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SIMILARITY AND MODEL DESIGNING IN A NONSCALAR DESCRIPTION OF AN EXAMINED PROCESS

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The problem of similarity and designing of a physical model of an examined process is considered on the basis of generalizations of Theorem π. It is shown that in cases of nonscalar description the similar model of a process could be designed on the basis of a special version of Theorem π. Similarity scales can be obtained only for the vector modulus, or for the components of vectors and tensors when special experimental conditions (described in this paper) are fulfilled.

Key words: similarity, dimensional analysis, physical modelling

1. Introduction

The well known notion of similarity has been used in scientific and engineering activities assessing the construction or investigation of a physical process. The basis for the establishment of the relations between observations of a process on a model and of processes of interest to us is created by the properties of the so called dimensional invariant and homogeneous function. The form of this function is produced by Theorem π. Accordingly, if the process is described by quantities $\widehat{Z}_0, \widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_s$ and we are interested in the identification of the functional relationship

$$\widehat{Z}_0 = \varPhi(\widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_s) \tag{1.1}$$

in which the arguments $\widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_m$ created the dimensional base, then

$$\widehat{Z}_{m+j} = \phi \prod_{i=1}^{m} \widehat{Z}_i^{a_{ji}} \qquad j = 1, 2, ..., r \qquad m + r = s \tag{1.2}$$

and

$$\widehat{Z}_0 = f(\phi_1, \phi_2, ..., \phi_r) \prod_{i=1}^{m} \widehat{Z}_i^{a_i} \tag{1.3}$$

In (1.2) and (1.3) the exponents $a_{ji}, a_i \in R$. We can get the values of a_{ji} and a_i if we take into account that the dimensions on the left- and right-hand side of (1.2) and (1.3) are the same (see Kasprzak *et al.* (1990) for details). The course of the process or functioning of the object may be tested on models constructed on an appropriate scale without the knowledge of the mathematical model. It should be noted that, in the original object as well as in the model, the same process, described by the same variables and the same function $\varPhi$ are investigated. Let us denote the quantities observed in the model by an asterisk. The realizations in the object are then described by (1.1) and (1.3) and in the model

$$\widehat{Z}_0^* = \varPhi(\widehat{Z}_1^*, \widehat{Z}_2^*, ..., \widehat{Z}_s^*) = f(\phi_1^*, \phi_2^*, ..., \phi_r^*) \prod_{i=1}^{m} \widehat{Z}_i^{*a_i} \tag{1.4}$$

In model designing we use scales

$$\begin{aligned} &\widehat{Z}_i^* = \lambda_i \widehat{Z}_i && \lambda_i \in R_+ && i = 1, 2, ..., m \\ &\widehat{Z}_{m+j}^* = \mu_j \widehat{Z}_{m+j} && \mu_j \in R_+ && j = 1, 2, ..., r \end{aligned} \tag{1.5}$$

We want to find the relationship between $\widehat{Z}_0^*$ and $\widehat{Z}_0$. The supposed property of homogeneity of the function $\varPhi$ guarantees that the quotient

$$\lambda = \frac{\widehat{Z}_0^*}{\widehat{Z}_0} \tag{1.6}$$

exists and belongs to R_+. Taking into account (1.3), (1.4) and $(1.5)_1$, we get

$$\frac{\widehat{Z}_0^*}{\widehat{Z}_0} = \lambda = \frac{f(\phi_1^*, \phi_2^*, ..., \phi_r^*)}{f(\phi_1, \phi_2, ..., \phi_r)} \prod_{i=1}^{m} \lambda_i^{a_i} \tag{1.7}$$

The real number

$$\lambda_\phi = \frac{f(\phi_1^*, \phi_2^*, ..., \phi_r^*)}{f(\phi_1, \phi_2, ..., \phi_r)} \tag{1.8}$$

cannot be determined if the function f is not known. It may, however, be easily seen that if we fulfill the so called similarity criteria

$$\phi_j^* = \phi_j \qquad j = 1, 2, ..., r \tag{1.9}$$

λ_ϕ will equal exactly 1, and from (1.6) we shall get

$$\lambda = \prod_{i=1}^{m} \lambda_i^{a_i} \qquad \text{and} \qquad \widehat{Z}_0 = \frac{\widehat{Z}_0^*}{\prod_{i=1}^{m} \lambda_i^{a_i}} \tag{1.10}$$

