

NUMERICAL AND EXPERIMENTAL COLLAPSE ANALYSIS
OF TUBULAR MULTI-MEMBER ENERGY ABSORBERS
UNDER LATERAL COMPRESSION

SEBASTIAN LIPA
MARIA KOTELKO

*Department of Strength of Materials and Structures, Technical University of Łódź
mkotelko@p.lodz.pl*

In the paper, the problem of collapse load and energy dissipation capacity of tubular multi-member energy absorbers subject to lateral crushing load is presented. The method of analytical solution of the problem of initial collapse load and load capacity at failure both for single-member and multi-member absorbers is discussed. A numerical Finite Element (FE) model of the multi-member absorber is presented and the parametric study of the influence of different absorber parameters (number of members, thickness of tubes, configuration of tubes) upon the collapse load and energy dissipation capacity is carried out. Results of numerical calculations are compared with those obtained from the experiment conducted by the authors. Both numerical and experimental results are presented in the form of load-deformation diagrams and diagrams of deformation patterns. Some final conclusions concerning the usefulness of multi-member tubular absorbers and indications into further investigations are derived.

Key words: impact crashworthiness, crushing load, energy dissipation, tubular energy absorbers

Notation

$D = 2R$	–	tube diameter
t	–	tube thickness
σ_0	–	yield stress
σ_{ult}	–	ultimate stress
E	–	Young's modulus
E_t	–	tangent modulus
ν	–	Poisson's ratio

1. Introduction

Increasing number of accidents related to impacts of many types like car crashes, collisions of ships with icebergs or rocks, collisions of ships themselves, etc., induced rapid development of the so-called *impact crashworthiness* dealing with impact engineering problems, particularly in the field of dynamic response of structures in the plastic range and the design of *energy absorbers*. Since public demands for safe design of vehicles, ships, etc. have substantially increased in the last few decades, a new challenge appeared that aims at designing special structural members which would dissipate impact energy in order to limit deceleration and finally to stop a moveable mass (e.g. vehicle) in a controlled manner. Such a structural member, termed the energy absorber, converts totally or partially kinetic energy into another form of energy. One of the possible design solutions is the conversion of kinetic energy of an impact into energy of plastic deformation of a thin-walled metallic structural member.

There are numerous types of energy absorbers of that kind that are cited in the literature (Alghamdi, 2001). Namely, there are steel drums, thin tubes or multi-corner columns subject to compression, compressed frusta (truncated circular cones), simple struts under compression, sandwich plates or beams (particularly honeycomb cells) and many others.

A separate type of a collapsible energy absorber is a system of moderately thin-walled tubes subject to lateral crushing. Such laterally loaded cylindrical clusters have been used in impact attenuation devices of vehicles. They are also employed as crash cushions in roadside safety applications. Some of these crash cushions are composed of clusters of metallic and non-metallic tubes (Wu and Carney, 1997). Another modification of that kind of energy absorbers are tubes (cylinders) stiffened diametrically in order to increase the collapse load without adding weight to the system (Wu and Carney, 1997).

In the early sixties of the 20th century, automotive safety regulations stimulated development of the new concept of a *crashworthy (safe) vehicle* that had to fulfil integrity and impact energy management requirements (Abramowicz, 2003). A new (*original*) concept of the impact attenuation device proposed in the design of the chassis frame of the *safe vehicle* is shown in Fig. 1. The impact energy absorber is a system of thin-walled elliptical tubes subject to lateral crushing.

A designer of any impact attenuation device must meet two main (sometimes contrary) requirements: the initial collapse load must not be too high in order to avoid unacceptably high impact velocities of the vehicle. On the other hand, the main requirement is possibly the highest energy dissipation

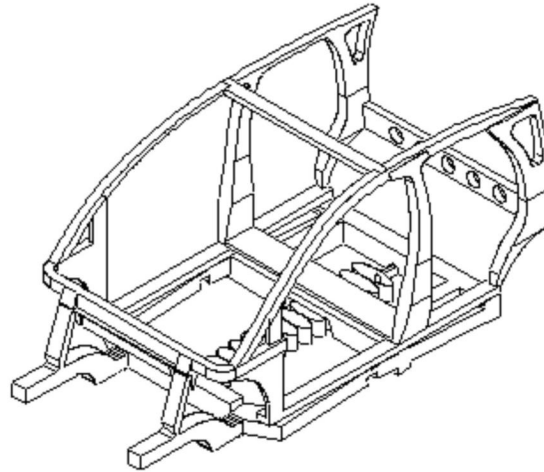


Fig. 1. The concept of the *safe chassis frame*

capacity, which may not be achieved if the collapse load of the impact device is too low. The latter may result in dangerously high occupant "ridedown" decelerations (Wu and Carney, 1997).

