JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
3, **39**, 2001

CONTACT PROBLEMS WITH FRICTION, ADHESION AND WEAR IN ORTHOPAEDIC BIOMECHANICS. PART I – GENERAL DEVELOPMENTS

Jerzy Rojek
Józef Joachim Telega

Institute of Fundamental Technological Research, Warsaw
e-mail: jrojek@ippt.gov.pl, jtelega@ippt.gov.pl

The present paper is a continuation of the contribution by Rojek and Telega (1999). An alternative adhesion law is used to the study of the bone-implant interface. Various problems related to the bone-implant interface are discussed.

Key words: bone-implant interface, contact, adhesion, friction

1. Introduction

Modelling of stress distribution and functioning of human (and animal) joints are inherently related to the study of complicated contact problems. One has to distinguish two major classes of such problems: the first class pertains to natural and diseased joints whilst the second class to artificial joints or joints after arthroplasty, cf Section 2 of this paper. The reader should be aware that understanding and modelling of human joints is far from being complete.

The aim of the present contribution is further development of the study of adhesion on bone-implant interface in human joints after arthroplasty, started in Rojek and Telega (1999), cf also Bednarz et al. (2000). An important role played by adhesion on the bone-implant interface has been confirmed by inspection of retrieved implants, cf Rojek and Telega (1999), Laffargue et al. (1998). In the paper by Rojek and Telega (1999) the model of adhesion was essentially based on the idea due to Frémond (1987, 1988). Then the set of unilateral constraints is nonconvex. A modified approach was developed by Raous and his coworkers, cf Bretelle et al. (2000), Cocu and Roca (2000), Cocu et al. (1997), Raous et al. (1999). Then one has to deal with two *convex* sets of constraints, one for a kinematic variable, and the second set for the density of adhesion β, cf Section 4.4.

In Rojek and Telega (1999), the influence of adhesion on the behaviour of a bone-femoral implant was investigated. In the present paper we shall study the tibial component of the condylar knee prosthesis, cf also Rojek et al. (1999), Bednarz et al. (2000). More realistic modelling of the interface bone-implant requires taking into account the influence of wear debris on the adhesion. This intriguing problem will be discussed in Part II (Rojek et al., 2001). However, the reader should be warned that a satisfactory model comprising friction, wear and adhesion and applicable to joints after arthroplasty is not available.

The plan of the paper is as follows. In Section 2 and 3 various biomechanical aspects of mainly knee joints after arthroplasty are briefly discussed. Theoretical modelling of adhesion is the subject of Section 4.

2. Knee joint: overview of selected biomechanical issues

The aim of this section is to discuss concisely some basic problems related to knee joints, mainly after arthroplasty. The hip joint will also incidentally be mentioned.

Implants, and particularly the knee prostheses, have already a long history, sometimes even dramatic, see the comprehensive review papers by Jansons (1987), Prendergast (2000), Walker and Blunn (1997) and the relevant references cited in Rojek and Telega (1999).

Knee models

Hefzy and Grood (1988) and Hefzy and Cooke (1996) reviewed various static and dynamic models of the human knee joint, including the models developed to determine the forces in the muscles and ligaments during various activities, cf Bull and Amis (1998), Seireg and Arvikar (1973), Blankenvoort and Huiskes (1991), Toutoungi et al. (1997), Zavatsky and O'Connor (1994), Zatsiorsky (1998). Bao and Willems (1999) developed an involute-on-plane model of the knee. The parameters appearing in this model are the position of the involute basic circle and the radius of this circle. A method has been proposed to estimate these parameters.

Determination of contact stresses

Early results concerning the contact stresses between femur and tibia are due to Chand et al. (1976). These authors studied a natural knee joint, and the unilateral contact problem between the femur and tibia was solved as a quadratic programming problem. Moreover, the stress-freezing photoelastic technique was used to show an irregular stress distribution in the contact region.

Blankenvoort et al. (1991) introduced the articular surface description which is based on a general theory for a thin elastic layer on a rigid foundation, cf also Blankenvoort and Huiskes (1996).

Heegaard (1993) proposed a frictionless discretized model of patello-femoral joint. The general procedure is valid for a moderate slip between deformable bodies or large slip contact with rigid obstacle, cf also Heegaard and Curnier (1993).

Waldman and Bryant (1997) assessed the contribution of ultra-high molecular weight polyethylene (UHMWPE) nonlinear viscoelastic properties in total knee replacement (TKR) from a developed mathematical rolling model. From such a model, predictions of the effects of kinematic conditions on contact stress and rolling resistance were possible. A key assumption was made regarding the equivalence of rolling and sliding with adhesive friction in the presence of lubricants. The hysteretic nature of UHMWPE caused a significant portion of the rolling energy to become unrecoverable, which could be dissipated as heat. Localised temperature rises within the region of contact can result in increased creep, wear, and oxidation of UHMWPE.

The paper in two parts by Lewis (1998) critically reviews the contact stress determination studies of cadaveric and artificial joints, with and without an associated TJR (total joint replacement), as well as physical systems that are deemed to represent idealized conditions at articular surfaces in TJRs. Analytical and numerical methods are typical for modern contact mechanics. The experimental methods discussed by this author include the method using a pressure-sensitive film, resistive ink sensor, transducers and photoelasticity.

Fuji film has been widely used in the studies of the contact mechanics of articular joints since 1980, cf Lewis (1998), Wu et al. (1998) and the references therein. The applications of Fuji Pressensor film range from the qualitative assessments of contact stress pattern to quantitative analysis of the contact pressure in articular joints using digital image techniques. Wu et al. (1998) studied the influence of inserting the Fuji Pressensor film into articular joints on the contact mechanics using the finite element method. The numerical tests showed that the contact pressure changes significantly when the Pressensor film is inserted into a joint.

Harris et al. (1999) developed an easy, reproducible and, as the authors claim, reliable technique to evaluate TKA (total knee arthroplasty) systems and to determine the influence of the design parameters on the contact area. The system used is a computerized contact area and a pressure measurement system, K-Scan 4000 (Tekscan, South Boston, USA). This system was compared with the Fuji pressure-sensitive film using a custom testing jig designed for freedom of movement. Contact area measurements using this jig demonstrated the limitations of Fuji films in determining the contact areas. The K-Scan system enabled the measurements of contact areas at different loads and flexion angles with the TKA system.

Herzog et al. (1998) determined the in situ functional and material properties of articular cartilage in an experimental model of joint injury and quantified the corresponding in situ joint contact mechanics. The cat ACL – transected knee was chosen as the experimental model (ACL – anterior cruciate ligament).

Stewart et al. (1998) developed a simplified method to analyse the contact problems for the composite *cushion knee joint replacement* problems which can be readily incorporated into the design cycle. These include predictions of the contact radius and the maximum contact pressure as well as the detailed stress distributions within different layers and the interfaces. Experimental measurement of the contact area has also been conducted in order to verify the theoretical predictions.

Parameters of knee design

Sathasivam and Walker (1999) determined the effects of four design parameters which vary to a large extent in the range of contemporary knee replacement designs. The parameters were the femoral frontal radius (30 or 70 mm), the difference between the femoral and tibial frontal radii (2 or 10 mm), the tibial sagittal radius (56 or 80 mm), and the posterior-distal transition angle (-8 or -20°), which is the angle at which the small posterior arc of the sagittal profile transfer to the larger distal arc.

