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# A METHOD OF POWER DISTRIBUTION IN THE POWER TRANSMISSION SYSTEM OF A REMOTELY OPERATED VEHICLE

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The paper addresses a method of distribution of propulsion for an unmanned underwater vehicle. The method is developed on basis of the decomposition of a configuration matrix describing the layout of thrusters in a power transmission system. The proposed solution of power distribution is worked out for the control system of the remotely operated vehicle "Ukwiał" designed and built for the Polish Navy. The algorithm has been tested for track-keeping control both for faultless work of thrusters and failure of one of them. Some computer simulations are provided to demonstrate the effectiveness and correctness of the approach.

*Key words:* underwater vehicle, power transmission system, distribution of propulsion

## 1. Introduction

Underwater Robotics has favoured an increasing interest for the last years. Currently, it is common to use unmanned underwater vehicles (UUVs) to accomplish missions such as inspection of coastal and off-shore structures, cable maintenance, as well as hydrographical and biological surveys. In the military field, they are employed in such tasks as surveillance, intelligence gathering, torpedo recovery and mine counter measures. The main benefits of usage of the UUVs can be the removal of men from dangers of the undersea environment and the reduction of cost of exploration of the underwater space.

There are various categories of the UUVs. The most often used underwater robot is a remotely operated vehicle (ROV). The ROV is usually connected to a surface ship by a tether, which all communication is wired through. The drag from the tether influences the vehicle motion and may represent significant disturbances and energy loss. The ROV is equipped in a power transmission system and controlled only by thrusters. Simultaneously, the spatial station-keeping or tracking of the underwater vehicle is a difficult task for a human operator, and hence supervisory control has been developed toward increasing its local intelligence and autonomy.

An automatic control of underwater vehicles is a difficult problem due to their strongly coupled and highly nonlinear dynamics. Moreover, the dynamics can change according to the alteration of configuration to be suited to the mission. In order to cope with those difficulties, the control system should be flexible. An interesting review of classical and modern techniques adopted to control the dynamic behaviour of unmanned underwater vehicles has been provided in Craven *et al.* (1998).

For the conventional ROVs, the basic motion is the movement in a horizontal plane with some variation due to diving. It operates in the crab-wise manner in 4 degrees of freedom (DOF) with small roll and pitch angles that can be neglected during normal operations. Therefore, it is purpose full to regard 3-dimensional motion of the vehicle as a superposition of two displacements: motion in the horizontal plane and motion in the vertical plane. It allows one to divide the vehicle power transmission system into two independent subsystems, i.e. the subsystem realizing the vertical motion and the subsystem responsible for motion in the horizontal plane. A general structure of such a system is shown in Fig. 1.

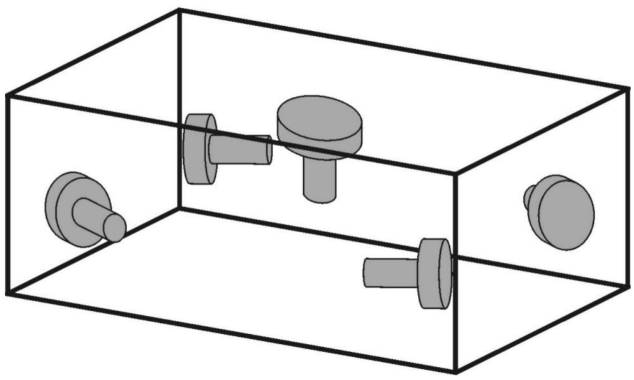

Fig. 1. A structure of power transmission system with 5 thrusters

The first subsystem usually consists of 1 or 2 thrusters generating a driving force acting along the vertical axis. In this subsystem, the distribution of propulsion is not a complicated task done in such a way that the thrust of a propeller or sum of thrusts of the propellers is equal to the commanded input. The second one consists of at least 3 thrusters assuring surge, sway and yaw motion. The most frequently applied solution is the use of 4 thrusters mounted askew in relation to the main axes of the vehicle symmetry (see Fig. 1). Demanded inputs, i.e. forces along the roll and lateral axes and moment around the vertical axis are linear combinations of propeller thrusts produced by all subsystem thrusters. Hence, from the operational point of view, the control system should have a procedure of power distribution among the thrusters. The procedure should include principles of distribution and determine such an allocation of thrusts of the propellers that the obtained values of the driving forces and moment would be equal to the desired input.

