JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
1, 40, 2002

ANALYSIS OF THERMAL PROCESSES OCCURING IN TISSUE WITH A TUMOR REGION USING BEM

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In the paper a numerical algorithm based on the boundary element method is used for temperature field computations in the non-homogeneous domain being a composition of healthy tissue and a tumor region. Thermophysical parameters of subdomains, in particular the perfusion coefficients and metabolic heat sources are different. From the mathematical point of view the problem is described by a system of two Poisson's equations with temperature-dependent source functions. These equations are supplemented by adequate boundary conditions. The discussed algorithm allows one to determine the temperature distribution on the surface of the skin. In this way it is possible to analyse the dependence between the geometrical features of the tumor region and the external thermal effects. The results can be useful as the information for noninvasive diagnostics. In the final part of the paper examples of computations are shown.

Key words: bio-heat transfer, boundary element method

1. Introduction

The body surface temperature is controlled by the blood perfusion, local metabolism and the boundary conditions determining the heat exchange between the skin and environment. It is well known that the apparition of a tumor region leads to an increase in the local blood perfusion and capacity

of the metabolic heat source (Liu and Xu, 2000). So, the change of the skin surface temperature can be the essential information about the menace. For instance, the temperature at the skin above a breast tumor has been found to be several degrees higher than that of the surrounding area (Liu and Xu, 2000; Miyakawa and Bolomey, 1996). The exact measurements of the skin temperature can be used for noninvasive diagnostics. The heat transfer processes in biological tissue are described by the Pennes equation (Majchrzak and Mochnacki, 1999). If we consider a steady-state problem, then this equation assumes a form of the Poisson one

$$x \in \Omega: \quad \lambda \nabla^2 T(x) + k[T_b - T(x)] + Q_m = 0 \tag{1.1}$$

where

λ – thermal conductivity
k – perfusion coefficient
T_b – blood temperature
Q_m – metabolic heat source.

If we assume that the local blood temperature is a constant value, and we consider the non-homogeneous domain healthy tissue-tumor region, then we obtain the following system of equations

$$x \in \Omega_e: \quad \lambda_e \nabla^2 T_e(x) - k_e T_e(x) + Q_e = 0 \tag{1.2}$$

where $e = 1, 2$ identifies the subdomains of tissue and tumor – see Figure 1, whereas

$$Q_e = k_e T_b + Q_{me} \tag{1.3}$$

Taking into account the shape of the domain, the symmetrical fragment is analysed (as in Figure 1).

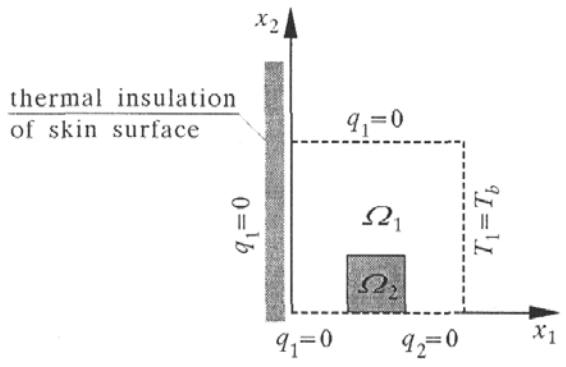

Fig. 1. Considered domain

On the surface between the tissue and tumor the ideal thermal contact is assumed

$$x \in \Gamma_c : \quad \begin{cases} T_1(x) = T_2(x) \\ q_1(x) = q_2(x) \end{cases} \tag{1.4}$$

where $q_e(x) = -\lambda_e \partial T(x)/\partial n$ is the heat flux, while $\partial T_e(x)/\partial n$ denotes the normal derivative at the considered boundary point.

On the remaining parts of the boundary the conditions marked in Figure 1 are accepted. It should be pointed out that the skin is covered with an insulating material which allows one to avoid the influence from the surrounding environment, and it determines the no-flux condition on this part of the boundary.

