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PREDICTION OF THE DYNAMIC CHARACTERISTICS AND CONTROL OF AIRCRAFT IN PRESCRIBED TRAJECTORY FLIGHT

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A unified and general approach to modeling and simulation of aircraft prescribed trajectory flight is presented in the paper. The program of motion is composed of a specified trajectory in space (two constraints on the aircraft position), a required fuselage attitude with respect to the trajectory, and optionally, a specified flight velocity. For an aircraft traditionally controlled by aileron, elevator and rudder deflections and thrust changes, a tangent realization of the trajectory constraints arises, which yields two additional constraints on the fuselage attitude (which thus becomes fully specified). The governing equations of the programmed motion are developed in the form of differential-algebraic equations, and a method of solving the equations is proposed. The solution consists of variations of the aircraft state variables and the required control that ensures realization of the prescribed motion program. This gives a unique opportunity to study simulated control strategies and evaluate feasibility of modeled aircraft missions. Some results of numerical simulations are reported.

Key words: inverse dynamics, inverse simulation control, prescribed trajectory flight

Notations

A_{KL} – matrix of transformation between (L) and (K) reference systems

ϕ_K, θ_K, ψ_K – Bryant's angles that orientate the reference system (K) with respect to the reference system (G) (Fig.1)

α, β – angle of attack and sideslip angle, respectively (Fig.2)

$\boldsymbol{\omega}_K$ – vector of absolute angular velocity of the reference system (K), $\boldsymbol{\omega}_K \equiv \boldsymbol{\omega}_{K/G}$

$\boldsymbol{\omega}_{K/L}$ – vector of angular velocity of the reference system (K) with respect to the non-inertial reference system (L)

$\mathbf{B}_A, \mathbf{B}_{A/B}$ – matrices defined in Eqs (2.2) and (2.4)

$\boldsymbol{\omega}_B^{(B)}$ – components of aircraft angular velocity expressed in the reference system (B), $\boldsymbol{\omega}_B^{(B)} = [p, q, r]^\top$

$\boldsymbol{v}$ – vector of aircraft velocity with respect to the air

$\boldsymbol{w}$ – wind velocity vector

θ_w, ψ_w – angles that orientate $\boldsymbol{w}$ with respect to axes of the systems (I) and (G) (Fig.8)

$\boldsymbol{v}_0$ – vector of aircraft absolute velocity (with respect to the ground), $\boldsymbol{v}_0 = \boldsymbol{v} + \boldsymbol{w}$

$\boldsymbol{\delta}$ – control surface deflections (aileron, elevator, rudder), $\boldsymbol{\delta} = [\delta_a, \delta_e, \delta_r]^\top$

T – jet thrust value

α_T – inclination angle between $\boldsymbol{T}$ and the Ox_B axis

d – distance between the point O and line of the force $\boldsymbol{T}$

J_T, ω_T – moment of inertia and angular velocity of rotating jet elements

$\mathbf{F}^{(A)}, \mathbf{N}^{(B)}$ – applied forces and moments, expressed in the systems (A) and (B)

$\mathbf{c}_F^{(A)}$ – aerodynamic force coefficients (drag, side, lift), $\mathbf{c}_F^{(A)} = [c_D, c_S, c_L]^\top$

$\mathbf{c}_N^{(B)}$ – aerodynamic moment coefficients (roll, pitch, yaw), $\mathbf{c}_N^{(B)} = [c_l, c_m, c_n]^\top$

m – aircraft mass

$\mathbf{J}$ – 3×3 matrix of aircraft moments of inertia in (B)

g – acceleration of gravity

S – lifting surface area

b – wing span

c_a – mean aerodynamic chord

ρ – air density

ρ_c – radius of trajectory curvature

$\mathbf{r}_O^{(I)}$ – position vector of aircraft (point O) in the reference system (I), $\mathbf{r}_O^{(I)} = [x, y, z]^\top$

s – arc length parameter
$\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{u}$ – vectors of algebraic, differential and control variables
t – time

1. Introduction

Inverse simulation techniques are computational methods in which control inputs to a dynamic system that produce desired system outputs are determined. The governing equations that result after coupling the equations of motion of the system with the equations of program constraints (= desired motion specifications) are differential-algebraic equations (DAEs) in the system state variables and control parameters. The numerical solution to these governing equations provides a basis for studying the system dynamic characteristics in the specified motion and evaluating the required control effort.

Though the inverse dynamics simulation is a legitimate analysis tool in problems concerned with flight maneuvering (Etkin, 1972), it is only recently, with the increased availability of powerful small computers and their associated software, that the technique has gained practical significance. The applications range from the theoretical studies on aircraft space maneuvers to the design of aircraft flight control systems (see e.g.: Kato and Sugiura, 1986; Lane and Stengel, 1988; Kato, 1990; Thomson and Bradley, 1990; Hess et al., 1991; Wang and Hu, 1993; Azam and Singh, 1994). Most of the attempts use fuselage attitude prescribed time histories as the starting point of their analysis, replaced sometimes by specified variations of the body-axis angular rates and supplemented by a flight velocity specification. Different four-element sets of program constraints of these types are then used to model the desired maneuvers of planned missions.

The program of motion involved in this contribution is based on specifying a flight path directly, which is the most natural way to prescribe flight maneuvers. A specified trajectory means two program constraints imposed on position in space of the aircraft centre of mass. An important observation stated by Parczewski and Blajer (1989) and then by Blajer (1997) is that the realization of these constraints is tangent. Namely, the primary effects of control surface deflections are the aerodynamic moments, and their direct influence on aerodynamic forces is usually small. The jet thrust induced by the fourth control parameter, the throttle setting, is normally nearly tangent to the trajectory (and is used to enforce realization of a possible flight velocity specification). In other words, the available control reactions cannot directly regulate the balance of the applied and inertial forces in the constrained (orthogonal to trajectory) directions, and as such cannot directly assure the realization of trajectory constraints (the orthogonal realization of the constraints is not feasible). The realization can however be achieved by appropriately adjusting the fuselage attitude with respect to the trajectory – the force balance in the constrained directions can then be assured by the induced changes in aerodynamic forces. Now, the required attitude variations can directly be regulated by control surface deflections.

The tangent realization of the trajectory constraints yields two additional requirements on the fuselage attitude – the trajectory constraints are thus "redoubled". Only two supplementary constraints can then be added to fully specify the aircraft motion. One of the constraints must specify the aircraft position on the trajectory, which is usually achieved by making use of the required time history of flight velocity. The other constraint is on the fuselage attitude, and most often it is the coordinated turn condition $(\beta = 0)$ or a prescribed bank angle variation. The realization of the supplementary constraints, as well as the fuselage attitude constraints consequent on tangent realization of the trajectory constraints, is orthogonal (see Parczewski and Blajer, 1989; Blajer and Parczewski, 1990; Blajer, 1997). Concluding, no more than four program constraints (including the two "redoubled" trajectory constraints) can be imposed on aircraft motion. The observation explains also the "paradox" that aircraft, a six-degree-of-freedom system, can be explicitly governed by four control inputs.

In this paper, a unified and general approach to the modeling and simulation of aircraft prescribed trajectory flight is developed. The solution to the derived equations of program motion consists of variations of the aircraft state variables and the required time histories of control parameters. This gives a unique opportunity to study control strategies of the modeled maneuvers and to evaluate their feasibility, which may be very valuable in analysing extreme flight conditions (e.g. aerobatic maneuvers) or planning missions of unmanned aerial vehicles (UAV). Moreover, the possible flight control systems based upon the non-linear inverse dynamics may offer the potential for providing improved levels of performance over the conventional flight control designs developed using linearizing assumptions.