The objective of the paper is to present a method of power distribution for the underwater vehicle. The proposed solution assures proper motion of the vehicle not only in the case of correct work of thrusters but also thruster failure.

The paper consists of the following six sections. A brief description of the dynamics and control system of the underwater vehicle is given in the next section. In Section 3 a thruster model is discussed. An algorithm of power distribution is presented in Section 4. Section 5 provides results of simulation study. The concluding remarks are given in Section 6. A model of the remotely operated vehicle is contained in Appendix A.

## 2. Dynamics and control system of the underwater vehicle

The general motion of a marine vehicle in 6 DOF can be described by the following vectors (see e.g. Fossen, 1994)

$$\begin{aligned} \boldsymbol{\eta} &= [x,y,z,\phi,\theta,\psi]^\top \qquad \boldsymbol{v} = [u,v,w,p,q,r]^\top \\ \boldsymbol{\tau} &= [X,Y,Z,K,M,N]^\top \end{aligned} \tag{2.1}$$

where

- $\boldsymbol{\eta}$ – position and orientation vector with elements in the earth-fixed coordinate system
- $\boldsymbol{v}$ – linear and angular velocity vector with elements in the body-fixed coordinate system
- $\boldsymbol{\tau}$ – describes the forces and moments acting on the vehicle in the body-fixed coordinate system.

The nonlinear dynamic equations of motion can be written in form

$$\mathbf{M}\dot{\boldsymbol{v}} + \mathbf{C}(\boldsymbol{v})\boldsymbol{v} + \mathbf{D}\boldsymbol{v} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \tag{2.2}$$

where

| | | |
|---|---|---|
| $\mathbf{M}$ | – | inertia matrix (including added mass) |
| $\mathbf{C}(\boldsymbol{v})$ | – | matrix of Coriolis and centripetal terms (including added mass) |
| $\mathbf{D}(\boldsymbol{v})$ | – | hydrodynamic damping and lift matrix |
| $\boldsymbol{g}(\boldsymbol{\eta})$ | – | vector of gravitational forces and moments. |

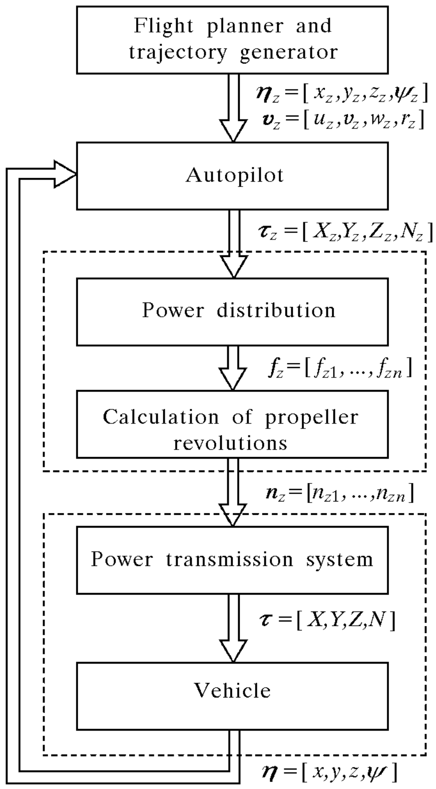


Fig. 2. Block diagram of the control system

Figure 2 presents majority elements of the vehicle control system. The flight planner and trajectory generator provide the desired vehicle position and orientation as functions of time. The autopilot computes desired vehicle forces and moment by comparing the desired vehicle position and orientation with their current estimates based on sensors measurements. The corresponding value of each thruster force is computed in the power distribution module. Then, the desired propeller revolution of each thruster is computed by using a mapping from the thrust demand to propeller revolution.

