JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
4, **39**, 2001

FATIGUE CRACK GROWTH PECULIARITIES AND MODIFICATIONS OF THE WHEELER RETARDATION MODEL (PART 1)

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What has been given consideration in the paper, are peculiarities of fatigue crack growth, ones referred to curves of a crack length vs. the number of cycles or the rate of propagation vs. the range of stress intensity factor, i.e. the curves typical of tests with overloads applied in cycles – for the PA7 aluminium alloy and 18G2A steel. Difficulties with the model-based description (based on the Wheeler retardation model) of fatigue crack growth have been shown as due to the peculiarities discussed.

Key words: crack growth, overloads, retardation, Wheeler model

1. Introduction

At present, it is generally accepted that fatigue tests under a constant amplitude or program loading insufficiently represent realistic service load histories. With respect to fatigue crack growth under a variable amplitude loading, the term load interaction effects (also referred to as load sequence or load history effects) is used to label the phenomenon that the crack growth increment in a given cycle can be different than that in a constant amplitude test under the same maximum and minimum stress intensity. Random load tests and a statistically equivalent program load test do not give the same results (Jacoby, 1970; Schijve, 1973). Moreover, the effect of alternative designs, materials, manufacturing techniques, etc. on the fatigue life under a variable amplitude loading can only be judged correctly from variable amplitude tests. In some cases, constant amplitude tests can even give qualitatively wrong answers with respect to the fatigue crack growth resistance (Bucci et al., 1980; Schulte et al., 1984). This, together with development in data procesing methods and equipment capabilities (computer controled servohydraulic testing machines), has caused the fatigue testing under a realistic, random loading to become widely appreciated nowadays.

Because a deeper interest in the effects of load interaction on fatigue crack growth was initiated by aircraft manufacturers, aircraft materials were oryginally of primary importance in fatigue tests under a variable amplitude loading. Reviews of these early experimental results obtained under simple variable amplitude sequences and also under a program and random loading were prepared by Schijve (1972, 1985). Later on, the same autor presented an ample survey of trends observed in the analysis fatigue crack growth under flight-simulation loadings (Schijve, 1985). Following aircraft, other industrial branches, e.g. pressure vessels, nuclear reactors, bridges and offshore structures have also placed more reliance on damage tolerance designs. Though introducing the latter philosophy in a wider scale has prompted investigations on load interaction effects in a variety of metals, the coresponding data bank still remains by far more meagre than in the case of aircraft materials.

Imposition of a single overload during baseline cycling causes a retardation of the crack growth effect. The severity of this effect is usually measured by the normalized number of retardation (delay) cycles, N_D/N_{CA} (Skorupa, 1996). Here, N_D (reffered to as the delay distance) is the number of cycles between the overload application at the crack length a_{ov} and the recommencement of the steady state fatigue crack growth rate after the overload-affected crack growth increment Δa_{ov}. The N_{CA} is the number of baseline cycles that would have elapsed over Δa_{ov} in the absence of the overload. The fatigue crack growth rate da/dN response to a single overload can vary depending on the load parameters. In general, higher overload ratios $k_{ov} = F_{ov}/F_{max}$ give larger retardation in the crack growth. Increasing the overload ratio results in an increase in N_D/N_{CA} and Δa_{ov} and yields a lower minimum da/dN value (Skorupa, 1996; Blom, 1989; Schin and Fleck, 1987). For high overloads (overload ratio $k_{ov} > 2.5$), the initial acceleration was absent and immediate retardation was observed in a Ti-alloy (Ward-Close et al., 1989) and in mild steel (Damri and Knott, 1991). The retardation decreases if the stress ratio R is increased (Damri and Knott, 1991). For a stainless steel the fatigue crack growth increment between the overload application and the occurrence of the minimum da/dN to decrease and the initial acceleration to vanish as R increased from 0.1 to 0.6 until, at $R = 0.65$, retardation became immediate (Skorupa, 1996; Shin and Hsu, 1993). The ratio between Δa_{ov} and the size of the plastic zone measured on the surface of specimens decreased for increasing R from values larger than 1 (3.5 at $R = 0$) to values ranging between 0.2 and 0.3 for $R > 0.45$. The influence of the stress intensity factor ranges at the baseline level, ΔK is complex since the N_D vs. ΔK and Δa_{ov} vs. ΔK plots are typically U-shaped curves, thus indicating the least retardation effect at some intermediate ΔK value (Skorupa, 1996; Ward-Close et al., 1989). For certain combination of loading parameters, crack may retard by several orders of magnitude or even arrest though ΔK is far above threshold (Skorupa, 1996; Bernard et al., 1977; Bertel et al., 1983).