For clarity reasons in presenting the principles of the mathematical model, it has been assumed in the paper that the planned maneuvers are performed in windless conditions. Then, in Appendix the formulation is extended to the case including the wind drift.

2. Reference coordinate systems

The following reference coordinate systems are used:

- (I) – inertial $O_I x_I y_I z_I$, with the $O_I z_I$ axis pointed vertically and downward
- (G) – gravitational $Ox_G y_G z_G$, with its origin at the mass center O of the aircraft and always parallel to the reference system (I)
- (A) – aerodynamic $Ox_A y_A z_A$, with the Ox_A axis directed along the flight velocity vector $\boldsymbol{v}$
- (B) – body-fixed $Ox_B y_B z_B$, with the $Ox_B z_B$ plane being the fuselage symmetry plane.

The three angles that orientate the reference systems (A) and (B) with respect to the system (G) are, traditionally, Bryant's angles ϕ_K, θ_K and ψ_K $(K = A, B)$, shown in Fig.1.

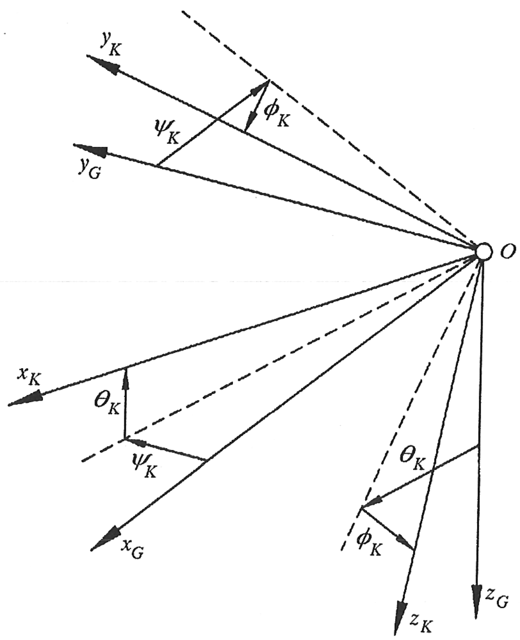

Fig. 1. Angular orientation of the reference systems (K) and (G)

The transformation matrix between the systems (K) and (G) is

$$\mathbf{A}_{KG}(\phi_K, \theta_K, \psi_K) = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix} \tag{2.1}$$

where the abbreviations $c_\theta = \cos\theta_K$, $s_\psi = \sin\psi_K$, ... are used for compactness. The absolute angular velocity of the system (K), $\boldsymbol{\omega}_K$ ("absolute" means here with respect to (G), i.e. $\boldsymbol{\omega}_K \equiv \boldsymbol{\omega}_{K/G}$), expressed in (K), is

$$\boldsymbol{\omega}_K^{(K)} = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & s_\phi c_\theta \\ 0 & -s_\phi & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} \dot\phi_K \\ \dot\theta_K \\ \dot\psi_K \end{bmatrix} \equiv \mathbf{B}_K(\phi_K, \theta_K) \begin{bmatrix} \dot\phi_K \\ \dot\theta_K \\ \dot\psi_K \end{bmatrix} \tag{2.2}$$

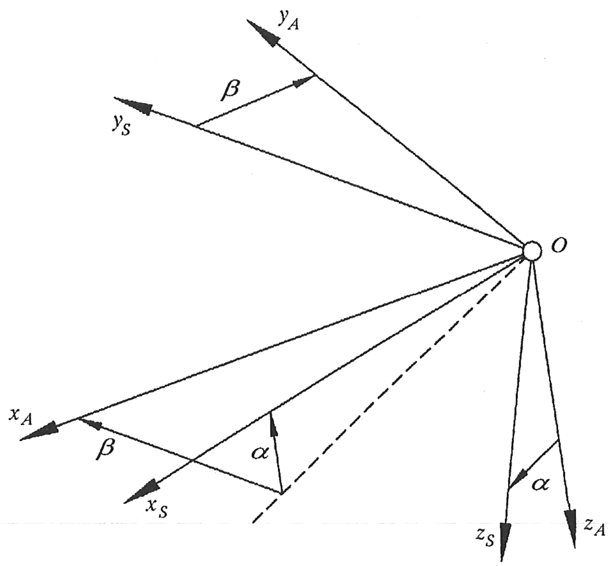

Fig. 2. Angular orientation of the reference systems (B) and (A)

The angular orientation of the reference systems (B) and (A) is described by the angles α and β (see Fig.2), and the transformation matrix between these systems is

$$\mathbf{A}_{BA}(\alpha, \beta) = \begin{bmatrix} c_\alpha c_\beta & -c_\alpha s_\beta & -s_\alpha \\ s_\beta & c_\beta & 0 \\ s_\alpha c_\beta & -s_\alpha s_\beta & c_\alpha \end{bmatrix} \tag{2.3}$$

where the abbreviations $c_\alpha = \cos\alpha$, $s_\beta = \sin\beta$, ... are used again. The angular velocity of the reference system (A) with respect to (B), $\boldsymbol{\omega}_{A/B}$, expressed in (A), is

$$\boldsymbol{\omega}_{A/B}^{(A)} = \begin{bmatrix} -s_\beta & 0 \\ -c_\beta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\alpha \\ \dot\beta \end{bmatrix} \equiv \mathbf{B}_{A/B}(\beta) \begin{bmatrix} \dot\alpha \\ \dot\beta \end{bmatrix} \tag{2.4}$$

Based on Eqs (2.1) ÷ (2.4) two other useful relations can be derived. The first one is

$$\mathbf{A}_{BG}(\phi_B,\theta_B,\psi_B) = \mathbf{A}_{BA}(\alpha,\beta)\mathbf{A}_{AG}(\phi_A,\theta_A,\psi_A) \tag{2.5}$$

and can be used to determine the classical roll ϕ_B, pitch θ_B and yaw ψ_B angles in terms of α, β, ϕ_A, θ_A, ψ_A. The other relation is the matrix representation of the vector formula for the aircraft absolute angular velocity, $\boldsymbol{\omega}_B = \boldsymbol{\omega}_A + \boldsymbol{\omega}_{B/A} = \boldsymbol{\omega}_A - \boldsymbol{\omega}_{A/B}$. Expressed in (B), the formula reads

$$\begin{aligned}\boldsymbol{\omega}_B^{(B)} \equiv \begin{bmatrix} p \\ q \\ r \end{bmatrix} &= \mathbf{A}_{BA}(\boldsymbol{\omega}_A^{(A)} - \boldsymbol{\omega}_{A/B}^{(A)}) = \\ &= \mathbf{A}_{BA}(\alpha,\beta)\Big(\mathbf{B}_A(\phi_A,\theta_A)\begin{bmatrix} \dot{\phi}_A \\ \dot{\theta}_A \\ \dot{\psi}_A \end{bmatrix} - \mathbf{B}_{A/B}(\beta)\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix}\Big)\end{aligned} \tag{2.6}$$

and can serve to determine the aircraft roll, pitch and yaw rates, p, q and r in terms of α, β, ϕ_A, θ_A, ψ_A and $\dot{\alpha}$, $\dot{\beta}$, $\dot{\phi}_A$, $\dot{\theta}_A$, $\dot{\psi}_A$.