Different possible configurations of members of the multi-member energy absorber shown in Fig. 1 (number of members, thickness of subsequent tubes, their shape, etc.) allow a designer to fulfil the requirements mentioned above. However, the main advantage of the impact attenuation device formed as a single tube or multi-member cluster of laterally crushed tubes is its load-deformation characteristics, i.e. its collapsing stroke approaching even 95% of original diameter of the tube. These unquestionable pros of such energy absorbers are main reasons of great interest in the design and research of these devices in recent years.

Crushing of a single circular, cylindrical tube between two rigid plates was investigated by De Runtz and Hodge (1963). They carried out experiments and approximate theoretical analysis of the problem, based on the lower-bound approach. Wu and Carney (1997) investigated crushing of single elliptical tubes, both braced (stiffened diametrically) and unbraced, subject to compression between two rigid plates. They theoretically analysed the collapse behaviour of the tube using the kinematical method (upper-bound approach) and also the static method (lower-bound approach). They compared the analytical results with Finite Element results obtained using ABAQUS software. The same authors compared their theoretical results with experimental ones (Wu and Carney, 1998).

The works mentioned above were devoted to the analysis of single tubular energy absorbers subject to lateral crushing load. Any results concerning collapse analysis of tubular multi-member devices of that kind have not been reported in the available literature so far. The energy dissipated by a multi-member absorber may be greater than the energy dissipated by a single tube subject to lateral crushing, the time of the absorber response during collision is longer in a multi-member device and finally, as mentioned above, a designer may shape such an absorber in many ways that allow one to fulfil different integrity and impact requirements. Since then, the present work is devoted to both theoretical and experimental analysis of a multi-member tubular absorber subject to lateral crushing load.

2. Structural problem

The subject of the present analysis is a multi-member energy absorber built from circular cylindrical tubes subject to lateral compressive load (Fig. 2.).

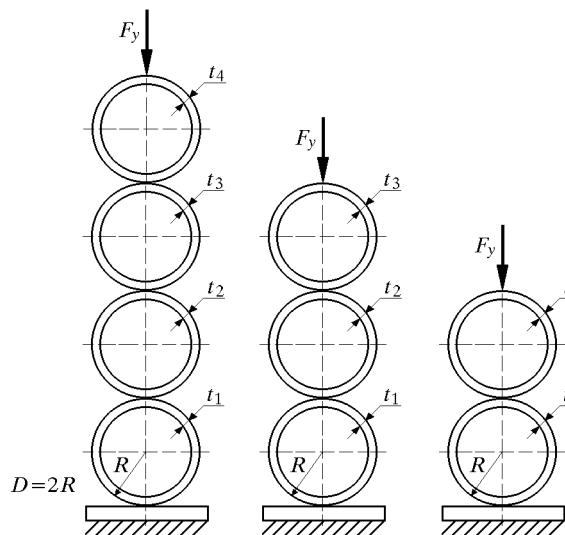


Fig. 2. Lay-out of the absorber

The aim of the analysis is the parametric study of the multi-member absorber, namely the investigation of the influence of the number, dimensions and lay-out of the absorber upon its collapse behaviour, load-capacity at collapse and energy dissipation capacity. The theoretical analysis is conducted using

the Finite Element Method (FEM). Simultaneously, results of experimental investigations of the absorber built from steel tubes are presented.

The theoretical parametric study is carried out for different dimensions and configurations of the absorber as shown in Table 1.

Table 1. Absorbers dimensions and lay-out

Absorber symbol	Number of members	t_1 [mm]	t_2 [mm]	t_3 [mm]	t_4 [mm]	D [mm]
AB2	2	5.5	5.5	–	–	44.25
AB3-1	3	5.5	5.5	5.5	–	44.25
AB3-2	3	4	3	2	–	42
AB3-3	3	2	3	4	–	42
AB3-4	3	4	2	3	–	42
AB3- t	3	$t_1 = t$	$t_2 = t$	$t_3 = t$	–	42
AB4	4	5.5	5.5	5.5	5.5	44.25

Lengths of all analysed tubular members are constant and amounts $l = 150$ mm. Each absorber member has the same external diameter.