For an equivalent reduction in K values, a multiple overload gives rise to a more severe fatigue crack growth retardation during subsequent baseline cycles than a single overload (Ward-Close et al., 1989). The retardation at the step-down in load occurs more rapidly than for a single overload, and even immediately if during an overload block the stationary fatigue crack growth rate value is attained (Skorupa, 1996; Ward-Close et al., 1989; Sehitoglu and McDiamid, 1980). The retardation effect is amplified by elevating the overload ratio value, similarily to a single overload case (Chang et al., 1981; Chehini et al., 1984). Increasing the number of overload cycles N_{ov} gives rise to a more severe retardation effect through extending the N_D period, while the Δa_{ov} distance remains the same as for a single overload (Skorupa, 1996; Chehini et al., 1984; Chen and Roberts, 1985).

An intermittent overload can produce either retardation or acceleration in fatigue crack growth rates depending on the combination of load and material parameters. With a relatively frequently applied single intermittent overload ($N_{ov} = 1$) crack growth acceleration has been observed by several authors. In stress controlled tests the acceleration phenomenon appeared to be more pronounced in the low ΔK region and to diminish or even pass into retardation at higher ΔK values. The longer the interval DN between single overloads, the more severe retardation is produced, as exemplified by Skorupa (1996), Zhang et al. (1987). For a sufficiently large DN period, each overload application is visualized by a discontinuity in the corresponding a vs. N (crack length vs. number of cycles) plot. For a multiple intermittent overload $(DN > 1)$ the authors observed the retardation to increase by N_{ov} when DN was kept constant. Further extending of N_{ov}, however, resulted in the acceleration effect. Like in the case of a single intermittent overload, the retardation effect for a multiple intermittent overload sequence was enhanced when the DN interval was increased (Skorupa, 1996; Zhang et al., 1987).

For engineering practice the difficulties of crack growth propagation analysis are related, among other things, with choice of adequate calculating model, well fitted to geometrical, loading and material requirements of analysed problems. A model-based (theoretical) description of fatigue crack growth is usually a problem of great complexity for the following reasons, at least:

- it calls for a properly determined stress intensity factor adequate to the way of applying loads to a component under tests, geometry of the component with a propagating crack, occurrence of internal stresses, effects of environment, e.g. of a corroding medium, upon the component, etc.
- it requires verifying whether coefficients of equations of propagation remain constant for various modes of loading (loading mode I, II, III; the random nature of the applied load), or how the coefficients change with changes in stress ratios, mean loading levels, frequency, temperature
- it needs taking into account the phenomena such as effects of retardation, creeping, etc., i.e. ones that accompany the fatigue cracks growth.

Moreover, a good description of a selected case of fatigue crack propagation does not ensure similar description for other one – in another condition of testing and type of the material. Investigators must keep this restriction in mind. Existence of some osculate elements should be worthy of notice in the aspect of developing the modelling in fracture mechanics. Such elements are in fact mechanisms contributing to crack propagation at the microstructural level. Unfortunately, testing such phenomena is difficult, laborious, time-consuming and expensive. Theoretically, the crack growth rate retardation models can be a consequence in this area.

The purpose of this paper is to present peculiarities of fatigue crack propagation for various materials and various load conditions (part 1). Moreover, modifications of the Wheeler crack growth retardation model are proposed to cover more possibilities of modelling the crack propagation (part 2). The extension presented have deals with (among other things) the peculiarities mentioned above.

In this paper results of fatigue tests of 18G2A steel and the aluminiun alloy PA7 are presented. The experiments were performed under cyclic loading with overloads. The results obtained during the examinations and presented in fatigue curves $a = f(N)$ and $da/dN = f(\Delta K)$ are analysed. The genesis of changes in the Wheeler model and features of the modified model describing the fatigue crack propagation in a wide range are illustrated.

2. Examination of PA7 duralumin

The Wheeler model (Wheeler, 1972; Fuchs and Stephens, 1980; Kocańda and Szala, 1985) is a pretty good tool to describe fatigue crack propagation in materials for which a change in the crack length (or crack opening displacement – COD), after the load has been applied, against a number of load cycles, shows the nature as shown in Fig. 1:

Fig. 1a – initially, the rate of crack growth (or increasing of COD) is more or less intensively retarded in comparison to the crack growth rate prior to the application of the load, then systematic (monotonic) growth can be observed up to reaching the crack growth rate equal to that from before the load application, or

Fig. 1b – intensive retardation of the crack length increase (no change of COD) for a long time after applying the load (or up to applying a subsequent load, at least).