Walker and Sathasivam (1999) claim that the ideal TKR would be the one where the bearing surfaces are anatomical in shape and where the cruciate ligaments are preserved. However, for a total knee replacements without one or both cruciate ligaments, the ideal situation is where the bearing surfaces provide the same kinematic function as the cruciates, in particular, providing for the anterior and posterior translations of the femur on the tibia. In addition, there should be a provision for internal-external rotation. Walker and Sathasivam (1999) investigated the feasibility of designing a knee replacement which would control and allow motion in this way. The scheme envisaged was to employ the usual type of partially conforming bearing surfaces for a fixed-bearing knee, or the closely-conforming surfaces of a mobile bearing knee, in combination with Guide Surfaces. A further requirement of the design was a sufficient femoral-tibial conformity for acceptable contact stresses.

Effect of anisotropy on stress distribution

Lewis et al. (1998) investigated the effect on the stresses in a construct comprising the tibial component and tibial insert of TKR anchored to the contiguous bones with cement. A two-dimensional model with perfect bonding at the interfaces was studied. Two cases were considered. In the first case all the materials were assumed to have linear isotropic properties. Heterogeneity in the values of the Young and Poisson coefficients was taken into account. In the second case, the cortical bone was taken to have transversely isotropic elastic properties, with the representation of the elastic properties of all other materials remaining unchanged. The results obtained are not surprising, at least for us, since a strong effect of the representation of the elastic properties of material on the stresses was observed. Lewis et al. (1998) investigated also an elastomeric metacarpophalangeal joint implant in two cases. In the first case the elastomer was taken to have linear elastic properties whilst in the second case it was treated as a hyperelastic Mooney-Rivlin material. Again, the differences observed are obvious.

Fatigue properties of HAPEX

Ton That et al. (2000) investigated fatigue properties of HAPEX (hydroxyapatite-reinforced polyethylene composite) developed for bone replacement. The uniaxial tests comprised tension, compression and torsion to failure in order to determine the ultimate tensile strength. The biaxial fatigue tests of the same material were performed using various combinations of axial and torsional stresses.

Micromovements

By now it is well established that micromovements of implants either stabilise, during the first postoperative year, or may contribute to aseptic loosening, cf the papers contained in Lewis and Galante (1985), Turner-Smith (1993). There are four regions of micromovement in relation to the development and use of prostheses: (i) micromovement predicted by finite element and other numerical studies, (ii) strain-induced micromovement at the bone/implant interface in cadavers, (iii) strain-induced micromovement at the bone/implant interface, measured intra-operatively, post-operatively or both, and (iv) micromovement observed in patients with the passage of time migration, cf the comments by Freeman in Turner-Smith (1993), pp. 93-98. Various measurement techniques of micromovement are also presented in Turner-Smith (1993), for instance, intra-operative measurement of tibial component micromovement in TKA. The paper by Berzins and Sumner (2000b) provides a useful review of some of the basic principles involved in performing *in vitro* tests of implant stability, cf also An and Draughn (2000). The technique described can measure both the inducible or recoverable motion and the nonrecoverable motion (often referred to as migration).

3. The bone-implant interface

Micromovements in fact reflect the bone-implant interface behaviour. We observe that the cemented fixation of prosthesis is nothing else but the bone-implant contact where an implant consists of two phases: metal and UHMW-PE. Earlier ideas and results on mechanical, biological, chemical and electrical factors in tissue response to implants are summarized in the Workshop Report edited by Lewis and Galante (1985), and in Boss et al. (1994). An important part of recent results on the bone-implant interface were summarized in the books edited by An and Draughn (2000), Helsen and Breme (1998).

The bone-implant interface behaviour is far from being fully recognised and understood; also one lacks reliable phenomenological models. Anyway, adhesion, wear and friction play an important role in modelling the bone-implant interface. A model comprising all these phenomena would be rather complicated and is not yet available. Therefore, in the next section we will focus on modelling the adhesion with friction. Primarily, however, we present some important ideas pertaining to the structure of the interface of interest.

3.1. Structure of interface

Causes of tissue reactions around alloplastic materials are identified as cell mediated hypersensitivity to an implant component and tissue modification due to the presence of wear particles, debris and corrosion products of the prostheses used, cf Hildebrand and Hornez (1998), Jallot et al. (1999b), Jones (1998), Thull (1998).

Refractory metals used for implants such as titanium, zirconium, niobium, tantalum and their alloys, as well as ceramic materials, are characterized by very low disintegration rates and justify the question of how the physiochemical communication between material surfaces and the extracellular matrix actually occurs, cf Fig. 1.

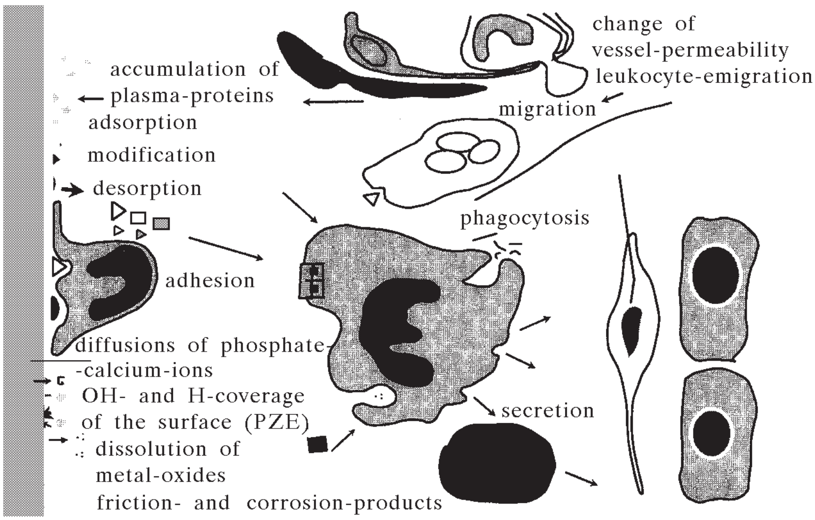

Fig. 1. Overview of the complex reactions at the surface of passivated metal biomaterials. The initial adsorption pattern and conformational changes to constituents of the formed biofilm will be influenced by the physiochemical properties of the implant surface, after Thull (1998)

In organic macromolecules, intramolecular and intermolecular bonds, together with oxygen bridge bonds, may break down giving rise to structural or conformational changes, or both. Conformational changes may arise as a result of an exchange of charge carriers between the surface of the biomaterial and the biological macromolecules.

In chronological order, the interaction bone-implant runs in the following steps:

(i) The implant surface builds an interface according to the properties of the solid state and the surrounding liquid phase.

(ii) A protein layer adsorbs and is structured in response to the physiochemical properties of the surface in the equilibrium state.

(iii) Cell recognise the protein film and react.

(iv) Tissue is structured according to the properties of the protein and cell layer on the surface.