2. Boundary element method for homogeneous domain

At first, we consider a homogeneous domain (e.g. healthy tissue), in which the heat transfer processes are determined by the equation

$$x \in \Omega : \quad \lambda \nabla^2 T(x) - kT(x) + Q = 0 \tag{2.1}$$

The boundary integral equation corresponding to the considered problem is of the form (Brebbia and Dominiquez, 1992; Banerjee, 1994; Majchrzak, 2001)

$$\begin{aligned} \xi \in \Gamma : \qquad & B(\xi)T(\xi) + \int_\Gamma q(x)T^*(\xi, x)\, d\Gamma = \\ & = \int_\Gamma T(x)q^*(\xi, x)\, d\Gamma + \iint_\Omega Q(x)T^*(\xi, x)\, d\Omega \end{aligned} \tag{2.2}$$

where $B(\xi)$ is the coefficient from the interval $(0, 1)$, $T^*(\xi, x)$ is the fundamental solution, ξ is the observation point, $q^*(\xi, x) = -\lambda \partial T^*(\xi, x)/\partial n$.

The fundamental solution to equation (2.1) is the following (Majchrzak, 2001)

$$T^*(\xi, x) = \frac{1}{2\pi\lambda} K_0\left(\sqrt{\frac{k}{\lambda}}\, r\right) \tag{2.3}$$

where r is the distance between the points ξ and x

$$r = \sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2} \tag{2.4}$$

while $K_0(\cdot)$ is the zero order modified Bessel function of the second kind (Abramowitz and Stegun, 1994).

Using formula (2.3) one can find the function q^*

$$q^*(\xi, x) = \frac{d}{2\pi r}\sqrt{\frac{k}{\lambda}}\, K_1\Big(\sqrt{\frac{k}{\lambda}}\, r\Big) \tag{2.5}$$

where

$$d = (x_1 - \xi_1)\cos\alpha_1 + (x_2 - \xi_2)\cos\alpha_2 \tag{2.6}$$

and $\cos\alpha_1$, $\cos\alpha_2$ are the directional cosines of the normal outward vector. In formula (2.5) $K_0(\cdot)$ is the first order modified Bessel function of the second kind (Abramowitz and Stegun, 1994).

At the stage of numerical computations the boundary of the domain as well as its interior is divided into a boundary and internal elements – see Figure 2.

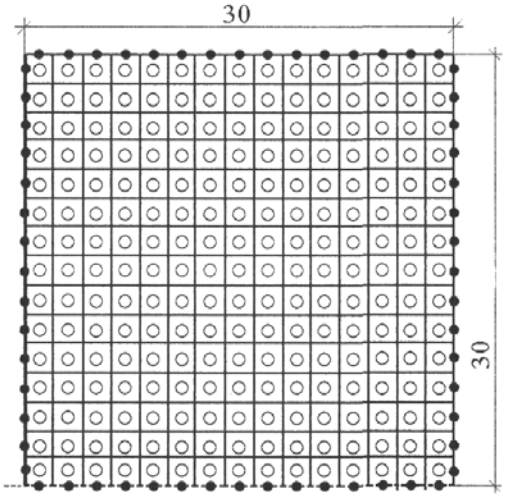

Fig. 2. Discretization

For constant boundary elements we assume that

$$x \in \Gamma_j : \quad \begin{cases} T(x) = T(x^j) = T_j \\ q(x) = q(x^j) = q_j \end{cases} \tag{2.7}$$

whereas for constant internal cells

$$x \in \Omega_l : \quad Q(x) = Q(x^l) = Q_l \tag{2.8}$$

So, the approximation of equation (2.2) takes th form

$$\frac{1}{2}T_i + \sum_{j=1}^{N} G_{ij}q_j = \sum_{j=1}^{N} \widehat{H}_{ij}T_j + \sum_{l=1}^{L} P_{il}Q_l \tag{2.9}$$

where N is the number of boundary nodes, L is the number of internal nodes, $i = 1, ..., N$.

