JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
4, 38, 2000

ON APPLICATION OF DYNAMICAL SYSTEMS THEORY INTO INVESTIGATION OF CRITICAL FLIGHT REGIMES OF FLYING VEHICLES

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Non-linear dynamics phenomena have become important for various aircraft motions. Manoeuvrability of an aircraft in critical flight regimes involves non-linear aerodynamics and inertial coupling. Dynamical systems theory provides a methodology for studying non-linear systems of ordinary differential equations. Bifurcation theory is a part of that theory which is considering changes in the stability, which lead to qualitatively different responses of the system. These changes are called bifurcations. The mathematical models used in the paper assume a rigid aircraft with movable control surfaces, and "individual blade" rotorcraft model. Aerodynamic model includes also a region of higher angles-of-attack including deep stall phenomena. In the present paper, the wing-rock oscillations, and helicopter spin (i.e. intensive spiral glide motion) was studied by means of checking the stability characteristics related to unstable equilibria. Numerical simulations were used to verify the predictions. Wing-rock oscillations were studied to observe the chaos phenomenon in post-stall manoeuvres. Unsteady aerodynamics for prediction of the airfoil loads was included, and the ONERA-type stall model was used.

Key words: flight mechanics, bifurcation theory, dynamical systems theory

1. Introduction

Aircraft is an inherently non-linear and time-depending system. Non-linear dynamics is crucial for several important aircraft motions, including roll-coupling and stall/spin phenomena. Linearized equations of motion can not be used to analyze these phenomena. Indeed, roll-coupling instabilities were first discovered in flight, often with fatal results, because the linearized equations of motion used for analysis at that time did not account for the instability (Jahnke and Culick, 1994).

There are many problems associated with flight dynamics for modern and advanced aircraft, which have not been solved (or solved rather unsatisfactorily) with traditional tools. A list of such problems includes, among others, flight control for agile and post-stall aircraft. The post-stall maneuverability has become one of the important aspects of military aircraft development. Such maneuvers are connected with a number of singularities, including "unexpected" aircraft motion. As the result, there is a danger of faulty pilot's actions. Therefore, it is necessary to investigate the aircraft flight phenomena at high and very high angles of attack.

The appearance of a highly augmented aircraft requires a study of its high angle of attack dynamics. The main aim of the paper is to discuss capabilities of the dynamical system theory methods as the tools for the analysis of such phenomena.

Dynamical system theory has provided a powerful tool for analysis of nonlinear phenomena of the aircraft behaviour. In application of this theory, numerical continuation methods (wiggins, 1990) and bifurcation theory (Ioos and Joseph, 1980) have been used to study roll-coupling instabilities and stall/spin phenomena of a number of aircraft models. Results of great interest have been reported in several papers (let us mention the papers by Jahnke and Culick, 1994; Avanzini and de Matteis, 1998; Carroll and Mehra, 1982; Guicheteau, 1990). Continuation methods are the numerical techniques for calculating the steady states of systems of ordinary differential equations and can be used to study the roll-coupling instabilities and high-angle of attack instabilities.

2. Non-linear equations of motion

For the study, the following aircraft model is used in the state space formulation (Sibilski, 1998b)

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{2.1}$$

with the state vector for a fixed wing aircraft

$$\boldsymbol{x} = [V, \alpha, \beta, p, q, r, \Phi, \Theta, \Psi, x_g, y_g, z_g]^\top \tag{2.2}$$

or, with the state vector for a rotorcraft

$$\begin{aligned} \boldsymbol{x} \;=\; & [u, v, w, p, q, r, , \dot{\beta}_1, ..., \dot{\beta}_n, \dot{\zeta}_1, ..., \dot{\zeta}_n, \Omega, \psi_1, ..., \psi_n, \beta_1, ..., \beta_n, \\ & \zeta_1, ..., \zeta_n, \Theta, \Phi, \Psi, x_g, y_g, z_g]^\top \end{aligned} \tag{2.3}$$