Both diagrams have resulted from fatigue tests on crack propagation. The tests were carried out using compact-tension (CT) specimens, made of the PA7 duralumin. The constant-amplitude loading with overloads (featured with the constant overload factor $k_{ov} = F_{ov}/F_{max}$) applied in cycles, i.e. every DN number of cycles, determined the testing conditions. An initial fatigue precrack, approximately 2 mm long as from the bottom of the notch, was made due to 30-40 thousands cycles of the constant amplitude $F_{min} - F_i$ (where F_i – maximum load applied at the stage of the precrack initiation) and stress ratio $R = 0.3$. The numbers of the applied load cycles and values of the crack opening displacement (converted then into the length of the propagating crack – with the compliance method) were recorded.

The Wheeler model describes fatigue crack propagation phenomena by the relationship

$$\frac{da}{dN} = C_p C(\Delta K)^m \tag{2.1}$$

and defines the retardation coefficient C_p in the following way

$$C_p = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^n \tag{2.2}$$

where

$r_{p,i}, r_{p,ov}$ – radii of plastic zones of current and overload cycles, respectively

a_i, a_{ov} – crack lengths of the current and overload cycles, respectively

n – exponent in the Wheeler model.

In the first case (Fig. 1a), corresponding to a relatively low value of the exponent n (if description made with the Wheeler model), the propagating

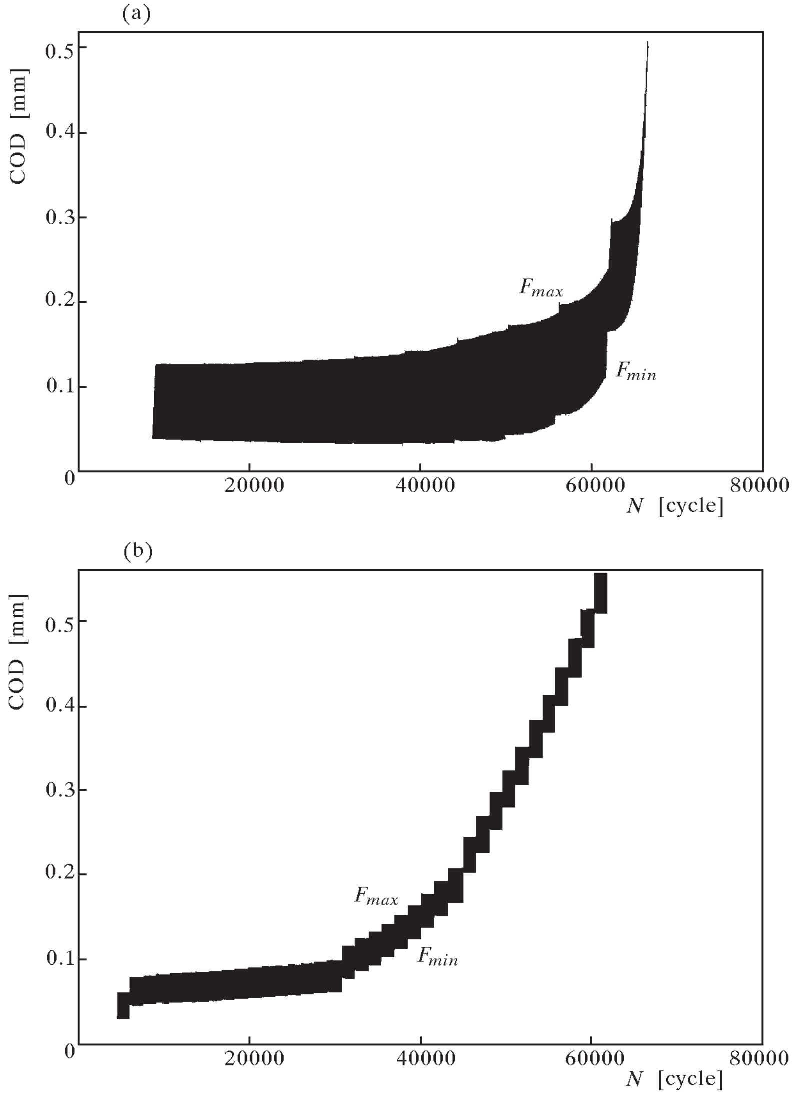

Fig. 1. Experimental records of crack opening displacement in CT specimen made of PA7 duralumin under fatigue tests with overloads applied in cycles

crack overcomes – partially or completely – the overload plastic zone, and completely escapes from the phase of retardation due to the overload – before the subsequent overload is applied.

In the second instance (Fig. 1b), corresponding to a relatively high value of the exponent n (the model-based description), the propagating crack does not surmount the overload plastic zone. It continues propagating under severe retardation conditions due to the overload, up to applying the subsequent overload. Considerable increments are gained in the course of applying the overload, rather than between the overloads.