Finally, the other important features of the transformation matrices introduced in Eqs (2.1) and (2.3) are

$$\begin{aligned}\mathbf{A}_{KL} &= \mathbf{A}_{LK}^{-1} = \mathbf{A}_{LK}^{\top} \\ \dot{\mathbf{A}}_{KL} &= \widetilde{\boldsymbol{\omega}}_{K/L}^{(K)}\mathbf{A}_{KL} = \mathbf{A}_{KL}\widetilde{\boldsymbol{\omega}}_{L/K}^{(L)}\end{aligned} \tag{2.7}$$

where the superscript $\widetilde{(\cdot)}$ in Eq $(2.7)_2$ denotes a skew-symmetric matrix (a cross product operator), which for the vector $\boldsymbol{\omega} = [\omega_x,\omega_y,\omega_z]^\top$ is defined as

$$\widetilde{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{2.8}$$

3. Dynamic equations

The aircraft is modeled as a rigid body of six degrees of freedom, symmetrical with respect to the Ox_Bz_B plane. The control parameters are the values of the control surface deflections $\boldsymbol{\delta} = [\delta_a,\delta_e,\delta_r]^\top$ and the value of the jet thrust T (active only if a specification on flight velocity is imposed). It is assumed that the main effect produced by the deflections $\boldsymbol{\delta}$ is on the aerodynamic moments, while their direct effect on the aerodynamic forces is small. The vector $\boldsymbol{T}$ is contained in the Ox_Bz_B plane.

For the purpose of the present formulation, it is convenient to use the equations of translatory motion expressed in the reference system (A), and the equations of rotational motion expressed in the reference system (B). In matrix form the equations are

$$
\begin{aligned}
m\dot{\mathbf{v}}_O^{(A)} + m\boldsymbol{\omega}_A^{(A)}\mathbf{v}_O^{(A)} &= \mathbf{F}^{(A)} \\
\mathbf{J}\dot{\boldsymbol{\omega}}_B^{(B)} + \boldsymbol{\omega}_B^{(B)}\mathbf{J}\boldsymbol{\omega}_B^{(B)} &= \mathbf{N}^{(B)}
\end{aligned}
\tag{3.1}
$$

where $\boldsymbol{v}_O^{(A)} = [1,0,0]^\top v$ is the representation of the absolute velocity of the point O in (A) equal to the aircraft air velocity for windless conditions. The components of the applied forces $\mathbf{F}^{(A)}$ and torques $\mathbf{N}^{(B)}$, in the reference systems (A) and (B), respectively, are

$$
\begin{aligned}
\mathbf{F}^{(A)} &= -\frac{1}{2}\rho S v^2 \begin{bmatrix} c_D \\ c_S \\ c_L \end{bmatrix} + T \begin{bmatrix} \cos(\alpha+\alpha_T)\cos\beta \\ -\cos(\alpha+\alpha_T)\sin\beta \\ -\sin(\alpha+\alpha_T) \end{bmatrix} + mg \begin{bmatrix} -\sin\theta_A \\ \sin\phi_A\cos\theta_A \\ \cos\phi_A\cos\theta_A \end{bmatrix} \\
\mathbf{N}^{(B)} &= \frac{1}{2}\rho S v^2 \begin{bmatrix} bc_l \\ c_a c_m \\ bc_n \end{bmatrix} + T \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} + J_T\omega_T \begin{bmatrix} -q\sin\alpha_T \\ p\sin\alpha_T - r\cos\alpha_T \\ q\cos\alpha_T \end{bmatrix}
\end{aligned}
\tag{3.2}
$$

It is assumed that ω_T and T are closely related, and the relationship $\omega_T = \omega_T(T)$ is known. Then, the coefficients of aerodynamic forces $\mathbf{c}_F^{(A)} = [c_D, c_S, c_L]^\top$ and moments $\mathbf{c}_N^{(B)} = [c_l, c_m, c_n]^\top$ are the following functions of state variables and controls

$$
\begin{aligned}
c_D &= c_D(\alpha,\beta,\delta_e) \qquad & c_l &= c_l(\alpha,\beta,p,r,\delta_a,\delta_r) \\
c_S &= c_S(\alpha,\beta,p,r,\delta_r) \qquad & c_m &= c_m(\alpha,q,\delta_e) \\
c_L &= c_L(\alpha,q,\delta_e) \qquad & c_n &= c_n(\alpha,\beta,p,r,\delta_a,\delta_r)
\end{aligned}
\tag{3.3}
$$

4. Equations of program constraints

4.1. Prescribed trajectory

A desired trajectory is defined in the reference system (I), Fig.3, and its most convenient representation has the following parametric form with the arc length s as the parameter

$$\mathbf{r}_O^{(I)} = \widehat{\mathbf{r}}_O^{(I)}(s) \equiv \begin{bmatrix} \widehat{x}(s) \\ \widehat{y}(s) \\ \widehat{z}(s) \end{bmatrix} \tag{4.1}$$

where the superscript $\widehat{(\cdot)}$ means "specified". For the purpose of this model, $\widehat{\mathbf{r}}_O^{(I)}(s)$ must be at least twice differentiable function. In applications, a trajectory is first sketched by a set of successive points in space, and then interpolated/approximated by spline functions of appropriate order. The foundations of such a procedure were described by Blajer and Parczewski (1990) and Blajer (1991). Trajectory specification (4.1) is equivalent to two program constraints on the aircraft (point O) position in space.

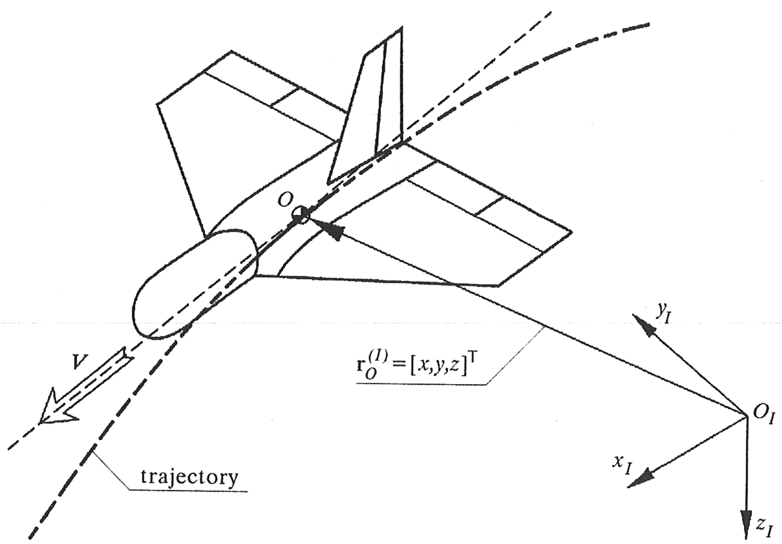

Fig. 3. Prescribed trajectory flight of the UAV

4.2. Specification on fuselage attitude

As mentioned previously, only one program constraint can originally be imposed on the fuselage attitude with respect to the trajectory (the two other constraints of this type consequent upon the tangent realization of the trajectory constraints, and will be introduced in the next section). In the modeling of specific maneuvers (see Blajer and Parczewski, 1990; Blajer, 1990, 1991);

the mentioned constraint is either

$$\beta = \widehat{\beta}(s) \qquad \text{or} \qquad \phi_A = \widehat{\phi}_A(s) \tag{4.2}$$

Most often the first of (4.2) is $\widehat{\beta} = 0$, and characterizes a wide range of maneuvers that assume coordinated turns, while $\widehat{\beta}(s) \neq 0$ can serve for the modeling of possible side-slipping flight phases. The other specification of Eq (4.2) can serve for the modeling of some special aerobatic maneuvers like roll or bunt (Blajer, 1991). The constructing of linked aerobatic maneuvers may require making use of both of requirements (4.2) alternately in different flight phases.

