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STABILIZATION OF BEAM PARAMETRIC VIBRATIONS BY MEANS OF DISTRIBUTED PIEZOELECTRIC ELEMENTS

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The paper concerns stabilization of an elastic beam subjected to a time-dependent axial forcing. The direct Liapunov method is proposed to establish criteria for the almost sure stochastic stability of the unperturbed (trivial) solution of the structure with closed-loop control. We construct the Liapunov functional as a sum of the modified kinetic energy and elastic energy of the structure. An influence of damping in the finite bonding layer is described by the effective retardation time of the Voigt-Kelvin model. The distributed control is realized by the piezoelectric sensor and actuator, with changing widths, glued to the upper and lower beam surfaces, respectively. The paper is devoted to the stability analysis of the closed-loop system described by the stochastic partial differential equation without a finite-dimensional approach. The effective stabilization conditions implying the almost sure stability are the main results. A fluctuating axial force is modelled by the physically realizable ergodic process. The rate velocity feedback is applied to stabilize the panel parametric vibrations. Calculations are performed for the Gaussian process with given mean value and variance as well as for the harmonic process with an amplitude A.

Key words: active damping, parametric excitation, distributed control

1. Introduction

Piezoelectric materials show great advantages as sensors and actuators in intelligent structures i.e. the structures with highly distributed actuators, sensors, and processor networks. Piezoelectric sensors and actuators have been applied successfully in the closed-loop control of beams (Bailey and Hubbard, 1985) and plates (Dimitriadis et al., 1991). A comprehensive static analysis of a piezoelectric actuator glued to a beam was given by Crawley and de-Luis (1987). The relationship between the static strains (both in the structure and actuator) and applied voltage across the piezoelectric was presented. A dynamic model of a simply supported beam with piezoelectric actuators perfectly glued to both surfaces was developed by Jie Pan and Hansen (1991). An extended dynamic model of beam, bonding layers, sensor and piezoactuators with emphasis on active damping was considered by Tylikowski (1994). Several authors (Tzou and Fu, 1992; Pourki, 1993; Tylikowski, 1995) analysed the influence of the shape and segmentation of actuators on the structure vibrations and the observability and controlability of the system. The direct Liapunov method was applied to the stabilization problem of the beam subjected to a wide-band axial time-dependent force (Tylikowski, 1995) and to a panel flutter problem (parametric excitation) (Tylikowski, 1997).

The purpose of the present paper is to solve an active control problem of beam parametric vibrations excited by the axial randomly fluctuating force. The problem is solved using the concept of distributed piezoelectric sensors and actuators with a sufficiently large value of velocity feedback. Real mechanical systems are subjected not only to nontrivial initial conditions but also to permanently acting excitations increasing the structure energy and the active vibration control should be modify in order to balance the energy supplied by external parametric excitation. The bonding layer is modelled as the Voigt-Kelvin viscoelastic material increasing a global passive damping of the beam. The applicability of active vibration control is extended to cover distributed systems with stochastic parametric excitation. The velocity feedback is applied to stabilize the beam parametric vibrations. Applying the direct Liapunov method the sufficient almost sure stability conditions for the beam with closed-loop control are derived. Almost sure stability domains in terms of the effective retardation time, feedback constant, force mean value and variance are obtained. The fluctuating axial force is modelled by the physically realizable ergodic process. The rate velocity feedback is applied to stabilize the panel parametric vibrations. Calculations are performed for the Gaussian and harmonic processes.