In equations (2.9) the following denotations are introduced

$$G_{ij} = \int_{\Gamma_j} T^*(\xi^i, x)\, d\Gamma_j \tag{2.10}$$

and

$$\widehat{H}_{ij} = \int_{\Gamma_j} q^*(\xi^i, x)\, d\Gamma_j \tag{2.11}$$

at the same time

$$P_{il} = \int\int_{\Omega_l} T^*(\xi^i, x)\, d\Omega_l \tag{2.12}$$

The system of algebraic equations (2.9) can be written in the form

$$\sum_{j=1}^{N} G_{ij} q_j = \sum_{j=1}^{N} H_{ij} T_j + \sum_{l=1}^{L} P_{il} Q_l \tag{2.13}$$

or

$$\mathbf{G}\boldsymbol{q} = \mathbf{H}\boldsymbol{T} + \boldsymbol{Q} \tag{2.14}$$

where

$$H_{ij} = \begin{cases} \widehat{H}_{ij} & i \neq j \\ \widehat{H}_{ij} - \dfrac{1}{2} & i = j \end{cases} \tag{2.15}$$

This system allows one to find the "missing" temperatures and heat fluxes at the boundary points. Next, the temperatures at the internal nodes are calculated on the basis of the formula $(i = N+1, N+2, ..., N+L)$

$$T_i = \sum_{j=1}^{N} H_{ij} T_j - \sum_{j=1}^{N} G_{ij} q_j + \sum_{l=1}^{L} P_{il} Q_l \tag{2.16}$$

3. Boundary element method for non-homogeneous domain

The algorithm presented in Section 2 constitutes the basis for BEM application to numerical modelling of the heat transfer process proceeding in the non-homogeneous domains (healthy tissue and tumor region).

For the needs of further considerations the following denotations are introduced (Figure 3):

- $\boldsymbol{T}_1$, $\boldsymbol{q}_1$ are the vectors of temperatures and heat fluxes on the surfaces Γ_1, Γ_2, Γ_4 of the domain Ω_1
- $\boldsymbol{T}_b$, $\boldsymbol{q}_b$ are the vectors of temperatures and heat fluxes on the surface Γ_3 of the domain Ω_1
- $\boldsymbol{T}_2$, $\boldsymbol{q}_2$ are the vectors of temperatures and heat fluxes on the surface Γ_5 of the domain Ω_2
- $\boldsymbol{T}_{12}$, $\boldsymbol{T}_{21}$, $\boldsymbol{q}_{12}$, $\boldsymbol{q}_{21}$ are the vectors of temperatures and heat fluxes on the contact surface Γ_c between the domains Ω_1 and Ω_2.

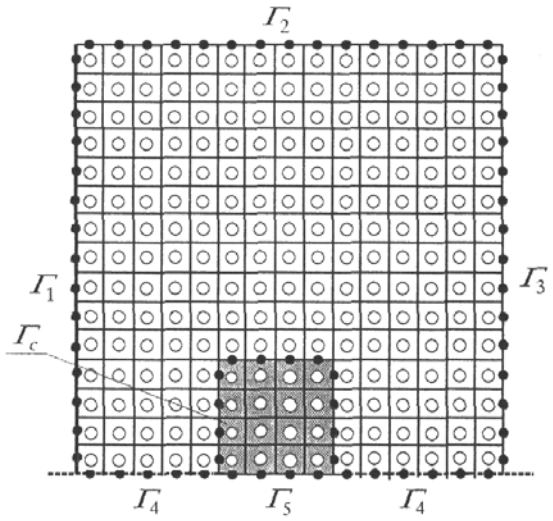

Fig. 3. Non-homogeneous domain

In this case one obtains the following systems of equations
— for the first domain

$$[\mathsf{G}_1, \mathsf{G}_{1b}, \mathsf{G}_{12}] \begin{bmatrix} \boldsymbol{q}_1 \\ \boldsymbol{q}_b \\ \boldsymbol{q}_{12} \end{bmatrix} = [\mathsf{H}_1, \mathsf{H}_{1b}, \mathsf{H}_{12}] \begin{bmatrix} \boldsymbol{T}_1 \\ \boldsymbol{T}_b \\ \boldsymbol{T}_{12} \end{bmatrix} + \mathsf{P}_1 \boldsymbol{Q}_1 \tag{3.1}$$

