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A CONTRIBUTION TO MODELLING OF COMPOSITE SOLIDS

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The aim of this paper is to prove that micro-periodic composites made of linear-elastic isotropic components and having a hexagonal representative cell with the triple axis of material symmetry are also isotropic in the macro-scale. The prove of this statement will be given on the basis of the tolerance averaging method of modelling, Woźniak and Wierzbicki (2000).

Key words: modelling, elastic composites, tolerance averaging

1. Introduction

There are two factors which significantly influence the description and modelling of observed real bodies: the scale and accuracy of measurements. For example, in one scale it can be assumed that the body is continuous, however in other, a smaller scale, it can be observed that the body is not continuous. In the case of displacements in the "lower precision" scale, the displacements can be interpreted as functions slowly varying together with their derivatives. In the "higher precision" scale the situation can be different. On these slowly varying displacements small oscillations are superimposed, which can be described by oscillating functions with a small period. These two scales can be called *macro-* and *micro-scale*, respectively. Modelling of the phenomena in these two scales can be performed with a certain accuracy. The term "accuracy" can be formalised in different ways. A useful tool for the description of the accuracy is the so called toleration relation or shortly tolerance, cf. Zeeman (1965), Woźniak (1983), Woźniak and Wierzbicki (2000).

If a nonhomogeneous elastic body is periodic in the micro-scale (a micro periodic body) than many difficulties arise during finding solutions to special problems. This situation often excludes application of analytical and computer methods to the analysis of the problems. Nonhomogeneous bodies which are periodic in the micro scale can be modelled in the macro-scale by applying one from the averaging methods. After the averaging the description of the body can be assumed as homogeneous. However, in the averaging model (macro-model) it is necessary to include the influence of the micro-nonhomogenity on the solution.

Among the macro-modelling methods of periodic media asymptotic the homogenisation described by Bensoussen et al. (1978), Sanchez-Palencia (1980), Jikov et al. (1994) can be used. However, in thas model it is impossible to describe such phenomena as the dispersion of waves and the existence of higher-order motions and higher free vibration frequencies. Another example of macro-modelling is the averaging applying nonstandard analysis, cf. Woźniak (1987). The model of a micro-periodic body obtained thas way helps consideration of the influence of the micro-nonhomogenity by introducing additional quantities, which are called microlocal parameters. This model was applied to the analysis of many equilibrium problems, but in dynamical problems is less useful; see for instance Woźniak and Wierzbicki (2000). To remove this drawback the tolerance averaging as given by Woźniak (1993, 1997), Woźniak and Wierzbicki (2000) was applied. The basic assumptions of this method will be presented in the next section.

In many cases micro-periodic bodies, such as composites, are isotropic in the micro-scale. In the macro model, i.e. after the averaging, the description of the body usually becomes anisotropic. Thus, the following question appears: *are there any isotropic micro-periodic bodies being in the macro-scale (i.e., after averaging) isotropic as well?*

The positive answer to this question in the framework of the asymptotic homogenisation theory and for the heat condition problem can be found in Jikov et al. (1994). So far, for the linear elasticity problem it is an open question, cf. Jikov et al. (1994), p. 380. This problem was also analysed in papers by Lewiński (1984, 1985, 1988), who considered bodies with a honeycomb structure using the Cosserat theory. More general approaches were analysed by Cielecka et al. (2000), Wierzbicki and Woźniak (2000).

The aim of this paper is to prove that micro-periodic composites made of linear-elastic isotropic components and having a hexagonal representative cell with the triple axis of material symmetry (cf. Fig. 1) are also isotropic in the macro-scale. The prove of this statement will be given on the basis of the tolerance averaging method of modelling, Woźniak and Wierzbicki (2000).

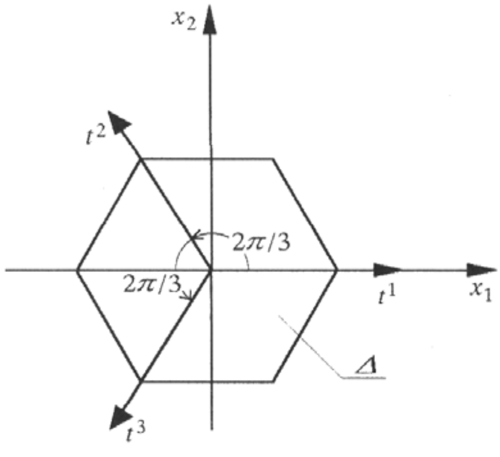

Fig. 1.