4.3. Velocity specification

The most natural specification of this type is

$$v_O = \widehat{v}_O(s) \tag{4.3}$$

which prescribes the aircraft motion along trajectory (4.1). For the purpose of this formulation, however, a more convenient form of the constraint is

$$s = \widehat{s}(t) \tag{4.4}$$

where $\widehat{v}_O = \dot{\widehat{s}}$ and $\dot{\widehat{v}}_O = \ddot{\widehat{s}}$. In the simple cases, e.g. $v_O = \text{const}$, the formulation of Eq (4.4), based on Eq (4.3), may be evident. In a more general case, Eq (4.4) can be obtained as a solution to the following integral equation

$$\int_0^{\widehat{s}(t)} \frac{ds}{\widehat{v}_O(s)} = t \tag{4.5}$$

As the analytical solution of Eq (4.5) may be quite complex, it is suggested that it should be solved numerically. It is then assumed that the specification of aircraft motion along the trajectory is given in the form of Eq (4.4), and $\widehat{s}(t)$ must be at least twice differentiable.

4.4. Programs of fully and partly specified flights

As it will be shown in the next section (see also Parczewski and Blajer, 1989; Blajer and Parczewski, 1990; Blajer, 1997), the tangent realization of trajectory constraints (4.1) yields two additional requirements on the fuselage attitude with respect to the trajectory path. Having given the required trajectory and complementary constraint (4.2) we fully specify the aircraft angular motions. The angular motion specifications are regulated by appropriate control surface deflections. The trajectory constraints originally impose two requirements on the aircraft (point O) position in space, and the realization of the induced requirements on the fuselage attitude assures the realization of the original constraints as well. Then, constraining the aircraft position on the trajectory path according to Eq (4.4) makes the aircraft translational motion fully specified, and the realization of constraint (4.4) is assured by changes of the thrust. However, while imposing complementary constraint (4.2) is necessary, demand (4.4) can be treated as optional. When the "velocity" constraint is imposed or not, we can respectively say that the programs are fully specified (FS) or partly specified (PS) and, accordingly, the thrust control is active or not. In the modeling of particular phases of an aircraft mission, the PS and FS programs can be used alternately. The definitions relating PS and FS motions are summarized in Table 1.

Table 1. Types of constraint realization

	Program constraints	Control
PS motion	2 constraints (4.1) 1 constraint (4.2)	$\delta_a, \delta_e, \delta_r$
FS motion	2 constraints (4.1) 1 constraint (4.2) 1 constraint (4.4)	$\delta_a, \delta_e, \delta_r, T$

5. Conditions induced by trajectory constraints

Differentiating trajectory constraint (4.1) with respect to time one obtains $\mathbf{v}_O^{(I)} = \dot{\hat{\mathbf{r}}}_O^{(I)} = \hat{\mathbf{r}}_O'^{(I)}\dot{s}$, where $(\cdot)'$ denotes differentiating with respect to s. This condition means that the aircraft absolute linear velocity vector $\boldsymbol{v}_O$ is tangent to the postulated trajectory. In windless flight conditions we have $\boldsymbol{v}_O = \boldsymbol{v}$, where $\boldsymbol{v}$ is the aircraft velocity with respect to the air, and $v_O = v = \dot{s}$ (see Appendix for the case including the wind drift). As in the case $\mathbf{v}_O^{(I)} = \mathbf{v}_O^{(G)} = \mathbf{A}_{GA}\mathbf{v}^{(A)}$ and $\mathbf{v}^{(A)} = [1, 0, 0]^\top v$, the tangency of $\boldsymbol{v}_O$ to the trajectory is equivalent to $\hat{\mathbf{r}}_O'^{(I)} = \boldsymbol{A}_{GA}^{(1)}$, where $\boldsymbol{A}_{GA}^{(1)}$ denotes the first column of $\mathsf{A}_{GA} = \mathsf{A}_{AG}^{\top}$ defined in Eq (2.1). In the full form the condition reads

$$\begin{bmatrix} \cos\theta_A \cos\psi_A \\ \cos\theta_A \sin\psi_A \\ -\sin\theta_A \end{bmatrix} = \begin{bmatrix} \widehat{x}' \\ \widehat{y}' \\ \widehat{z}' \end{bmatrix} \tag{5.1}$$

As $\theta_A \in (-\pi/2, \pi/2)$ and $\psi_A \in (0, 2\pi)$, Eq (5.1) serves for the explicit determination of the angles θ_A and ψ_A that orientate the vector $\boldsymbol{V}_O$ with respect to the reference systems (G) and (I). The above can thus be interpreted as conditions imposed on the angles.

Differentiating the trajectory constraint at the "velocity level" $(\mathsf{A}_{GA}\mathsf{v}^{(A)} = \widehat{\mathsf{r}}_O^{'(I)})$ with respect to time once more, we obtain the trajectory constraint condition at the "acceleration level" which, after using Eq $(2.7)_2$, takes the following form

$$\mathsf{A}_{GA}(\dot{\mathsf{v}}_O^{(A)} + \boldsymbol{\omega}_A^{(A)}\mathsf{v}_O^{(A)}) = \widehat{\mathsf{r}}_O^{'(I)}\ddot{s} + \widehat{\mathsf{r}}_O^{''(I)}\dot{s}^2 \tag{5.2}$$

Then, premultiplying Eq (5.2) by A_{AG}, and using the dynamic equation $(3.1)_1$, we arrive at

$$-\mathsf{F}^{(A)} + m\mathsf{A}_{AG}(\widehat{\mathsf{r}}_O^{'(I)})\ddot{s} + \widehat{\mathsf{r}}_O^{''(I)})\dot{s}^2) = \mathbf{0} \tag{5.3}$$

The right-hand side of Eq (5.2) is the representation of the tangent $\boldsymbol{a}_r$ $(\mathsf{a}_r^{(G)} = \widehat{\mathsf{r}}_O^{'(I)}\ddot{s})$ and normal $\boldsymbol{a}_n$ $(\mathsf{a}_n^{(G)} = \widehat{\mathsf{r}}_O^{''(I)}\dot{s}^2)$ accelerations, both with respect to the trajectory path.

It is easy to ascertain that $\widehat{\mathsf{r}}_O^{'(I)} = [\widehat{x}', \widehat{y}', \widehat{z}']^{\top}$ is the representation of the unit vector $\widehat{\boldsymbol{r}}'_O$ $(|\widehat{\boldsymbol{r}}'_O| = \sqrt{\widehat{x}'^2 + \widehat{y}'^2 + \widehat{z}'^2} = 1$ pointed along the direction of $\boldsymbol{v}_O$. According to Eq (5.1), in windless conditions $\widehat{\mathsf{r}}_O^{'(I)}$ is also equivalent to the first row of A_{AG} (the first column of A_{GA}), and as such $\widehat{\boldsymbol{r}}'_O$ is pointed along the first axis of the reference system (A) as well (see Appendix for the case including the wind drift). Then, $\widehat{\mathsf{r}}_O^{''(I)} = [\widehat{x}'', \widehat{y}'', \widehat{z}'']^{\top}$ is the representation of the vector $\widehat{\boldsymbol{r}}''_O$ pointed at the center of the trajectory curvature, and its value is $(|\widehat{\boldsymbol{r}}''_O| = \sqrt{\widehat{x}''^2 + \widehat{y}''^2 + \widehat{z}''^2} = \rho_c^{-1}$, where ρ_c is the curvature radius at a given position on the trajectory. The vector $\widehat{\boldsymbol{r}}''_O$ is thus represented in the plane orthogonal to the trajectory, which, for the case in hand, is equivalent to the Oy_Az_A plane of the reference system (A). On this basis we can write

$$\mathsf{A}_{AG}\widehat{\mathsf{r}}_O^{'(I)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathsf{A}_{AG}\widehat{\mathsf{r}}_O^{''(I)} = \begin{bmatrix} 0 \\ \times \\ \times \end{bmatrix} \tag{5.4}$$

where $(\times)$ denotes a non-zero element.