## 3. Thruster model

The relationship between the vector of forces and moments $\boldsymbol{\tau}$ acting on the vehicle and the control input of thrusters $\boldsymbol{u}$ is a complicated function depending on density of water, tunnel length and its cross-section area, propeller diameter and revolutions and the vehicle velocity vector $\boldsymbol{v}$

$$\boldsymbol{\tau} = f(\boldsymbol{v}, \boldsymbol{u}) \tag{3.1}$$

where $f(\cdot)$ is a nonlinear function. A detailed analysis of the thruster dynamics can be found e.g. in Healey *et al.* (1995) or Charchalis (2001).

In many practical applications, model (3.1) is approximated by the so called affine model (see Fossen, 1994), e.g. a system being linear in its input

$$\boldsymbol{\tau} = \mathbf{B}\boldsymbol{u} \tag{3.2}$$

where $\mathbf{B}$ is a known non-square constant matrix.

For the affine model, $\boldsymbol{u}$ can be computed as

$$\boldsymbol{u} = \mathbf{B}^*\boldsymbol{\tau} \tag{3.3}$$

where $\mathbf{B}^*$ is the matrix pseudoinverse to the matrix $\mathbf{B}$, i.e. $\mathbf{B}^* = \mathbf{B}^\top(\mathbf{B}\mathbf{B}^\top)^{-1}$.

The condition of usability of the above dependence is the proper work of all thrusters. If any is non-operational, it can not be used. It is a main disadvantage of the solution based on equation (3.3). To cope with this problem, special algorithms should be implemented in the control system. In the next section one of the possible approaches is proposed.

Farther in the paper, considerations are restricted to motion of the vehicle in the horizontal plane. This limitation results from the construction of the propulsion system. As mentioned in Section 1, for most of the ROVs, it is composed of two subsystems. Only the distribution of propulsion in the subsystem responsible for motion in the horizontal plane is complex. Usually, this subsystem consists of 4 thruster layout symmetrical around the gravity centre and it assures linear motion in the $X$ and $Y$ axes and rotational motion around the $Z$ axis. Such a solution, presented in Fig. 3, requires a specialized procedure for allocation of thrust among the thrusters.

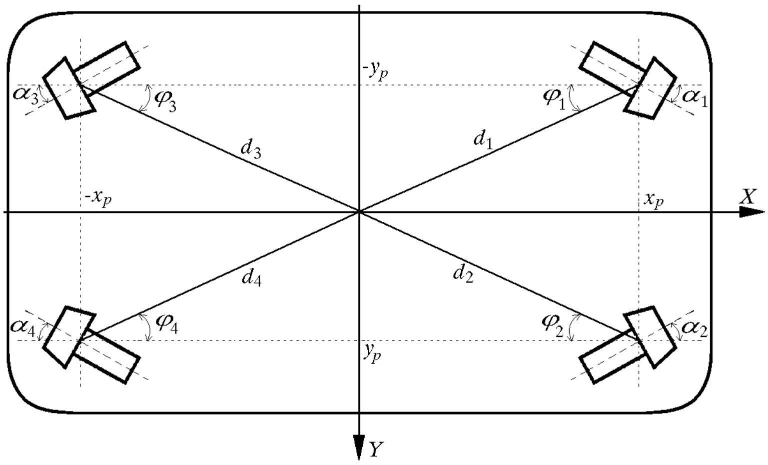


Fig. 3. Layout of thrusters in the subsystem responsible for horizontal motion

Transforming (3.2), the vector of forces and moment $\boldsymbol{\tau}$ acting on the vehicle in the horizontal plane can be described in terms of thrusts of the propellers by the following expression (Garus, 2003)

$$\boldsymbol{\tau} = \mathbf{TP}\boldsymbol{f} \tag{3.4}$$

where $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^\top$; $\tau_1$ is the force in the $X$ direction, $\tau_2$ – force in the $Y$ direction, $\tau_3$ – moment around the $Z$ axis and

$\mathbf{T}$ – thruster configuration matrix

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \\ \boldsymbol{t}_3 \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 & \cos\alpha_4 \\ \sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 & \sin\alpha_4 \\ d_1\sin\gamma_1 & d_2\sin\gamma_2 & d_3\sin\gamma_3 & d_4\sin\gamma_4 \end{bmatrix}$$

$$\gamma_i = \alpha_i - \varphi_i \qquad i = 1, 2, 3, 4$$

$\alpha_i$ – angle between the roll axis and direction of the propeller thrust $f_i$

