

ON A THERMODYNAMIC MODEL OF FLUID-SATURATED POROUS MEDIUM

WITOLD KOSIŃSKI

*Polish-Japanese Institute of Information Technology; Center of Mechanics and Information
Technology, IPPT PAN Warsaw; Department of Environmental Mechanics, Bydgoszcz University
e-mail: wkos@pjwstk.waw.pl*

JÓZEF KUBIK
MIECZYŚLAW CIESZKO
MARIUSZ KACZMAREK

*Department of Environmental Mechanics, Bydgoszcz University
e-mail: kubik@rose.man.poznan.pl*

In order to describe isotropic pore structure of a solid skeleton saturated with a fluid, two internal state variables are introduced: a volume porosity f_v and a structural parameter κ . The internal friction in the skeleton material is incorporated by a tensor-valued internal state variable α . Thermal properties are described by a single entropy density function η and two thermal state variables β^s and β^f . In the energy balance law, both heat fluxes appear and an added mass effect is incorporated to manifest the influence of the pore structure of the skeleton on the fluid motion. Consequences of the second law of thermodynamics are formulated in the form of four potential relations for stresses and heat flux vectors together with a representation of interaction forces.

Key words: fluid-saturated porous solid, entropy, heat flux

1. Introduction

Constitutive modelling of fluid-saturated porous solids has been a subject of wide discussion through the last two decades. Nonlinear models of such materials are based mostly upon the fundamental notions of the *classical mixture theory* (Bowen, 1982), and its reformulated form – the *theory of interacting*

continua (Green and Naghdi, 1965; Bowen, 1984) in which a fluid filled porous medium is treated as a superposition of two immiscible continua: solid and fluid, characterized by two independent velocity fields: \boldsymbol{v}^s and \boldsymbol{v}^f . In such approach the microstructure of solid-fluid mixture is not taken into account in the formulation of balance equations and constitutive relations. However, it has been observed that in a number of typical multiphase media, consisting of an identifiable porous matrix and a fluid filling its pores, the internal geometrical pore structure strongly influences the behaviour of phases inducing the inhomogeneity of micro-velocity fields. This effect is regarded to be of prime importance in understanding the acoustic properties of porous media saturated with fluids.

Taking into account the fact that individual physical properties of immiscible constituents play important role in both the transport phenomena and deformation processes of porous media, in the presented approach it is assumed that in the constitutive modelling each constituent shall obey the constitutive relations for that constituent alone. On the other side, interaction forces appearing in linear momentum balance laws shall depend on gradients of structural state variables and the relative velocity vector.

In the authors' previous publications as well as in a number of other papers (cf Biot, 1972; Bowen, 1982, 1984; Cieszko and Kubik, 1996a,b; Kaczmarek and Kubik, 1985; Kubik, 1986, 1992; Kubik and Cieszko, 1987; Szefer, 1978) devoted to modelling of fluid-saturated porous solids, the immiscibility effect has been incorporated into description by introducing a parameter of volume porosity characterizing the volume fraction of the fluid constituent. Thermodynamic modelling of fluid-saturated porous solids is still being focused on. Some recent contributions to this subject has been made by Svendsen and Hutter (1995), and Wilmański (1995).

The main aim of this paper is to propose a thermodynamic framework to the previously developed mechanical model (cf Cieszko and Kubik, 1996a,b) of a fluid-saturated porous solid. In order to consider micro-inhomogeneities resulting from the pore architecture influence, an additional pore structure characteristic is necessary (cf Kubik, 1992). In order to describe the structure of a porous solid saturated with fluid in an isotropic case, two scalar dimensionless internal state variables are introduced: a pore volume fraction f_v or a volume porosity, and a structural parameter κ . The second variable describes an inhomogeneity of the fluid micro-velocity fields caused by the pore structure of the skeleton.

To describe the internal friction in the skeleton, a tensor-valued internal state variable $\boldsymbol{\alpha}$ is introduced. Thermal properties are incorporated by a sin-

gle entropy density function η and two thermal state variables β^s and β^f , following the approach of Kosiński (1998). In the previous publication (see Cieszko and Kubik, 1996a,b) an isothermal theory of fluid-saturated porous solid with isotropic pore structure undergoing pure mechanical large deformations was developed within the Eulerian formalism. Here a thermodynamic model is proposed within which the energy balance law contains both the heat fluxes and an added mass effect. The added mass effect represents the dynamic coupling between constituents caused by the influence of pore structure of skeleton on the fluid motion. This leads to the complete representation of the kinetic energy by macroscopic quantities. The second law of thermodynamics in the inequality form is used. Consequences of this inequality are formulated in the form of four potential relations for stresses and heat flux vectors. An extra thermomechanical coupling effect related to the spatial gradient of the scalar internal state variables β^s and β^f is obtained. Moreover the interaction force is determined in terms of the spatial gradient of both structural state variables and the relative velocity vector.