— for the second domain

$$[\mathsf{G}_2, \mathsf{G}_{21}] \begin{bmatrix} \boldsymbol{q}_2 \\ \boldsymbol{q}_{21} \end{bmatrix} = [\mathsf{H}_2, \mathsf{H}_{21}] \begin{bmatrix} \boldsymbol{T}_2 \\ \boldsymbol{T}_{21} \end{bmatrix} + \mathsf{P}_2 \boldsymbol{Q}_2 \tag{3.2}$$

Condition (1.4) on the contact surface between the sub-domains can be written in the form

$$x \in \Gamma_{12}: \quad \begin{cases} \boldsymbol{T}_{12} = \boldsymbol{T}_{21} = \boldsymbol{T} \\ \boldsymbol{q}_{12} = -\boldsymbol{q}_{21} = \boldsymbol{q} \end{cases} \tag{3.3}$$

Coupling together systems (3.1) and (3.2), and taking into account the remaining boundary conditions, we find the final form of the system

$$\begin{bmatrix} -\mathbf{H}_1 & \mathbf{G}_{1b} & -\mathbf{H}_{12} & \mathbf{G}_{12} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{H}_{21} & -\mathbf{G}_{21} & -\mathbf{H}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_1 \\ \boldsymbol{q}_b \\ \boldsymbol{T} \\ \boldsymbol{q} \\ \boldsymbol{T}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1b}\boldsymbol{T}_b + \mathbf{P}_1\boldsymbol{Q}_1 \\ \mathbf{P}_2\boldsymbol{Q}_2 \end{bmatrix} \tag{3.4}$$

The internal values of T_1 and T_2 can be determined on the basis of formulas (2.16), for Ω_1 and Ω_2, separately.

4. Examples of computations

The dimensions [mm] and discretization of the tissue domain are shown in Figures 2 and 3. The following input data are assumed (Liu and Xu, 2000, Torvi and Dale, 1994): thermal conductivity of the tissue $\lambda_1 = 0.75\,\mathrm{W/mK}$, thermal conductivity of the tumor $\lambda_2 = 0.75\,\mathrm{W/mK}$, metabolic heat source: $Q_{m1} = 420\,\mathrm{W/m^3}$, $Q_{m2} = 4200\,\mathrm{W/m^3}$, perfusion coefficient: $k_1 = 1198.1$, $k_2 = 7992.4$, blood temperature $T_b = 37^\circ\mathrm{C}$.

In Figure 4 positions of isotherms in the domain of the healthy tissue are shown. It is visible that the isotherms are parallel to the skin surface.

Figures 5, 6, 7 and 8 illustrate the temperature field in the domain of tissue with a tumor. Different positions and sizes of the tumor region are taken into account. Figure 9 shows the temperature distribution on the skin surface. The effect of the tumor region is clearly visible. Because of the small size of the tumor region and assumed values of k_2 and Q_{m2}, the differences of the surface temperature are small too, but measurable even in the case of simple instruments.

It seems that the algorithm presented in this paper can be used as a source of essential information for thermal diagnostics in medical practice. In the future the authors are going to identify an approximate size and position of