3. Tests of the 18G2A steel

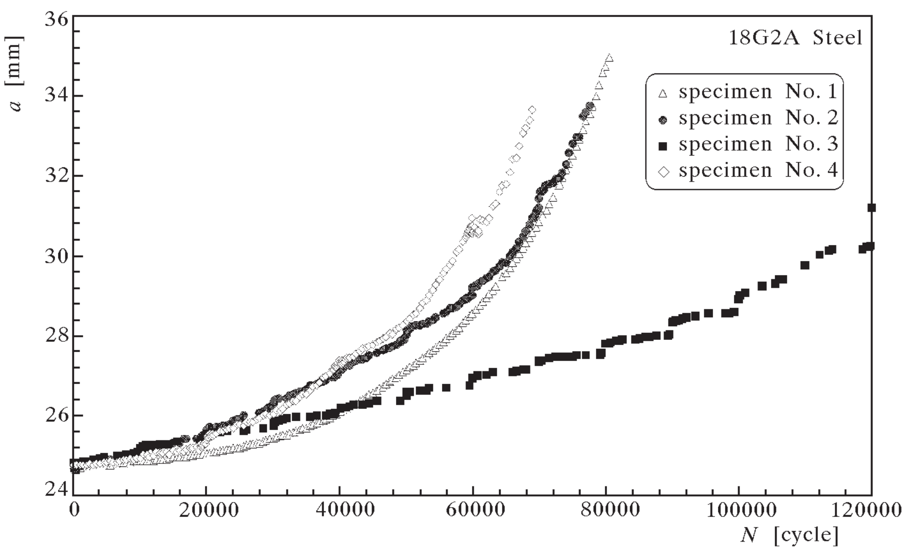

Fig. 2. Crack length a vs. number of cycles N for CT specimens made of 18G2A steel under fatigue tests with overloads applied in cycles

For comparison, Fig. 2. shows some results of tests aimed at investigating fatigue crack propagation in the 18G2A steel and, Fig. 3, a sample of their theoretical description. The tests were conducted using CT specimens, as well, under a cyclic constant-amplitude loading, without overloads at all, and with ones applied in cycles. The testing conditions and numbers of cycles up to the failures of the selected test pieces have been presented in Table 1.

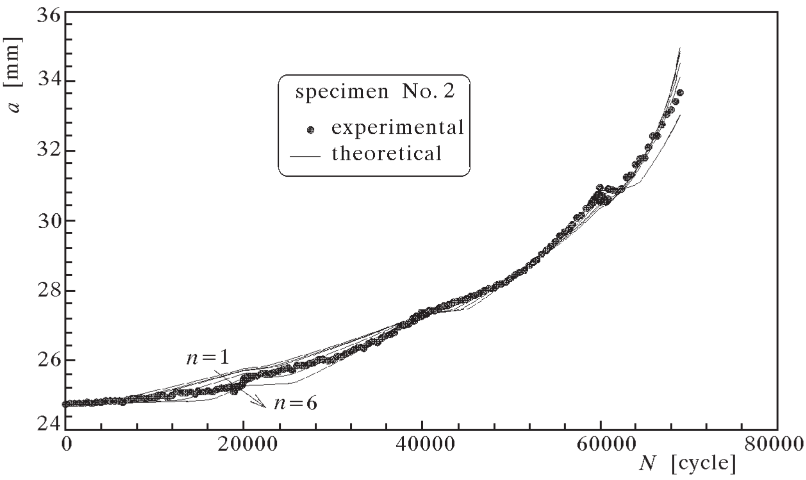

Fig. 3. Theoretical description of crack propagation in CT specimens made of 18G2A steel

Table 1. Testing conditions and results of tests with specimens made of 18G2A steel

Specimen No.	$F_{min} - F_{max}$ [kN]	F_{ov} [kN]	DN [cycle]	$2N_f$ [cycle]
1	2.02-6.72	–	–	80500
2	2.02-6.72	8.40	20000	69000
3	2.02-6.72	8.40	10000	77500
4	2.02-6.72	10.08	10000	120000

The similarity between the plots of $a = f(N)$ relationships (i.e. crack length against a number of cycles) as well as between the theoretical descriptions of specimens 1, 2, and 3 are peculiar to the instances of fatigue crack propagation under loads of a constant amplitude with small overloads applied (up to approx. $k_{ov} = 1.2$, as in the cases under discussion) or with overloads applied within long time intervals (e.g. longer than 20 per cent of the fatigue life typical of a given range of the loads $F_{max} - F_{min}$), although it is not the rule and depends on the material tested. These are the cases of rather monotone crack propagation or of propagation when the effects of retardation fade away before a subsequent overload is applied (analogous to the case illustrated in Fig. 1a). In the cases of application of larger overloads (e.g. $k_{ov} > 1.4$) (like in specimen No. 4), the relationships $a = f(N)$ do not show a systematic (or fully monotonic) increase. Characteristic are the step-by-step increments in the crack lengths at every moment of overload occurrences, whereas between the overloads the crack growth is explicitly retarded (analogous to the case illustrated in Fig. 1b). The case shown in Fig. 3 presents a result of theoretical description of crack propagation, with retardation models incorporated, (for various values of the retardation exponent n) which reveals the averaging of the $a = f(N)$ relationship – arriving at the minimum of the sum of squares of deviations in experimental and computational values for values of the exponent n approaching zero or unity, rather than of matching it up to the real (retarded) crack growth. The asterisk marks the instance of the minimum of the sum of squares of deviations in the experimental and computational values.