2. Preliminaries

For every micro-periodic nonhomogeneous body in the reference position Ω a representative volume element Δ with the characteristic diameter l can be assumed. For every $\boldsymbol{x} \in \Omega$ define $\Delta(\boldsymbol{x}) = \boldsymbol{x} + \Delta$. The set of these $\boldsymbol{x} \in \Omega$ for which $\Delta(\boldsymbol{x}) \subset \Omega$ is denoted by Ω_Δ. Let $\varphi(\cdot)$ be any integrable function defined in Ω. The Δ-averaging of $\varphi(\cdot)$, which is denoted by $\langle\varphi\rangle(\cdot)$, is defined on Ω_Δ as

$$\langle\varphi\rangle(\boldsymbol{x}) = \frac{1}{|\Delta|} \int\limits_{\Delta(\boldsymbol{x})} \varphi(\boldsymbol{y})\, d\boldsymbol{y} \tag{2.1}$$

Let us denote by $\mathcal{F}(\overline{\Omega})$ a class of real valued functions defined and bounded in $\overline{\Omega}$ together with all their derivatives. Let $\varepsilon(\cdot)$ be a functional defined on $\mathcal{F}(\overline{\Omega})$ a real number assigning to every function $F(\cdot) \in \mathcal{F}(\overline{\Omega})$, $\varepsilon(F) > 0$. $\varepsilon(F)$ defines the accuracy of computation of values of $F(\cdot)$, and is called the tolerance parameter. Hence, the tolerance $\cong$ is given by

$$F(\boldsymbol{x}) \cong F(\boldsymbol{y}) \Leftrightarrow |F(\boldsymbol{x}) - F(\boldsymbol{y})| \leqslant \varepsilon(F) \tag{2.2}$$

The triple element consisting of the class of the functions $\mathcal{F}(\overline{\Omega})$, the tolerance functional $\varepsilon(\cdot)$ and the characteristic diameter l of the representative element Δ will be denoted by $\mathcal{T}$ and referred to as *the tolerance system*, cf. Woźniak and Wierzbicki (2000)

$$\mathcal{T} = (\mathcal{F}(\overline{\Omega}), \varepsilon(\cdot), l) \tag{2.3}$$

The bounded real valued function $F(\cdot) \in \mathcal{F}(\overline{\Omega})$ will be called *slowly varying* if for every $\boldsymbol{x} \in \Omega_\Delta$ condition $\boldsymbol{y}_1, \boldsymbol{y}_2 \in \Delta(\boldsymbol{x})$ implies $F(\boldsymbol{y}_1) \cong F(\boldsymbol{y}_2)$, where the tolerance $\cong$ is defined by the tolerance parameter $\varepsilon(F)$. The space of the functions slowly varying together with all their derivatives that occur in the problem under consideration, is denoted by $SV(\mathcal{T})$. Every continuous function $\varphi(\cdot) \in \mathcal{F}(\overline{\Omega})$ is called *a periodic like function* if for every $\boldsymbol{x} \in \Omega_\Delta$ there exists Δ-periodic function $\varphi_x(\cdot)$ such that for every $\boldsymbol{x} \in \Delta(\boldsymbol{x})$ relation $\varphi(\boldsymbol{y}) \cong \varphi_x(\boldsymbol{y})$ is satisfied, where the tolerance $\cong$ is defined by the tolerance parameter $\varepsilon(\varphi)$. The space of the periodic-like functions with all their derivatives is denoted by $PL(\mathcal{T})$. The function $\varphi_x(\cdot)$ is called the *Δ-periodic approximation* of the function $\varphi(\cdot)$.

Let $\rho(\cdot)$ be a positive valued Δ-periodic function. Every function $\varphi(\cdot) \in PL(\mathcal{T})$ is called the oscillating periodic-like function (with the weight ρ) if for every $\boldsymbol{x} \in \Omega_\Delta$ the relation $\langle \rho\varphi \rangle \cong 0$ holds.

The concept of functions which are Δ-periodic, slowly varying, periodic-like and the concept of the Δ-periodic approximation of the function are used in the tolerance averaging, which will be realised subsequently. For detailes the reader is referred to Woźniak and Wierzbicki (2000).