We observe that the equilibrium state of the material surface in the biological environment is characterized by identical rates of adsorption and desorption of ions and constituents of, for instance, the extracellular matrix, cf Thull (1998).

The molecules making up an interface are different from those of the bulk phase because they are not surrounded by bulk phase molecules. The bonding energy of the surface molecule is less than that associated with a bulk phase molecule, and the energy of the surface molecule is therefore higher than that of molecules in the bulk phase. The force in the surface that attempts to keep the surface area at a minimum is called the *surface tension* and is usually given in $\mathrm{mJ\,m^{-2}}$. Indications in the literature show that the critical surface tension should be of the order of 60-120 $\mathrm{mJ\,m^{-2}}$ for good tissue adhesion, cf Fig. 2.

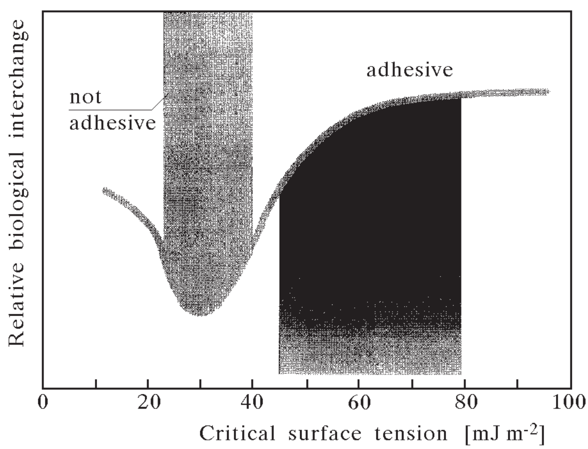

Fig. 2. Critical surface tension leading to adhesion or non-adhesion of the adjacent tissue, after Thull (1998)

Remark 3.1. For a study of the behaviour of bone replacement-implant, such as hydroxyapatite, the reader is referred to Benhayoune et al. (1999), Jallot (1998), Jallot et al. (1999a), and the references therein.

3.2. Mechanical testing of bone-implant interface

The contact which will be studied in Section 4 is purely mechanical. However, the material coefficients reflect macroscopic properties of interface.

Let us now briefly discuss the relevant experimental techniques. Tests of interfacial bonding strength have been pursued using a variety of test methods, implant geometries, and surface configurations. One type of test is a torque test, frequently involving screw-shaped implants in the rabbit tibia or femur, cf An and Draughan (2000), Berzins and Sumner (2000b), Nakamura and Nishoguchi (2000), and the references therein.

Another types of tests are pushout and pullout tests, cf Berzins and Sumner (2000a), Berzins et al. (1997). The most common applications of pushout and pullout tests include testing for the effects of implant material, surface texture, cross-sectional geometry, porosity, and surface composition in the context of cementless fixation by bone ingrowth or bone apposition to the implant. A few studies have used pushout tests to test either the cement-bone or cement-metal interfaces in cemented replacements. Berzins and Sumner (2000a) provided a practical guide to conducting these tests and the extensive list of citations. With both pushout and pullout tests, a load is applied to the implant via a device connected to the cross-head of the materials testing machine and a force-displacement curve is recorded, cf Fig. 3.

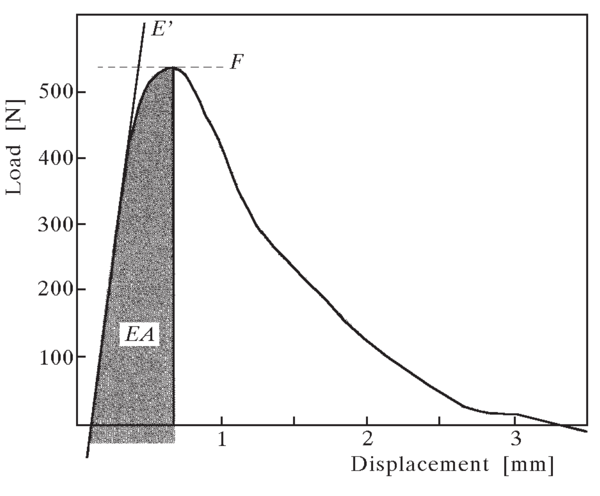

Fig. 3. A typical load-displacement curve from a pushout or pullout test, F – the maximum forces applied to the implant during the test, E' – apparent shear stiffness (the slope of the load-displacement curve in its linear region); EA – energy adsorption to failure, after Berzins and Sumner (2000a)

Another methods for investigating interfacial strength is the tensile test, in which loads are applied in a direction normal to the tissue-biomaterial interface, cf Nakamura and Nishoguchi (2000).

For a lot of other problems and methods related to mechanical testing of the bone-implant interface, the reader is referred to McKoy et al. (2000), Davis et al. (2000), Dhert and Jansen (2000), Wang et al. (2000).

According to Breme (1998), the microstructure of the implant interface is of great importance, cf also Jallot (1998), Jallot et al. (1999b). For instance, Young's modulus value of the material must be lowered in order to decrease the stiffness of the implant in the direction of the bone. As can be seen from Fig. 4a, a drastic change in Young's modulus from the implant to the bone should be avoided because the difference in the elastic deformation can cause delamination stresses in the interface. In contrast, a gradual transition in Young's modulus from the bone to the implant helps to avoid such a drastic change of stresses, cf Fig. 4b.

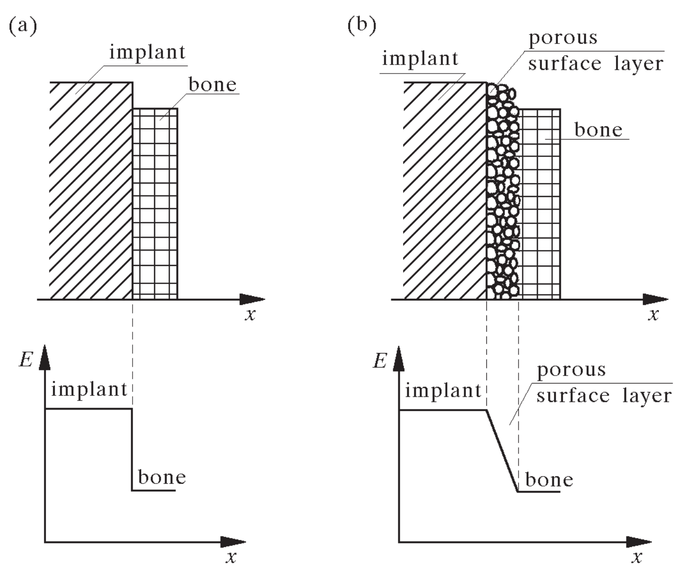

Fig. 4. Scheme of change in Young's modulus at the bone-implant interface with a smooth and a porous implant surface, respectively, after Breme (1998)

4. Modelling unilateral contact with adhesion and friction

In the present section we shall introduce the description of adhesion with friction. Here we follow Raous et al. (1999). This approach constitutes a modification of the theory primarily proposed by Frémond (1987, 1988), cf also Rojek and Telega (1999).