2. Main assumptions

In this section we repeat the main model assumptions of the authors of the previous paper (Cieszko and Kubik, 1996a,b). On the macroscopic level, porous skeleton saturated with a fluid is modeled by:

– two partial mass densities

$$\bar{\rho}^s \quad \text{and} \quad \bar{\rho}^f$$

– two effective mass densities

$$\rho^s \quad \text{and} \quad \rho^f$$

related to partial densities by

$$\bar{\rho}^f = f_v \rho^f \quad \text{and} \quad \bar{\rho}^s = (1 - f_v) \rho^s$$

here $f_v \in [0, 1]$ is a fluid volume fraction or the volume porosity

– two average (particle) velocities of the constituents

$$\mathbf{v}^s \quad \text{and} \quad \mathbf{v}^f$$

– two continuity equations

$$\frac{\partial \bar{\rho}^s}{\partial t} + \operatorname{div}(\bar{\rho}^s \mathbf{v}^s) = 0 \quad \frac{\partial \bar{\rho}^f}{\partial t} + \operatorname{div}(\bar{\rho}^f \mathbf{v}^f) = 0 \quad (2.1)$$

In order to include, on the phenomenological level, the influence of the skeleton pore structure on the kinetic energy, one incorporates an added mass effect in the balance of mass. Thus, it is assumed that

- There are two virtual components:
 - 1st component – porous skeleton and fluid associated with it, moving with velocity $\overset{1}{\mathbf{v}}$ equal to \mathbf{v}^s
 - 2nd component moving with velocity $\overset{2}{\mathbf{v}}$ equal to $\mathbf{v}^s + (\mathbf{v}^f - \mathbf{v}^s)/\kappa$ here $\kappa \in [0, 1]$ is a structural parameter.

The partial mass densities and velocities of the physical components and that of virtual ones are related as follows

$$\begin{aligned} \overset{1}{\rho} &= \bar{\rho}^s + (1 - \kappa)\bar{\rho}^f & \overset{2}{\rho} &= \kappa\bar{\rho}^f \\ \mathbf{v}^f &= (1 - \kappa)\overset{1}{\mathbf{v}} + \kappa\overset{2}{\mathbf{v}} & \mathbf{v}^s &= \overset{1}{\mathbf{v}} \end{aligned} \quad (2.2)$$

- The total density of energy E of the medium is

$$E = \bar{\rho}^s e^s + \bar{\rho}^f e^f + \frac{1}{2}(\overset{11}{\rho\mathbf{v}} \cdot \overset{1}{\mathbf{v}} + \overset{22}{\rho\mathbf{v}} \cdot \overset{2}{\mathbf{v}})$$

While $\bar{\rho}^s \mathbf{v}^s + \bar{\rho}^f \mathbf{v}^f = \overset{11}{\rho\mathbf{v}} + \overset{22}{\rho\mathbf{v}}$, in general, due to the added mass effect we have

$$\bar{\rho}^s \mathbf{v}^s \cdot \mathbf{v}^s + \bar{\rho}^f \mathbf{v}^f \cdot \mathbf{v}^f \neq \overset{11}{\rho\mathbf{v}} \cdot \overset{1}{\mathbf{v}} + \overset{22}{\rho\mathbf{v}} \cdot \overset{2}{\mathbf{v}}$$

Remark 1. Notice that the particle velocity vectors \mathbf{v}^f , \mathbf{v}^s are the convex combinations of the velocities of virtual components $\overset{1}{\mathbf{v}}$, $\overset{2}{\mathbf{v}}$ in (2.2).