3. Theoretical collapse of tubular energy absorbers under lateral compression

The analysis of the collapse behaviour of the absorber and particularly, of its load-capacity at collapse may be achieved in two ways: by applying the *lower-bound* estimation of the load-capacity with the implementation of the static method or using the *upper-bound* estimation – with the implementation of the kinematical method. In the first case the condition of equilibrium and the yield condition have to be fulfilled, which may be accomplished using e.g. the method of statically permissible stress fields (Szczepiński and Szlagowski, 1985; Gill, 1976). In the latter case, a kinematically permissible *plastic mechanism of failure* has to be analysed (Kotelko, 2000). The load-capacity at collapse is then determined using the Principle of Virtual Velocities

$$P\dot{\delta} = \int_V \sigma_{ij} \dot{\epsilon}_{ij}^p(\boldsymbol{\beta}, \boldsymbol{\chi}) dV \tag{3.1}$$

where

- δ – generalised displacement
- $\dot{\delta}$ – rate of change of the generalised displacement

- P – generalised force
- β – vector of kinematical parameters of the plastic mechanism
- χ – vector of geometrical parameters of the plastic mechanism
- $\dot{\varepsilon}_{ij}^p$ – rate of change of the plastic strain tensor.

The first step in the solution to the problem defined above is to determine the geometry of the plastic mechanism of the absorber, thus to formulate the vectors χ and β in equation (3.1). They should be determined in through theoretical identification of kinematically permissible fields of displacements or velocities using FEM, and have to be verified by experimental tests.

3.1. Crushing of laterally loaded tube (lower-bound estimation)

A metal cylindrical tube under lateral load (Fig. 3) forms a plastic mechanism of failure when the load reaches the value associated with the creation of four plastic hinges located by 90° from each other.

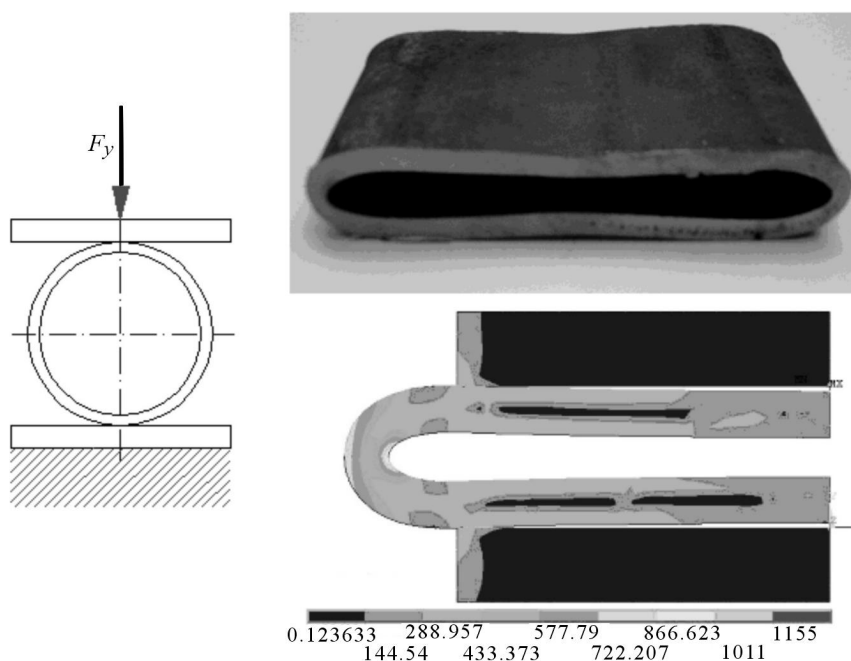


Fig. 3. Crushing of a tube between two rigid plates

Figure 3 shows a characteristic deformation form of the steel tube at the final stage of failure – a "figure of eight" – and the corresponding deformation pattern obtained from FE analysis.

De Runtz and Hodge (1963) obtained the complete lower-bound solution for a thin circular tube subject to lateral crushing between two parallel rigid plates on the assumption that the material is rigid-perfectly plastic. They evaluated the initial yield load of the tube using both Huber-von Mises and Tresca yield criteria.

The initial yield load is given by the following formula

$$P_0 = \frac{4M_0}{R} \tag{3.2}$$

According to the Huber-von Mises yield criterion, $M_0 = \sigma_0 t^2/4$ is a fully plastic moment at the plastic hinge (Kotelko, 2000). De Runtz and Hodge derived the following load-displacement relation

$$P = \frac{P_0}{\sqrt{1 - \left(\frac{u}{D}\right)^2}} \tag{3.3}$$

where u is the displacement of the load application point. Figure 4 shows the comparison of the results obtained from de Runtz and Hodge formula (3.3), FE analysis and the experiment performed by the authors within this study.

