\qquad u_z = -w_z + U \tag{3.26}$$

The fact, that the vorticity vector ζ has the form $(0, \zeta, 0)$ gives (cf Ramkissoon and Majumdar, 1976)

$$\zeta R = L_1 \Psi \tag{3.27}$$

which allows us to write the boundary conditions $(2.3)_2$ for the microrotation ϖ vector as

$$\frac{1}{2r}(L_1\Psi^1 + \delta\Psi^2) = \alpha_i \frac{1}{2r} L_1(\Psi^2 + \Psi^1) \tag{3.28}$$

The Eqs (3.26) and (3.28) can be easily inverted and integration can be performed using the Hankel transforms. It is

$$\begin{aligned}
\mathcal{E}^0(\alpha, z_0) &= -\int\limits_0^\infty t \sum_{n=1}^\infty [B_n\mathcal{B}_{rn}(t, z_0) + A_n\mathcal{A}_{rn}(t, z_0) + D_n\mathcal{D}_{rn}(t, z_0)] J_1(\alpha t)\, dt \\
\mathcal{F}^0(\alpha, z_0) &= -\int\limits_0^\infty t \sum_{n=1}^\infty [B_n\mathcal{B}_{zn}(t, z_0) + A_n\mathcal{A}_{zn}(t, z_0) + D_n\mathcal{D}_{zn}(t, z_0)] J_0(\alpha t)\, dt \\
\mathcal{G}^0(\alpha, z_0) &= -\int\limits_0^\infty t \sum_{n=1}^\infty [(1-\alpha_1)B_n\mathcal{B}_{on}(t, z_0) + (1-\alpha_1)A_n\mathcal{A}_{on}(t, z_0)] J_0(\alpha t)\, dt
\end{aligned} \tag{3.29}$$

These integrals for some functions can be calculated analytically. Let us denote by

$$\begin{aligned}
\mathcal{B}^*_{rn} &= -\int\limits_0^\infty t\mathcal{B}_{rn}(t, z_0) J_1(\alpha t)\, dt = -\frac{1}{n!}\Big(\frac{\alpha|z_0|}{z_0}\Big)^{(n-1)} \mathrm{e}^{-\alpha|z_0|} \\
\mathcal{B}^*_{zn} &= -\int\limits_0^\infty t\mathcal{B}_{zn}(t, z_0) J_0(\alpha t)\, dt = -\frac{\alpha^{(n-1)}}{n!}\Big(\frac{|z_0|}{z_0}\Big)^n \mathrm{e}^{-\alpha|z_0|} \\
\mathcal{D}^*_{rn} &= -\int\limits_0^\infty t\mathcal{D}_{rn}(t, z_0) J_1(\alpha t)\, dt = \\
&= -\frac{1}{n!}\Big(\frac{\alpha|z_0|}{z_0}\Big)^{(n-3)} \mathrm{e}^{-\alpha|z_0|}[(2n-3)\alpha|z_0| - n(n-2)]
\end{aligned} \tag{3.30}$$

$$
\begin{aligned}
\mathcal{D}^*_{zn} &= -\int_0^\infty t\mathcal{D}_{zn}(t,z_0)J_0(\alpha t)\,dt = \\
&= -\frac{\alpha^{(n-3)}}{n!}\Big(\frac{\alpha|z_0|}{z_0}\Big)^n \mathrm{e}^{-\alpha|z_0|}[(2n-3)\alpha|z_0| - (n-1)(n-3)]
\end{aligned}
$$

Using this notation it is obvious that the functions $\mathcal{A}^*_{zn}$, $\mathcal{A}^*_{rn}$ calculated numericaly. Then the components w_r, w_z can be now rewritten as

$$
\begin{aligned}
w_r &= \sum_{n=2}^{\infty} B_n \mathcal{W}\mathcal{B}_{rn} + D_n \mathcal{W}\mathcal{D}_{rn} + A_n \mathcal{W}\mathcal{A}_{rn} \\
w_z &= \sum_{n=2}^{\infty} B_n \mathcal{W}\mathcal{B}_{zn} + D_n \mathcal{W}\mathcal{D}_{zn} + A_n \mathcal{W}\mathcal{A}_{zn}
\end{aligned}
\tag{3.31}
$$

The functions $\mathcal{W}\mathcal{B}_{rn}$, $\mathcal{W}\mathcal{D}_{rn}$, $\mathcal{W}\mathcal{B}_{zn}$, $\mathcal{W}\mathcal{A}_{zn}$ are listed in Appendix A2.