3. Balance laws

The corresponding continuity equations for densities of virtual components are

$$\frac{\partial \overset{1}{\rho}}{\partial t} + \operatorname{div}(\overset{11}{\rho\mathbf{v}}) = g \quad \frac{\partial \overset{2}{\rho}}{\partial t} + \operatorname{div}(\overset{22}{\rho\mathbf{v}}) = -g \quad (3.1)$$

with the mass exchange intensity

$$g = \bar{\rho}^s \frac{D^1}{Dt} \left[(1 - \kappa) \frac{\bar{\rho}^f}{\bar{\rho}^s} \right]$$

where

$$\frac{D^n}{Dt}(\cdot) = \left(\frac{\partial}{\partial t} + \mathbf{v}^n \text{grad} \right)(\cdot)$$

Let \mathbf{T}^s and \mathbf{T}^f be Cauchy stress tensors of both physical constituents then Cauchy stresses of the virtual components are

$$\overset{1}{\mathbf{T}} = \mathbf{T}^s + (1 - \kappa)\mathbf{T}^f \quad \overset{2}{\mathbf{T}} = \kappa\mathbf{T}^f \quad (3.2)$$

and the motion equations

$$\begin{aligned} \overset{1}{\rho} \frac{D^1}{Dt} \overset{1}{\mathbf{v}} &= \text{div} \overset{1}{\mathbf{T}} + \overset{1}{\rho} \mathbf{b} + \boldsymbol{\pi}^1 + \frac{1}{2}g(\overset{2}{\mathbf{v}} - \overset{1}{\mathbf{v}}) \\ \overset{2}{\rho} \frac{D^2}{Dt} \overset{2}{\mathbf{v}} &= \text{div} \overset{2}{\mathbf{T}} + \overset{2}{\rho} \mathbf{b} + \boldsymbol{\pi}^2 + \frac{1}{2}g(\overset{2}{\mathbf{v}} - \overset{1}{\mathbf{v}}) \end{aligned} \quad (3.3)$$

where $\boldsymbol{\pi}^1 = -\boldsymbol{\pi}^2 =: \boldsymbol{\pi}$ interaction forces.

Remark 2. Notice that the representation (3.2) is the consequence of relation (2.2) and requirement that the fluxes of powers of mechanical forces of physical and virtual components are equal

$$\overset{1}{\mathbf{T}}\overset{1}{\mathbf{v}} + \overset{2}{\mathbf{T}}\overset{2}{\mathbf{v}} = \mathbf{T}^s\mathbf{v}^s + \mathbf{T}^f\mathbf{v}^f \quad (3.4)$$

Now we formulate the balance law of energy for the system, which extends the version proposed by Cieszko and Kubik (1996a,b), including terms related to thermal phenomena: heat fluxes $\mathbf{q}^s, \mathbf{q}^f$ and body heat supplies $\bar{\rho}^s r^s$ and $\bar{\rho}^f r^f$

$$\begin{aligned} \bar{\rho}^s \frac{D^1}{Dt} e^s + (1 - \kappa) \bar{\rho}^f \frac{D^1}{Dt} e^f + \kappa \bar{\rho}^f \frac{D^2}{Dt} e^f &= \\ = \overset{1}{\mathbf{T}} \cdot \overset{1}{\mathbf{L}} + \overset{2}{\mathbf{T}} \cdot \overset{2}{\mathbf{L}} + \boldsymbol{\pi} \cdot (\overset{2}{\mathbf{v}} - \overset{1}{\mathbf{v}}) - \text{div}(\mathbf{q}^s + \mathbf{q}^f) + \bar{\rho}^s r^s + \bar{\rho}^f r^f \end{aligned} \quad (3.5)$$

where $\overset{1}{\mathbf{L}} := \text{grad} \overset{1}{\mathbf{v}}$ and $\overset{2}{\mathbf{L}} := \text{grad} \overset{2}{\mathbf{v}}$.

4. Consequences of the second law of thermodynamics

In the paper, a Clausius-Duhem inequality is assumed for the formulation of the second law of thermodynamics

$$\frac{\partial}{\partial t}(\bar{\rho}^s \eta + \bar{\rho}^f \eta) + \operatorname{div} \left(\bar{\rho}^s \eta \mathbf{v}^s + \bar{\rho}^f \eta \mathbf{v}^f + \frac{\mathbf{q}^s}{\vartheta^s} + \frac{\mathbf{q}^f}{\vartheta^f} + \mathbf{h} \right) \geq \bar{\rho}^s \frac{r^s}{\vartheta^s} + \bar{\rho}^f \frac{r^f}{\vartheta^f}$$

where $\vartheta^s := \partial e^s / \partial \eta$ and $\vartheta^f := \partial e^f / \partial \eta$ are temperatures of skeleton and fluid constituents, respectively, and \mathbf{h} is an extra entropy flux.