Subsequently, we shall deal with micro-periodic linear elastic composites. The considered problem is assumed to be plane in the Carthesian coordinate system $(x_\alpha) \in \Omega$, $\alpha = 1,2$, and hence Ω will be treated as a plane region. It is assumed that all Greek subscripts run over 1,2 being related to the coordinates x_α.

Let us denote the displacement vector by u_α and the body force by b_α, $\alpha = 1,2$, which depend on the points $\boldsymbol{x} = (x_\alpha) \in \Omega$ and time t. The mass density will be denoted by $\rho = \rho(\boldsymbol{x})$.

The constitutive relations are assumed in the well known form

$$\sigma_{\alpha\beta} = B_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta}$$

where $B_{\alpha\beta\gamma\delta}$ is the tensor of the elastic module, $\sigma_{\alpha\beta}$ is the stress tensor and $\varepsilon_{\alpha\beta}$ is the strain tensor. The local relation describing the dynamics of the body is

$$(B_{\alpha\beta\gamma\delta}u_{\gamma,\delta})_{,\beta} - \rho\ddot{u}_\alpha + \rho b_\alpha = 0 \tag{2.4}$$

Relation (2.4) together with the respective boundary, initial and continuity conditions represent a system of equations for the displacements. In the case of microperiodic nonhomogeneous bodies the components of the tensor field $B_{\alpha\beta\gamma\delta}(\cdot)$ and the mass density scalar field $\rho(\cdot)$ are Δ-periodic functions. For the formulation of the tolerance averaged model it is enough to use the mathematical concepts as described in Section 2 using certain tolerance system (2.3) and the heuristic assumption, which states that for every t *the displacements* $u_\alpha(\cdot,t)$ *in the problem under consideration are periodic like functions*;

$u_\alpha(\cdot,t) \in PL(\mathcal{T})$. As the consequence of this assumption it is possible to represent the displacements in the form

$$u_\alpha(\cdot,t) = u^0_\alpha(\cdot,t) + v_\alpha(\cdot,t) \tag{2.5}$$

where $u^0_\alpha(\cdot,t) \in SV(\mathcal{T})$ and $v_\alpha(\cdot,t) \in PL^\rho(\mathcal{T})$. The prove of this fact was given by Woźniak and Wierzbicki (2000). In this reference it is also shown that for u^0_α we obtain the equation

$$(\langle B_{\alpha\beta\gamma\delta}\rangle u^0_{\gamma,\delta} + \langle B_{\alpha\beta\gamma\delta} v_{\gamma,\delta}\rangle)_{,\beta} - \langle\rho\rangle \ddot{u}^0_\alpha + \langle\rho b_\alpha\rangle = 0 \tag{2.6}$$

and the following variational periodic cell problem: find in every $\overline{\Delta}(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$ a Δ-periodic function $v_{\boldsymbol{x}\alpha}(\boldsymbol{y},t)$, $\boldsymbol{y} \in \Delta(\boldsymbol{x})$, such that $\langle\rho v_{\boldsymbol{x}\alpha}\rangle(\boldsymbol{x}) = 0$ and the condition

$$\begin{aligned}
&\langle B_{\alpha\beta\gamma\delta} v_{\boldsymbol{x}\gamma,\delta} v^*_{\alpha,\beta}\rangle(\boldsymbol{x},t) + \langle\rho \ddot{v}_{\boldsymbol{x}\alpha} v^*_\alpha\rangle(\boldsymbol{x},t) = \\
&= \langle B_{\alpha\beta\gamma\delta} v^*_{\gamma,\delta}\rangle(\boldsymbol{x},t) u^0_{\alpha,\beta}(\boldsymbol{x},t) - \langle\rho b_\alpha v^*_\alpha\rangle(\boldsymbol{x},t)
\end{aligned} \tag{2.7}$$

holds for every Δ-periodic test function v^* satisfying $\langle\rho v^*_\alpha\rangle = 0$.