2. Dynamics equation

A continuous mechanical system (beam or plate under cylindrical bending) uni-axially loaded in the middle plane by time-dependent force $S = S_0 + S(t)$ is considered. The dynamics equation of structure motion includes both an

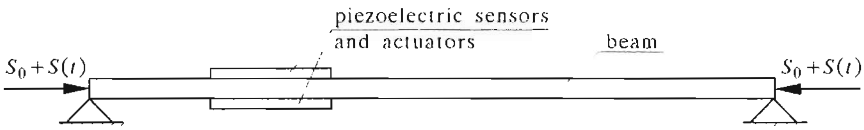

Fig. 1. Beam bonded with piezoelectric layers

internal passive damping due to viscoelastic properties and an active damping. Finite thickness piezoelectric patches are mounted on the opposite sides of the structure. It is assumed that the transverse motion dominates the in-plane plate vibrations. The sensing and actuating effects of piezoelectric layers are used to extract mechanical energy and as a final result to stabilize both the free vibrations due to initial disturbances and the parametric vibrations excited by the time-dependent axial force. We assume a negligible stiffness of the sensor in comparison with that of the structure and reduce the influence of the piezoelectric actuator on the structure to shear forces in the bonding layers distributed over the structure surface.

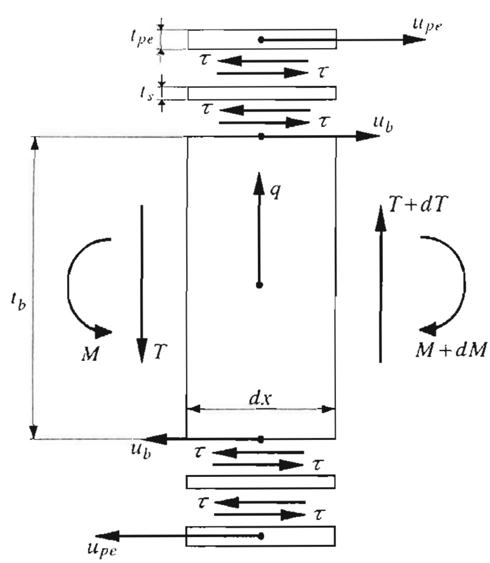

Fig. 2. Geometry of the beam element with bonding layers and piezoelements

Let us consider two opposite phase piezoelectric elements bonded by a finite-thickness bonding layer to an elastic beam. The beam is described by the Bernoulli-Euler theory. In order to derive the dynamic equations the beam element shown in Fig.2 is examined. Under the assumption of pure one-dimensional shear in the bonding layer treated as the Voigt-Kelvin material, pure extensional strains in the piezoactuators and negligible inertia of the piezoactuator and bonding layer, the equations have the form

$$
\begin{aligned}
&N_{pe,x} - b\tau = 0 \\
&\rho_b t_b b w_{,tt} = T_{,x} - \Big(S_0 + S(t)\Big) w_{,xx}
\end{aligned}
\tag{2.1}
$$

where

w	–	transverse beam displacement
u_{pe}	–	axial piezoactuator displacement
x	–	axial coordinate
ℓ	–	beam length
A	–	piezoelectric layer cross-section, $A = t_{pe} b$
ρ_b	–	beam density.

The axial force in piezoelectrics N_{pe}, interlayer shear stress τ, and beam transverse force T and bending moment M can be found from the classical formulae

$$
\begin{aligned}
&N_{pe} = A E_{pe} u_{pe,x} \\
&\tau = \frac{G}{t_s}\Big[u_{pe} + \frac{t_b}{2} w_{,x} + \alpha\Big(u_{pe,t} + \frac{t_b}{2} w_{,xt}\Big)\Big] \\
&M = -E_b J_b (w_{,xx} + \lambda_b w_{,xxt}) \\
&T = -E_b J_b (w_{,xxx} + \lambda_b w_{,xxxt}) + t_b b \tau
\end{aligned}
\tag{2.2}
$$

where

t_b	–	beam thickness
b	–	beam constant width
E_b	–	beam Young modulus
J	–	cross-sectional moment of inertia
α, λ_b	–	retardation times of bonding layer and beam, respectively.