Using peculiarities (5.4), the first relation of matrix equation (5.3) is

$$\frac{1}{2}\rho S v^2 c_D - T\cos(\alpha+\alpha_T)\cos\beta + mg\sin\theta_A + m\ddot{s} = 0 \tag{5.5}$$

and it expresses the balance of the applied and inertial forces in the direction tangent to the trajectory. In the PS motion case, Eq (5.5) denotes the aircraft motion equation on the trajectory, and T is an inactive control parameter (can be an arbitrary function of time and state variables). In the FS motion case we have $v = \hat{\dot{s}}(t)$ and $\ddot{s} = \hat{\ddot{s}}(t)$, and Eq (5.5) expresses the condition imposed by specification (4.4) on the aircraft motion on the trajectory. The realization of the requirement is assured by appropriate changes of value of T (which is now the active control parameter). Note also that, according to Eq (14), condition (5.5) is influenced by δ_e as well, but the induced drag force effects are usually small (negligible).

The two other relations of matrix Eq (5.3) are

$$\begin{aligned}
&\frac{1}{2}\rho S v^2 c_S + T\cos(\alpha+\alpha_T)\sin\beta - mg\sin\phi_A\cos\theta_A + \\
&+m\dot{s}^2[\hat{x}''(\sin\phi_A\sin\theta_A\cos\psi_A - \cos\phi_A\sin\psi_A) + \\
&+\hat{y}''(\sin\phi_A\sin\theta_A\sin\psi_A + \cos\phi_A\cos\psi_A) + \hat{z}''\sin\phi_A\cos\theta_A] = 0 \\
&\frac{1}{2}\rho S v^2 c_L + T\sin(\alpha+\alpha_T) - mg\cos\phi_A\cos\theta_A + \\
&+m\dot{s}^2[\hat{x}''(\cos\phi_A\sin\theta_A\cos\psi_A + \sin\phi_A\sin\psi_A) + \\
&+\hat{y}''(\cos\phi_A\sin\theta_A\sin\psi_A - \sin\phi_A\cos\psi_A) + \hat{z}''\cos\phi_A\cos\theta_A] = 0
\end{aligned} \tag{5.6}$$

and they express the balance of the applied and inertial (centrifugal) forces along the Oy_A and Oz_A axes, i.e. they are the conditions of a vanishing motion in the plane orthogonal to the trajectory. For both the PS or FS motion cases, the conditions cannot be explicitly governed by the available control reactions – the direct influence of the control surface deflections δ_r and δ_e on the side and lift aerodynamic forces is small (negligible) and T (the effect of which on Eqs (5.6) is also small) is either inactive or set aside for controlling condition (5.5). Therefore, the only way to meet the requirements is to appropriately change the fuselage attitude with respect to the trajectory, by controlling the angles α and, according to which of (4.2) is used, ϕ_A or β (note that θ_A and ψ_A follow from Eq (4.4) and do not relate the fuselage attitude with the trajectory). Conditions (5.6) can thus be treated as two further restrictions on the aircraft angular orientation. It can also be interpreted that the trajectory constraints are "redoubled" due to their tangent realization (Blajer, 1997).

The required variations of the fuselage attitude can now be directly controlled by adequate control surface deflections, and the realization of conditions (5.6) ensures the realization of trajectory constraints (4.1). In this indirect way the trajectory constraints can be realized, and the six-degree-of-freedom aircraft can be controlled by the four control inputs.

6. Equations of program motion

The governing equations of aircraft prescribed trajectory flight can conveniently be written in the following differential-algebraic form

$$\begin{aligned} \mathbf{0} &= \mathbf{F}(\mathbf{y},\mathbf{z},\mathbf{u},t) \\ \mathbf{z} &= \mathbf{G}(\mathbf{y},\dot{\mathbf{y}}) \\ \dot{\mathbf{z}} &= \mathbf{H}(\mathbf{y},\mathbf{z},\mathbf{u},t) \end{aligned} \tag{6.1}$$

where $\mathbf{y}$, $\mathbf{z}$ and $\mathbf{u}$ are the vectors of algebraic (position), differential (velocity), and control variables, respectively. As shown in the following, Eqs $(6.1)_1$ are the algebraic equations induced by program constraints, Eqs $(6.1)_2$ are the kinematic differential relations, and Eqs $(6.1)_3$ are the dynamic equations. The index of differential-algebraic equations (DAEs) (6.1) is three (Brenan et al., 1989; Blajer, 1997).

The above general DAE form is common for both the PS and FS motion cases. However, the explicit forms of the equations as well as the definitions of $\mathbf{y}$, $\mathbf{z}$ and $\mathbf{u}$ vectors are slightly different for the two cases.

6.1. Equations of PS motion

The state variables $\mathbf{y}$ and $\mathbf{z}$, and the control variables $\mathbf{u}$ of the PS motion are

$$\begin{aligned} \mathbf{y} &= [\alpha,\beta,\phi_A,\theta_A,\psi_A,s]^\top \\ \mathbf{z} &= [v,p,q,r]^\top \\ \mathbf{u} &= [\delta_a,\delta_e,\delta_r]^\top \end{aligned} \tag{6.2}$$

and Eqs (6.1) are thirteen DAEs composed of:

- Five algebraic equations
  - One specification (4.2) on β or ϕ_A

- Eq (5.1), which denotes two requirements on θ_A and ψ_A
- Two conditions (5.6), which can be written symbolically as

$$F_y(\alpha, \beta, \phi_A, \theta_A, \psi_A, s, v, p, r, \delta_r, t) = 0$$
$$F_z(\alpha, \phi_A, \theta_A, \psi_A, s, v, q, \delta_e, t) = 0$$

- Four kinematical equations
  - One obvious equation, $v = \dot{s}$
  - Three equations (2.6), the symbolic form of which is

$$[p, q, r]^\top = \mathbf{G}_\omega(\alpha, \beta, \phi_A, \theta_A, \dot{\alpha}, \dot{\beta}, \dot{\phi}_A, \dot{\theta}_A, \dot{\psi}_A)$$

- Four dynamic equations
  - One equation of the aircraft motion on the trajectory, according to Eq (5.5)

$$\dot{v} = H_x(\alpha, \beta, \theta_A, , s, v, \delta_e, t)$$

  - Three equations of rotational motions, see Eq $(3.1)_2$

$$[\dot{p}, \dot{q}, \dot{r}]^\top = \mathbf{H}_\omega(\alpha, \beta, s, v, p, q, r, \delta_a, \delta_e, \delta_r, t)$$