the control vector for a fixed wing aircraft

$$\boldsymbol{u} = [\alpha_{zH}, \delta_H, \delta_A, \delta_T]^\top \tag{2.4}$$

or, the control vector for a rotorcraft

$$\boldsymbol{u} = [\theta_0, \kappa, \eta, \phi_T]^\top \tag{2.5}$$

where: V – velocity of the aircraft; α – angle of attack; β – slip angle; p, q, r – roll, pitch and yaw rate, respectively; $\Phi, \Theta, \Psi, x_g, y_g, z_g$ – the parameters describing the aircraft position; u, v, w – linear velocities of the centre of fuselage mass in the co-ordinate system fixed with the fuselage; β_i – flap angle of the ith blade; ζ_i – lead angle of the ith blade; α_{zH} – longitudinal decalage; $\delta_H, \delta_A, \delta_V, \delta_T$ – control surfaces deflections; θ_0 – angle of collective pitch of the main rotor; κ – control angle in the longitudinal motion, η – control angle in the lateral motion, and ϕ_T – angle of collective pitch of the tail rotor. Equations of motion are completed with equation of engine rotation and equation of thrust (Sibilski, 1998b). The engine model should be adapted to the code, data, and flow charts provided by the engine manufacturer. The aerodynamic characteristics and derivatives, both for steady and unsteady aerodynamic models (including deep stall conditions), are calculated using the modified wing section theory (Sibilski, 1998b). Non-linear airfoil characteristics, which were used, were calculated using the ONERA deep stall model. More exact and particular description of the methods and algorithms used is numerical algorithms and computer codes predicted for calculations of aircraft aerodynamics loads are found by Sibilski (1998b), Narkiewicz (1994).

3. Theoretical background

Dynamical systems theory (DST) provides a methodology for studying systems of ordinary differential equations (2.1). More information on DST can be found in the book of Wiggins (1990). The most important ideas of DST used in the paper will be introduced in the following sections.

The first step in the DST approach is to calculate the steady states of the system and their stability. Steady states can be found by setting all time derivatives equal to zero and solving the resulting system of algebraic equations. The Hartman-Grobman theorem (Wiggins, 1990) proves that the local stability of a steady state can be determined by linearizing the equations of motion about the steady state and calculating the eigenvalues.

The implicit function theorem (Ioos and Joseph, 1980) proves that the steady states of a system are continuous functions of the parameters of the system at all steady states in which the linearized system is non-singular. A singular linearized system is characterised by a zero eigenvalue. Thus, the steady states of the equations of motion for the aircraft are continuous functions of the control surface deflections. Stability changes can occur as the parameters of the system are varied in such a way that the real parts of one or more eigenvalues of the linearized system change sign. Changes in the stability of a steady state lead to qualitatively different responses for the system and are called bifurcations. Stability boundaries can be determined by searching for steady states, which have one or more eigenvalues with zero real parts. There are many types of bifurcations and all they have different effects on the aircraft response. Qualitative changes in the response of the aircraft can be predicted by determining how many and what types of eigenvalues have zero real parts at the bifurcations point. Bifurcations for which one real eigenvalue is zero lead to the creation or destruction of two or more steady states. Bifurcations for which one pair of complex eigenvalues has zero real parts, can lead to the creation or destruction of a periodic motion. Bifurcations for which more than one real eigenvalue or more than one pair of complex eigenvalues have zero real parts, lead to a very complicated behaviour.

3.1. Continuation technique and methodology scheme

Continuation methods are a direct result of the implicit function theorem, which proves that the steady states of a system are continuous functions of the parameters of the system at all steady states except for such steady states at which the linearized system is singular. The general technique consists in fixing all parameters except one and analysing the steady states of the system as functions of this parameter. If one steady state of the system is known, a new steady state can be approximated by linear extrapolation from the known steady state (Guckenheimer and Holmes, 1983; Troger and Steindl, 1991). The slope of the curve at the steady state can be determined by taking the derivative of the equation given by setting all the time derivatives equal to zero. If two steady states are known, a new steady state can be approximated by linear extrapolation through the two known steady states. The stability of each steady state can be determined by calculating the eigenvalues of the linearized system. Any changes in stability from one steady state to the next one will signify a bifurcation. Taking into account the experience of many researchers, one can formulate the following three-step methodology scheme (based on the bifurcation analysis and continuation technique) for the investigation of nonlinear aircraft behaviour:

- During the first step it is supposed that all parameters are fixed. The main aim is to search for all possible equilibria and closed orbits, and to analyze their local stability. This study should be as thorough as possible. The global structure of the state space (or *phase portrait*) can be revealed after determining the asymptotic stability regions for all the discovered attractors (stable equilibria and closed orbits). An approximate graphic representation plays an important role in the analysis of the calculated results.