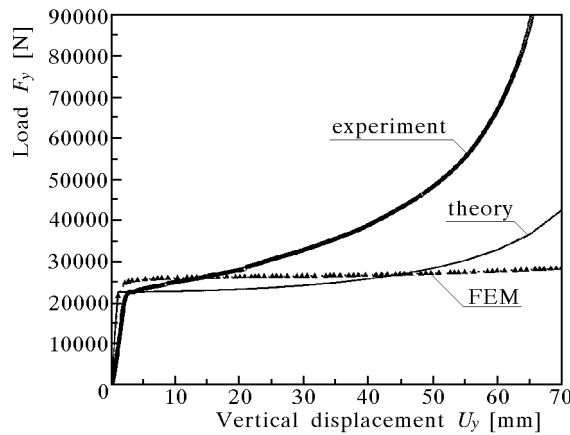


Fig. 4. Load-displacement characteristics of a single tube laterally crushed between two rigid plates

The agreement of the initial yield load P_0 is very good for all diagrams. Both theoretical Hodge and de Runtz results as well as FE results are underestimated above the initial yield point. In the case of the analytical solution (Hodge and de Runtz), it comes from the negligence of the strain-hardening effect, in the latter case this factor is taken into account but the material characteristic is still very approximate (bilinear).

3.2. Kinematical method – upper-bound estimation

In the case of multi-member absorber consisting of several tubes, the upper-bound collapse load in terms of the displacement parameter δ can be derived from general relation (3.1), which in that particular case assumes the form

$$P\dot{\delta} = \sum_{i=1}^n M_0\dot{\beta}_i \quad (3.4)$$

where β_i is an angle of relative rotation of two arcs of a particular tube at the i th plastic hinge and n is the total number of plastic hinges in the n -hinge plastic mechanism. A theoretical model of the plastic mechanism, i.e. a number of plastic hinges, their situation and geometrical relations between subsequent angles of rotation β_I and displacement δ (a control parameter of the analysis) should be established on the basis of both experimental tests performed on metal absorbers and FE analysis.

This analysis exceeds the scope of the present paper and will be the subject of a separate study. The objective of this estimation is to elaborate a relatively simple analytical-numerical solution and a computer program based on the kinematic approach, which will allow a designer to calculate the load-capacity at collapse for a multi-member tubular absorber.

Such a "quick" analysis, much less time consuming than FE calculations, would be of aid in the initial stage of the design process.

4. Finite element model

The finite element model of the analysed absorber was loaded as shown in Fig. 1. The software package ANSYS 7.1 has been used for numerical calculations. Since the problem was considered as plane one, plane rectangular and triangular elements were applied, namely PLANE183 and PLANE42. The physically non-linear problem (non-linear constitutive relations applied) has been solved using the bi-linear material characteristics with the linear strain hardening beyond the yield point and the geometrically non-linear problem as well. Calculations were conducted step by step in an incremental procedure. The control parameter was the displacement of the force application point. The concentrated force was applied to the top point of the model. The force application in the model of the absorber tested experimentally was different and is described in the next section. Material parameters applied in the

calculations carried out for the absorbers of dimensions shown in Table 1 were as follows

$$\begin{aligned}
 E &= 2 \cdot 10^5 \text{ MPa} & \nu &= 0.3 \\
 \sigma_0 &= 250 \text{ MPa} & E_t &= 2500 \text{ MPa}
 \end{aligned}$$

The contact pairs have been used in the FE model in order to represent various 2D "target" surfaces for the associated contact elements. The contact elements themselves overlaid the solid elements describing the boundary of the deformable body and were potentially in contact with the target surface. The target surface was discretized by a set of target segment elements and was paired with its associated contact surface.

Numerical calculations were carried out for all absorbers indicated in Table 1, including absorber AB3-*t* with four different thicknesses of tubes. Separate calculations were conducted for the absorber under experimental investigation. Dimensions and material parameters of the absorber subject to experimental investigation that were taken into account in its FE model are given in the next section.

Figure 5 shows deformation patterns and maps of equivalent stresses calculated according to the Huber-von Mises yield criterion for absorber AB3-1 for three different load values.