The approximate solution to the above variational cell problem will be obtained by the orthogonalization method and assumed in the form

$$v_{\boldsymbol{x}\alpha}(\boldsymbol{y},t) = h^A_\beta(\boldsymbol{y}) W^A_{\alpha\beta}(\boldsymbol{x},t) \qquad A = 1,2,...,N \tag{2.8}$$

where $h^A_\beta(\boldsymbol{y})$ are the given periodic shape functions and $W^A_{\alpha\beta}(\boldsymbol{x},t)$ are the unknowns which are assumed to be slowly varying functions, $W^A_{\alpha\beta}(\cdot,t) \in SV(\mathcal{T})$. From (2.5) and (2.6) we finally obtain

$$\langle B_{\alpha\beta\gamma\delta}\rangle u^0_{\gamma,\delta} + \langle B_{\alpha\beta\gamma\delta} h^A_{\mu,\delta}\rangle W^A_{\gamma\mu,\beta} - \langle\rho\rangle \ddot{u}^0_\alpha + \langle\rho b_\alpha\rangle = 0$$

$$\langle B_{\alpha\beta\gamma\delta} h^A_{\mu,\gamma} h^B_{\nu,\delta}\rangle W^B_{\beta\nu} + \langle\rho h^A_\mu h^B_\nu\rangle \ddot{W}^B_{\alpha\nu} = -\langle B_{\alpha\beta\gamma\delta} h^A_{\mu,\delta}\rangle u^0_{\beta,\gamma} - \langle\rho h^A_\mu b_\alpha\rangle$$

The above equations can be also written in the form

$$\begin{aligned}
&S_{\alpha\beta,\beta} - \langle\rho\rangle \ddot{u}^0_\alpha + \langle\rho b_\alpha\rangle = 0 \\
&\langle\rho h^A_\mu h^B_\nu\rangle \ddot{W}^B_{\alpha\nu} + H^A_{\mu\alpha} + \langle\rho h^A_\mu b_\alpha\rangle = 0
\end{aligned} \tag{2.9}$$

where

$$\begin{aligned}
S_{\alpha\beta} &= \langle B_{\alpha\beta\gamma\delta}\rangle u^0_{\gamma,\delta} + \langle B_{\alpha\beta\gamma\delta} h^A_{\mu,\delta}\rangle W^A_{\gamma\mu} \\
H^A_{\mu\alpha} &= \langle B_{\alpha\beta\gamma\delta} h^A_{\mu,\gamma} h^B_{\nu,\delta}\rangle W^B_{\beta\nu} - \langle B_{\alpha\beta\gamma\delta} h^A_{\mu,\delta}\rangle u^0_{\beta,\gamma}
\end{aligned} \tag{2.10}$$

The above system of equations for the unknown functions $u^0_\alpha(\boldsymbol{x},t)$, $W^A_{\alpha\beta}(\boldsymbol{x},t)$ has constant coefficients, and together with the conditions

$$u^0_\alpha(\cdot,t) \in SV(\mathcal{T}) \qquad W^A_{\alpha\beta}(\cdot,t) \in SV(\mathcal{T}) \tag{2.11}$$

and the formula

$$u_\alpha \cong u^0_\alpha + h^A_\beta W^A_{\alpha\beta} \tag{2.12}$$

represents the tolerance averaged model (macro-model) of the linear micro-periodic bodies.

3. Analysis

Now assume that:

- components of the micro-periodic composite are made of isotropic materials,
- representative volume element Δ can be taken in the form of a hexagonal shown in Fig. 1,
- material properties of the hexagonal representative element Δ in Fig. 1 are invariant under rotation by the angle $2\pi n/3$ for $n = \pm 1, \pm 2, ...$, i.e. Δ has the triple axis of material symmetry.

It follows that

$$B_{\alpha\beta\gamma\delta}(\boldsymbol{x}) = \lambda(\boldsymbol{x})\delta_{\alpha\beta}\delta_{\gamma\delta} + \mu(\boldsymbol{x})(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) \tag{3.1}$$

where the Lame modulae $\lambda(\cdot)$, $\mu(\cdot)$ as well as the mass density $\rho(\cdot)$ in Δ are invariant under rotation of the coordinates $0x_1x_2$ (Fig. 1) by $2\pi n/3$. Hence, variational cell problem (2.7) has to be invariant under the aforementioned rotations. It follows that also shape functions (2.8) have to satisfy the above invariance condition and will be assumed in the form (here and afterwards summation over $\alpha = 1,2,3$ holds)

$$h_\alpha(x_1,x_2) = g^a(x_1,x_2)t^a_\alpha \qquad \alpha = 1,2 \qquad a = 1,2,3 \tag{3.2}$$

which is related to the case $N = 1$ (i.e. instead of h^1_α we write h_α), and t^a_α are components of the three unit vectors $\boldsymbol{t}^a$ shown in Fig. 1. The scalar functions g^a are assumed to be known. Vectors $\boldsymbol{t}^a$ have components: $\boldsymbol{t}^1 = [1,0]$,