4. Analysis of results and their theoretical description

The matching made at values of the exponent n approaching zero or unity means in practice nothing but curvilinear regression of a low order (with no point of inflexion); therefore, one cannot recognised the matching that correctly reflects the experimentally determined relationship. There is no chance to simultaneously provide a severe retardation in the crack propagation between the overloads and considerable crack growth for the overload cycle (those step-by-step increments shown in Fig. 1 and Fig. 2) using the classical Wheeler model (because of its mathematical and physical fundamentals). To make the increment for one overload cycle be one or two orders of magnitude higher than that reached in the course of several thousands of cycles between the overloads, the exponent n must assume a high value. It would make the C_p coefficient reach some low value (which, in turn, would result in some considerable retardation in the crack increasing between the overloads); the C and m coefficients should be at the same time high enough to ensure this substantial increment in the crack length for a single overload cycle (when $C_p = 1$). However, the cracks within the tested specimens with no overloads (or at least within the range of up to the first overload, or within all adequate growth periods when $C_p = 1$) would then show the crack growth rate 10 up to 100 times as high as in the case of cracks within the overloaded specimens (at the utmost, the specimens with no overloads would show fatigue life many times shorter). In practice, it does not take place on such a large scale.

Numerical analysis of the already obtained results has been carried out to determine the parameters of equations that describe propagation rate of fatigue cracks within the tested material. Mathematical models described by

Kłysz (1991) have been used, as well, with the Paris equation $da/dN = C_pC(\Delta K)^m$ employed. Owing to the fact that the initial fatigue precracks within the tested specimens were started up with applying the load higher than the base load of the test ($F_i = 9\,\mathrm{kN}$ instead of $F_{max} = 6.72\,\mathrm{kN}$), the model-based description takes also into account the effect of retardation of the crack growth due to a change (decrease) in the level of the loads applied between the stage of precracking and the exact propagation test (in particular, for the specimen tested with no overloads applied in cycles). Values of the coefficients C and m of the Paris equation have been determined (for the pre-set value of n), ones matched with the least squares method up to the $a = f(N)$ relationships found experimentally for individual specimens. Initially, assuming values of the exponent n of the Wheeler model to be expressed with integers, the following values of the coefficients of interest have been arrived at – see Table 2. The S_n column includes values of the sum of squares of deviations of the least squares method, ones suitable for individual matches.

Fig. 3 show the essence of the model-based (according to the Wheeler model) description of crack propagation with the effect of retardation due to overloads taken into account (starting up with the extreme instance for $n = 0$, when no retardation, $C_p = 1$ is considered). What has become evident from Table 2 and the figures, is that both the values of the coefficients C and m as well as the approximating curves show pretty wide divergence. The visible mismatch for some values of the exponent n proves that there is an optimum value of the exponent, for which the sum S_n reaches the minimum, and values of C and m adequate to the values of the exponent n for which the mismatch has been observed give the best ever possible – with a given retardation model assumed – theoretical description of the experimental data.

It is also evident that the optimum values of the coefficients n, C, m are different for particular specimens (testing conditions). However, this is not an optimistic statement, because such a model-based description of the fatigue crack propagation and the effect of retardation in the crack growth (for a given material) would not be uniform. It would depend on loading conditions. What is observed for specimens No. 1, 2, and 3 are some monotonic changes in values of the coefficients C and m depending on the exponent n of the retardation model, which is not the case with specimen No. 4. Besides, the changes in values of these coefficients for the specimen No. 4 are different.

Table 2. Coefficients in the Paris equation describing experimental data for CT specimens made of 18G2A steel, for various values of the exponent n of the Wheeler retardation model

Specimen No.	n	C $(\cdot 10^{-13})$	m	S_n
1	0	0.271	4.59	67.90
	1	0.345	4.53	62.80
	2	0.579	4.39	56.31
	3	1.096	4.21	47.80
	4	3.071	3.93	37.82
	5	13.033	3.53	28.53
	6	82.802	3.03	24.51
	7	947.062	2.35	31.50
2	0	0.598	4.39	29.65
	1	1.061	4.24	27.06
	2	1.992	4.07	23.56
	3	5.198	3.81	18.95
	4	16.504	3.50	14.00
	5	50.931	3.21	10.91
	6	352.378	2.69	18.63
3	0	3.965	3.81	15.68
	1	7.539	3.65	15.28
	2	17.378	3.43	15.76
	3	48.398	3.16	19.66
	4	196.043	2.79	26.50
	5	534.804	2.54	34.82
4	0	112.992	2.64	11.30
	1	154.190	2.67	10.06
	2	129.298	2.86	13.63
	3	169.516	2.93	28.65
	4	40.695	3.51	60.24
	5	0.086	5.48	86.33