The dimensionless quantities ϕ_j (1.9) are well known invariants of the similarity. From relationships (1.2) and (1.9) we shall get

$$\frac{\widehat{Z}_{m+j}}{\prod_{i=1}^{m} \widehat{Z}_i^{a_{ji}}} = \frac{\mu \widehat{Z}_{m+j}}{\prod_{i=1}^{m} (\lambda_i \widehat{Z}_i)^{a_{ji}}} \tag{1.11}$$

r conditions imposed on the $m+r$ scales. This so called similarity criterion can be expressed by using scales (1.5)

$$\frac{\mu_j}{\prod_{i=1}^{m} \lambda_i^{a_{ji}}} = 1 \qquad j = 1, 2, ..., r \tag{1.12}$$

Obviously, we can get (1.10) when the process is described by the scalar $\widehat{Z}_0$. In many cases, vector and tensor quantities are used to describe a process, especially in the mechanics of continuous media. In recent years, for example, the explanation of aerodynamic performance of flapping biofoils has been one of the most interesting tasks in biophysics (the flying force of an insect is produced by complicated motion of the wings, see, for example Lehman (1999)). We shall consider a similar problem, on the base of two generalized versions of Theorem π in which quantities modelled by tensors will be used.

2. Generalizations of Theorem π

We shall examine two approaches to the description of a process with some nonscalar quantities by the generalizations of Theorem π

- the first refers only to scalar quantities as arguments of the function Φ, but the process will be described by complex functions for every tensor component,
- in the second, we shall use dimensional quantities with internal geometry and Theorem π satisfying postulates of invariance in relation to groups of rotations.

Generalized Theorem π was shown for both cases after the works Kasprzak *et al.* (1990), Kasprzak *et al.* [2], Rybaczuk (1987).

Theorem. *Theorem π for the Complex Dimensional Function. If in a dimensionally homogeneous and invariant function*

$$\widehat{Z} = \Phi(\widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_s, \widehat{X}_1, \widehat{X}_2, ..., \widehat{Z}_q) \tag{2.1}$$

the arguments $\widehat{Z}_i$, $i = 1, 2, ..., m$ are dimensionally independent (dimensional base), and the dimensionally dependent arguments can be written in the base

$$\widehat{X}_p = \xi_p \prod_{i=1}^{m} \widehat{Z}_i^{b_{pi}} \qquad p = 1, 2, ..., q \tag{2.2}$$

$$\widehat{Z}_{m+j} = \Phi(\widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_m, \widehat{X}_1, \widehat{X}_2, ..., \widehat{X}_q) = \phi(\xi_1, \xi_2, ..., \xi_q) \prod_{i=1}^{m} \widehat{Z}_i^{a_{ji}} \tag{2.3}$$

$$j = 1, 2, ..., r \qquad m + r = s$$

then the function Φ has the form

$$\widehat{Z} = f(\phi_1, \phi_2, ..., \phi_r, \xi_1, \xi_2, ..., \xi_q) \prod_{i=1}^{m} \widehat{Z}_i^{a_i} \tag{2.4}$$

where $\phi_1, \phi_2, \dots, \phi_r, \xi_1, \xi_2, \dots, \xi_q \in R_+$; $a_i, a_{ji}, b_{pi} \in R$. Conversely, every function of form (2.4) is dimensionally homogeneous and invariant.

This solution, similar to classic Theorem π, although it presents the possibility of describing processes occurring in a field or in a material continuum, cannot be considered satisfactory. Physics, for instance, requires appropriate symmetries and invariances in relation to certain transformations. For further deliberations let us differentiate a class of processes where the set $\widehat{X}_p$ will be restricted to

$$\begin{aligned} &\widehat{\boldsymbol{X}} = (\widehat{X}_1, \widehat{X}_2, \widehat{X}_3) \qquad [\widehat{X}_p] = [length] \qquad p = 1, 2, 3 \\ &[\widehat{X}_4] = [\widehat{t}] = [time] \end{aligned} \tag{2.5}$$

Formulas (2.2) will assume the form

$$\xi_p = \frac{\widehat{X}_p}{\prod_{i=1}^{m} \widehat{Z}_i^{b_i}} \qquad p = 1, 2, 3 \qquad \tau = \frac{\widehat{t}}{\prod_{i=1}^{m} \widehat{Z}_i^{t_i}} \tag{2.6}$$

Finally, function (2.4) will become

$$\widehat{Z} = f(\phi_1, \phi_2, ..., \phi_r, \xi_1, \xi_2, \xi_3, \tau) \prod_{i=1}^{m} \widehat{Z}_i^{a_i} \tag{2.7}$$