$\boldsymbol{t}^2 = [-1/2, \sqrt{3}/2]$, $\boldsymbol{t}^3 = [-1/2, -\sqrt{3}/2]$. Taking into account (3.2), formulae (2.8), (2.12) assume the form

$$u_\alpha(\boldsymbol{y}, t) \cong u^0_\alpha(\boldsymbol{y}, t) + g^a(\boldsymbol{y}) t^a_\beta W_{\alpha\beta}(\boldsymbol{y}, t) \tag{3.3}$$

It is easy to prove that

$$\operatorname{grad} g^2 = \Big[-\frac{1}{2}g^1{}_{,1} + \frac{\sqrt{3}}{2}g^1{}_{,2}\,,\ \frac{\sqrt{3}}{2}g^1{}_{,1} - \frac{1}{2}g^1{}_{,2}\Big]$$

$$\operatorname{grad} g^3 = \Big[-\frac{1}{2}g^1{}_{,1} - \frac{\sqrt{3}}{2}g^1{}_{,2}\,,\ -\frac{\sqrt{3}}{2}g^1{}_{,1} - \frac{1}{2}g^1{}_{,2}\Big]$$

Taking into account these relations the expression $g^a{}_{,\alpha} t^a_\beta$ can be presented as

$$g^a{}_{,\alpha} t^a_\beta = \frac{3}{2}g^1{}_{,1} - \frac{3}{2}g^1{}_{,2}\varepsilon_{\alpha\beta}$$

where

$$\varepsilon_{\alpha\beta} = \begin{cases} 0 & \text{for} \quad \alpha = \beta \\ 1 & \text{for} \quad \alpha = 1 \qquad \beta = 2 \\ -1 & \text{for} \quad \alpha = 2 \qquad \beta = 1 \end{cases}$$

and hence

$$h_{\beta,\alpha} = \frac{3}{2}g^1{}_{,1}\delta_{\alpha\beta} - \frac{3}{2}g^1{}_{,2}\varepsilon_{\alpha\beta} \tag{3.4}$$

Substituting (3.4) and (3.1) into $(2.10)_1$ we obtain

$$\begin{aligned} S_{\alpha\beta} = {} & \langle\lambda\rangle\delta_{\alpha\beta}u^0_{\gamma,\gamma} + \langle\mu\rangle(u^0_{\alpha,\beta} + u^0_{\beta,\alpha}) + \langle\lambda^1\rangle\delta_{\alpha\beta}W_{\gamma\gamma} + \\ & + \langle\mu^1\rangle(W_{\alpha\beta} + W_{\beta\alpha}) - \langle\mu^2\rangle(\varepsilon_{\beta\sigma}W_{\alpha\delta} + \varepsilon_{\alpha\sigma}W_{\beta\delta}) - \langle\lambda^2\rangle\delta_{\alpha\beta}\varepsilon_{\delta\sigma}W_{\delta\sigma} \end{aligned} \tag{3.5}$$

where $\langle f^i\rangle = (3/2)\langle f g^1{}_{,i}\rangle$ for $f = \lambda, \mu$ and $i = 1, 2$. Substituting (3.4) and (3.1) into $(2.10)_2$ we obtain

$$\begin{aligned} H_{\alpha\xi} = {} & \langle\lambda^1\rangle u^0_{\alpha,\xi} - \langle\lambda^2\rangle\varepsilon_{\gamma\xi}u^0_{\alpha,\gamma} + \langle\mu^1\rangle(u^0_{\xi,\alpha} + \delta_{\alpha\xi}u^0_{\gamma,\gamma}) - \\ & - \langle\mu^2\rangle(\varepsilon_{\beta\xi}u^0_{\beta,\xi} + \varepsilon_{\alpha\xi}u^0_{\gamma,\gamma}) + \Big[\langle\lambda^{11}\rangle W_{\alpha\zeta} + \\ & + \langle\mu^{11}\rangle(W_{\xi\alpha} + \delta_{\alpha\zeta}W_{\gamma\gamma}) + \langle\lambda^{22}\rangle W_{\alpha\xi} + \langle\mu^{22}\rangle(\varepsilon_{\alpha\xi}\varepsilon_{\beta\eta}W_{\beta\eta} + \varepsilon_{\beta\xi}\varepsilon_{\alpha\eta}W_{\beta\eta})\Big] - \\ & - \langle\mu^{12}\rangle(\delta_{\alpha\xi}\varepsilon_{\beta\eta}W_{\beta\eta} + \varepsilon_{\alpha\xi}W_{\beta\beta} + \varepsilon_{\alpha\eta}W_{\xi\eta} + \varepsilon_{\beta\xi}W_{\beta\alpha}) \end{aligned} \tag{3.6}$$