$d_i$ – distance of the $i$th thruster from the centre of gravity

$\varphi_i$ – angle between the lateral axis and line connecting the centre of gravity with the $i$th thruster centre of symmetry

$\boldsymbol{f}$ – thrust vector, $\boldsymbol{f} = [f_1, f_2, f_3, f_4]^\top$

$\mathbf{P}$ – diagonal matrix of thrusters readiness

$$p_{ii} = \begin{cases} 0 & \text{for } i\text{th thruster off} \\ 1 & \text{for } i\text{th thruster active} \end{cases}$$

Let us note that the elements of the thruster configuration matrix $\mathbf{T}$ are geometry dependent and can be obtained for each vehicle in advance.

## 4. Procedure of power distribution

It can be proved (see Kiełbasinski and Schwetlick, 1992; Proskuryakov, 1978) that for every matrix $\mathbf{A} = [a_{ij}]_{m\times n}$ there exists such orthogonal matrices $\mathbf{U} = [u_{ii}]_{m\times m}$ and $\mathbf{V} = [v_{jj}]_{n\times n}$ that

$$\mathbf{U}^\top\mathbf{A}\mathbf{V} = \mathbf{S} = \operatorname{diag}(\sigma_1, ..., \sigma_l) \tag{4.1}$$

where

$$\begin{aligned} &l = \min(m, n) && r = \operatorname{rank}(\mathbf{A}) \\ &\sigma_1 \geqslant \sigma_2 \geqslant ... \geqslant \sigma_r > 0 && \sigma_{r+1} = ... = \sigma_l = 0 \end{aligned}$$

The numbers $\sigma_1, ..., \sigma_l$ are called singular values of the matrix $\mathbf{A}$. Transforming (4.1) and replacing $\mathbf{A}$ by $\mathbf{T}$, the following expression for the matrix $\mathbf{T}$ is obtained

$$\mathbf{T} = \mathbf{U}\mathbf{S}\mathbf{V}^\top \tag{4.2}$$

where $\mathbf{U}$, $\mathbf{V}$ are the orthogonal matrices of the dimension $3\times 3$ and $n\times n$, respectively.

$$\mathbf{S} = [\boldsymbol{S}_T \,|\, \mathbf{0}] = \left[\begin{array}{ccc|c} \sigma_1 & 0 & 0 & \\ 0 & \sigma_2 & 0 & \mathbf{0} \\ 0 & 0 & \sigma_3 & \end{array}\right]$$

$\mathbf{S}_T$ is the diagonal matrix, dim $3\times 3$ and $\mathbf{0}$ – null matrix, dim $3\times n-3$.

Decomposition of the matrix $\mathbf{T}$ allows one to work out a computationally convenient procedure to calculate the thrust vector $\boldsymbol{f}$. The procedure will be regarded for two cases:

1) all thrusters are operational $(\mathbf{P} = \mathbf{I})$

2) one of the thrusters is off due to a fault $(\mathbf{P} \neq \mathbf{I})$.

### 4.1. Algorithm for all thrusters active

Let us denote: $\boldsymbol{\tau}_z = [\tau_{z1}, \tau_{z2}, \tau_{z3}]^\top$ – required input vector, $\boldsymbol{f} = [f_1, f_2, ..., f_n]^\top$ – thrust vector necessary to generate the input vector $\boldsymbol{\tau}_z$ and $n$ – number of thrusters.

Substituting (4.2) into equation (3.4) gives

$$\boldsymbol{\tau}_z = \mathbf{TP}\boldsymbol{f} = \mathbf{USV}^\top\mathbf{P}\boldsymbol{f} \tag{4.3}$$

Multiplying both sides by $\mathbf{U}^{-1}$ yields

$$\mathbf{U}^{-1}\boldsymbol{\tau}_z = \mathbf{SV}^\top\mathbf{P}\boldsymbol{f} \tag{4.4}$$