Now constitutive assumptions are made in which the so-called equipresence principle (Bowen, 1982) is applied separately for each phase.

- The fluid component is thermo-barotropic

$$\begin{aligned} e^f &= e^f(\eta, \bar{\rho}^f, f_v, \nabla \beta^f) \\ \mathbf{T}^f &= \mathbf{T}^f(\eta, \bar{\rho}^f, f_v, \nabla \beta^f) \\ \mathbf{q}^f &= \mathbf{q}^f(\eta, \bar{\rho}^f, f_v, \nabla \beta^f) \end{aligned}$$

- The solid skeleton is thermo-viscoelastic

$$\begin{aligned} e^s &= e^s(\eta, \mathbf{F}, \bar{\rho}^s, f_v, \kappa, \nabla \beta^s, \boldsymbol{\alpha}) \\ \mathbf{T}^s &= \mathbf{T}^s(\eta, \mathbf{F}, \bar{\rho}^s, f_v, \kappa, \nabla \beta^s, \boldsymbol{\alpha}) \\ \mathbf{q}^s &= \mathbf{q}^s(\eta, \mathbf{F}, \bar{\rho}^s, f_v, \kappa, \nabla \beta^s, \boldsymbol{\alpha}) \end{aligned}$$

where \mathbf{F} denotes the deformation gradient of the skeleton, and the tensor-valued internal state variable $\boldsymbol{\alpha}$ is responsible for internal friction of the solid skeleton and satisfies the evolution equation

$$\frac{\partial}{\partial t}(\bar{\rho}^s \boldsymbol{\alpha}) + \operatorname{div}(\bar{\rho}^s \boldsymbol{\alpha} \otimes \mathbf{v}^s) = \bar{\rho}^s \mathbf{A}$$

- Both structural parameters f_v and κ are internal state variables and satisfy two equations

$$\begin{aligned} \frac{\partial}{\partial t}(\bar{\rho}^f f_v) + \operatorname{div}(\bar{\rho}^f f_v \mathbf{v}^f) &= \bar{\rho}^f H \\ \frac{\partial}{\partial t}(\bar{\rho}^s \kappa) + \operatorname{div}(\bar{\rho}^s \kappa \mathbf{v}^s) &= \bar{\rho}^s K \end{aligned}$$

- Two scalar thermal variables β^s and β^f are internal state variables and satisfy two equations

$$\begin{aligned}\frac{\partial}{\partial t}(\bar{\rho}^f \beta^f) + \operatorname{div}(\bar{\rho}^f \beta^f \mathbf{v}^f) &= \bar{\rho}^f F^f \\ \frac{\partial}{\partial t}(\bar{\rho}^s \beta^s) + \operatorname{div}(\bar{\rho}^s \beta^s \mathbf{v}^s) &= \bar{\rho}^s F^s\end{aligned}$$

- Both temperatures are equal and the flux \mathbf{h} vanishes

$$\vartheta^s = \vartheta^f = \vartheta \quad \text{and} \quad \mathbf{h} = \mathbf{0} \quad (4.1)$$

- Functions \mathbf{A} , H , F^s , F^f , H and K depend at most on the following set of variables: $\bar{\rho}^s$, $\bar{\rho}^f$, f_v , κ , η , \mathbf{F} , $\boldsymbol{\alpha}$, β^s , β^f , $\nabla\beta^s$, $\nabla\beta^f$, and their forms are to be determined on the basis of physical interpretations and experimental measurements.

Thermodynamic postulate: every smooth solution to all balance, constitutive and evolution equations should satisfy the second law of thermodynamics.

Theorem. The necessary and sufficient conditions of fulfilling the postulate are:

1. Four potential relations

$$\begin{aligned}\frac{1}{\mathbf{T}} + (1 - \kappa)(\bar{\rho}^f)^2 \frac{\partial e^f}{\partial \bar{\rho}^f} \mathbf{1} &= \bar{\rho}^s \frac{\partial e^s}{\partial \mathbf{F}} + \frac{1}{\vartheta} (\tau^f \nabla \beta^f \otimes \mathbf{q}^f (1 - \kappa) + \tau^s \nabla \beta^s \otimes \mathbf{q}^s) \\ \frac{2}{\mathbf{T}} &= -(\bar{\rho}^f)^2 \frac{\partial e^f}{\partial \bar{\rho}^f} \kappa \mathbf{1} + \frac{1}{\vartheta} (\tau^f \nabla \beta^f \otimes \mathbf{q}^f \kappa)\end{aligned} \quad (4.2)$$

$$\mathbf{q}^f = -\bar{\rho}^f \vartheta (\tau^f)^{-1} \frac{\partial e^f}{\partial \nabla \beta^f}$$

$$\mathbf{q}^s = -\bar{\rho}^s \vartheta (\tau^s)^{-1} \frac{\partial e^s}{\partial \nabla \beta^s}$$