4.1. Adhesion: some basic notions

Adhesion between two solids may be due to ionic, covalent, metallic, hydrogen or van der Waals forces; for more details the reader is referred to Maugis (1982), Maugis and Barquins (1980), Breme et al. (1998), Possart (1988). To cut these bonds and to separate the two solids 1 and 2 in contact on a unit area, energies γ_1 and γ_2 are needed to create the unit areas 1 and 2, whereas the excess energy γ_{12} (interfacial energy) is recovered. The quantity

$$w = \gamma_1 + \gamma_2 - \gamma_{12} \tag{4.1}$$

is the *Dupré energy of adhesion* or the *thermodynamic work of adhesion*.

The separation never occurs as a whole, but by progression of a crack. During this propagation, the interface bonds are broken, elastic energy is released and irreversible work is dissipated at the crack tip.

Work of adhesion is a useful quantity because it distinguishes the two states, contact and separation. This work is done over a very small distance for van der Waals forces, 99% of the work is achieved when the surfaces are pulled 1 nm apart. For other types of bonding, such as ionic and covalent, even smaller distances are involved. Thus, the precise shape of the force separation curve need not be known to understand many phenomena. Indeed, the precise shape may even not be measurable because of the instability of the spontaneous jumping of smooth surfaces into contact.

For other problems related to adhesion in biomechanics the reader is referred to Rojek and Telega (1999), and the references therein.

4.2. Formulation of the initial–boundary value problem

Let us consider two deformable bodies (Fig. 5) occupying a domain $\Omega = \Omega^{(1)} \cup \Omega^{(2)}$ of $\mathbb{R}^3$ in their undeformed configuration. Let $\Gamma^{(\alpha)} = \partial\Omega^{(\alpha)}$ $(\alpha = 1,2)$ consist of three disjoint parts: Γ_0^α, Γ_1^α and Γ_c^α such that

$$\Gamma^{(\alpha)} = \overline{\Gamma}_0^\alpha \cup \overline{\Gamma}_1^\alpha \cup \overline{\Gamma}_c^\alpha \tag{4.2}$$

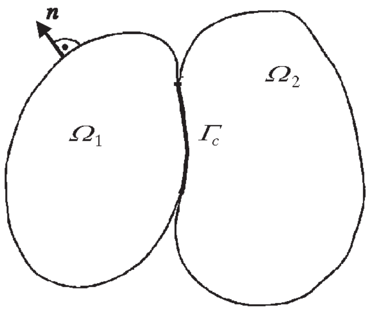

Fig. 5. Two deformable bodies in contact

Here the bar denotes the closure of set, and $\Gamma_c = \Gamma_c^{(1)} = \Gamma_c^{(2)}$ is the contact surface of the two bodies

$$\Gamma c = \Gamma^{(1)} \cap \Gamma^{(2)} \tag{4.3}$$

We assume that the considered bodies are initially in contact, that is

$$\Gamma c \neq \emptyset \tag{4.4}$$

Moreover, the boundaries of the deformed bodies $\Gamma^{(\alpha)}$, $\alpha = 1,2$, possess a unique outward normal $\boldsymbol{n}^{(\alpha)}$ at each of their points. At the common part of the boundary, Γ_c, we have

$$\boldsymbol{n}^{(1)} = -\boldsymbol{n}^{(2)} \tag{4.5}$$

The motion of the two bodies is described by the following system of equations

$$\begin{aligned} &\sigma_{ij,j}^{(\alpha)} + f_i^{(\alpha)} = \rho \ddot{u}_i^{(\alpha)} && \text{in} \quad \Omega^{(\alpha)} \\ &u_i^{(\alpha)} = \overline{u}_i^{(\alpha)} && \text{on} \quad \Gamma_0^{(\alpha)} \\ &\sigma_{ij}^{(\alpha)} n_j^{(\alpha)} = g_i^{(\alpha)} && \text{on} \quad \Gamma_1^{(\alpha)} \\ &\sigma_{ij}^{(\alpha)} = a_{ijkl}^{(\alpha)} e_{ij}(\boldsymbol{u}^{(\alpha)}) \end{aligned} \tag{4.6}$$

where $\alpha = 1,2$. All the static and kinematic quantities appearing in Eqs (4.6) depend on spatial variables and time $t \in (0,T)$. Here and below there is no summation over α. The initial conditions for the displacement fields are specified by

$$\begin{aligned} &\boldsymbol{u}^{(\alpha)}(0,\boldsymbol{x}^{(\alpha)}) = \boldsymbol{u}_0^{(\alpha)}(\boldsymbol{x}^{(\alpha)}) \\ &\dot{\boldsymbol{u}}^{(\alpha)}(0,\boldsymbol{x}^{(\alpha)}) = \boldsymbol{u}_1^{(\alpha)}(\boldsymbol{x}^{(\alpha)}) \qquad \boldsymbol{x}^{(\alpha)} \in \Omega^{(\alpha)} \end{aligned} \tag{4.7}$$

The elastic moduli a_{ijkl} are functions from $L^\infty(\Omega)$ and satisfy the following condition

$$\exists c_1 \geqslant c_0 > 0 \quad \forall \boldsymbol{\epsilon} \in \mathbb{E}^3_s \qquad c_0 \epsilon_{ij}\epsilon_{ij} \leqslant a_{ijkl}(\boldsymbol{x})\epsilon_{ij}\epsilon_{kl} \leqslant c_1 \epsilon_{ij}\epsilon_{ij} \tag{4.8}$$

for almost every $\boldsymbol{x} \in \Omega$. Here $\mathbb{E}^3_s$ stands for the space of symmetric 3×3 matrices. Obviously, the functions $\overline{\boldsymbol{u}}^{(\alpha)}$, $\boldsymbol{g}^{(\alpha)}$, $\boldsymbol{f}^{(\alpha)}$, $\boldsymbol{u}_0^{(\alpha)}$, and $\boldsymbol{u}_1^{(\alpha)}$ are prescribed. Our assumptions allow for the body to be made of an anisotropic material.

The set of equations (4.6) and (4.7) has to be supplemented with contact conditions on Γ_c.

4.3. Conditions for contact with adhesion

We define the relative displacement $[\![\boldsymbol{u}]\!]$ on the contact surface Γ_c as

$$[\![\boldsymbol{u}]\!] = \boldsymbol{u}^{(1)} - \boldsymbol{u}^{(2)} \tag{4.9}$$

which can be decomposed into the normal and tangential components, $\boldsymbol{u}_n$ and $\boldsymbol{u}_T$, respectively

$$\boldsymbol{u}_n = u_n \boldsymbol{n} \qquad\qquad \boldsymbol{u}_T = [\![\boldsymbol{u}]\!] - u_n \boldsymbol{n} \tag{4.10}$$

where

$$u_n = [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n} \tag{4.11}$$

and the unit normal vector $\boldsymbol{n}$ is taken as exterior to Ω_1

$$\boldsymbol{n} = \boldsymbol{n}^{(1)} = -\boldsymbol{n}^{(2)} \tag{4.12}$$

The interaction between the two bodies can be represented by the contact traction vectors, $\boldsymbol{R}^{(1)}$ and $\boldsymbol{R}^{(2)}$, which satisfy the following relations

$$R_i^{(1)} = \sigma_{ij}^{(1)} n_j^{(1)} \qquad\qquad R_i^{(2)} = \sigma_{ij}^{(2)} n_j^{(2)} \tag{4.13}$$

By the Newton's third law, we have

$$\boldsymbol{R}^{(1)} = -\boldsymbol{R}^{(2)} \tag{4.14}$$

We take $\boldsymbol{R} = \boldsymbol{R}^{(1)}$ and similarly to (4.10) and (4.11), decompose $\boldsymbol{R}$ into the normal and tangential components, $\boldsymbol{R}_n$ and $\boldsymbol{R}_T$, respectively

$$\boldsymbol{R} = \boldsymbol{R}_n + \boldsymbol{R}_T = R_n \boldsymbol{n} + \boldsymbol{R}_T$$

In the standard unilateral contact formulation no tensile normal contact forces are allowed

$$R_n \leqslant 0 \tag{4.15}$$

In the present formulation contact forces can be either compressive or tensile (due to adhesion). The adhesion can generate reactions both in the normal and tangential directions. Possibility of decohesion (partial or total) is taken into account, the bond restitution will not be allowed, however.