6.2. Equations of FS motion

The variables $\mathbf{y}$, $\mathbf{z}$ and $\mathbf{u}$ are now

$$\begin{aligned} \mathbf{y} &= [\alpha, \beta, \phi_A, \theta_A, \psi_A]^\top \\ \mathbf{z} &= [p, q, r]^\top \\ \mathbf{u} &= [\delta_a, \delta_e, \delta_r, T]^\top \end{aligned} \tag{6.3}$$

and Eqs (6.1) are twelve DAEs composed of:

- Six algebraic equations
  - Eqs (4.2) and (5.1) as before (= three equations)
  - Three conditions (5.5) and (5.6), which can now be written as

$$F_x(\alpha, \beta, \theta_A, T, \delta_e, t) = 0$$
$$F_y(\alpha, \beta, \phi_A, \theta_A, \psi_A, p, r, T, \delta_r, t) = 0$$
$$F_z(\alpha, \phi_A, \theta_A, \psi_A, q, T, \delta_e, t) = 0$$

- Three kinematical equations (2.6)

$$[p,q,r]^\top = \mathbf{G}_\omega(\alpha,\beta,\phi_A,\theta_A,\dot{\alpha},\dot{\beta},\dot{\phi}_A,\dot{\theta}_A,\dot{\psi}_A)$$

- Three dynamic equations $(3.1)_2$

$$[\dot{p},\dot{q},\dot{r}]^\top = \mathbf{H}_\omega(\alpha,\beta,p,q,r,T,\delta_a,\delta_e,\delta_r,t)$$

7. Numerical procedure

The solution to DAEs (6.1) are the program variations of the state variables $\mathbf{y}(t)$ and $\mathbf{z}(t)$, and the variation of the control signal $\mathbf{u}(t)$ that ensures the realization of an assumed program of motion. Various DAE solvers can be used to solve the governing equations; see e.g. Brenan et al. (1989), Gear and Petzold (1984), Petzold (1990). Most of the methods originate from Gear's approach. Using the simplest Euler's backward difference approximation method for demonstrative reasons, the procedure is as follows. Having given $\mathbf{y}_n$, $\mathbf{z}_n$ and $\mathbf{u}_n$ at time t_n, the values $\mathbf{y}_{n+1}$, $\mathbf{z}_{n+1}$ and $\mathbf{u}_{n+1}$ at time $t_{n+1} = t_n + \Delta t$ can be found as a solution to the following set of non-linear algebraic equations

$$\begin{aligned}
\mathbf{0} &= \mathbf{F}(\mathbf{y}_{n+1},\mathbf{z}_{n+1},\mathbf{u}_{n+1},t_{n+1})\\
\mathbf{z}_{n+1} &= \mathbf{G}\Big(\mathbf{y}_{n+1},\frac{\mathbf{y}_{n+1}-\mathbf{y}_n}{\Delta t}\Big)\\
\frac{\mathbf{z}_{n+1}-\mathbf{z}_n}{\Delta t} &= \mathbf{H}(\mathbf{y}_{n+1},\mathbf{z}_{n+1},\mathbf{u}_{n+1},t_{n+1})
\end{aligned} \tag{7.1}$$

In this way the solution can be advanced from time t_n to t_{n+1}. In order to improve numerical accuracy, the Euler method is usually replaced by higher-order backward difference approximation methods (Brenan et al., 1989; Gear and Petzold, 1984; Petzold, 1990).

It may be worth noting that the first three algebraic equations $(6.1)_1$, composed of Eqs (4.2) and (5.1), can be solved independently for $\beta(t)$ (or $\phi_A(t)$), $\theta_A(t)$ and $\psi_A(t)$. An important observation is also that there is only a weak coupling by the control variables $\mathbf{u}$ between the remaining algebraic equations and dynamic equations $(6.1)_3$. Namely, the pivotal effect of the control surface deflections δ is manifested in dynamic equations $(3.1)_2$ of rotational motions, included in Eq $(6.1)_3$, while, in the case of FS motion, the control T affects chiefly condition (5.5), included in Eq $(6.1)_1$. Confining ourselves to the FS motion case, the solutions ϕ_A (or $\beta(t)$) and $\alpha(t)$ are then consequent mainly on algebraic equations (5.6), and the solutions $p(t)$, $q(t)$ and $r(t)$ are determined from equation of kinematics (2.6) based on the program variations $\alpha(t)$, $\beta(t)$, $\phi_A(t)$, $\theta_A(t)$ and $\psi_A(t)$. Finally, $T(t)$ and $\delta(t)$ mainly originate from condition (5.5) and dynamic equations of rotational motions $(3.1)_2$, respectively, after using the program variations of the state variables. This means the exact numerical solution for $\mathbf{y}(t)$ at a given time, and the approximated numerical solutions for $\mathbf{z}(t)$ and $\mathbf{u}(t)$. Note also that neither the numerical error accumulates in simulation time nor the current approximations in the obtaining of $\mathbf{z}(t)$ and $\mathbf{u}(t)$ influence the subsequent solution of the governing equations of motion. In the case of PS motion the situation differs in one respect only – $v(t)$ and $s(t)$ are integrated using the dynamic equations given by Eq (5.5). The numerical errors may thus accumulate during the simulation time, which will influence the solution with respect to the other variables in the governing equations of PS motion.

8. Case study

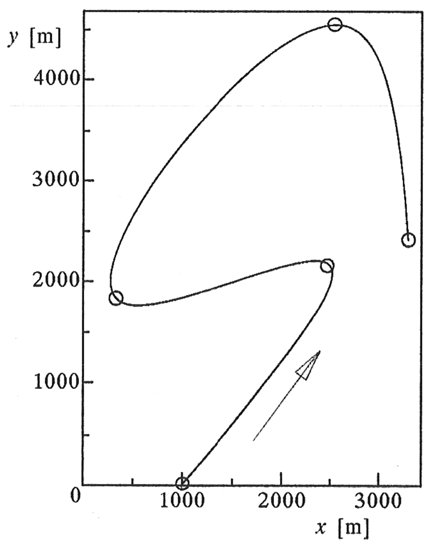

Fig. 4. Path points and the specified trajectory

The mission of an Unmanned Aerial Vehicle (UAV), outlined in Fig.3, was to pass successively through five way points indicated in Fig.4. Based on the points, the required trajectory has been interpolated using cubic spline functions; see Blajer (1990, 1991) for details. The UAV was then forced to fly along a modeled trajectory at the constant speed $v = 180\,\mathrm{km/h}$, and the condition of coordinated turns $(\beta = 0)$ was assumed.