- During the second, step the system behaviour is predicted by using the information about the evolution of the portrait with the parameter variations. The knowledge of the type of the encountered bifurcation and current position with respect to the stability regions of other steady motions is helpful for the prediction of further motion of the aircraft. The rates of parameters variations are also important for such a forecast. The faster the parameter change, the better the difference between steady state solution and transient motion can be observed.

- Finally, the numerical simulation is used for checking the obtained predictions and obtaining the transient characteristics of the system dynamics for large amplitude state variable disturbances and parameter variations.

The set of ODE (2.1) was solved using the continuation and bifurcation software AUTO97 (available at ftp://ftp.cs.concordia.ca/pub/doedel/auto). This very useful free software gives all the necessary bifurcation points for different values of control vector components.

4. Results

Fig.1 show the steady states of advanced jet trainer aircraft at the medium and high angles of attack.

The steady states represented in those figures show longitudinal trim conditions and spirally divergent motions. These figures show that for elevator deflections greater than -10.1 deg, the trim conditions of the aircraft are stable. The trim conditions for a given elevator deflection can be determined by

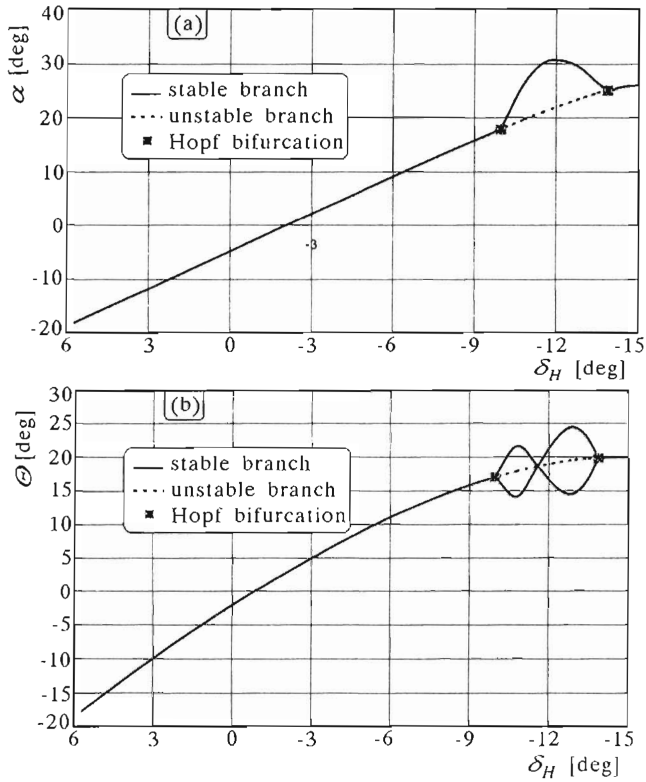

Fig. 1. Steady states at medium and high angles of attack

drawing the vertical line representing the desired elevator deflection on each plot; each intersection of this line with the curve of steady states gives a possible steady state of the aircraft. For the elevator deflection between -10.1 deg and -13.7 deg, the steady state trim conditions of the aircraft are unstable as a result of two Hopf bifurcations. Hopf bifurcations can lead to periodic motions, so it is possible that for elevator deflections between $-13.7°$ and $-10.1°$, the aircraft will undergo periodic motion.

The rotor blade stall affects the limiting condition of operation of the helicopter. Stall on the rotor blades limits the high-speed possibilities of the helicopter. This is understandable, when one considers that the retreating blade of the helicopter rotor encounters lower velocities as the forward speed is increased. The retreating blade must produce its portion of the lift, therefore, as the velocity decreases with forward speed, the blade angle of attack must be increased. It follows that at some forward speed, the retreating blade will stall. In forward flight, the angle of attack distribution along the blade is far from uniform, so it must be expected that some portion of the blade will stall before the rest.