The formulation presented here employs a general framework for the study of contact problems with adhesion developed by Frémond (1987, 1988) and used in Rojek and Telega (1999), cf also Alves and Kikuchi (1998), Vena (1998). Some of the concepts presented by Raous et al. (1999) have also been adopted, cf also Cocu and Rocca (2000), Cocu et al. (1997), Bretelle et al. (2000). The main idea consists in introducing the *intensity of adhesion* $\beta(\boldsymbol{x}, t)$, where $\boldsymbol{x} \in \mathit{\Gamma}_c$ and t stands for the time variable.

The intensity of adhesion β is such that:

(i) if $\beta = 1$, all the bonds are active,

(ii) if $\beta = 0$, all the bonds are broken or the adhesion is absent,

(iii) if $0 < \beta < 1$, a part β of the the bonds remains active, the remaining bonds are broken, the adhesion is partial.

Initial adhesive bonds are given by

$$\beta(0, \boldsymbol{x}) = \beta_0(\boldsymbol{x}) \qquad \boldsymbol{x} \in \mathit{\Gamma}_c \tag{4.16}$$

which supplements the set of equations (4.6) and (4.7).

4.4. Constitutive model of the contact interface

Let us consider viscous contact with adhesion and friction. We shall formulate a constitutive model for such a contact interface allowing us to evaluate the contact reaction $\boldsymbol{R}$. The contact traction will be split into reversible, R_n^e and $\boldsymbol{R}_T^e$, and irreversible parts R_n^i and $\boldsymbol{R}_T^i$ (on $\mathit{\Gamma}_c$). As the state variables on the contact interface we take u_n, $\boldsymbol{u}_T$ and β. Then the conjugate thermodynamic forces are reversible parts of the contact traction, R_n^e and $\boldsymbol{R}_T^e$, as well as the thermodynamic force of decohesion G_β.

We define the generalized potential $\varphi(\beta, u_n, \boldsymbol{u}_T)$ in the following form

$$\varphi(\beta, u_n, \boldsymbol{u}_T) = \frac{k_n}{2} u_n^2 \beta^2 + \frac{k_T}{2} \|\boldsymbol{u}_T\|^2 \beta^2 - w\beta + I_P(\beta) + I_K(u_n) \tag{4.17}$$

where k_n and k_T are non-negative constants characterizing the interface stiffness, w stands for Dupré's energy, I_P and I_K are indicator functions of the sets K and P

$$K = \{v \mid v \geqslant 0\} = \mathbb{R}^+ \qquad\qquad P = \{\gamma \mid 0 \leqslant \gamma \leqslant 1\} \tag{4.18}$$

defined as, cf Rockafellar and Wets (1998)

$$
\begin{aligned}
I_K(v) &= \begin{cases} 0 & \text{if } \ v \in K \\ +\infty & \text{otherwise} \end{cases} \\
I_P(\gamma) &= \begin{cases} 0 & \text{if } \ \gamma \in P \\ +\infty & \text{otherwise} \end{cases}
\end{aligned}
\tag{4.19}
$$

Here $\mathbb{R}^+$ denotes the set of non-negative reals. The indicator functions impose appropriate constraints on u_n (impenetrability) and β. In Rojek and Telega (1999) only one indicator function is involved, nonconvex with respect to the couple $(\beta, \boldsymbol{u})$, cf Frémond (1987, 1988). Here we follow a modified approach primarily used by Raous et al. (1999). The nonsmooth potential (4.17), sometimes called a pseudo-potential (Panagiotopoulos, 1993), has a convenient feature, since it incorporates two *convex* indicator functions, I_P and I_K. The extended function φ is obviously nondifferentiable. Consequently, the state laws are written in the form of partial subdifferentials

$$
\begin{aligned}
&R_n^e \in \partial_{u_n} \varphi(\beta, u_n, \boldsymbol{u}_T) \\
&\boldsymbol{R}_T^e \in \partial_{\boldsymbol{u}_T} \varphi(\beta, u_n, \boldsymbol{u}_T) \\
&-G_\beta \in \partial_\beta \varphi(\beta, u_n, \boldsymbol{u}_T)
\end{aligned}
\tag{4.20}
$$

where, for instance, $\partial_{u_n}\varphi$ denotes the subdifferential of the the potential φ with respect to the variable u_n.

Subdifferentials (4.20) are defined as follows, cf Panagiotopoulos (1993), Rockafellar and Wets (1998)

$$
\begin{aligned}
&\partial_{u_n}\varphi(\beta, u_n, \boldsymbol{u}_T) = \{Q_n \mid \varphi(\beta, v_n, \boldsymbol{u}_T) - \varphi(\beta, u_n, \boldsymbol{u}_T) \geqslant Q_n(v_n - u_n) \quad \forall v_n\} \\
&\partial_{\boldsymbol{u}_T}\varphi(\beta, u_n, \boldsymbol{u}_T) = \{\boldsymbol{Q}_T \mid \varphi(\beta, u_n, \boldsymbol{v}_T) - \varphi(\beta, u_n, \boldsymbol{u}_T) \geqslant \boldsymbol{Q}_T \cdot (\boldsymbol{v}_T - \boldsymbol{u}_T) \quad \forall \boldsymbol{v}_T\} \\
&\partial_\beta\varphi(\beta, u_n, \boldsymbol{u}_T) = \{Q_\beta \mid \varphi(\eta, u_n, \boldsymbol{v}_T) - \varphi(\beta, u_n, \boldsymbol{u}_T) \geqslant Q_\beta(\eta - \beta) \quad \forall \eta\}
\end{aligned}
\tag{4.21}
$$

Explicit subdifferentiation (4.21) leads to the following relations, cf Raous et al. (1999)

$$
u_n \geqslant 0 \qquad -R_n^e + k_n u_n \beta^2 \geqslant 0 \qquad (-R_n^e + k_n u_n \beta^2) u_n = 0 \tag{4.22}
$$

$$
\boldsymbol{R}_T^e = k_T \boldsymbol{u}_T \beta^2 \tag{4.23}
$$

$$
\begin{aligned}
&G_\beta \geqslant w && \text{if} \quad \beta = 0\\
&G_\beta = w - (k_n u_n^2 + k_T \|\boldsymbol{u}_T\|^2)\beta && \text{if} \quad \beta \in]0,1[\\
&G_\beta \leqslant w - (k_n u_n^2 + k_T \|\boldsymbol{u}_T\|^2) && \text{if} \quad \beta = 1
\end{aligned}
\tag{4.24}
$$

We observe that conditions (4.22) express generalized Signorini's conditions for the contact with adhesion.