where $\langle f^{ij}\rangle = (9/4)\langle f g^1{}_{,i} g_{1,j}\rangle$ for $f = \lambda, \mu$ and $(i, j) = (1, 1), (1, 2), (2, 2)$.

The first one from Eqs. (2.9) retains its form

$$S_{\alpha\beta,\beta} - \langle\rho\rangle\ddot{u}_\alpha\langle\rho b_\alpha\rangle = 0 \tag{3.7}$$

while the second equation, under denotations $W_{\alpha\eta} = W^1_{\alpha\eta}$, $H_{\alpha\xi} = H^1_{\alpha\xi}$, takes the form

$$\langle\rho h_\xi h_\eta\rangle\ddot{W}_{\alpha\eta} + H_{\alpha\xi} = \langle\rho h_\xi b_\alpha\rangle \tag{3.8}$$

By means of (3.2) we have $\langle\rho h_\xi h_\eta\rangle = \langle\rho g^a g^b\rangle t^a_\xi t^b_\eta$ where summation over $1, 2, 3$ for a and b holds. Let us observe that

$$\langle\rho g^1 g^1\rangle = \langle\rho g^2 g^2\rangle = \langle\rho g^3 g^3\rangle$$
$$\langle\rho g^1 g^2\rangle = \langle\rho g^2 g^3\rangle = \langle\rho g^3 g^1\rangle$$

It follows that

$$\langle\rho h_\xi h_\eta\rangle = \langle\rho(g^1)^2\rangle t^a_\xi t^b_\eta + 2\langle\rho g^1 g^2\rangle\Big(t^1_{(\xi}t^2_{\eta)} + t^2_{(\xi}t^3_{\eta)} + t^3_{(\xi}t^1_{\eta)}\Big)$$

it can be shown that

$$t^a_\xi t^a_\eta = \frac{3}{2}\delta_{\xi\eta}$$
$$t^1_{(\xi}t^2_{\eta)} + t^2_{(\xi}t^3_{\eta)} + t^3_{(\xi}t^1_{\eta)} = -\frac{3}{4}\delta_{\xi\eta}$$

and hence

$$\langle\rho h_\xi h_\eta\rangle = \widetilde{\rho}\delta_{\xi\eta}$$

where we have denoted

$$\widetilde{\rho} = \frac{3}{2}\langle\rho g^1(g^1 - g^2)\rangle$$

It follows that the second one from equations (2.9) can be reduced to the final form

$$\widetilde{\rho}\ddot{W}_{\alpha\xi} + H_{\alpha\xi} - \langle\rho h_\xi b_\alpha\rangle = 0 \tag{3.9}$$

Equations (3.5)-(3.9) constitute the special case of equations (2.9) and (2.10), which takes place for the class of composites specified at the begining of this section, and on the assumption that the shape functions are assumed in the form (3.2).

4. Conclusion

On the grounds of the above analysis we conclude that the tolerance averaged model of the composite under consideration is described by equations (3.5)-(3.9) together with conditions (2.11), (2.12). It can be seen that the obtained model of the composite is isotropic. It means that micro-periodic composites made of linear-elastic components and having the hexagonal representative cell with the triple axis of material symmetry are isotropic also in the macro-scale. The above statement holds on the assumption that the shape function in the tolerance averaging are taken in the form (3.2). The application of Eqs. (3.5)-(3.9) to certain special dynamic problems will be given in a separate paper.

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Modelowanie kompozytów z trójosiową symetrią

Streszczenie

W pracy rozważa się kompozyty sprężyste o mikroperidycznej strukturze, których elementy reprezentatywne mają trójosiową symetrię. Wykazuje się, że takie ciała mają w modelu otrzymanym w wyniku zastosowania techniki uśredniania tolerancyjnego własności ciał izotropowych.

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