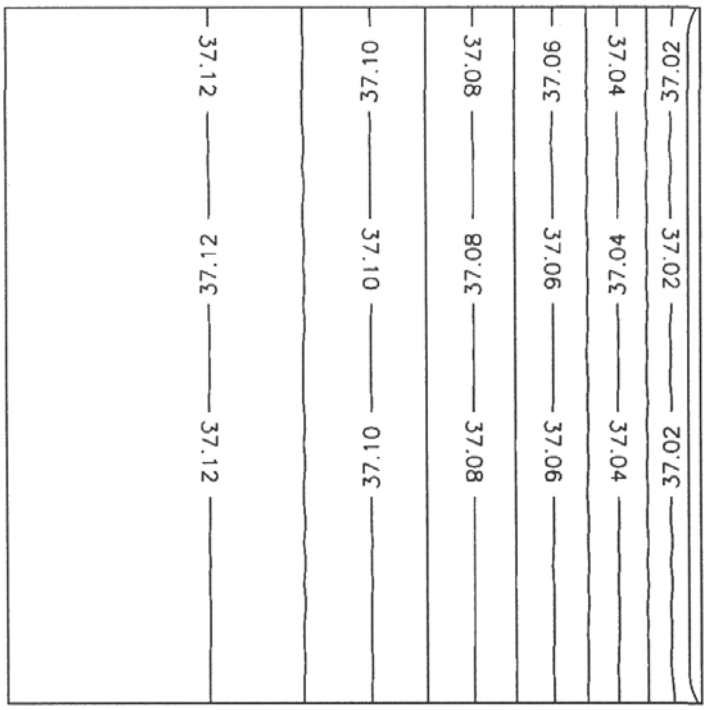

Fig. 4. Temperature distribution in healthy tissue

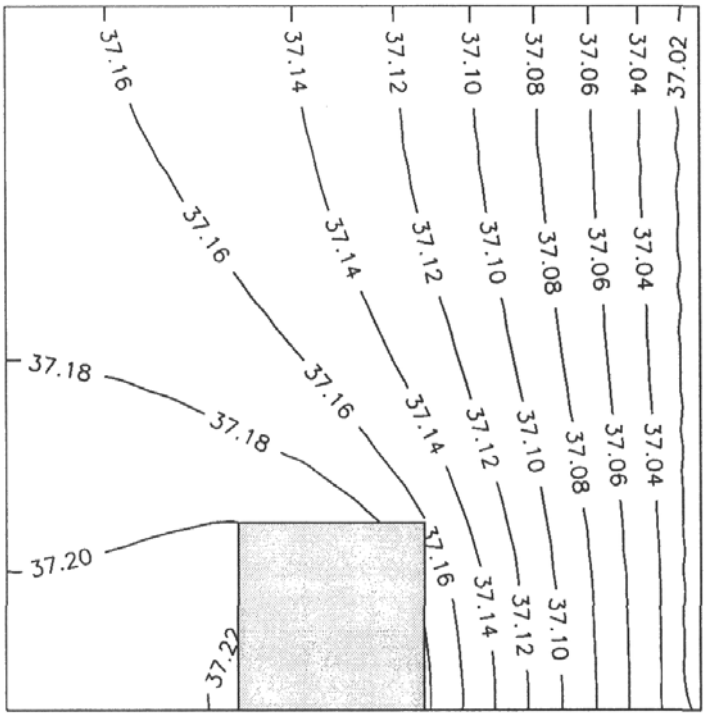

Fig. 5. Temperature distribution in the tissue with a tumor (1st variant)