Using Eqs (2.2) the dynamics equations of beam motion can be written in the form

— for $x \in (x_1, x_2)$

$$
\begin{aligned}
&-E_{pe}Au_{pe,xx} + \frac{Gb}{t_s}\Big[u_{pe} + \frac{t_b}{2}w_{,x} + \alpha\Big(u_{pe,t} + \frac{t_b}{2}w_{,xt}\Big)\Big] = 0 \\
&\rho_b t_b b w_{,tt} + E_b J_b(w_{,xxxx} + \lambda_b w_{,xxxxt}) + \\
&-\frac{Gbt_b}{2t_s}\Big[u_{pe} + t_b w_{,x} + \alpha\Big(u_{pe_t} + \frac{t_b}{2}w_{,xt}\Big)\Big]_{,x} + \Big(S_0 + S(t)\Big)w_{,xx} = 0
\end{aligned}
\tag{2.3}
$$

— for $x \in (0, x_1) \cup (x_2, \ell)$

$$
\rho_b t_b b w_{,tt} + E_b J_b(w_{,xxxx} + \lambda_b w_{,xxxxt}) + \Big(S_0 + S(t)\Big)w_{,xx} = 0 \tag{2.4}
$$

We assume the simply supported boundary conditions imposed on the solution of Eq (2.4) at $x = 0$ and $x = \ell$, continuity of deflection, slope, curvature and transverse force for $x = x_1$ and $x = x_2$.

In view of Eq $(2.2)_4$ the conditions corresponding to the continuity of transverse forces can be written down as

$$
\begin{aligned}
E_b J_b w_{,xxx}(x_1^-) &= E_b J_b w_{,xxx}(x_1^+) - t_b b\tau(x_1^+) \\
E_b J_b w_{,xxx}(x_2^+) &= E_b J_b w_{,xxx}(x_2^-) - t_b b\tau(x_2^-)
\end{aligned}
\tag{2.5}
$$

The piezoelectric displacement should satisfy the free edge conditions for x_1 and x_2, which can be written in the form

$$
u_{pe,x} = \Lambda \tag{2.6}
$$

The dynamics equations with zero initial conditions, and unactivated actuators have a trivial solution, which corresponds to an undeflected structure equilibrium.

Vibration damping of the visco-elastic beam with parametric excitation can be examined by differentiating the total energy of the beam with piezoelements (Tylikowski, 1995). The rate of energy extraction indicates that for the sufficiently large gain factor it is possible to stabilize parametric vibrations. However, the result does not provide effective quantitive estimation of the minimal active damping coefficients stabilizing the parametric vibrations.

If the parametric excitation is a realizable ergodic stochastic process the dynamics equation should be understood as a partial differential equation with a random parameter.

3. Dynamics equation with distributed feedback

Consider the Bernoulli-Euler beam axially loaded by a time-dependent force with piezoelectric layers mounted on each of the two opposite sides of the beam. The piezoelectric layers are assumed to be bonded on the beam surfaces and the mechanical properties of the bonding material are represented by the effective retardation time λ_b of the beam treated as a laminated one (Jones, 1975). The effective retardation time is a linear function of both the beam and bonding layer retardation times. It is assumed that the transverse motion dominates axial vibrations of the beam. The thicknesses of actuator and sensor are denoted by t_a and t_s, respectively. The sensing and actuating effects of piezoelectric layers are used to stabilize both the free vibrations due to initial disturbances and the parametric vibrations excited by the axial force. Assuming a negligible stiffness of the sensor in comparison with that of the beam and reducing the influence of the piezoelectric actuator on the beam to a bending moment M distributed along the beam, Eq $(2.3)_2$ for the beam with distributed sensor and actuator layers can be rewritten in the form, $x \in (0,\ell)$

$$\rho b t_b w_{,tt} + E_b J(w_{,xxxx} + \lambda_b w_{,xxxxt}) + \Big(S_0 + S(t)\Big) w_{,xx} + M_{,xx} = 0 \tag{3.1}$$

Using the constitutive equation of piezoelectric materials and integrating over the sensor area define the total charge

$$Q = -d_s E_s \frac{t_b + t_s}{2} \int\limits_0^\ell b_s(x) w_{,xx}\, dx \tag{3.2}$$

where

d_s – sensor piezoelectric constant
b_s – sensor width
E_s – sensor Young modulus.