2. Residual internal dissipation inequality

$$\begin{aligned}\left[-\frac{1}{\vartheta} \tau^f \nabla \beta^f \otimes \mathbf{q}^f + (\bar{\rho}^f)^2 \frac{\partial e^f}{\partial \bar{\rho}^f} \mathbf{1} \right] \cdot (\overset{2}{\mathbf{v}} - \overset{1}{\mathbf{v}}) \otimes \operatorname{grad} \kappa - \boldsymbol{\pi} \cdot (\overset{2}{\mathbf{v}} - \overset{1}{\mathbf{v}}) + \\ -\bar{\rho}^s \frac{\partial e^s}{\partial f_v} \kappa \operatorname{grad} f_v \cdot (\overset{2}{\mathbf{v}} - \overset{1}{\mathbf{v}}) - \bar{\rho}^s \frac{\partial e^s}{\partial \kappa} K - \bar{\rho}^s \frac{\partial e^s}{\partial \boldsymbol{\alpha}} \cdot \mathbf{A} - \bar{\rho}^s \frac{\partial e^s}{\partial f_v} H + \\ -\bar{\rho}^f \frac{\partial e^f}{\partial f_v} H + \frac{1}{\vartheta} \tau^s \frac{\partial F^s}{\partial \beta^s} \mathbf{q}^s \cdot \nabla \beta^s + \frac{1}{\vartheta} \tau^f \frac{\partial F^f}{\partial \beta^f} \mathbf{q}^f \cdot \nabla \beta^f \geq 0\end{aligned} \quad (4.3)$$

where

$$\tau^s := \left(\frac{\partial F^s}{\partial \vartheta} \right)^{-1} \quad \tau^f := \left(\frac{\partial F^f}{\partial \vartheta} \right)^{-1}$$

Corollary. If the interaction force π is a linear combination of $\overset{2}{\boldsymbol{v}} - \overset{1}{\boldsymbol{v}}$, $\text{grad } \kappa$ and $\text{grad } f_v$, i.e.

$$\pi = a_v(\overset{2}{\boldsymbol{v}} - \overset{1}{\boldsymbol{v}}) + \mathbf{A}_\kappa \text{grad } \kappa + a_{f_v} \text{grad } f_v \quad (4.4)$$

with some coefficients a_v , \mathbf{A}_κ and a_{f_v} , then the residual dissipation inequality (4.3) is satisfied iff the following relations hold

$$a_v \leq 0 \quad a_{f_v} = -\bar{\rho}^s \frac{\partial e^s}{\partial f_v} \kappa \quad (4.5)$$

$$\mathbf{A}_\kappa = -\frac{1}{\vartheta} \tau^f \nabla \beta^f \otimes \mathbf{q}^f + (\bar{\rho}^f)^2 \frac{\partial e^f}{\partial \rho^f} \mathbf{1}$$

and

$$-\bar{\rho}^s \frac{\partial e^s}{\partial \kappa} K - \bar{\rho}^s \frac{\partial e^s}{\partial \boldsymbol{\alpha}} \cdot \mathbf{A} - \bar{\rho}^s \frac{\partial e^s}{\partial f_v} H - \bar{\rho}^f \frac{\partial e^f}{\partial f_v} H \geq 0 \quad (4.6)$$

is satisfied.

5. Conclusions

Notice that the last relations (4.5) lead to a more general representation of the interaction force π ; in our previous publications particular cases of this representation have been given, however, with vanishing coefficients a_v and \mathbf{A}_κ (Cieszko and Kubik, 1996a,b).

In the further research extra dependence of the internal energy functions on $\text{grad } \kappa$ will be investigated together with the full representation of the entropy flux, i.e. with $\mathbf{h} \neq \mathbf{0}$, following the suggestions of Wang and Hutter (1999).

Acknowledgement

Several discussions with Professor Kolumban Hutter, the winner of the Alexander-von-Humboldt price of the Foundation for the Polish Science, on the subject related to the paper, is deeply acknowledged. The research on this paper has been conducted under the grant from the State Committee for Scientific Research (KBN, Poland) No. 7T07A 05 115.