Consistently with this interpretation, to each point P' with the coordinates $\widehat{X}_1, \widehat{X}_2, \widehat{X}_3$ of a dimensional space belonging to the set D' (interpreted as a configuration of the examined continuum immersed in a physical space of the Euclidean space structure) while

$$D' = X_{p=1}^{3}(\widehat{X}_p^L, \widehat{X}_p^R) \tag{2.8}$$

is mapped – using formulas (2.6) – by point $P \subset D$ of the Euclidean three-dimensional space. The set D will be obtained, of course, from set (2.8) by formulas (2.6)

$$D = X_{p=1}^{3}(\xi_p^L, \xi_p^R) \tag{2.9}$$

It is evident that, utilizing formulas (2.6) from the set D, we may – changing the values $\widehat{Z}_i$, $i = 1, 2, ..., m$ as required – generate an entire family of D' configurations. This is so because each point in the set D in the family of D' configurations conforms with a hyper-surface satisfying the equations

$$\xi_p = \frac{\widehat{X}_p}{\prod_{i=1}^{m} \widehat{Z}_i^{b_i}} = \text{const} \qquad\qquad p = 1, 2, 3 \tag{2.10}$$

The cognizance of f in the set D is decisive in the knowledge of the dimensional description of the function $\varPhi$ in all generated D' configurations – for fixed $\phi_1, \phi_2, ..., \phi_r, \tau$ parameters, of course. Considering the variables ξ_1, ξ_2, ξ_3, function (2.7) describes the scalar field. If we regard that $\phi_1, \phi_2, ..., \phi_r$ are also functions of $\xi_1, \xi_2, \xi_3, \tau$ in this field, then (2.7) can be written differently

$$\widehat{Z} = f^*(\xi_1, \xi_2, \xi_3, \tau) \prod_{i=1}^{m} \widehat{Z}_i^{a_i} \tag{2.11}$$

Physics and technology operate with vectors and tensors; their components are elements of a dimensional space of the same dimension. Thus, for example, for a vector or tensor with the components $\widehat{Z}^1, \widehat{Z}^2, \widehat{Z}^3$: $[\widehat{Z}^1] = [\widehat{Z}^2] = [\widehat{Z}^3]$. Each of these components $\widehat{Z}^\nu$ is expressed by formulas (2.7) or (2.11), i.e.

$$\begin{aligned} \widehat{Z}^\nu &= f^\nu(\phi_1, \phi_2, ..., \phi_r, \xi_1, \xi_2, \xi_3, \tau) \prod_{i=1}^{m} \widehat{Z}_i^{a_i} \\ \widehat{Z}^\nu &= f^{*\nu}(\xi_1, \xi_2, \xi_3, \tau) \prod_{i=1}^{m} \widehat{Z}_i^{a_i} \end{aligned} \tag{2.12}$$

Corollary 1. *When the functions ϕ_j, $j = 1, 2, ..., r$ of the variables $\xi_1, \xi_2, \xi_3, \tau$ are known and have been fixed, then the function f^* and functions $f^{*\nu}$ describe, on the basis of the similarity relations, the process in every D' (2.8) configuration generated through (2.6) from subset (2.9).*

This essential conclusion allows us to transfer obtained experimental results to geometrically similar spatial configurations (fields of a similar geometry and similar geometrical bodies). Let us return to the formulation of Theorem π for the dimensional function with nonscalar values and arguments.

Theorem *(Theorem π). Let arguments and values of a dimensional function belong to generalized dimensional spaces presented by Rybaczuk (1987) and the function be invariant with respect to the action of the rotation group, then the form of this function, according to Kasprzak et al. (1990), Rybaczuk (1987) (see also Rychlewski, 1978, 1991), can be presented as*

$$\begin{aligned}\widehat{Z}_0 &= \Phi(\widehat{Z}_1, \widehat{Z}_2, ., \widehat{Z}_s) = \\ &= \sum_{i=1}^{l} f_i(\phi_{p(i)+1}, \phi_{p(i)+2}, ..., \phi_r; \gamma_{k=1}^{r}) \times \widehat{g}^i(\widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_s) \prod_{t=1}^{p(i)} |I_t^i|^{b_{ti}}\end{aligned} \tag{2.13}$$

where

- $\widehat{g}_i$ *– are well known generators supplied by a suitable representation theorem for* $O(3)$ *– invariant function (Wang, 1971a,b,c),*
- I_q *are formed from* $\widehat{Z}_1, \widehat{Z}_2, ..., \widehat{Z}_s$ *scalar invariants,*
- ϕ_w, $w = 1, 2, ..., r$ *are dimensionless numbers (invariants of the gauge group) formed from the scalars* I_q,
- $\gamma_{k=1}^{r}$ *denotes the sequence of signs of the scalars* I_q,
- $p(i)$ *is the numerator of a dimensional base (chosen from* I_q*) for* f_i.