Using the standard equation for capacitance the voltage $\mathcal{V}_s$ produced by the sensor is

$$\mathcal{V}_s = -\frac{d_s E_s (t_b + t_s) t_s}{2\epsilon_{33} A_s} \int\limits_0^\ell b_s(x) w_{,xx}\, dx \tag{3.3}$$

where ϵ_{33} represents the permittivity of sensor material. Using the velocity feedback control (Balas, 1979) the voltage applied to the actuator is

$$\mathcal{V}_a = K_a \frac{d\mathcal{V}_s}{dt} \tag{3.4}$$

The control bending moment can be expressed by the actuator stress σ_a, moment arm $t_b + t_a$, and cross section area $t_a b(x)$ of the actuator in the following way

$$M = \sigma_a t_a \frac{t_b + t_a}{2} b_a(x) \tag{3.5}$$

Introducing the feedback gain factor K_a the control moment is related with the time derivative of beam curvature as follows

$$M = \frac{K_a(t_b + t_s)(t_b + t_a)t_s d_s d_a E_s E_a}{4\epsilon_{33} A_s} b_a(x) \int\limits_0^L b_s(x) w_{,xxt}\, dx \tag{3.6}$$

In dimensionless variables: the time $t \to \frac{t}{\ell^2}\sqrt{\frac{E_b J}{\rho b t_b}}$, the coordinate $x \to x/\ell$ and the displacement $w \to w/\ell$ Eq (3.1) becomes

$$w_{,tt} + \Big(f_0 + f(t)\Big) w_{,xx} + w_{,xxxx} + \lambda w_{,xxxxt} + m_{,xx} = 0 \qquad x \in (0,1) \tag{3.7}$$

where the reduced axial load is given by

$$f_0 + f(t) \to \frac{\ell^2}{E_b J}\Big[S_0 + S\Big(t\sqrt{\frac{\rho b t_b \ell^4}{E_b J}}\Big)\Big]$$

and the bending moment produced by the piezoelectric actuator is as follows

$$m = 2\beta_a \chi_a \int\limits_0^1 \chi_s w_{,xxt}\, dx \tag{3.8}$$

The internal (passive) damping coefficient λ and the active damping coefficient β_a are given, respectively

$$\lambda \to \frac{\lambda_b \ell^2}{2\sqrt{\rho b t_b E_b J}}$$

$$\beta_a \to \frac{K_a(t_b + t_s)(t_b + t_a)t_s}{8\epsilon_{33} A_s} d_s d_a E_s E_p \frac{1}{\ell\sqrt{\rho b t_b E_b J}}$$

The functions $\chi_s(x) \equiv b_s(x\ell)$ and $\chi_a(x) \equiv b_a(x\ell)$ denote the sensor and actuator widths, respectively, in the range $x \in (0,1)$. In the case when the bending moment is equal to zero at the beam ends, the beam can be treated as simply supported. Thus, we assume that the beam transverse displacement satisfies the following boundary conditions

$$
\begin{aligned}
w(0,t) &= w(1,t) = 0 \\
w_{,xx}(0,t) &= w_{,xx}(1,t) = 0
\end{aligned}
\tag{3.9}
$$

Eq (3.7) with zero initial conditions $w(x,0) = w_{,t}(x,0) = 0$ has possess a trivial solution $w(x,t) = 0$, which corresponds to an undeflected beam axis.