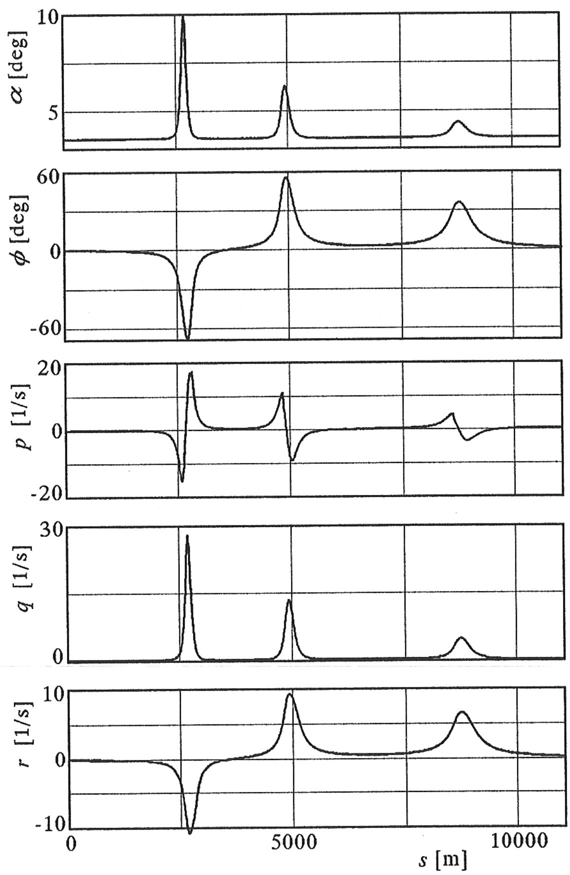

Fig. 5. Variation of the aircraft state variables

The results of numerical simulation are shown in Fig.5 and Fig.6. As can be seen in the graphs, the programmed flight is characterized by five phases of practically steady motion, separated by three highly non-linear phases in the neighborhood of the inner way points. The extreme flight conditions at those places partly result from the fact that cubic spline functions have been used to interpolate the trajectory. The produced variations of the trajectory curvature can be seen in Fig.7, and their maximal values are at the inner way points. The obtained program variations of the state variables and the control

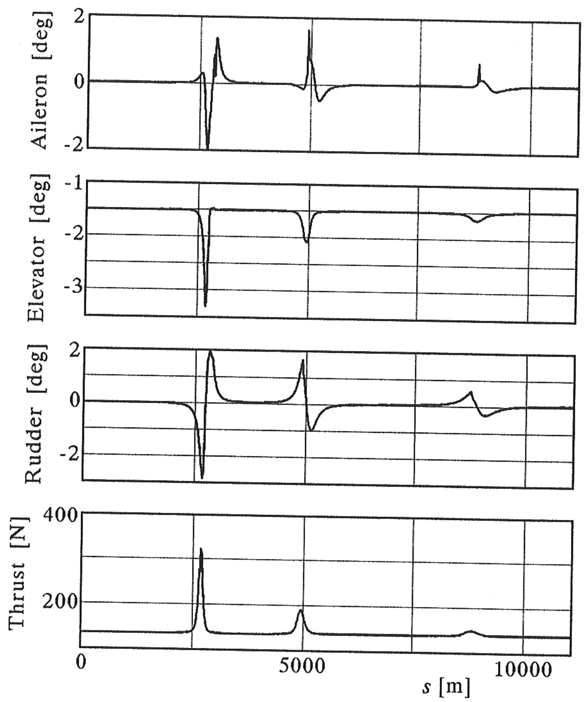

Fig. 6. Required control parameters

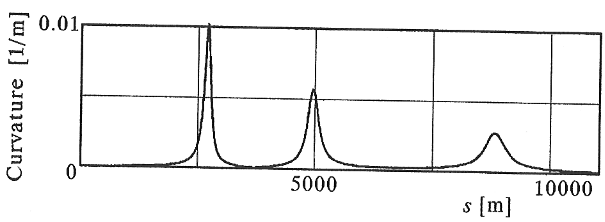

Fig. 7. Changes of the trajectory curvature

parameters are consequent on the trajectory setting. The using of fifth order splines to construct the trajectory would smooth the trajectory curvature and, consequently, the changes of the state variables and the control parameters. Similar effects can be achieved by setting additional points to sketch the trajectory more "rounded", and using additional points as the approximation points (Blajer, 1990, 1991). All these improvements would make the results more realistic, however this was not the purpose of this study.

9. Conclusion

The paper presents a concise and general mathematical model of aircraft prescribed trajectory flight. The program of motion is introduced in the most natural way, setting the required trajectory path directly. It is then demonstrated that the realization of the trajectory constraints is tangent, and this yields two additional constraints on the fuselage attitude. The observation is very interesting from the theoretical point of view – the trajectory constraints can be interpreted as "redoubled" constraints, and the realization of the induced attitude constraints assures the realization of the original constraints as well. In this way, the aircraft, a six-degree-of-freedom system, can explicitly be governed by three or four control parameters (respectively the case of PS or FS motion is considered).

The developed governing equations make it possible to predict the variations of the aircraft state variables and the desired control in the programmed motion. Such results can be useful for at least two reasons:

- The nature and feasibility of a wide range of aircraft missions, including aerobatic maneuvers and extreme flight conditions, can be studied. On this basis the missions can be improved/optimized.
- Flight control systems based upon the non-linear inverse dynamics can be developed, providing an improved level of safety and performance over the conventional designs.

A. Conditions induced by trajectory constraints – the case including the wind drift

It is assumed that the wind velocity and its direction are constant in the area of a given maneuver, $\boldsymbol{w} =$ const. Having defined angles θ_w and ψ_w that orientate $\boldsymbol{w}$ with respect to the reference systems (I) and (G) (see Fig.8), we obtain

$$\mathbf{w}^{(I)} = \mathbf{w}^{(G)} = \begin{bmatrix} \cos\theta_w \cos\psi_w \\ \cos\theta_w \sin\psi_w \\ -\sin\theta_w \end{bmatrix} w \tag{A.1}$$

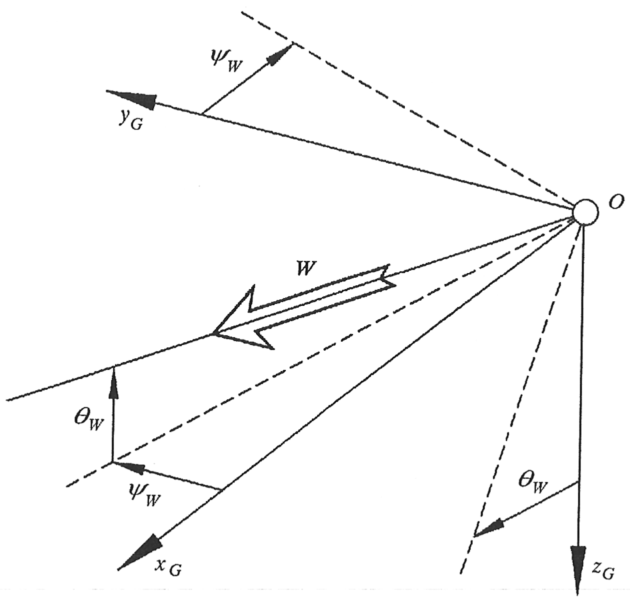

Fig. 8. Orientation of the wind velocity vector

The vector of aircraft absolute velocity (with respect to the ground) is now $\boldsymbol{v}_O = \boldsymbol{v} + \boldsymbol{w}$, which, when expressed in the reference system (G), is

$$\begin{bmatrix} \widehat{x}' \\ \widehat{y}' \\ \widehat{z}' \end{bmatrix} \dot{s} = \begin{bmatrix} \cos\theta_A \cos\psi_A \\ \cos\theta_A \sin\psi_A \\ -\sin\theta_A \end{bmatrix} v + \begin{bmatrix} \cos\theta_w \cos\psi_w \\ \cos\theta_w \sin\psi_w \\ -\sin\theta_w \end{bmatrix} w \tag{A.2}$$

For the given value $\dot{s} = v_O$, using $\widehat{x}'^2 + \widehat{y}'^2 + \widehat{z}'^2 = 1$, Eq (A.2) can be resolved for v

$$v = \sqrt{\dot{s}^2 - 2\dot{s}w(\widehat{x}' \cos\theta_w \cos\psi_w + \widehat{y}' \cos\theta_w \sin\psi_w - \widehat{z}' \sin\theta_w) + w^2} \tag{A.3}$$