To describe the irreversible parts R_n^i and $\boldsymbol{R}_T^i$ we introduce a positive function $\varPhi$, the so-called pseudo-potential of dissipation. Following Raous et al. (1999) we choose the following form for the extended function $\varPhi$

$$
\varPhi(R_n, u_n, \beta; \dot{\boldsymbol{u}}_T, \dot{\beta}) = \mu |R_n - k_n u_n \beta^2| \|\dot{\boldsymbol{u}}_T\| + \frac{b}{p+1} |\dot{\beta}|^{p+1} + I_{\mathbb{R}^-}(\dot{\beta}) \tag{4.25}
$$

where μ is the Coulomb friction coefficient, whilst the parameters b and p characterize viscous properties of adhesive bonds and $\mathbb{R}^-$ stands for the set of nonpositive reals. The indicator function $I_{\mathbb{R}^-}(\dot{\beta})$ imposes the constraint $\dot{\beta} \leqslant 0$, which means that adhesive bonds can only weaken and cannot be restituted. The first term on the right-hand side of Eq. (4.25) characterizes the dissipation due to friction, while the second term represents a viscous-type dissipation. The form of the friction dissipation in Eq. (4.25) corresponds to the Coulomb law of friction; other friction models are possible. From the mathematical point of view, the space of functions with variation bounded in time is an appropriate space for βs, cf references in Rojek and Telega (1999).

The complementary laws defining irreversible parts of contact traction R_n^i and $\boldsymbol{R}_T^i$, and the thermodynamic force G_β can be written as

$$
\begin{aligned}
&R_n^i = 0\\
&\boldsymbol{R}_T^i \in \partial \varPhi_{\dot{\boldsymbol{u}}_T}(R_n, u_n, \beta; \dot{\boldsymbol{u}}_T, \dot{\beta})\\
&G_\beta \in \partial \varPhi_{\dot{\beta}}(R_n, u_n, \beta; \dot{\boldsymbol{u}}_T, \dot{\beta})
\end{aligned}
\tag{4.26}
$$

Differential inclusion $(4.26)_2$ leads to the following condition

$$
\|\boldsymbol{R}_T^i\| \leqslant \mu |R_n - k_n u_n \beta^2| \tag{4.27}
$$

The partial subdifferential $\partial \varPhi_{\dot{\beta}}$ of the function $\varPhi(R_n, u_n, \beta; \dot{\boldsymbol{u}}_T, \cdot)$ is a sum of two subdifferentials, cf Rockafeller and Wets (1998)

$$
\partial \varPhi_{\dot{\beta}}(R_n, u_n, \beta; \dot{\boldsymbol{u}}_T, \dot{\beta}) = b|\dot{\beta}|^p \partial |\dot{\beta}| + \partial I_{\mathbb{R}^-}(\dot{\beta})
$$

where

$$
\partial I_{\mathbb{R}^-}(\dot{\beta}) = \begin{cases} 0 & \text{if} \quad \dot{\beta} < 0\\ \mathbb{R}^- & \text{if} \quad \dot{\beta} = 0\\ \emptyset & \text{otherwise} \end{cases}
$$

Here $\emptyset$ denotes the empty set. Thus we finally get

$$\begin{aligned} & G_\beta = -b|\dot{\beta}|^p && \text{if} \quad \dot{\beta} < 0 \\ & G_\beta \in \mathbb{R}^- && \text{if} \quad \dot{\beta} = 0 \end{aligned} \tag{4.28}$$

or

$$\dot{\beta} = -\Big(-\frac{G_\beta^-}{b}\Big)^{1/p}$$

since

$$\partial|\dot{\beta}| = \begin{cases} -1 & \text{if} \quad \dot{\beta} < 0 \\ [-1,1] & \text{if} \quad \dot{\beta} = 0 \\ +1 & \text{if} \quad \dot{\beta} > 0 \end{cases}$$

Condition (4.27) describes two situations:
(i) stick

$$\|\boldsymbol{R}_T^i\| < \mu|R_n - k_n u_n \beta^2| \qquad \Rightarrow \qquad \dot{\boldsymbol{u}}_T = \boldsymbol{0} \tag{4.29}$$

(ii) slip

$$\|\boldsymbol{R}_T^i\| = \mu|R_n - k_n u_n \beta^2| \qquad \Rightarrow \qquad \exists\lambda \geqslant 0 \qquad \dot{\boldsymbol{u}}_T = -\lambda \boldsymbol{R}_T^i \tag{4.30}$$

The stick/slip conditions (4.29) and (4.30) can be written in the equivalent form

$$\begin{aligned} & \phi \leqslant 0 \qquad\qquad \lambda \geqslant 0 \qquad\qquad \phi\lambda = 0 \\ & \dot{\boldsymbol{u}}_T + \lambda \boldsymbol{R}_T^i = \boldsymbol{0} \end{aligned} \tag{4.31}$$

where

$$\phi = \|\boldsymbol{R}_T^i\| - \mu|R_n - k_n u_n \beta^2| \tag{4.32}$$

Relation (4.28) can be combined with (4.24) to eliminate the thermodynamic force G_β

$$\begin{aligned} & \dot{\beta} = -\Big\{-\frac{1}{b}[w - (k_n u_n^2 + k_T\|\boldsymbol{u}_T\|^2)\beta]^-\Big\}^{1/p} && \text{if} \quad \beta \in [0,1[\\ & \dot{\beta} \leqslant -\Big\{-\frac{1}{b}[w - (k_n u_n^2 + k_T\|\boldsymbol{u}_T\|^2)]^-\Big\}^{1/p} && \text{if} \quad \beta = 1 \end{aligned} \tag{4.33}$$

Summarizing we can state that the complete set of contact conditions that must be taken into account consists of conditions for normal contact with adhesion (4.22), relations for tangential contact with adhesion and friction (4.23), (4.31), as well as the evolution law for the adhesion intensity β given by relations (4.33).

Remark 4.1. In the sense of the theory of hemivariational inequalities (Panagiotopoulos, 1993), the interface laws involving adhesion are of nonmonotone type, cf Mistakidis and Panagiotopoulos (1997), Tzaferopoulos and Panagiotopoulos (1994). In other words, the interface laws could be formulated in the form of a hemivariational inequality.

Licht and Michaille (1997) studied a rather general case of bonding of two nonlinear elastic bodies, separated by a thin adhesive layer, cf also the references cited therein.

Acknowledgment

The authors were supported by the State Committee for Scientific Research (KBN, Poland) through the grant No. 8 T11F 017 18.