4. Almost sure stability analysis

The purpose of the present paper is to derive the criteria for solving the following problem: will the deviations of beam axis from the unperturbed state (trivial solution) be sufficiently small, in some mathematical sense in the case when the axial force is the stochastic process. The beam buckles dynamically when the axial force becomes so large that the beam with closed-loop control does not oscillate (vibrations of beam with closed-loop control do not decay) and a new increasing mode of oscillations occurs. To estimate a perturbed solution of Eq (3.7) it is necessary to introduce a measure of distance $\|\cdot\|$ before the solution with nontrivial initial conditions and the trivial one. The most common stability definition used in continuum mechanics, states that an equilibrium state is stable whenever in the motion following any sufficiently small initial disturbances the displacement w and the velocity $w_{,t}$ are arbitrarily small everywhere for $t > 0$. In order to investigate the behavior of the solutions of stochastic equations a modification of the Liapunov stability definition is needed. The equilibrium state of Eq (3.7) is said to be almost sure stochastically stable (Kozin, 1972), if

$$
P\Big\{\lim_{t\to\infty} \|w(\cdot,t)\| = 0\Big\} = 1 \tag{4.1}
$$

In the present paper the direct Liapunov method is proposed to establish the criteria for the almost sure stability of the unperturbed (trivial) solution of the structure with closed-loop control.

The crucial point of the Liapunov method is construction of a suitable functional, which is positive-definite along any motion of the beam with closed-loop control. We construct the Liapunov functional as a sum of the modified kinetic energy and the elastic energy of the beam

$$V = \frac{1}{2}\int_0^1 (w_{,t}^2 + 2\lambda w_{,t} w_{,xxxx} + 2\lambda^2 w_{,xxxx}^2 + w_{,xx}^2 - f_0 w_{,x}^2)\, dx \tag{4.2}$$

If the classical condition for the static buckling is fulfilled, functional (4.2) satisfies the desired positive-definiteness condition, and the measure of distance between the perturbed solution and the trivial one can be chosen as the square root of the functional $\|\cdot\| = V^{1/2}$.

If trajectories of the processes are physically realizable the clasical calculus is applied to calculation of the time-derivative of Eq (4.2)

$$\begin{aligned}\frac{dV}{dt} = \int_0^1 \Big[&-\lambda w_{,xxt}^2 - \lambda w_{,xxxx}^2 + \lambda(f_0 + f(t)) w_{,xxx}^2 + \\ &+ w_{,xx}^2 - f(t) w_{,t} w_{,xx} - m_{,xx}(w_{,t} + \lambda w_{,xxxx})\Big]\, dx\end{aligned} \tag{4.3}$$

Let us focus our attention on the following particular shapes of piezoelectric elements. The sensor and actuator are described by the sine function with diffrent maximum widths b_s, b_a, respectively

$$\chi_s(x) = b_s \sin \pi x \qquad\qquad \chi_a(x) = b_a \sin \pi x \tag{4.4}$$

The beam motion can be expanded into the following sine Fourier series

$$w(x,t) = \sum_{n=1}^{\infty} T_n(t) \sin n\pi x \tag{4.5}$$

satisfying the boundary conditions (3.9). We are now in a position to calculate the spatial integral involved in the bending moment (3.8) with the shape functions (4.4). We rewrite the time-derivative (4.3) of the functional in the form

$$\frac{dV}{dt} = -2\lambda V + 2U \tag{4.6}$$

where the auxiliary functional U is as follows

$$\begin{aligned} U &= \int_0^1 \Big[-\lambda w_{,xxt}^2 - \lambda w_{,xxxx}^2 + \lambda(f_0 + f(t)) w_{,xxx}^2 + w_{,xx}^2 + \\ &- f(t) w_{,t} w_{,xx} - \gamma \dot{T}_1 \sin \pi x (w_{,t} + \lambda w_{,xxxx}) + \\ &+ \lambda(w_{,t}^2 + 2\lambda w_{,t} w_{,xxxx} + 2\lambda^2 w_{,xxxx}^2 + w_{,xx}^2 - f_0 w_{,x}^2)\Big]\, dx \end{aligned} \tag{4.7}$$

The gain factor γ is calculated from Eq (3.8) substituting Eq (4.4).