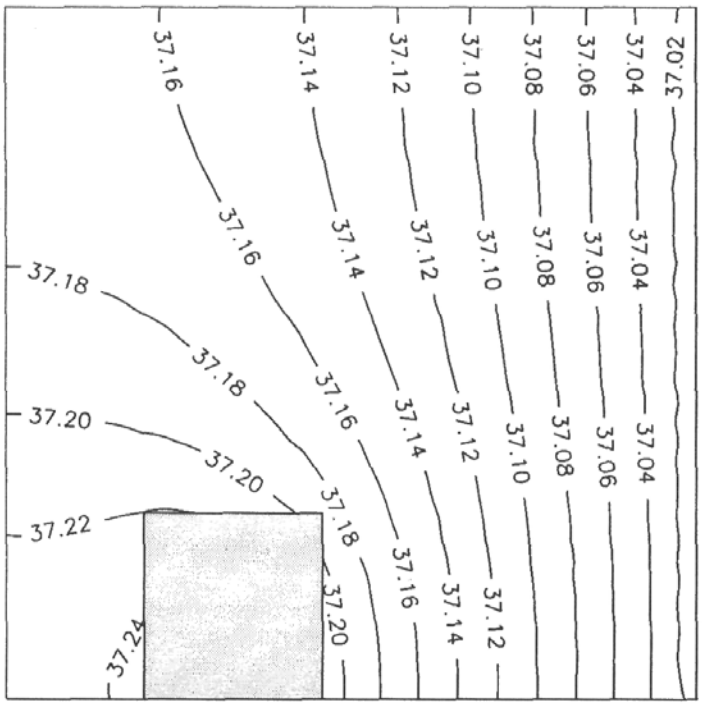

Fig. 6. Temperature distribution in the tissue with a tumor (2nd variant)

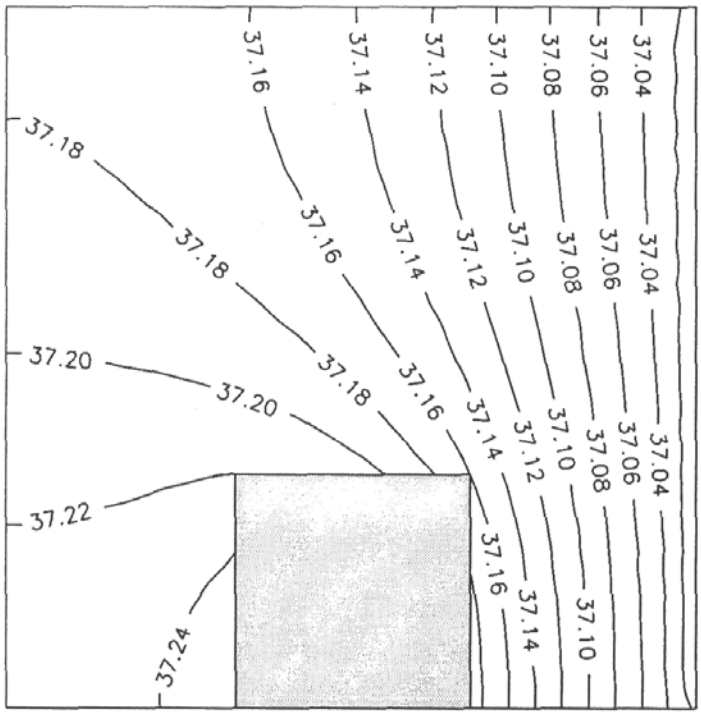

Fig. 7. Temperature distribution in the tissue with a tumor (3rd variant)

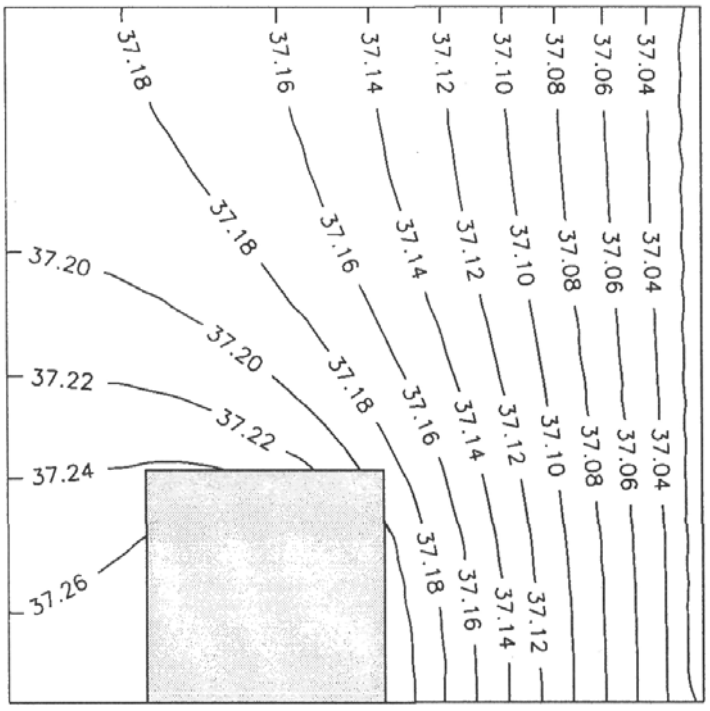

Fig. 8. Temperature distribution in the tissue with a tumor (4th variant)