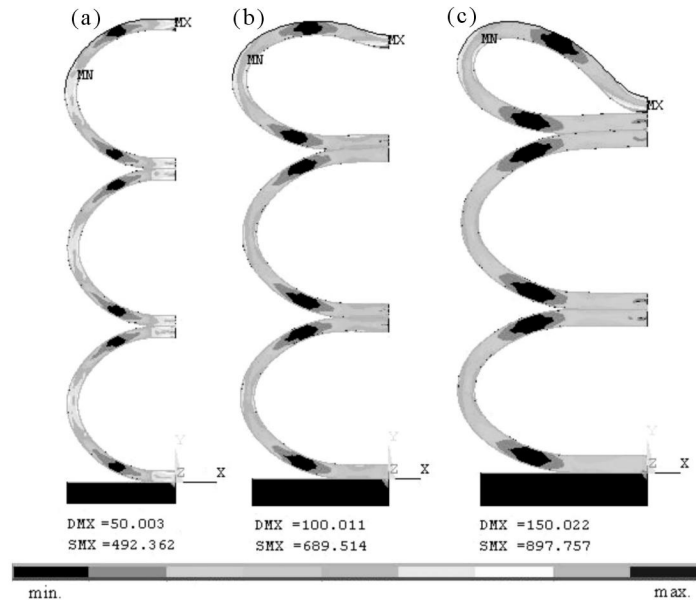


Fig. 5. Deformation patterns and equivalent stress fields of absorber AB3-1; (a) $F_y = 15.04 \text{ kN}$, (b) $F_y = 17.8 \text{ kN}$, (c) $F_y = 21.13 \text{ kN}$

5. Experimental analysis

The experimental test has been carried out on a model of a three-member absorber of equal wall thickness of each tube. Dimensions and material parameters of the tested model are shown in Table 2. The model was manufactured from steel tubes without welding seam. The tubes were spot-welded along their generating lines.

The experiment was conducted on the testing machine Instron of the loading range 200 kN. The tested model was subject to compression between two rigid, thick steel plates. The compressive force was applied through the prismatic steel bar. Both the compressive force and model deformation (displacement of the upper crosshead beam of the testing machine) were recorded using an integrated, computer aided measurement system of the testing machine. The velocity of loading was relatively low so that the test was of the quasi-static character. The experimental stand is shown in Fig. 6.

Table 2. Dimensions and material parameters of the tested model

Material parameters	Dimensions
$\sigma_{f0} = 335 \text{ MPa}$	$D_{ext} = 43.75 \text{ mm}$
$\sigma_{ult} = 440 \text{ MPa}$	$t = 5 \text{ mm}$
$E = 192000 \text{ MPa}$	$l = 150 \text{ mm}$
$E_t = 1050 \text{ MPa}$	

Simultaneously with the experimental test, FE analysis of the tested model was performed. The FE model was loaded in the same way as the tested specimen, i.e. through the rigid plate completed with the prismatic "inset". The base of the model was a rigid thick plate. The deformation pattern of the tested model in the initial phase of the loading process and the corresponding FE map of equivalent stresses are presented in Fig. 7.

The deformation scenario observed during the test was insignificantly different from the deformation pattern obtained from FE analysis. In the tested model, the deformation and yielding of the first member was observed first, then the same happened with the last member and finally – with the intermediate (compare the circular shape of the lower and elliptical shape of the intermediate tube in Fig. 7), while the theoretical deformation maps (Fig. 5) indicate subsequent deformation of the first, second and last member. It was induced by initial imperfections of the tested model on the contrary to the ideal model assumed in the theoretical calculations.