Thus, substituting the above formulae into Eqs (3.24) and (3.25) one obtains the velocity fields v_r, v_z and microrotation ϖ, however, still in terms of unknown coefficients B_n, D_n, A_n

$$
\begin{aligned}
v_r &= \sum_{n=2}^{\infty} B_n(\mathcal{B}_{rn} + \mathcal{W}\mathcal{B}_{rn}) + D_n(\mathcal{D}_{rn} + \mathcal{W}\mathcal{D}_{rn}) + A_n(\mathcal{A}_{rn} + \mathcal{W}\mathcal{A}_{rn}) \\
v_z &= \sum_{n=2}^{\infty} B_n(\mathcal{B}_{zn} + \mathcal{W}\mathcal{B}_{zn}) + D_n(\mathcal{D}_{zn} + \mathcal{W}\mathcal{D}_{zn}) + A_n(\mathcal{A}_{zn} + \mathcal{W}\mathcal{A}_{zn}) - U \\
\varpi &= \sum_{n=2}^{\infty} B_n(\mathcal{B}_{on} + \mathcal{W}\mathcal{B}_{on}) + D_n(\mathcal{D}_{on} + \mathcal{W}\mathcal{D}_{on}) + A_n(\mathcal{A}_{on} + \mathcal{W}\mathcal{A}_{on})
\end{aligned}
\tag{3.32}
$$

In order to obtain a unique solution, we should combine the boundary conditions imposed on the moving particle (2.2) with Eqs (3.32) at a finite number of discrete points on the sphere. Then, after truncating the infinite series we solve the derived equations for B_n, D_n, A_n.

This step finishes the procedure of finding solutions to the considered problem. We can see that the three material constants of micropolar fluid; i.e., μ, κ, γ and the form of boundary conditions on microrotation vector ϖ affect values of the coefficients written above.

From the relations (3.32) it is possible to observe that the velocities and microrotation in the flow field depend on the three material constants μ, κ, γ and on the boundary conditions for the microrotation as well.

The drag force acting on the sphere can be obtained by the use of a standard integral formula, but evaluation of it is often troublesome problem. Ramkissoon and Majumdar (1976) derived the elegant formula for a drag force acting on an axially symetric body and we employ it in the paper. We have

$$D = 4\pi(2\mu + \kappa) \lim_{\rho\to\infty} \frac{\rho\Psi}{r^2} \tag{3.33}$$

Using this formula one can find the drag on any axially symmetric body from the knowledge of the stream function and a simple limiting process.

Using the formula for Ψ given by Eqs (3.18), (3.10) to (3.33) as a result we obtain

$$D = 2\pi U(2\mu + \kappa)D_2 \tag{3.34}$$

It should be noted that the drag is affected by the sphere radius, translation velocity, all material constants of the micropolar fluid and form of the boundary conditions to be satisfied by the microrotation vector on the wall. The latter can be observed from the solution procedure for obtaining the unknown constants.

The drag force acting on the sphere moving towards the wall can be also expressed using the drag correction factor dc as

$$dc = \frac{D}{DU} \tag{3.35}$$

where

$$DU = -\frac{6\pi Ua(2\mu + \kappa)(\mu + \kappa)(1 + a\lambda)}{\kappa + 2\mu + 2a\lambda\mu + 2a\lambda\kappa} \tag{3.36}$$

is the drag force acting on a sphere in unbounded region, derived by Ramkissoon and Majumdar (1976).

4. Numerical results and conclusions

The algorithm was implemented in Fortran and run on an IBM Pentium 160 computer. The unique feature of the approach consists in the fact that the convergence and accuracy of the solution could be controlled simply by selecting the proper set of test points on the surface of the spheres. Because the "best" collocation criterion does not exist a priori (cf Kim and Karilla, 1991), the scheme for arrangement of the collocation points on the surface of the sphere follows Ganatos et al. (1980) in which the coresponding problem for the Newtonian fluid is studied. More, the calculations of the wall correction factor dc in this study have been made using the set of points: 0°, 45°, 90°, 132°, 145°, 175°, 177°, 180°.