Another peculiarity of the fatigue crack propagation has been shown in subsequent figures, which represent experimentally determined curves of propagation, ones gained using the SEN (single-edge notch) specimens made of the 18G2A steel, also tested under a load of constant amplitude (stress ratio $R = 0.3$), with overloads applied in cycles, i.e. every DN number of cycles, on the levels of $k_{ov} = 1.2,\ 1.6,\ 1.75$ (Fig. 4a to Fig. 4c, respectively). The

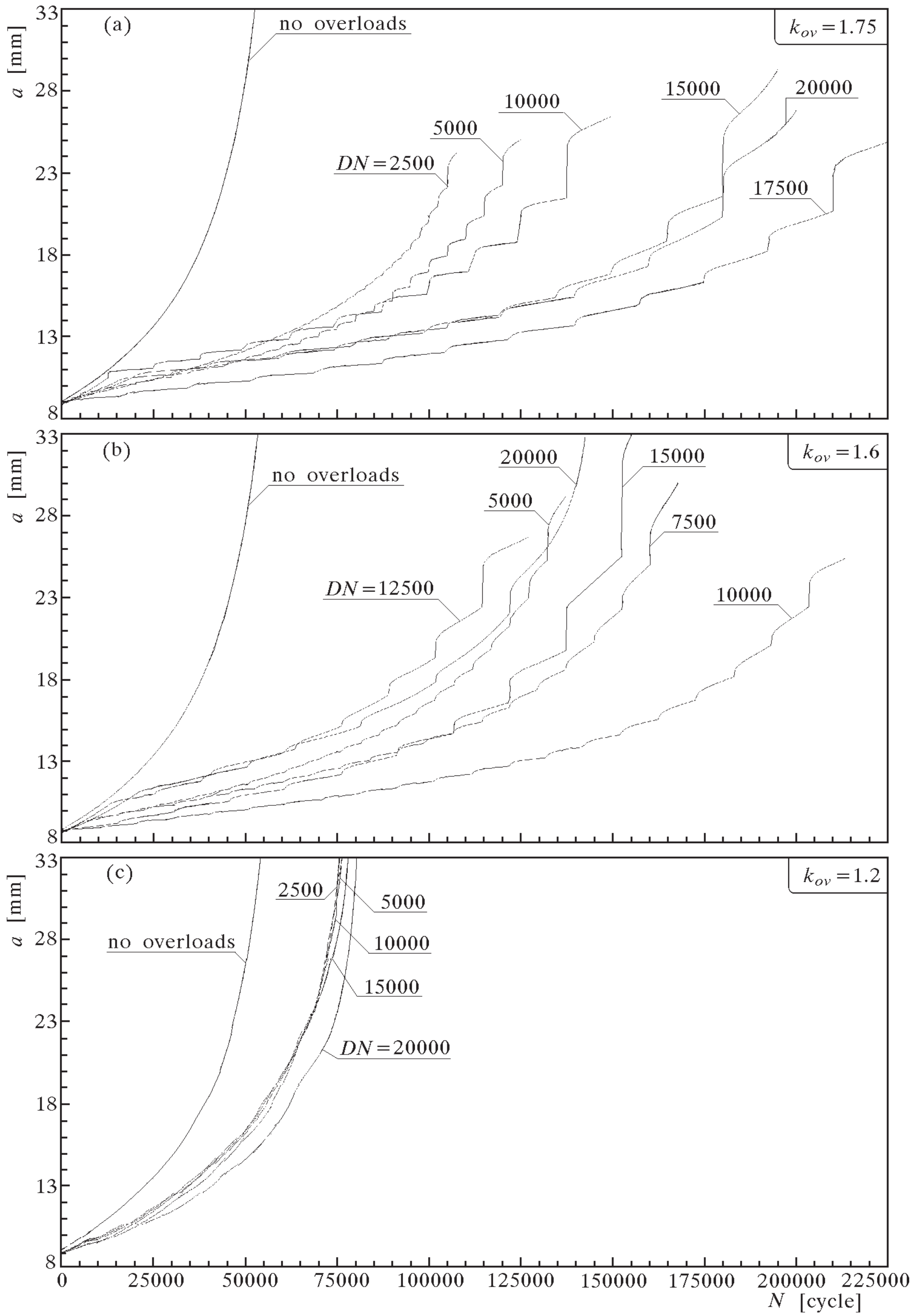

Fig. 4. $a = f(N)$ relationship for specimen tested with overload $k_{ov} = 1.75$, 1.6, 1.2