Due to the orthogonality of $\sin n\pi x$ we have

$$V = \sum_{n=1}^{\infty} V_n \qquad\qquad U = \sum_{n=1}^{\infty} U_n \tag{4.8}$$

and we can study the associated problem

$$U_n < \chi_n V_n \tag{4.9}$$

which implies

$$U < \chi V \qquad\qquad \frac{dV}{dt} \leqslant -2(\lambda - \chi)V \tag{4.10}$$

Inequality $(4.10)_2$ is equivalent to the solution of quadratic inequalities for $n = 1, 2, ...,$ where δ_{ij} is the Kronecker delta

$$\begin{aligned}
&\Big[\lambda(n\pi)^4 + (\chi_n - \lambda) + \delta_{1n}\gamma\Big]\dot{T}_n^2 + \\
&+\Big[2(\chi_n - \lambda)(n\pi)^4 - f(t)(n\pi)^2 + \lambda\gamma\pi^4\delta_{1n}\Big]\dot{T}_n T_n + \\
&+\Big\{\lambda(n\pi)^4\Big[(n\pi)^2 - f_0 - f(t)\Big] + \\
&+(\chi_n - \lambda)\Big[2\lambda^2(n\pi)^6 + (n\pi)^2 - f_0\Big]\Big\}(n\pi)^2 T_n^2 > 0
\end{aligned} \tag{4.11}$$

Calculating the maximum of χ_n we solve the almost sure stability problem for a class of ergodic processes in the following form

$$\lambda \geqslant \Big\langle \max_n \chi_n(t) \Big\rangle \tag{4.12}$$

where $\langle\cdot\rangle$ denotes an operator of mathematical expectation.

Inequalities (4.11) and (4.12) offer us a possibility to obtain minimal effective retardation times implying the almost sure asymptotic stability, called critical retardation times. A domain where the retardation times are greater than the critical ratardation times is called the stability region. The stability regions as functions of constant component of the axial force F_0, axial loading variance σ^2, effective retardation time λ, and gain factor γ are calculated numerically. First, discrete values of the force f are chosen and Eq (4.11) is solved.

Then the largest value χ corresponding to the given value of force is determined and the expectation is calculated numerically integrating the product of χ by the probability density function of loading. This is made for various values of parameters by choosing the variance and varying the retardation time until inequality (4.12) is satisfied. Numerical calculations are performed for the Gaussian process with the mean F_0 and variance σ^2 and for the harmonic process with an amplitude A. In order to compare both processes the variance of harmonic process $\sigma^2 = A^2/2$ is used. The almost sure stability region of the beam axially loaded by the zero mean Gaussian process is shown in Fig.3.

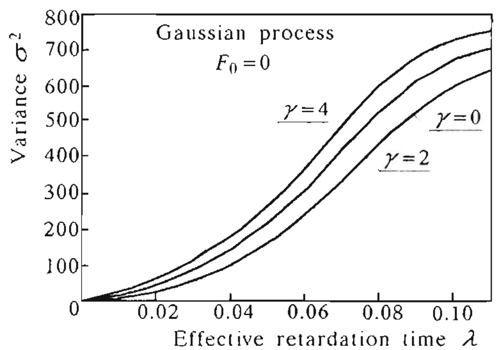

Fig. 3. Stability regions of beam for the zero mean Gaussian force

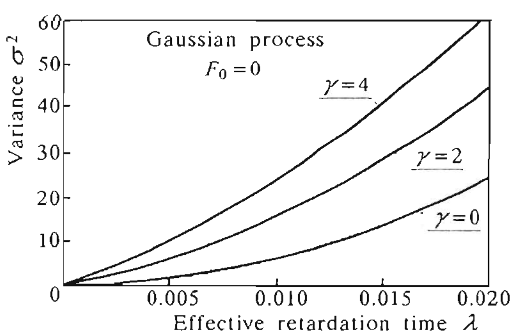

Fig. 4. Details of stability regions

It is seen that the stability regions enlarge as the gain factor increases. Details of the stability regions for smaller values of the variance are shown in Fig.4. The influence of the feedback gain factor is much more pronaunced for small values of force variance. Fig.5 compares the stability regions for the beam with the zero mean force loaded by the Gaussian and harmonic processes, respectively. It is visible that the influence of the class of excitation is not substantial. Fig.6 shows the critical variance versus the retardation time for the beam loaded by the constant force close to the static buckling load π^2. The increase of critical variances is much more slower in comparison to the zero mean loading.