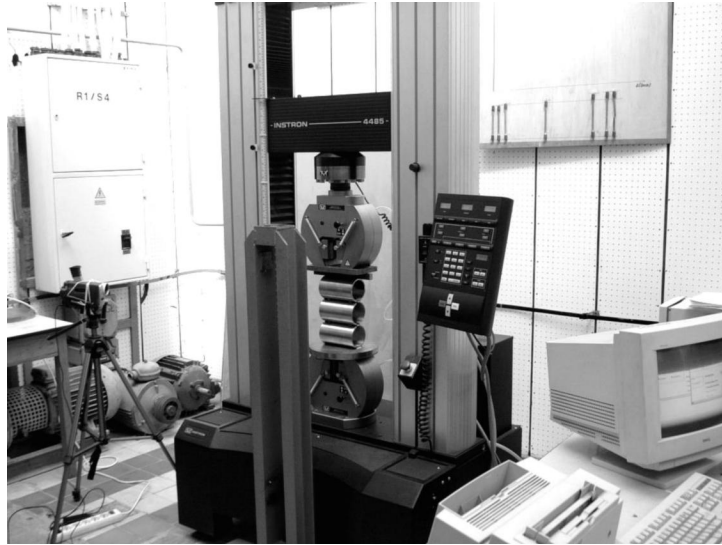


Fig. 6. General view of the experimental stand

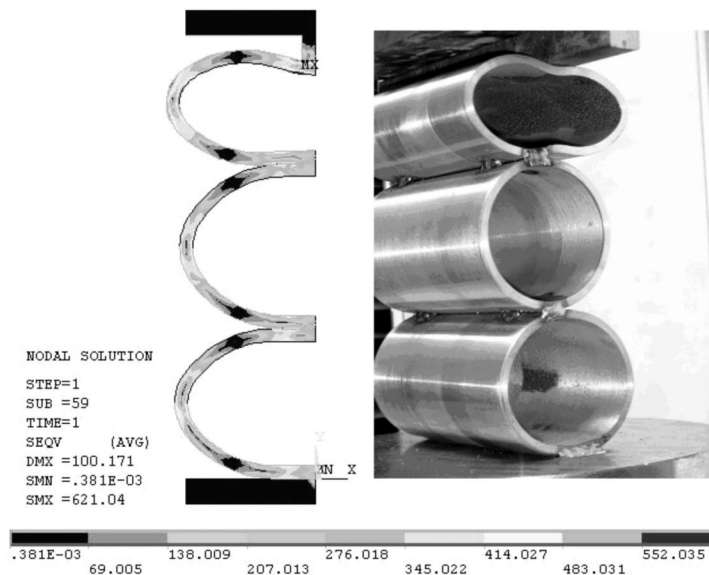


Fig. 7. Deformation of the tested model. Comparison with FE results

6. Numerical and experimental results

6.1. Comparison of theoretical and experimental results

A comparative diagram of the compressive load in terms of the displacement u_y of the load application point is shown in Fig. 8. Two diagrams obtained from the experiment and FE analysis, respectively, are superimposed here.

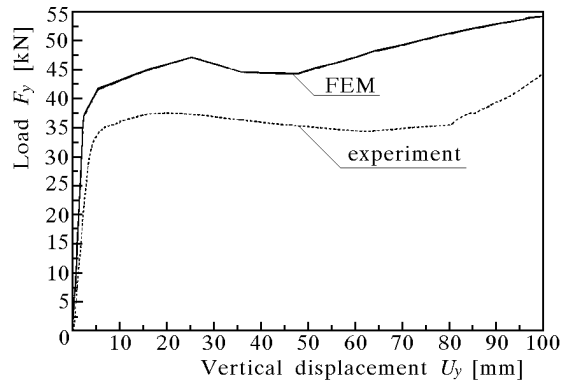


Fig. 8. Load-deformation characteristics of the absorber tested experimentally

The character of both diagrams is similar. The agreement of the values of the initial yield-load is satisfactory. Results of FE analysis are overestimated since they concern the ideal theoretical model without any initial imperfections.

6.2. Parametric study of multi-member tubular absorbers under lateral compression

In this section, results of FE parametric analysis of the absorber behaviour in a wide range of loading from zero up to the "jamming" load are presented. The results are shown in the form of load-deformation diagrams, from which both the initial yield loads and subsequent crushing loads corresponding to yield points in subsequent absorber members can be evaluated. The area below the load-deformation curve is equivalent to the amount of the energy accumulated by the absorber.

Load-deformation diagrams for three absorbers with different numbers of members but the same thickness of each member are presented in Fig. 9.

It is interesting that the gradient of the crushing load beyond the first yield point is nearly the same for two- and three-member absorbers, and it

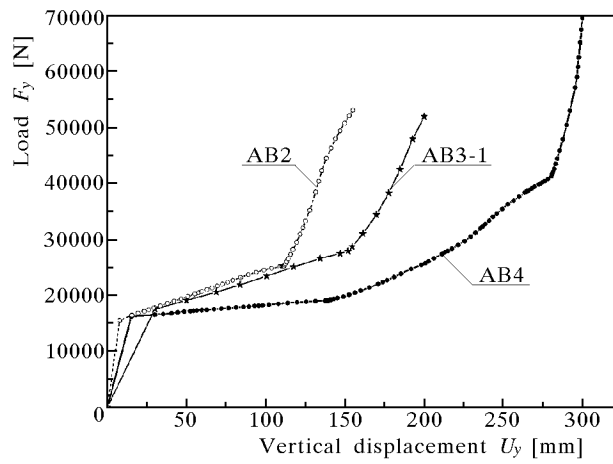


Fig. 9. Load-deformation characteristics of multi-member absorbers of constant tube thickness

diminishes for the four-member one. The four-member absorber is "jammed" at the deformation much greater than other absorbers under investigation. Thus, if there are no contraindications from the structural point of view, a multi-member absorber (of member number greater than 3) is more effective since it is more ductile and the energy dissipated is larger than in the case of two- or three-member absorbers. One can also see in Fig. 9 that the initial yield loads for all absorbers are nearly the same.