plots confirm the generally known properties of fatigue cracks propagation under overloading conditions: both single overloads and those applied in cycles increase the fatigue life as compared to the life gained with no overloads imposed at all. In general, the fatigue life increases with increase in the level of the applied overload (at least in the range of the discussed levels of the overload). There is, however, an optimum value of the time interval between the overloads, at which the life reaches the maximum (i.e. overloads applied too often or too rarely, as compared to this optimum time interval, do not give such strong effect of retardation of the crack growth).

However, there are distinct differences between particular plots of the crack propagation: dynamics of crack length growth, convexity of the curves over the sections between the application of the overloads are completely different in subsequent figures (comparing to the previous figures). For the loads on the level of $k_{ov} = 1.2$ the increment of the crack length are almost monotonic (only at final segments some collapses are observed during occurrences of the overloads). In the case when larger overloads are applied, the plots $a = f(N)$ represent (with their collapses) nearly every overload, but in a very distinct manner. The increments in the crack lengths within the exact cycles of overload application are greater, as well. Characteristic are also the plots of the corresponding relationships of the crack propagation rate da/dN against the range of the stress intensity factor ΔK. The experimentally found and recorded relationships $da/dN = f(\Delta K)$ can give – in the course of tests with overloads applied in cycles – a plot as shown in Fig. 5a. Between subsequent overloads and with systematically increasing ΔK (together with increasing of the crack length) the crack growth rate decreases – see the vertical columns in Fig. 5a with slightly negative inclination towards the X axis – which seems to contradict the Paris relationship. The shown plot corresponds to the overload $k_{ov} = 1.75$ applied every $DN = 15000$ cycles. The propagation rate decreases as much as by 2-3 orders of magnitude. The decrease in the crack growth rate (its level and range of the occurrence) depends mainly on the value of the applied overload. Subsequent figures show relationships analogous to that in Fig. 5a, but arrived at with specimens subjected to tests at smaller overloads, i.e. $k_{ov} = 1.6,\ 1.2$, respectively. It is evident that with the level of the overload getting lower, the degree of retardation growth gets reduced (the rate of propagation does not get so much reduced); the crack starts escaping the retardation phase: for the overload $k_{ov} = 1.6$ the statement is true for higher values of the crack length, whereas for the overload $k_{ov} = 1.2$ even for evident predominance of the non-retarded growth between the overloads. Obviously, the level of retardation and the range of its occurrence substan-