3. Similarity and model designing according to generalized Theorem π

Let us examine the similarity and model designing according to Theorem π formulated for a nonscalar description of a process. The equivalent expression for the scale λ, if $\widehat{Z}^{\nu}$ is a vector or tensor component, could be written in the form

$$\lambda_{\nu} = \prod_{i=1}^{m} \lambda_i^{a_i} \tag{3.1}$$

when conditions $(1.5)_1$ and (1.9) are fulfilled. The scale for the vector modulus (if the configurations of all acting vectors is similar – meaning that all angles between the vectors are respectively the same) according to (2.4) formulation of Theorem π could be expressed as

$$\lambda = \sqrt{\frac{\sum\limits_{\nu=1}^{3} f_{\nu}^2 \prod_{i=1}^{m} \lambda_i^{2a_i}}{\sum\limits_{\nu=1}^{3} f_{\nu}^2}} = \prod_{i=1}^{m} \lambda_i^{a_i} \tag{3.2}$$

When $\widehat{Z}^{\nu}$ is a tensor component and formula (2.13) is used as the presentation of Theorem π, we shall obtain a quite different result. For the description of the object we shall get the expression as in (2.13). The value of the component $\widehat{Z}_0^{\nu}$ in the model can be expressed as

$$\widehat{Z}_0^{\nu *} = \sum_{i=1}^{l} f_i^{\nu}(\phi_{p(i)+1}^{(i)}, \phi_{p(i)+2}^{(i)}, ..., \phi_r^{(i)}; \gamma_{k=1}^{r}) \times g_{\nu}^{(i)} \lambda_{g_{\nu}^{(i)}} \prod_{t=1}^{p(i)} |I_t \lambda_t|^{b_{ti}} \tag{3.3}$$

The scale λ of the investigated process could be assumed only for the tensor component or, if $\widehat{Z}_0$ is a vector, for its modulus (specifically for $\sqrt{Z_0^2}$). Let us investigate the value of the scale

$$\lambda^{(i)} = \lambda_{g_{\nu}^{(i)}} \prod_{t=1}^{p(i)} \lambda_t^{b_{ti}} \tag{3.4}$$

It is easy to notice that the dimensions in (3.4) and (2.13)

$$\Big[g_{\nu}^{(i)} \prod_{t=1}^{p_i} |I_t|^{b_{ti}}\Big] = \Big[g_{\nu}^{i} \lambda_{g_{\nu}^{(i)}} \prod_{t=1}^{p(i)} \lambda_t^{b_{ti}} |I_t|^{b_{ti}}\Big] = \Big[\widehat{Z}_0^{\nu}\Big] \tag{3.5}$$

irrespective of the value of the index i. From (3.5) we can see, after having noticed relations (1.12) and done some algebraic transformation, that $\lambda^{(i)}$ (3.4)

$$\lambda^{(i)} = \prod_{i=1}^{m} \lambda_i^{a_i} \tag{3.6}$$

It is easy to show that in this case the dimension of $[Z^\nu]$ can be expressed in the dimensional base exactly as in (1.3), and the scale λ_ν is equal to the scale λ in (1.10). We can get formally the similarity conditions for the nonscalar model (2.13) if we divide, as was previously stated, instead of vectors or tensors, vector moduli or values of vector and tensor components. We shall present the results of three different approaches to the model construction on the base of Theorem π in versions (1.3), (2.4) and (2.13) (numerically the same), using a process well known in the field of applied mechanics.