References

1. Alves M.K., Kikuchi N., 1998, Damage Theory Applied to the Analysis of Composite Materials Subjected to a Loss of Adherence Between its Constituents, *Eur. J. Mech., A/Solids*, **17**, 53-69

2. An Y.H., Draughn R.A., Edit., 2000, *Mechanical Testing of Bone and the Bone-Implant Interface*, CRC Press, Boca Raton

3. Bao H., Willems P.Y., 1999, On the Kinematic Modeling and the Parameter Estimation of the Human Knee Joint, *J. Biomech. Eng.*, **121**, 406-413

4. Bednarz P., Rojek J., Telega J.J., Kowalczewski J., 2000, Contact Problems in Knee Joint after Arthroplasty, *Acta Bioeng. Biomech.*, **1**, Supplement 1, 73-78

5. Benhayoune H., Jallot E., Frayssinet P., Balossier G., Bonhomme P., 1999, Etude de la Resorption de L'Hydroxyapatite dense après Implementation par Microscopie Electronique à Balayage et Microanalyse X, *Innov. Tech. Biol. Med.*, **20**, 181-186

6. Berzins A., Sumner D.R., 2000a, Implant Pushout and Pullout Tests, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 463-476, CRC Press, Boca Raton

7. Berzins A., Sumner D.R., 2000b, In Vitro Measurements of Implant Stability, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 515-526, CRC Press, Boca Raton

8. BERZINS A., SHAH B., WEINANS H., SUMNER D.R., 1997, Nondestructive Measurements of Implant-Bone Interface Shear Modulus and Effects of Implant Geometry in Pull-out Tests, *J. Biomed. Mater. Res.*, **34**, 337-340

9. BLANKENVOORT L., HUISKES R., 1991, Ligament-Bone Interaction in a Three-Dimensional Model of the Knee, *J. Biomech. Eng.*, **113**, 263-269

10. BLANKENVOORT L., HUISKES R., 1996, Validation of a Three-Dimensional Model of the Knee, *J. Biomechanics*, **29**, 955-961

11. BLANKENVOORT L., KUIPER J.H., HUISKES R., GROOTENBOER H.J., 1991, Articular Contact in a Three-Dimensional Model of the Knee, *J. Biomechanics*, **24**, 1019-1031

12. BOSS J.H., SHAJRAWI I., MENDES D.G., 1994, The Nature of Bone-Implant Interface, *Med. Prog. Technol.*, **20**, 119-142

13. BREME H.J., 1998, Surfaces, Surface Modification and Tailoring, In: J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 153-175, John Wiley & Sons, Chichester

14. BREME H.J., BARBOSA M.A., ROCHA L.A., 1998, Adhesion to Ceramics, In: J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 219-264, John Wiley & Sons, Chichester

15. BRETELLE A.S., COCU M., MONERIE Y., 2000, Formulation du Contact Avec Adhérence en Elasticité Non Linéaire Entre Deux Solides Déformables, *C.R. Acad. Sci. Paris, Série IIb*, **328**, 203-208

16. BULL A.M., AMIS A.A., 1998, Knee Joint Motion: Description and Measurement, *Proc. Inst. Mech. Engrs.*, **212**, Part H, 357-372

17. CHAND R., HAUG E., RIM K., 1976, Stress in the Human Knee Joint, *J. Biomechanics*, **9**, 417-422

18. COCU M., CANGEMI L., RAOUS M., 1997, Approximation Results for a Class of Quasistatic Contact Problems Including Adhesion and Friction, In: *Proc. IUTAM Symp. 'Variations of Domain and Free Boundary Problems in Solid Mechanics'*, Paris, April 22-25, 1997

19. COCU M., ROCCA R., 2000, Existence of Results for Unilateral Contact Problems with Friction and Adhesion, *Math. Modelling Numer. Anal.*, **34**, 981-1001

20. DAVIS J.R., LOFTHOUSE R., JINNAH R.H., 2000, In Vitro Testing of the Stability of Acetabular Components, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 527-540, CRC Press, Boca Raton

21. DHERT W., JANSEN J., 2000, The Validity of a Single Pushout Test, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 477-488, CRC Press, Boca Raton

22. Frémond M., 1987, Adhérence des Solides, *J. Méc. Théorique Appl.*, **6**, 383-407

23. Frémond M., 1988, Contact with Adhesion, In: J. Moreau and P.D. Panagiotopoulos, Edit., *Nonsmooth Mechanics and Applications*, 177-221, Springer-Verlag, Wien

24. Harris M.L., Morberg P., Bruce W.J.M., Walsh W.R., 1999, An Improved Method for Measuring Tibiofemoral Contact Areas in Total Knee Arthroplasty: A Comparison of K-Scan Sensor and Fuji Film, *J. Biomechanics*, **32**, 951-958

25. Heegaard J.-H., 1993, Large Slip Contact in Biomechanics: Kinematics and Stress Analysis of the Patello-Femoral Joint, Thèse No. 1113, Ecole Polytechnique Fédérale de Lausanne

26. Heegaard J.-H., Curnier A., 1993, An Augmented Lagrangian Method for Discrete Large-Slip Contact Problems, *Int. J. Num. Meth. Eng.*, **36**, 569-593

27. Hefzy M.S., Cooke T.D.V., 1996, Review of Knee Models: 1996 Update, *Appl. Mech. Reviews*, **49**, Part 2, S187-S193

28. Hefzy M.S., Grood E., 1988, Review of Knee Models, *Appl. Mech. Reviews*, **41**, 1-13

29. Helsen J.A., Breme H., Edit., 1998, *Metals as Biomaterials*, John Wiley & Sons, Chichester

30. Herzog W., Diet S., Suter E., Mayzus P., Leonard T.R., Müller C., Wu J.Z., Epstein M., 1998, Material and Functional Properties of Articular Cartilage and Patellofemoral Contact Mechanics in an Experimental Model of Osteoarthritis, *J. Biomechanics*, **31**, 1137-1145

31. Hildebrand H.F., Hornez J.-C., 1998, Biological Response and Biocompatibility, In: J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 265-290, John Wiley & Sons, Chichester

32. Jallot E., 1998, Correlation Between Hydroxyapatite Osseointegration and Young's Modulus, *Medical Eng. Physics*, **20**, 697-701

33. Jallot E., Irigaray J.L., Weber G., Frayssinet P., 1999a, In Vivo Characterization of the Interface Between Cortical Bone and Biphasic Calcium Phosphate by the PIXE Method, *Surface and Interface Annals*, **27**, 648-652

34. Jallot E., Benhayoune H., Kilian L., Balossier G., Bonhomme P., Oudadesse H., Irigaray J.L., 1999b, Ultrastructural Characterization of Soft Tissues Surrounding Implanted Hip Prosthesis by Complementary PIXE and TEM Methods. *Nuclear Instruments Meth. Phys. Res.*, **B155**, 315-321

35. Jansons H.A., 1987, Development of Endoprosthesis in the U.S.S.R. *CRC Crictical Reviews in Biocompatibility*, **3**, 87-192

36. Jones D.B., 1998, Cells and Metals, In: J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 317-335, John Wiley & Sons, Chichester

37. Laffargue P., Breme H.J., Helsen J.A., Hildebrand H.F., 1998, Retrieval Analysis, In: J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 467-501, John Wiley & Sons, Chichester