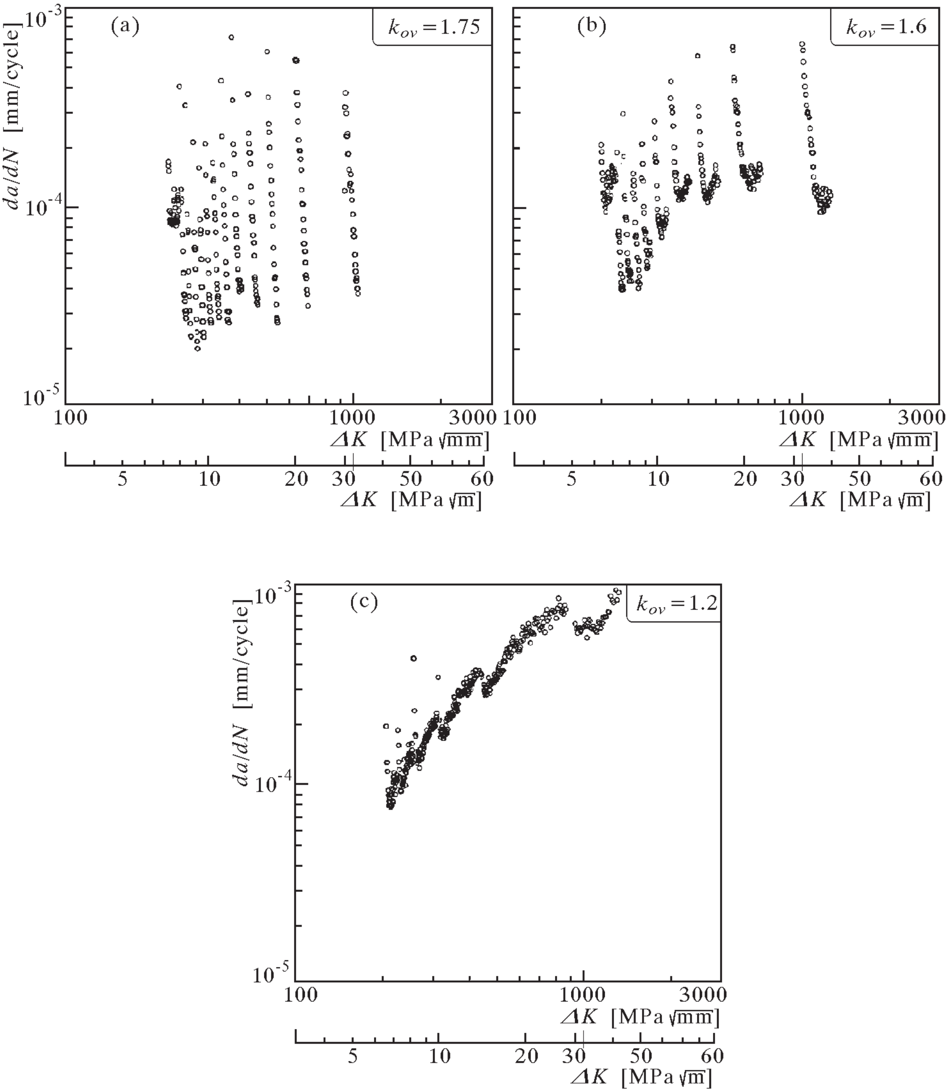

Fig. 5. $da/dN = f(\Delta K)$ relationship for specimen tested with overload $k_{ov} = 1.75$, 1.6, 1.2

tially affect the final fatigue life of the specimens under tests. In this case, with even (i.e. every DN number of intervals between the overload application) the fatigue life was changing for particular levels of the applied overloads as 150000:120000:100000:71560 cycles. It has become apparent that the analysis of fatigue crack growth can be carried out from many and various points of view; there are, however, no unequivocal settlements – in particular, when various kinds of materials and effects of specific kinds of the overloads upon them are analysed.

5. Conclusion

In most practical engineering situation the applied stresses fluctuate (often in a random manner) and, under this condition, the failure occurs at lower stress levels than it would be expected when a steady stress were applied. This phenomenon is called the fatigue and causes the vast majority of in-service failures. The service life of a structure is limited by the critical element whose fatigue life falls either to low cycle fatigue, high cycle fatigue, thermo-mechanical fatigue or creep damage. It is essential to understand how their greating is related with the fatigue life limit and the rate of approaching it. This paper is concerned with corresponding aspects of theoretical description of fatigue crack evaluation under the condition of overloads occurrence.

The experimentally determined basic relationships of fatigue crack growth $a = f(N)$ and $da/dN = f(\Delta K)$ are analysed in the paper. They show different features for both different and the same materials. It refers, first of all, to dynamics of the crack length growth convexity of the curves after application of the overloads. This is especially interesting because they contradict each other under the same test conditions. They depend on many different factors, and investigators do not know all relations between them as well as the crack growth rate or fatigue life. Admitteally, very important problems, which could put some light and explain this characteristic are conected with some microstructural aspects and mechanisms of the fatigue crack growth – which are not described in this paper.

The above mentioned features make creation of uniform theoretical description rather difficult. They sometimes preclude simple mathematical models from being applicable to the involved problems. The Wheeler retardation model is one of the oldest and simplest used in the analysis of crack growth phenomena in specimens undergoing overloads. It is very useful in many cases but not for all purposes.

To avoid in engineer practice the unacceptable risk of a catastrophic failure it is necessary to monitor the fatigue life of the critical elrments and retire them from service before their allocated life has been exceeded. Because the ability of structural elements to resist the effects of failure mechanisms is a function of material properties, design and operating conditions, then thorough undestanding of the failure mechanisms is essential if the failure modes, the safe-life and the service life of the elements are to be accurately determined and safely monitored. Therefore, in part 2 of this paper the modification of the Wheeler model improving the description of experimental results is presented. Because of its good mathematical and physical fundamentals it seems that more complete description of different behaviour of cracks in various materials.

Usually, results of analysis of fatigue crack growth are used to identify the critical locations of a fracture in the analysed elements.

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Osobliwości rozwoju pęknięć zmęczeniowych i modyfikacja modelu opóźnień Wheelera – część 1

Streszczenie

Przedstawiono osobliwości przebiegu procesu pęknięć zmęczeniowych w odniesienu do krzywych długości pęknięcia w funkcji liczby cykli lub prędkości propagacji w funkcji zakresu współczynnika intensywności naprężeń – to jest krzywych typowych dla badań z przeciążeniami zadawanymi cyklicznie. Badano stop aluminium PA7 i stal 18G2A. Omówiono trudności w opisie modelowym propagacji pęknięć (w oparciu o model opóźnień Wheelera) w związku z omawianymi osobliwościami.

Manuscript received March 19, 1999; accepted for print May 9, 2001