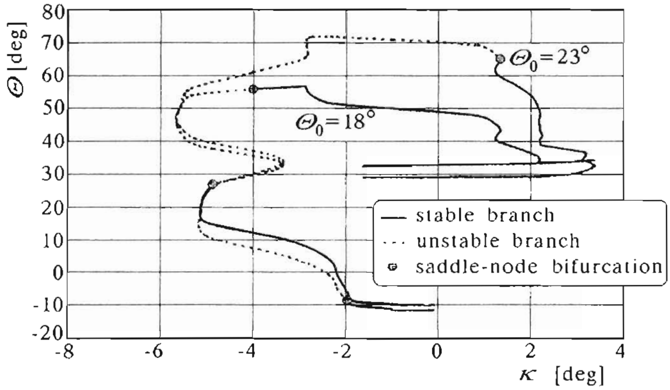

Fig. 2. Steady states for longitudinal manoeuvres

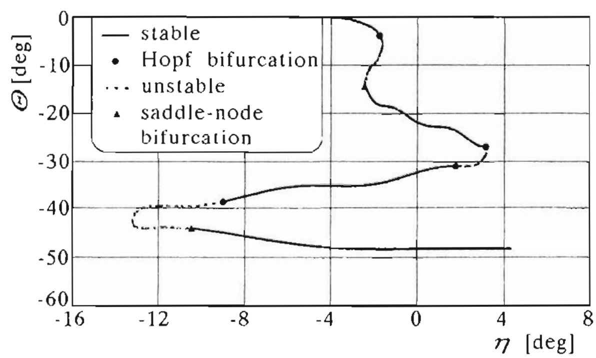

Fig. 3. Steady states for lateral manoeuvres

Fig.2 and Fig.3 show the steady states for the PZL "SOKÓŁ" utility helicopter as a function of the swash plate deflection. Three variants of calculations were considered. The first and the second one are for longitudinal manoeuvres, and the third one is for the lateral manoeuvre. For the first variant of calculations it is assumed that the collective pitch of main rotor blades equals $18°$, the collective pitch of tail rotor equals $17°$, and lateral swash plate deflection equals zero. For the second variant of calculations it is assumed that the collective pitch of main rotor blades is $23°$, the collective pitch of tail rotor is $19°$, and lateral swash plate deflection is equal to zero. And for the 3rd variant of calculations it is assumed that collective pitch of main rotor blades $\Theta = 24°$, collective pitch of tail rotor $\Theta_T = 21°$, and swash plate is deflected backwards (angle of deflection $\kappa = -2.5°$). These figures show that multiple steady states exist for most swash plate deflections. For example, the segment of unstable steady states contains the trim conditions between the lateral swash plate deflections at $-1.9°$ and $-2.1°$; $1.9°$ and $3.2°$; $-9.1°$ and $-10.3°$ because of six Hopf or saddle-node bifurcations. For example, one recalls that Hopf bifurcation can lead to periodic motions, so it is possible that for swash plate deflections between $1.9°$ and $3.2°$ the helicopter will undergo periodic motion.

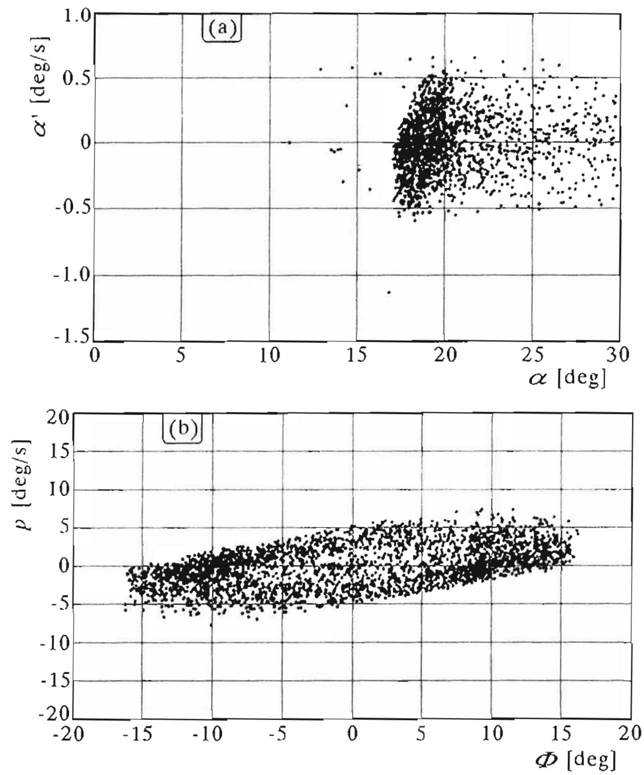

Fig. 4. Poincaré map – wing-rock instability

Fig.5 shows Poincaré maps and time simulation in which the elevator deflection is changed from $-9°$ to $-10.5°$, putting the aircraft in a region of unstable steady states. These figures show that slowly developing wing oscillations grow slowly and have a period of approximately 3 s. Note that magnitude and frequency of those oscillations are irregular and have a chaotic character.