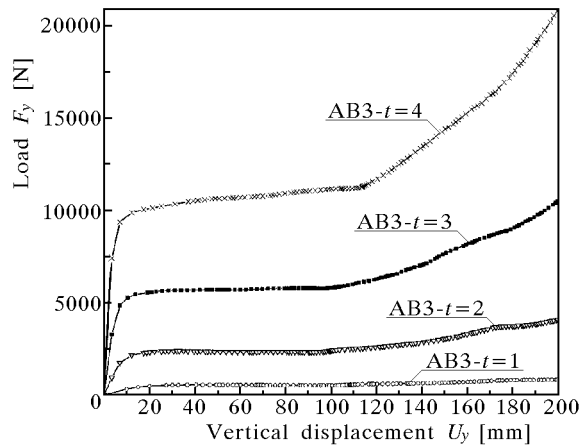


Fig. 10. Load-deformation characteristics of multi-member absorbers of constant tube thickness for different values of the member thickness

Figure 10 shows the load-deformation behaviour of the three-member absorber of the same thickness of each member for different values of the member thickness. Interesting is that the displacement corresponding to the initial yield load is nearly the same and does not depend on the tube thickness. We can also state that the thicker the tubes are, the more distinctive is the point of the "transition" of plastic deformation from the first member to the next one, and the larger is the gradient of the crushing load in terms of the displacement.

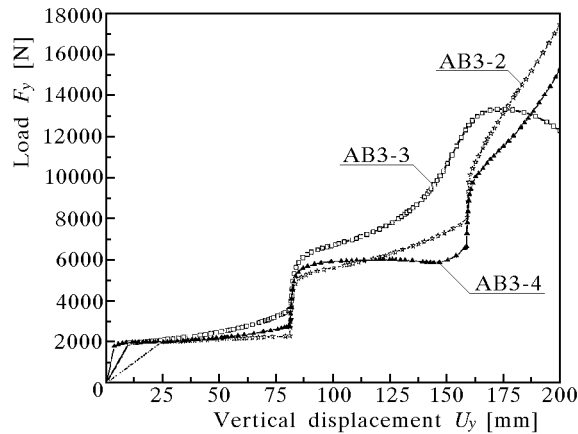


Fig. 11. Load-deformation characteristics of three-member absorbers of different tube thicknesses and for different tube configurations

In Fig. 11, load-deformation diagrams of three-member absorbers of different tube thicknesses and for different tube configurations are presented. Like in Fig. 9, for all absorbers of different tube configurations, the initial yield load is the same (about 2 kN), but the corresponding deformation is different, i.e. the configuration of tubes of different thickness influences "sensitivity" of the absorber to deformation. The most rigid is absorber AB3-4, the most ductile – AB3-2. If deformation of the absorber is limited due to certain structural reasons than the most effective (as far as the amount of the dissipated energy is concerned) is AB3-3, if there are no limitations on the final deformation, one can apply AB3-4 or AB3-2.

Deformation patterns of three-member absorbers of different tube thicknesses and different tube configurations corresponding to diagrams shown in Fig. 11 are presented in Fig. 12.