Consequently, Eq (5.1) modifies to

$$\begin{bmatrix} \cos\theta_A \cos\psi_A \\ \cos\theta_A \sin\psi_A \\ -\sin\theta_A \end{bmatrix} = \frac{1}{v}\left(\begin{bmatrix} \widehat{x}' \\ \widehat{y}' \\ \widehat{z}' \end{bmatrix} \dot{s} - \begin{bmatrix} \cos\theta_w \cos\psi_w \\ \cos\theta_w \sin\psi_w \\ -\sin\theta_w \end{bmatrix} w \right) \tag{A.4}$$

Due to the wind drift, neither the Ox_A axis is in general tangent nor the Oy_Az_A plane is orthogonal to required trajectory (4.1). As a consequence, relations (5.4) do not hold true any more, and expressing Eq (5.2) in the reference system (A), which leads to Eqs (5.5) and (5.6) for windless flight conditions, is useless. In that case, Eq (5.2) should be expressed in the trajectory reference system (T), the axes of which are defined by the versor $\boldsymbol{\tau}$ tangent to the trajectory $\boldsymbol{\tau}^{(G)} = \widehat{\mathbf{r}}_O^{'(I)}$, the normal versor $\boldsymbol{n}$ pointed at the center of the trajectory curvature $\mathbf{n}^{(G)} = \rho_c \widehat{\mathbf{r}}_O^{''(I)}$, and the binormal versor $\boldsymbol{b} = \boldsymbol{\tau} \times \boldsymbol{n}$, $\mathbf{b}^{(G)} = \widetilde{\boldsymbol{\tau}}^{(G)}\mathbf{n}^{(G)} = \rho_c \widetilde{\widehat{\mathbf{r}}}_O^{''(I)}\widehat{\mathbf{r}}_O^{''(I)}$, where

$$\rho_c^{-1}(s) = \sqrt{\widehat{x}''^2 + \widehat{y}''^2 + \widehat{z}''^2} \tag{A.5}$$

The transformation matrix $\mathbf{A}_{TG}$ between the reference systems (G) and (T) is

$$\mathbf{A}_{TG} = \begin{bmatrix} (\boldsymbol{\tau}^{(G)})^\top \\ (\mathbf{n}^{(G)})^\top \\ (\mathbf{b}^{(G)})^\top \end{bmatrix} = \begin{bmatrix} \widehat{x}' & \widehat{y}' & \widehat{z}' \\ \rho_c \widehat{x}'' & \rho_c \widehat{y}'' & \rho_c \widehat{z}'' \\ \rho_c(\widehat{y}'\widehat{z}'' - \widehat{z}'\widehat{y}'') & \rho_c(\widehat{z}'\widehat{x}'' - \widehat{x}'\widehat{z}'') & \rho_c(\widehat{x}'\widehat{y}'' - \widehat{y}'\widehat{x}'') \end{bmatrix} \tag{A.6}$$

Using the above definitions, we can then write

$$\mathbf{A}_{TG}\widehat{\mathbf{r}}_O^{'(I)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{A}_{TG}\widehat{\mathbf{r}}_O^{''(I)} = \begin{bmatrix} 0 \\ \rho_c^{-1} \\ 0 \end{bmatrix} \tag{A.7}$$

Now, according to dynamic equation $(3.1)_1$, Eq (5.2) can first be manipulated to $\mathbf{A}_{GA}\mathbf{F}^{(A)} = m(\widehat{\mathbf{r}}_O^{'(I)}\ddot{s} + \widehat{\mathbf{r}}_O^{''(I)}\dot{s}^2)$, and then, premultiplying the obtained relation by $\mathbf{A}_{TG}$, we arrive at $\mathbf{F}^{(T)} \equiv \mathbf{A}_{TG}\mathbf{A}_{GA}\mathbf{F}^{(A)} = m[a_\tau, a_n, 0]^\top$, where $a_\tau = \ddot{s}$ and $a_n = \dot{s}^2/\rho_c$ are the tangent and normal (with respect to the trajectory) accelerations. The corresponding three scalar equations can finally be written as

$$\begin{aligned} &-(\widehat{\mathbf{r}}_O^{'(I)})^\top \mathbf{A}_{GA}\mathbf{F}^{(A)} + m\ddot{s} = 0 \\ &-\rho_c(\widehat{\mathbf{r}}_O^{''(I)})^\top \mathbf{A}_{GA}\mathbf{F}^{(A)} + m\dot{s}^2\rho_c^{-1} = 0 \\ &-\rho_c(\widetilde{\widehat{\mathbf{r}}}_O^{'(I)}\widehat{\mathbf{r}}_O^{''(I)})^\top \mathbf{A}_{GA}\mathbf{F}^{(A)} = 0 \end{aligned} \tag{A.8}$$

which correspond to Eqs (5.5) and (5.6) derived for windless flight conditions. For brevity, the explicit forms of Eqs (A.8) are not reported here.

Equation $(A.8)_1$ expresses the balance of the applied and inertial forces in the direction tangent to the trajectory, and the case of PS or FS motion is considered, respectively – it can be used as the equation of dynamics transformed to

$$\dot{v} = H_x(\alpha, \beta, \phi_A, \theta_A, \psi_A, s, v, p, q, r, \delta_e, \delta_r, t)$$

or the algebraic equation

$$F_x(\alpha, \beta, \phi_A, \theta_A, \psi_A, p, q, r, T, \delta_e, \delta_r, t) = 0$$

Then, Eqs $(A.8)_{2,3}$ express the balance of the applied and inertial forces in the plane orthogonal to the trajectory (along the $\boldsymbol{n}$ and $\boldsymbol{b}$ directions), and should replace the algebraic equations $F_y = 0$ and $F_z = 0$ in the equations of the programmed motion. For the PS motion the algeraic equations are functions of α, β, ϕ_A, θ_A, ψ_A, s, v, p, q, r, δ_e, δ_r and t, while for the FS motion they are functions of α, β, ϕ_A, θ_A, ψ_A, p, q, r, T, δ_e, δ_r and t.

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Predykcja własności dynamicznych i sterowania samolotem w ruchu programowym po założonej trajektorii

Streszczenie

W pracy przedstawione jest uogólnione i zmodyfikowane podejście do modelowania i symulacji programowego ruch samolotu. Program ruchu budowany jest za pomocą założonej trajktorii przestrzennej (dwa warunki więzów nakładane na położenie samolotu), narzuconych zmian konfiguracji płatowca względem tej trajektorii oraz, opcjonalnie, zmian prędkości lotu. W przypadku samolotu sterowanego klasycznie za pomocą wychyleń powierzchni sterowych (lotek oraz sterów wysokości i kierunku) oraz zmiany siły ciągu, realizacja więzów trajektorii lotu jest styczna, co implikuje dwa dodatkowe warunki na zmiany konfiguracji płatowca względem trajektorii. W ten

sposób zmiany konfiguracji kątowej płatowca stają się w pełni określone (zaprogramowane). Równania ruchu programowego generowane są w postaci równań różniczkowo-algebraicznych. Proponowana jest metoda numerycznego rozwiązania tych równań. Jako rozwiązania są zmiany w czasie zmiennych stanu ruchu samolotu oraz przebiegi sterowania samolotem wymagane dla ścisłej realizacji więzów programowych. Otwiera to nowe możliwości analizy symulowanych manewrów samolotu oraz oceny ich realizowalności. Przytaczane są wybrane wyniki symulacji numerycznej.

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