References

1. BIOT M.A., 1972, Theory of Finite Deformations of Porous Solids, *Ind. Univ. Math. J.*, **21**, 7, 597-620
2. BOWEN R.M., 1982, Compressible Porous Media Models by Use of the Theory of Mixtures, *Int. J. Engng Sci.*, **20**, 697-735
3. BOWEN R.M., 1984, Diffusion Models Implied by the Theory of Mixtures, in: *Rational Thermodynamics*, C. Truesdell (edit.), 2ed Edition, 237-263, Springer, New York
4. CIESZKO M., KUBIK J., 1996a, Constitutive Relations and Internal Equilibrium Condition for Fluid-Saturated Porous Solids. Nonlinear Theory, *Arch. Mech.*, **48**, 5, 893-910
5. CIESZKO M., KUBIK J., 1996b, Constitutive Relations and Internal Equilibrium Condition for Fluid-Saturated Porous Solid. Linear Description, *Arch. Mech.*, **48**, 5, 911-923
6. GREEN A.E., NAGHDI P.M., 1965, A Dynamical Theory of Interacting Continua, *Int. J. Engng. Sci.*, **3**, 231-241
7. KACZMAREK M., KUBIK J., 1985, Determination of Material Constants of Fluid-Saturated Porous Solid, (in Polish) *Engng Trans. (Rozprawy Inżynierskie)*, **33**, 4, 589-609
8. KOSIŃSKI W., 1998, Hyperbolic Framework for Thermoelastic Materials, *Arch. Mech.*, **50**, 3, 423-450
9. KUBIK J., 1982, Large Elastic Deformations of Fluid-Saturated Porous Solid, *Journal de Mécanique Théorique et Appliquée*, 203-218
10. KUBIK J., 1986, On Internal Coupling in Dynamic Equations of Fluid-Saturated Porous Solid, *Int. J. Engng Sci.*, **24**, 981-989
11. KUBIK J., 1992, Pore Structure in Dynamic Behaviour of Saturated Materials, *Transport in Porous Media*, **9**, 15-24
12. KUBIK J., CIESZKO M., 1987, O oddziaływaniach wewnętrznych w ośrodku nasyconym cieczą, *Rozp. Inż.*, **35**, 55-70
13. KUBIK J., CIESZKO M., KACZMAREK M., 2000, Podstawy nasyconych ośrodków porowatych, (Fundamentals of Dynamics of Saturated Porous Media), IPPT PAN, Warszawa-Poznań-Bydgoszcz, in print
14. SVENDSEN B., HUTTER K., 1995, On the Thermodynamics of a Mixture of Isotropic Materials with Constraints, *Int. J. Engng Sci.*, **33**, 14, 2021-2054
15. SZEFER G., 1978, Nonlinear Problems of Consolidation Theory, in *Problems de Reologie*, Symposium Franco-Polonaise, Cracovie 1977, 585-604, PWN Warszawa

16. WANG Y., HUTTER K., 1999, Comparison of Two Entropy Principles and Their Applications in Granular Flows With/Without Fluid, *Arch. Mech.*, **51**, 5, 605-632
17. WILMAŃSKI K., 1995, Lagrangian Model of Two-Phase Porous Material, *J. Non-Equilibrium Therm.*, **20**, 1, 50-77

Termodynamiczny model ośrodka porowatego nasyconego cieczą

Streszczenie

Zaproponowano prosty model termodynamicznego ośrodka porowatego nasyconego cieczą. Dla opisu izotropowej struktury porów stałego szkieletu nasyconego cieczą zostały wprowadzone dwie skalarne zmienne stanu: objętościowa porowatość f_v i parametr strukturalny κ . Lepkość wewnętrzna materiału szkieletu została uwzględniona poprzez tensorową zmienną stanu α . Własności termiczne zostały opisane przez pojedynczą funkcję gęstości entropii η i dwie termiczne zmienne stanu β^s i β^f . W prawie bilansu energii występują oba strumienie ciepła, zaś uwzględnienie efektu masy dołączonej odzwierciedla wpływ struktury porów na ruch cieczy. Sformułowano konsekwencje drugiego prawa termodynamiki w postaci 4 związków potencjalnych dla naprężeń i wektorów strumienia ciepła wraz z reprezentacją sił oddziaływania.

Manuscript received March 30, 2000; accepted for print July 14, 2000