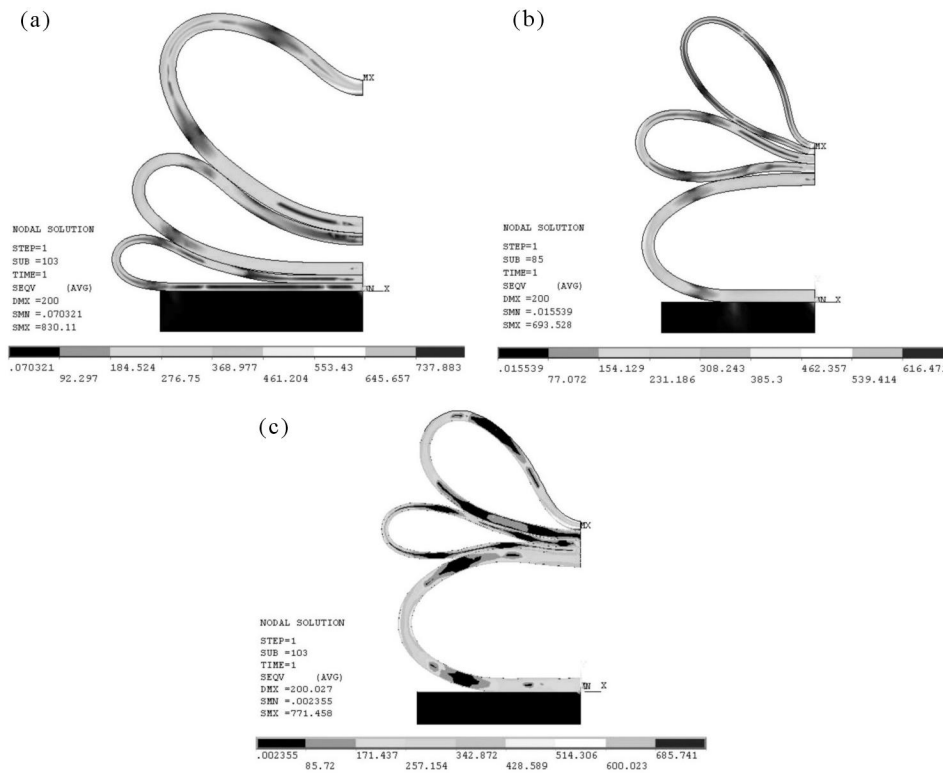


Fig. 12. Deformation patterns of three-member absorbers of different tube thickness and for different tube configurations; (a) AB3-3, (b) AB3-2, (c) AB3-4

7. Final remarks

A satisfactory agreement between FE results and results of the test performed on the absorber made of three steel tubes has been obtained. This good agreement confirms that the FE model used in numerical calculations was adequate to the problem.

The results of parametric study based on using the FE method indicate the usefulness of the application of multi-member tubular absorbers subject to lateral crushing load, mainly due to definitely higher energy dissipation capacity than in the case of a single member absorber of that kind. In the process of design of a multi-member tubular absorber one can "reconcile" contrary requirements: limitation of the initial collapse load due to impact velocity of the vehicle and – on the other hand – possibly high energy dissipation capacity, using the most adequate configuration as well as number and thickness

of particular tubular members (see comments regarding Fig. 10 and Fig. 11). Simultaneous fulfilment of both requirements mentioned above is much more difficult or even impossible in the case of a single tubular member absorber.

Further investigation on the elaboration of an analytical-numerical solution and a computer program based on the kinematic approach, leading to a relatively simple algorithm of calculation of the load-capacity at collapse and energy dissipation capacity for the multi-member tubular absorber, should be continued. The theoretical solution has to take into account the strain-hardening effect in the material of the absorber.

The presented numerical and experimental results concern static analysis only, and are of preliminary character. Thus, further experimental tests performed on absorbers subject to dynamic (impact) load would be of aid in better understanding of the character of the impact process of multi-member tubular absorbers. Also an implicit dynamic FE analysis is planned in order to complete the study on the dynamic response of the absorbers.

The present study does not concern the stability analysis of the system. In the case of relatively thin tubular members ($t/d \lesssim 0.015$) or a larger number of members, the analysis of possible equilibrium paths (stable and unstable) has to be carried out. A similar analysis has to be performed for systems with initial imperfections. These problems, however, are beyond the scope of the present work.

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Numeryczna i eksperymentalna analiza zniszczenia rurowych wieloczłonowych absorberów energii poddanych bocznemu zgniotowi

Streszczenie

Artykuł przedstawia zagadnienia nośności w fazie zniszczenia i zdolności rozproszenia energii, wieloczłonowych, rurowych absorberów energii poddanych bocznemu zgniotowi. Zaprezentowane zostały metody analizy wstępnej fazy zniszczenia i ilości energii dysypowanej przez absorber jedno- i wieloczłonowy. Przedstawiono numeryczny model elementów skończonych (FE) absorberów wieloczłonowych, jak również omówiono wpływ różnych parametrów absorbera, takich jak ilość członów, grubość elementów rurowych oraz konfiguracja tych elementów na fazę zniszczenia i ilość energii dysypowanej. Wyniki obliczeń numerycznych zostały porównane z wynikami uzyskanymi przez eksperyment autorów. Zarówno wyniki numeryczne, jak i eksperymentalne są przedstawione w formie wykresów obciążenia – odkształcenia i map odkształceń. Przedstawiono konkluzje dotyczące zastosowania wieloczłonowych absorberów rurowych oraz wskazówki do dalszych badań.

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