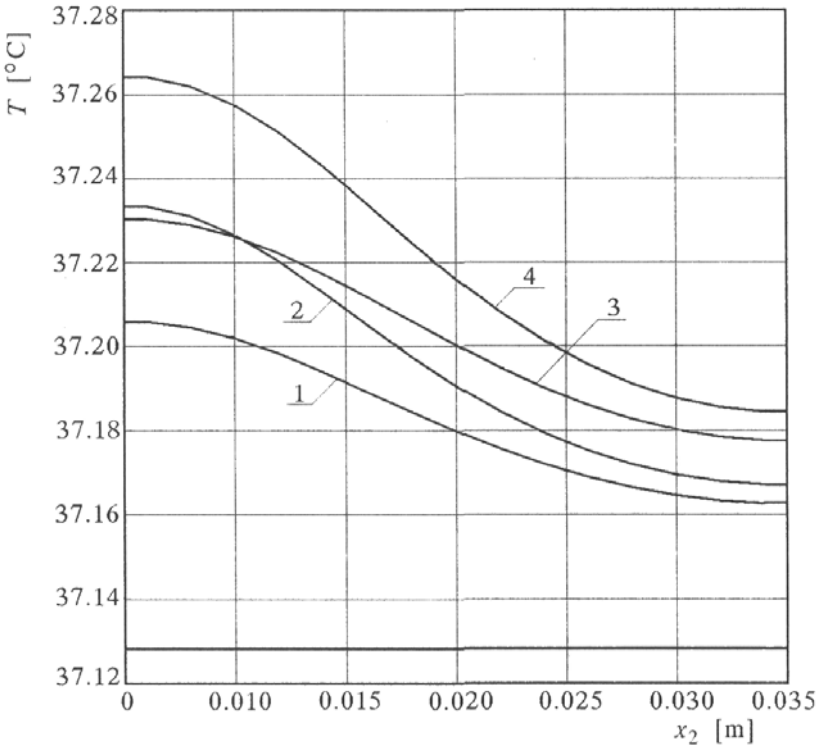

Fig. 9. Temperature distribution on the skin surface (variants 1-4)

the tumor region on the basis of the knowledge of skin surface temperature, briefly, to construct an algorithm for solving the inverse problem.

This paper has been sponsored by the State Committee for Scientific Research (KBN) under grant No. 8 T11F 004 19.

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Wykorzystanie MEB do analizy procesów cieplnych w tkance zaatakowanej nowotworem

Streszczenie

Algorytm numeryczny bazujący na metodzie elementów brzegowych wykorzystano do obliczeń rozkładów temperatury w obszarze niejednorodnym, składającym się ze zdrowej tkanki i podobszaru nowotworowego. Parametry termofizyczne tych

podobszarów są zróżnicowane, a w szczególności różnią się współczynniki perfuzji krwi oraz składniki źródłowe związane z metabolizmem. Z matematycznego punktu widzenia rozważane zagadnienie opisane jest układem dwóch równań z zależnymi od temperatury składnikami źródłowymi. Równania te uzupełniają odpowiednie warunki brzegowe. Przedstawiony algorytm pozwala określić rozkład temperatury na powierzchni tkanki skórnej. W ten sposób można analizować zależności między wielkością, kształtem i położeniem podobszaru nowotworowego, a zewnętrznymi efektami termicznymi. Uzyskane wyniki mogą być przydatne w diagnostyce nieinwazyjnej. W końcowej części artykułu przedstawiono przykłady obliczeń.

Manuscript received September 12, 2001; accepted for print October 26, 2001