It is worthwhile to note that owing to the nature of the problem each boundary point represents a ring. To study this algorithm a series of calculations of the force D, see Eq (3.34), for various rheological parameters of the fluid and the nondimensional distances $dis = c/a$ between the sphere and wall has been made.

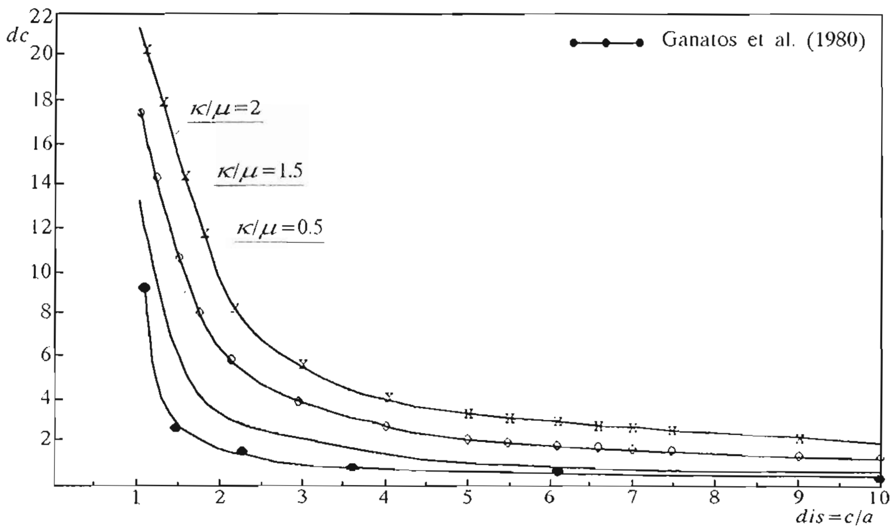

Fig. 2. Comparison between the results for different values of parameter κ/μ and Newtonian fluid (Ganatos et al. 1980); $\alpha_1 = \alpha_2 = 0.3$, $\gamma = 1$

The results of calculations are presented and compared with those obtained for a corresponding problem for the Newtonian fluid (cf Ganatos et al., 1980). Because the corrector factor dc is given only for selected parameters dis (cf Ganatos et al., 1980), the same values of dis were used in this test. The results are plotted in the Fig.2 and Fig.3. The curves show the force D acting on the translating sphere versus its position dis.

To study the effect of the fluid reological parameters on the wall correction factor the results in Fig.2 are plotted for variable ratio κ/μ and constant parameters γ and $\alpha_1 = \alpha_2$. In Fig.3 the wall corector factor is depicted for the constant ratio κ/μ and variable parameter γ. From the qualitative point of view, when we take into account the distance of the sphere to the wall, it can be noticed that the wall corrector factor reveals a similar behaviour

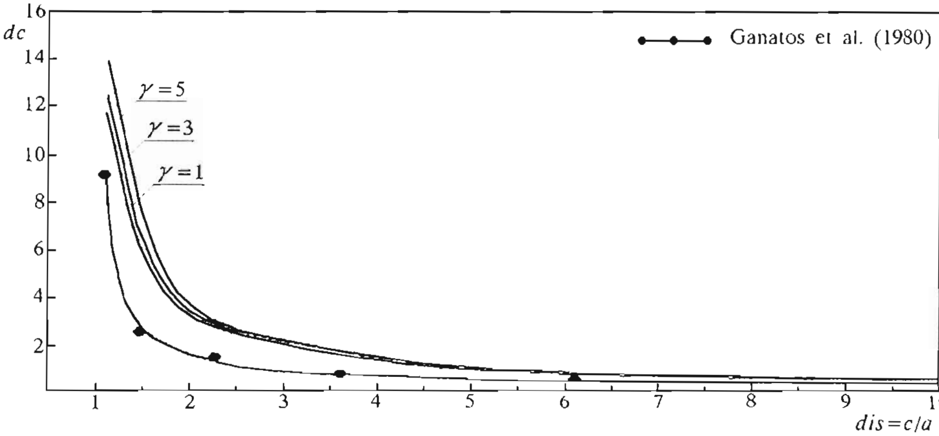

Fig. 3. Comparison between the results for different values of parameter γ and Newtonian fluid (Ganatos et al. 1980); $\alpha_1 = \alpha_2 = 0.3$, $\kappa/\mu = 0.5$