38. Lewis G., 1998, Contact Stress at Articular Surfaces in Total Joint Replacements. Part I: Experimental Methods, *Bio-Medical Mat. Eng.*, **8**, 91-110; Part II: Analytical and Numerical Methods, ibidem, 259-278

39. Lewis G., Jonathan V., Kambhampati S., 1998, Effect of Material Property Representation on Stress in Endoprostheses, *Bio-Medical Mat. Eng.*, **8**, 11-23

40. Lewis J.L., Galante J.O., Edit., 1985, *The Bone-Implant Interface*, Workshop Report, American Academy of Orthopaedic Surgeons

41. Licht C., Michaille G., 1997, A Modelling of Elastic Adhesive Bonded Joints, *Adv. Math. Sci. Appl.*, **7**, 711-740

42. Maugis D., 1982, Adherence of Solids, In: J. Georges, Ed., *Microscopic Aspects of Adhesion and Lubrication*, 221-252, Elsevier, Amsterdam

43. Maugis D., Barquins M., 1980, Fracture Mechanics and Adherence of Viscoplastic Solids, In: H.-L. Lee, Edit., *Adhesion and Adsorption of Polymers, Part A*, 203-277, Plenum Publishing Corporation, New York

44. McKoy B., An Y.H., Friedman R., 2000, Factors Affecting the Strength of the Bone-Implant Interface, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 439-462, CRC Press, Boca Raton

45. Mistakidis E.S., Panagiotopoulos P.D., 1997, Numerical Treatment of Problems Involving Nonmonotone Boundary or Stress-Strain Laws, *Comput. Struct.*, **64**, 553-565

46. Nakamura T., Nishiguchi S., 2000, Tensile Testing of Bone-Implant Interface, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 489-497, CRC Press, Boca Raton

47. Panagiotopoulos P.D., 1993, *Hemivariational Inequalities: Applications in Mechanics and Engineering*, Springer-Verlag, Berlin

48. Possart W., 1998, Adhesion of Polymers, In: J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 197-218, John Wiley & Sons, Chichester

49. Prendergast P.J., 2000, Bone Prostheses and Implants, In: S.C. Cowin, Ed., *Bone Mechanics*, (35-1)-(35-29), CRC Press, Boca Raton

50. Raous M., Cangemi L., Cocu M., 1999, A Consistent Model Coupling Adhesion, Frictional Unilateral Contact, *Comput. Meth. Appl. Mech. Eng.*, **177**, 383-399

51. Rockafellar R., Wets R.-B., 1998, *Variational Analysis*, Springer, Berlin

52. Rojek J., Telega J.J., 1999, Numerical Simulation of Bone-Implant Systems Using a More Realistic Model of Contact Interfaces with Adhesion, *J. Theor. Appl. Mech.*, **37**, 659-686

53. Rojek J., Telega J.J., Bednarz P., 1999, Adhesion in Hip and Knee Implants: Modelling and Numerical Analysis, *Acta Bioeng. Biomech.*, **1**, Supplement 1, 377-380

54. Rojek J., Telega J.J., Stupkiewicz S., 2001, Contact Problems with Friction, Adhesion and Wear in Orthopaedic Biomechanics. Part II: Numerical Implementation and Application to Implanted Knee Joints. *J. Theor. Appl. Mech.*, **39**, 2001

55. Sathasivam S., Walker P.S., 1999, The Conflicting Requirements of Laxity and Conformity in Total Knee Replacements, *J. Biomechanics*, **32**, 239-247

56. Seireg A., Arvikar R.J., 1973, A Mathematical Model for Evaluation of Forces in Lower Extremities of the Musculo-Skeletal System, *J. Biomechanics*, **6**, 313-326

57. Stewart T., Jim Z.M., Fisher J., 1998, Analysis of Contact Mechanics for Composite Cushion Knee Joint Replacement, *Proc. Inst. Mech. Engrs.*, **212**, Part H, 1-10

58. Thull R., 1998, Tissue-Implant Interaction, In J.A. Helsen and H.J. Breme, Edit., *Metals as Biomaterials*, 291-315, John Wiley & Sons, Chichester

59. Ton That P. T., Tanner K., Bonfield W., 2000, Fatigue Characterization of a Hydroxyapatite-Reinforced Polyethylene Composite. I. Uniaxial Fatigue, *J. Biomed. Mat. Res.*, **51**, 453-460; II. Biaxial Fatigue, ibidem, 461-468

60. Toutoungi D.E., Zavatsky A.B., O'Connor J.J., 1997, Parameter Sensitivity of a Mathematical Model of the Anterior Cruciate Ligament, *Proc. Inst. Mech. Engrs.*, **211**, Part H, 235-246

61. Turner-Smith A., Ed., 1993, *Micromovement in Orthopaedics*, Oxford Medical Engineering Series, **10**, Clarendon Press, Oxford

62. Tzaferopoulos M.A., Panagiotopoulos P.D., 1994, A Numerical Method for a Class of Hemivariational Inequalities, *Comput. Mechanics*, **15**, 233-248

63. Vena P., 1998, Shape Optimization of a Femoral Head Endoprosthesis Allowing for Interface Behaviour, PhD Thesis, Politecnico di Milano

64. Waldman S.D., Bryant J.T., 1997, Dynamic Contact Stress and Rolling Resistance Model for Total Knee Arthroplasties, *J. Biomech. Eng.*, **119**, 254-260

65. Walker P.S., Blunn G.W., 1997, Biomechanical Principles of Total Knee Replacement Design, In: V.C. Mow and W.C. Hayes, Edit., *Basic Orthopaedic Biomechanics*, 461-493, Lippincott-Raven Publishers, Philadelphia

66. Walker P.S., Sathasivam S., 1999, The Design of Guide Surfaces for Fixed-Bearing and Mobile-Bearing Knee Replacements, *J. Biomechanics*, **32**, 27-34

67. Wang X., Athanasiou K.A., Agrawal C.M., 2000, Fracture Toughness Tests of the Bone-Implant Interface, In: Y.H. An and R.A. Draughn, Edit., *Mechanical Testing of Bone and the Bone-Implant Interface*, 499-514, CRC Press, Boca Raton

68. Wu J.Z., Herzog W., Epstein M., 1998, Effects of Inserting a Pressensor Film into Articular Joints on the Actual Contact Mechanics, *J. Biomech. Eng.*, **120**, 655-659

69. Zatsiorsky V.M., 1998, *Kinematics of Human Motion, Human Kinetics*, Leeds

70. Zavatsky A.B., O'Connor J.J., 1994, Three-Dimensional Geometrical Models of Human Knee Ligaments, *Proc. Inst. Mech. Engrs.*, **208**, 229-240

Zagadnienia kontaktowe z tarciem, adhezją i zużyciem w biomechanice ortopedycznej. Część I – Rozważania ogólne

Streszczenie

Niniejsza praca stanowi kontynuację wcześniejszej pracy autorów, por. Rojek i Telega (1999). Zastosowano alternatywny model adhezji dla opsu interfazy kość-implant. Przedyskutowano również szereg zagadnień związanych z tą interfazą.

Manuscript received February 2, 2001; accepted for print May 23, 2001