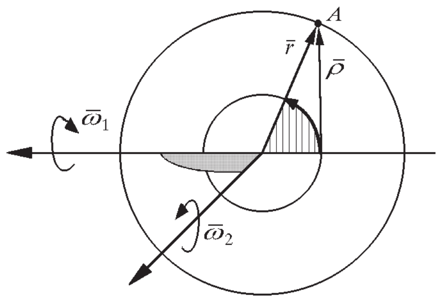

Fig. 1. Relative angular motion $\overline{\omega_2}$ of point A on the disk rotating disk with angular velocity $\overline{\omega}_1$

Let us investigate the relative motion accelerations presented in Figure 1. Accordingly, point A rotates with the constant angular velocity $\overline{\omega}_2$ along the edge of the disk. The disk is in angular motion $\overline{\omega}_1$ around its diameter. We assume that the acceleration $\widehat{Z}$ depends on

$$\widehat{Z} = \varPhi(\overline{\omega}_1, \overline{\omega}_2, t, \overline{r}, \overline{\rho}) \tag{3.7}$$

The dimensions of the function $\varPhi$ value and the arguments in the SI system of units are: $[\widehat{Z}] = [ms^{-2}]$, $[\omega_1] = [\omega_2] = [s^{-1}]$, $[t] = [s]$, $[r] = [\rho] = [m]$, $\rho = r \sin \omega_2 t$. If we wish to design the model on the base of Theorem π expressed by formula (1.3), accepting the dimensional base r, ω_1 (modulus of vectors r, ω_1) and the scales for the variables in the model

$$\begin{aligned} &\omega_1^* = \lambda_1 \omega_1 \quad r^* = \lambda_2 r \qquad\qquad \omega_2^* = \mu_1 \omega_2 \\ &t^* = \mu_2 t \qquad\ \rho^* = \mu_3 \rho \end{aligned} \tag{3.8}$$

we shall get for the process

$$\widehat{Z} = f\Big(\frac{\omega_2}{\omega_1}, \frac{\rho}{r}, t\omega_1\Big) r\omega_1^2 \tag{3.9}$$

and for the model

$$\widehat{Z}^* = f\Big(\frac{\mu_1\omega_2}{\lambda_1\omega_1}, \frac{\mu_3\rho}{\lambda_2 r}, \mu_2 t\lambda_1\omega_1\Big)\lambda_2 r\lambda_1^2\omega_1^2 \tag{3.10}$$

The scale

$$\lambda = \frac{\widehat{Z}^*}{\widehat{Z}} = \lambda_2\lambda_1^2 \tag{3.11}$$

if

$$\mu_1 = \lambda_1 \qquad \mu_2 = \frac{1}{\lambda_1} \qquad \mu_3 = \lambda_2 \tag{3.12}$$

We shall get the same result using formula (2.4) (of course, for the vector modulus, or the vector or tensor component). Let us now investigate the process description according to formulation (2.13) of Theorem π using variables as in (3.7). We shall get the scalar invariants

$$\overline{\omega}_1^2,\ \overline{\omega}_2^2,\ \overline{\omega}_1\overline{\omega}_2,\ t,\ \overline{r}^2,\ \overline{\rho}^2,\ \overline{r\rho},\ \overline{\omega}_1\overline{r},\ \overline{\omega}_2\overline{r},\ \overline{\omega}_1\overline{\rho},\ \overline{\omega}_2\overline{\rho} \tag{3.13}$$

and generators

$$\begin{aligned} &\overline{g}_1 = \overline{\omega}_1 \times (\overline{\omega}_\times\overline{\rho}) \qquad \overline{g}_2 = \overline{\omega}_2 \times (\overline{\omega}_2 \times \overline{r}) \\ &\overline{g}_3 = \overline{\omega}_1 \times (\overline{\omega}_2 \times \overline{r}) \end{aligned} \tag{3.14}$$

Let us accept the dimensional base as previously $\big(\sqrt{\omega_1^2}, \sqrt{r^2}\big)$ and scales (3.8), we shall get for (3.4)

$$\begin{aligned} &\overline{Z}^* = f_1^*\lambda_1^2\mu_3\overline{g}_1^* + f_2^*\mu_1^2\lambda_2\overline{g}_2^* + f_3^*\lambda_1\mu_1\lambda_2\overline{g}_3^* \\ &f_i^* = f_i^*\Big(\frac{\mu_1^2\omega_2^2}{\lambda_1^2\omega_1^2}, \frac{\lambda_1\mu_1\omega_1\omega_2}{\lambda_1^2\omega_1^2}, \mu_2\mu_1 t\omega_2, \frac{\mu_3^2\rho^2}{\lambda_2^2 r^2}, \frac{\lambda_2\mu_3 r\rho}{\lambda_2^2 r^2}, \frac{\mu_1\lambda_2\omega_2 r}{\lambda_1\lambda_2\omega_1 r}, \frac{\lambda_1\omega_1\mu_3\rho}{\lambda_1\omega_1\lambda_2 r}, \frac{\mu_1\omega_2\mu_3\rho}{\lambda_1\omega_1\lambda_2 r}\Big) \\ &\overline{g}_i^* = \overline{g}_i\lambda_g^i \qquad i = 1,2,3 \end{aligned} \tag{3.15}$$