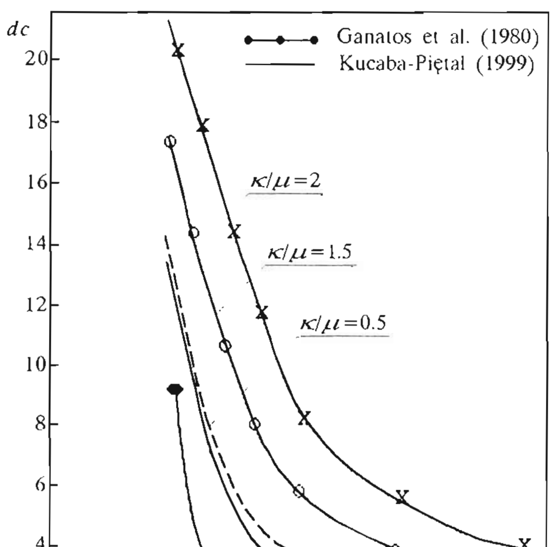

Fig. 4. Comparison between the results for different values of parameter κ/μ and Newtonian fluid (Ganatos et al. 1980) as well as asymptotic solution (Kucaba-Piętal, 1999)

for both the micropolar and classical fluid. Clearly, the influence of the fluid rheological properties on the force can be observed, i.e., the force D increases as the value of the ratio $K = \kappa/\mu$ grows.

In spite of the fact that the plane wall can be considered as the limit of the sphere with infinite radius, the results are compared with those obtained by Kucaba-Piętal (1999) for converging spheres. They are shown in Fig.4. The differencies can be explained by the fact, that the asymptotic expression for the force correct to $O(\epsilon \ln \epsilon)$ term contains only the two material constants of micropolar fluid μ, κ. When the distances dis are small, the influence of γ can be neglected.

The following conclusions can be drawn:

- The collocation method presented in the paper yields the solution of the flow field for the sphere moving towards the wall. It can be easily extended to cover the problem of creeping flow past a sphere moving between parallel walls.

- The material constants μ, κ, γ of micropolar fluid and the form of boundary conditions to be satisfied by the microrotation vector ϖ on the rigid surfaces have a considerable influence on the force acting on the sphere.

- The force appearing in a micropolar fluid is greater than that in a classical fluid and increases with κ/μ. This phenomenon was also reported by Ramkissoon and Majumdar (1976).

Appendix

A1. Functions $\mathcal{B}_{rn}$, $\mathcal{D}_{rn}$, $\mathcal{B}_{zn}$, $\mathcal{D}_{zn}$, $\mathcal{E}^i(\alpha, z)$, $\mathcal{F}^i(\alpha, z)$ which appear in Eqs (3.24) and (3.25) can be written as follows

$$\mathcal{A}_{rn} = \frac{1}{r^2+z^2} I_{n-1/2} \frac{1}{r} I_{n+1}\Big(\frac{z}{\sqrt{r^2+z^2}}\Big) - \frac{2}{r^2+z^2} I_{n+3/2} \frac{2}{r} I_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)$$

$$\mathcal{B}_{rn} = \frac{n+1}{\sqrt{(r^2+z^2)^n}} \frac{1}{r} I_{n+1}\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)$$

$$\mathcal{D}_{rn} = \frac{n+1}{\sqrt{(r^2+z^2)^{n-2}}} \frac{1}{r} I_{n+1}\Big(\frac{z}{\sqrt{r^2+z^2}}\Big) - \frac{2}{\sqrt{(r^2+z^2)^{n-1}}} \frac{2}{r} I_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)$$