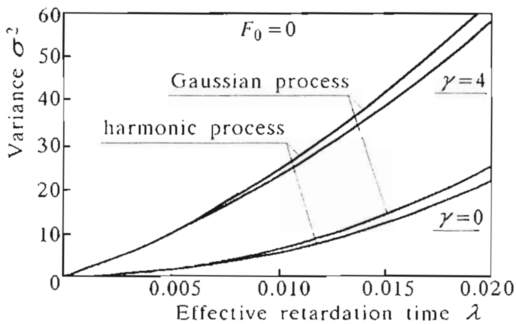

Fig. 5. Comparison of stability regions for the Gaussian and harmonic loadings

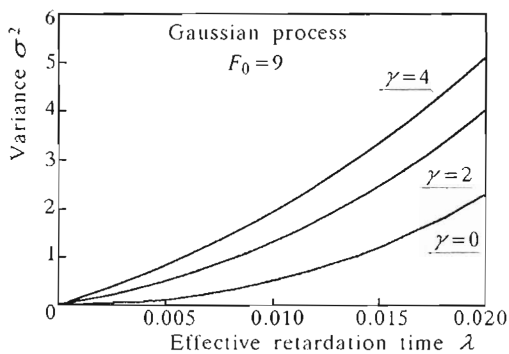

Fig. 6. Influence of constant compresive force on stability regions for the Gaussian loading

It is necessary to emphasise that the results are obtained just for the sinusoidal shapes of distributed sensor and actuator.

5. Conclusions

By means of the direct Liapunov method the active stabilization of a vibrating beam with distributed piezoelectric sensor, actuator, and the velocity feedback has been studied. The elastic beam is simply supported and subject to a compressive axial force randomly fluctuating. Without any passive damping and control, the beam motion is unstable due to the parametric excitation.

The stabilization of stochastic parametric vibrations needs sufficiently large active damping coefficient proportional to the gain factor. Admissible variances of loading depend strongly on the feedback gain factor. The stability regions do not change qualitatively when going from the Gaussian process to the harmonic one, but the Gaussian loading needs smaller critical retardation time than the harmonic loading.

For no axial force, this is the case of free vibration due to the nontrivial initial conditions. As long as the active or passive damping is present, the system is stable and oscillations decay.

Acknowledgments

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Stabilizacja drgań parametrycznych belki za pomocą rozłożonych elementów piezoelektrycznych

Streszczenie

Praca dotyczy stabilizacji belki sprężystej poddanej działaniu siły ściskającej zależnej od czasu. Warunek prawie pewnej stateczności stochastycznej wyprowadzono za pomocą bezpośredniej metody Lapunowa. Jako funkcjonał Lapunowa przyjęto sumę zmodyfikowanej energii kinetycznej i energii potencjalnej belki. Tłumienie w warstwie łączącej elementy piezoelektryczne z belką opisano efektywnym współczynnikiem tłumienia modelu Voigta-Kelvina. Sterowanie jest realizowane za pomocą rozłożonych piezoelektrycznych elementów pomiarowych i wykonawczych o zmiennej szerokości przyklejonych do powierzchni belki. Zasadniczym celem pracy jest analiza stabilności układu ciągłego ze sprzężeniem zwrotnym opisanym równaniem o pochodnych cząstkowych. Głównym wynikiem są efektywne warunki stabilizacji zapewniające prawie pewną stabilność stochastyczną belki. Zmienna siła modelowana jest jako fizycznie realizowalny stochastyczny proces ergodyczny. Prędkościowe sprzężenie zwrotne zapewnia stabilizację drgań parametrycznych. Obliczenia są przeprowadzone dla procesu Gaussowskiego o danej średniej i wariancji oraz procesu harmonicznego o danej amplitudzie.

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