(please notice that at this stage we know the values of f_i^*, $f_1 = f_1^* = f_2 = f_2^* = 1$, $f_3 = f_3^* = 2$), and for object (2.13)

$$\begin{aligned} &\overline{Z} = f_1\overline{g}_1 + f_2\overline{g}_2 + f_3\overline{g}_3 \\ &f_i = f_i\Big(\frac{\omega_2^2}{\omega_1^2}, \frac{\omega_1\omega_2}{\omega_1^2}, t\omega_1, t\omega_2, \frac{\rho^2}{r^2}, \frac{\rho r}{r^2}, \frac{\omega_2 r}{\omega_1 r}, \frac{\omega_1\rho}{\omega_1 r}, \frac{\omega_2\rho}{\omega_1 r}\Big) \qquad i = 1,2,3 \end{aligned} \tag{3.16}$$

Dividing the arguments of f_i^* by f_i for the same i, we shall get

$$\frac{\mu_1^2}{\lambda_1^2} = 1 \qquad \mu_2\lambda_1 = 1 \qquad \frac{\mu_3^2}{\lambda_2^2} = 1 \tag{3.17}$$

and $\mu_1 = \lambda_1$, $\mu_2 = 1/\lambda_1$, $\mu_3 = \lambda_2$ as in (3.12). Now, we can calculate the scales for the generators:

- for $\overline{g}_1$ we get $\lambda_g^{(1)} = \lambda_1^2\mu_3 = \lambda_1^2\lambda_2$
- for $\overline{g}_2$ we get $\lambda_g^{(2)} = \lambda_1^2\mu_3 = \lambda_1^2\lambda_2$
- for $\overline{g}_3$ we get $\lambda^{(3)} = \lambda_1\mu_1\lambda_2 = \lambda_1^2\lambda_2$

At the end we come to the following conclusion:

- the construction of a model of the investigated process produces in all considered formulations of Theorem π the same results for the values of the scales,
- the cognitive possibilities of experimental investigations on the base of Theorem π expressed by (1.3) or (2.4) are quite different in comparison to Theorem π formulated by formula (2.13).

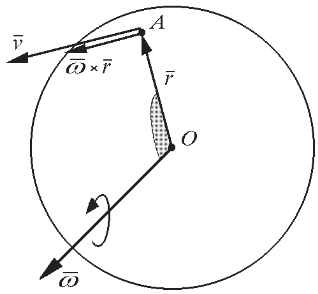

Fig. 2. Relative motion of point A with velocity $\overline{v}$ on the rotating plane

In the last case, we shall get all generators (in our example all accelerations) without any empirical investigations. If, for example, we do not know that there is – in the relative motion a centripetal acceleration and Coriolis acceleration, it is difficult to obtain pertinent knowledge on an experimental basis only. In some cases we can measure only the sum of all accelerations as was shown in Figure 2, where point A moves with a constant velocity v on a plane which rotates around point O with a constant angular motion ω – in this case $\overline{\omega}\times\overline{v}$ is parallel to $\overline{\omega}\times(\overline{\omega}\times\overline{r})$. The main difficulties in investigations on the model of a process, when the investigated quantity is modelled by a vector or tensor, are connected with measurements, exactly speaking with the knowledge about the proper direction of the acting generator. Such knowledge can only be obtained from formulation (2.13) of Theorem π (in the cases, when we are unable to get such information from the theory of the investigated process).

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Podobieństwo i projektowanie modeli dla procesów zależnych od nieskalarnych zmiennych

Streszczenie

Znana z literatury teoria podobieństwa modelowego i algorytmy projektowania modeli opracowano przy założeniu, że proces zależy od zmiennych, które są skalarami. W pracy podaje się rozwiązanie tych problemów dla nieskalarnych zmiennych,

od których zależy przebieg procesu. Podaje się odpowiednią wersję twierdzenia π dla funkcji wymiarowych zależnych od zmiennych, które mogą być tensorami i sposób wyznaczania skal. Pokazuje się, że otrzymuje się takie same rezultaty, jak dla modeli skalarnych, ale otrzymuje się pełną informację o składowych tensora opisującego badany proces, a więc pełne informacje pozwalające na opracowanie projektu badań empirycznych (nie otrzymywano ich w tradycyjnym modelu).

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