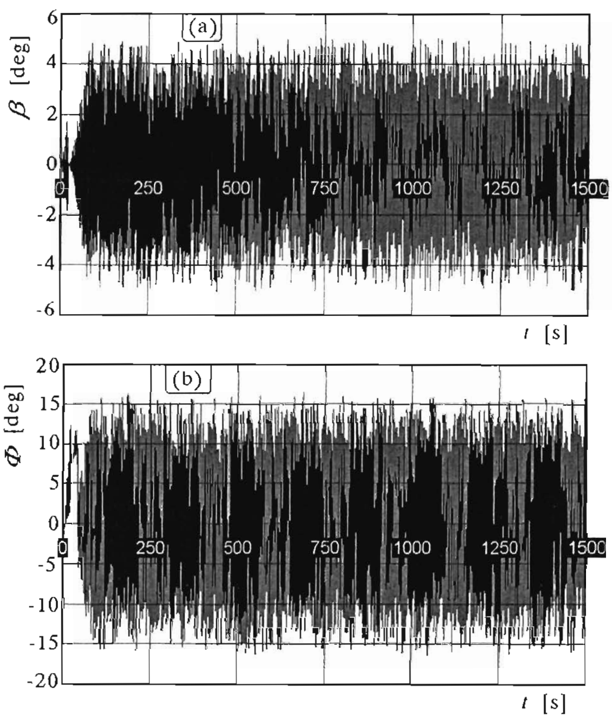

Fig. 5. Simulation of wing rock instability, variation of selected flight parameters

Fig.6 and Fig.7 show an attempted spin entry using only the lateral swash plate deflection. A small perturbation of swash plate deflection (near the initial value at $\eta_0 = -9.1°$) causes the rotorcraft to enter spin with a negative roll rate. During the spin, all flight parameters increase their values. In terms of the continuation methods, the spin is unstable because of the Hopf bifurcation that occurred at $\eta = -9.1°$. The Poincaré maps of selected state parameters are shown in Fig.6. It can be stated that taking into consideration unsteady rotor-blade aerodynamic model and hysteresis of aerodynamic coefficients, one encounters significant irregularities in solution of the equations of motion, that are characteristic for chaotic motion. When the condition for the onset of chaotic motion is satisfied, both the flapping and pitching motions appear to have chaotic oscillations.

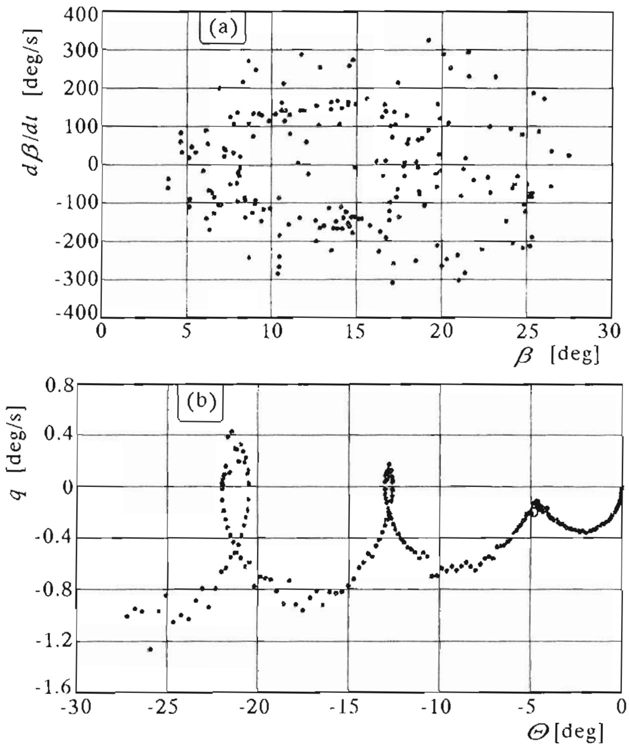

Fig. 6. Poincaré map – helicopter spin

5. Conclusions

The main aim of the study was to apply modern methods to investigation of non-linear problems in flight dynamics. Based on the investigation described above, the following conclusions can be drawn:

- The present results show the advantages which follow from using the continuation and bifurcation methods for analyzing the equations of aircraft motion;

- The efficiency of the methods makes it possible to analyze complicated aerodynamic models, using the complete equations of motions for the whole range of control surface deflections;

- Knowledge of such deflections which cause bifurcation, allows us to select

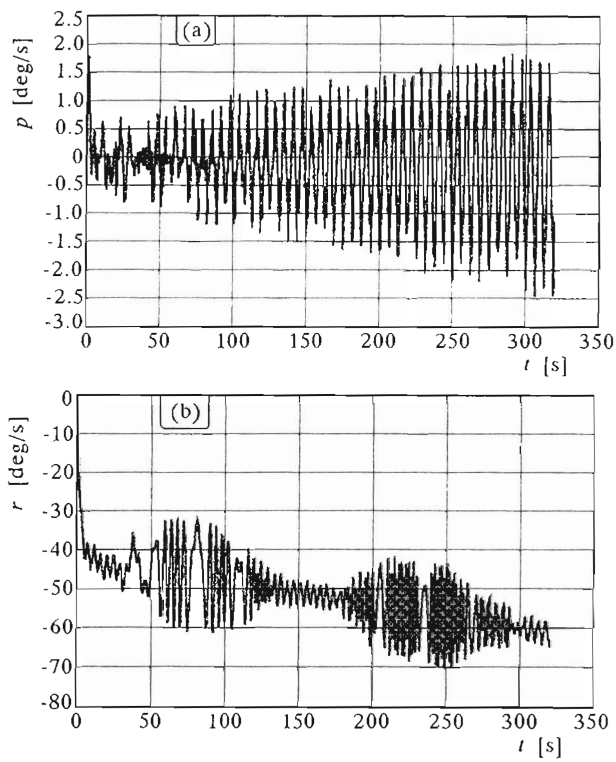

Fig. 7. Helicopter spin (intensive spiral-glide motion) variation of selected flight parameters

the most probable scenario of occurrences before the accident, and to avoid from risky motions;

- The need for a precise description of aerodynamic loads is a fundamental cause of difficulties;
- The presented approach can be applied to the prediction of space behaviour of an aircraft. Therefore, it can be also applied to modification of the aircraft dynamic characteristics.

Acknowledgments

The paper was prepared as a part of the project (Grant No. 9 T12C 002 15) financed by the State Committee of Scientific Research.

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Zastosowanie teorii systemów dynamicznych do badania krytycznych stanów lotu statków powietrznych

Streszczenie

Ruch statku powietrznego jest opisywany za pomocą układu silnie nieliniowych równań różniczkowych zwyczajnych. Zlinearyzowane równania ruchu nie mogą być zastosowane do opisu wielu zagadnień dynamiki lotu. Teoria systemów dynamicznych pozwala na efektywne badania nieliniowych równań różniczkowych. Teoria bifurkacji, będąca częścią teorii systemów dynamicznych, umożliwia badanie zmian stateczności, które prowadzą do jakościowo różnych odpowiedzi systemu. Założono, że statek powietrzny jest nieodkształcalny. Uwzględniono stopnie swobody ruchomych powierzchni sterowych oraz łopat wirnika nośnego. Przyjęty model aerodynamiczny umożliwia uwzględnienie zjawiska głębokiego przeciągnięcia dynamicznego oraz niestacjonarności opływu (histereza współczynników aerodynamicznych). Za pomocą metodyki teorii systemów dynamicznych rozpatrzono osobliwości niestateczności typu *wing-rock* i tzw. "korkocigu" śmigłowca. W celu zweryfikowania przewidzianych niestabilności przeprowadzono cyfrową symulację tych ruchów. Zaobserwowano nieregularność rozwiązań charakterystycznych dla ruchów chaotycznych.

Manuscript received March 31, 2000; accepted for print June 6, 2000