$$
\begin{aligned}
\mathcal{A}_{zn} &= \frac{2}{r^2+z^2} I_{n-1/2} I_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big) + \frac{1}{\sqrt{r^2+z^2}} I_{n+3/2} P_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{B}_{zn} &= \frac{1}{\sqrt{(r^2+z^2)^{n+1}}} P_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{D}_{zn} &= \frac{2}{\sqrt{(r^2+z^2)^{n-1}}} I_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big) + \frac{1}{\sqrt{(r^2+z^2)^{n-1}}} P_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{E}(\alpha,z) &= B\mathrm{e}^{-z} - D(1-\sigma)\mathrm{e}^{-z} + Gx\mathrm{e}^{-x}\\
\mathcal{F}(\alpha,z) &= B\mathrm{e}^{-z} + D\sigma\mathrm{e}^{-z} + G\mathrm{e}^{-x}\\
\mathcal{G}(\alpha,z) &= B\mathrm{e}^{-z} + G\mathrm{e}^{-x}
\end{aligned}
$$

where: $\sigma = \alpha c$, $x = \sqrt{\alpha^2+\lambda^2}$ and

$$
\begin{aligned}
B &= \frac{-2u_1\sigma + u_2(\sigma-1+x\sigma) + u_3(\sigma-1-x\sigma)}{\sigma-1+x\sigma}\mathrm{e}^{-\sigma}\\
D &= \frac{-u_2 + u_3(x-1) + u_1}{\sigma-1+x\sigma}\mathrm{e}^{-\sigma}\\
G &= \frac{-u_2\sigma - u_3(x-1) + u1\sigma}{-\sigma-1+x\sigma}\mathrm{e}^{-2\sigma-x}
\end{aligned}
$$

and u_1 denotes $\mathcal{E}(\alpha,-c)$, u_2 denotes $\mathcal{F}(\alpha,-c)$, u_3 denotes $\mathcal{G}(\alpha,-c)$ from Eqs (3.29) for $z_0 = -c$.

A2. The functions which appear in Eqs (3.31) read

$$
\begin{aligned}
\mathcal{WB}_{rn} &= -\int\limits_0^\infty \Big[(1-\sigma)\mathrm{e}^{-\sigma}\mathcal{B}^*_{rn} + \sigma\mathrm{e}^{-\sigma}\mathcal{B}^*_{zn}\alpha\Big] J_1(\alpha r)\,d\alpha\\
\mathcal{WB}_{zn} &= -\int\limits_0^\infty \Big[-\sigma\mathrm{e}^{-\sigma}\mathcal{B}^*_{rn} + (\sigma+1)\mathrm{e}^{-\sigma}\mathcal{B}^*_{zn}\alpha\Big] J_0(\alpha r)\,d\alpha\\
\mathcal{WD}_{rn} &= -\int\limits_0^\infty \Big[(1-\sigma)\mathrm{e}^{-\sigma}\mathcal{D}^*_{rn} + \sigma\mathrm{e}^{-\sigma}\mathcal{D}^*_{zn}\alpha\Big] J_1(\alpha r)\,d\alpha\\
\mathcal{WD}_{zn} &= -\int\limits_0^\infty \Big[-\sigma\mathrm{e}^{-\sigma}\mathcal{D}^*_{rn} + (\sigma+1)\mathrm{e}^{-\sigma}\mathcal{D}^*_{zn}\alpha\Big] J_0(\alpha r)\,d\alpha
\end{aligned}
$$

Acknowledgements The Author would like to express her gratitude to Prof. N.P.Migoun from the Belarussian Academy of Science for his valuable, helpful comments and remarks concerning this work.

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Przepływ wokół cząstki kulistej poruszającej się w kierunku ścianki w płynie mikropolarnym

Streszczenie

Rozwiązane zostało zagadnienie wyznaczenia pola przepływu i działającej siły na cząstkę kulistą poruszającą się w kierunki ścianki w płynie mikropolarnym. Skonstruowano metodę analityczno-numeryczną w oparciu o metodę kolokacji. Rozpatrzono przepływ quasistacjonarny w przybliżeniu Stokesa.

Wykazano, że istotny wpływ na wartość siły mają zarówno stałe materiałowe płynu mikropolarnego μ, κ, γ jak i postać warunków brzegowych, jakie spełnia wektor mikrorotacji na ograniczającej powierzchni.

Manuscript received December 15, 1998; accepted for print January 8, 1999