By denoting $\mathbf{S}^* = [\boldsymbol{S}_T^{-1}, \mathbf{0}]^\top$ and taking into account that $\mathbf{P} = \mathbf{I}$ and $\mathbf{U}^{-1} = \mathbf{U}^\top$, Eq. (4.4) can be written in the form

$$\mathbf{S}^*\mathbf{U}^\top\boldsymbol{\tau}_z = \mathbf{V}^\top\boldsymbol{f} \tag{4.5}$$

Taking advantage of the orthogonal matrix property that $\mathbf{V}^{-\top} = \mathbf{V}$, the following simple expression for calculation of the thrust vector is obtained

$$\boldsymbol{f} = \mathbf{VS}^*\mathbf{U}^\top\boldsymbol{\tau}_z = \mathbf{V}\begin{bmatrix}\mathbf{S}_T^{-1}\\ \mathbf{0}\end{bmatrix}\mathbf{U}^\top\boldsymbol{\tau}_z \tag{4.6}$$

### 4.2. Algorithm for a non-operational thruster

As in the above, let us denote the required input vector by $\boldsymbol{\tau}_z$, the thrust vector by $\boldsymbol{f}$ and assume that the $k$th thruster is off. It means that $f_k = 0$ and element $p_{kk} = 0$. The other elements of $\boldsymbol{f}$ will be calculated using formula (4.3).

Defining

$$\begin{aligned}
&\boldsymbol{f}' = [f_1, ..., f_{k-1}, f_{k+1}, ...f_n]^\top\\
&\mathbf{V}^* = \mathbf{V}^\top\mathbf{P} = [\boldsymbol{v}_1^*, ..., \boldsymbol{v}_{k-1}^*, \mathbf{0}, \boldsymbol{v}_{k+1}^*, ..., \boldsymbol{v}_n^*]\\
&\mathbf{V}_f^* = [\boldsymbol{v}_1^*, ..., \boldsymbol{v}_{k-1}^*, \boldsymbol{v}_{k+1}^*, ..., \boldsymbol{v}_n^*]
\end{aligned}$$

expression (4.3) can be written in the form

$$\boldsymbol{\tau}_z = \mathbf{USV}_f^*\boldsymbol{f}' \tag{4.7}$$

The matrices $\mathbf{U}$ and $\mathbf{SV}_f^*$ have dimensions $3 \times 3$ and $3 \times m$ $(m = n - 1)$, so the vector $\boldsymbol{f}'$ can be computed by means of the equation

$$\boldsymbol{f}' = [(\mathbf{SV}_f^*)^\top\mathbf{SV}_f^*]^{-1}(\mathbf{SV}_f^*)^\top\mathbf{U}^\top\boldsymbol{\tau}_z \tag{4.8}$$

The since values of the thrust vector $\boldsymbol{f}$ are obtained as follows

$$\boldsymbol{f} = [f_1', ..., f_{k-1}', 0, f_k', ...f_m']^\top \tag{4.9}$$

If $n = 4$ than (4.8) is simplified to the form

$$\boldsymbol{f}' = (\mathbf{SV}_f^*)^{-1}\mathbf{U}^\top\boldsymbol{\tau}_z \tag{4.10}$$

## 5. Simulation study

The described method of power distribution is worked out to be implemented in the control system of the ROV called "Ukwiał". The vehicle is duty in the fleet of Polish minesweepers. It is an open frame robot controllable in 4 DOF, being 1.5 m long and having a propulsion system consisting of 6 thrusters. Motion in the horizontal plane is realised by means of 4 thrusters which can generate a force up to $\pm 750\,\mathrm{N}$ assuring the speed up to $\pm 1.2\,\mathrm{m/s}$ and $\pm 0.6\,\mathrm{m/s}$ in the $X$ and $Y$ direction, respectively.

Simulation study has been performed for the vehicle dynamical model set forth in Appendix A. Simulation experiments were made for tracking control under interaction of sea current disturbance (speed $0.3\,\mathrm{m/s}$, direction $135°$). Maximum velocities were also determined by the length of the tether. The vehicle was assumed to follow the trajectory beginning from the position and orientation (10 m, 10 m, 0°), passing target waypoints (10 m, 90 m, 90°), (30 m, 90 m, 0°), (30 m, 10 m, 270°), (60 m, 10 m, 0°) and ending at (60 m, 90 m, 90°). The autopilot calculated command signals were $\tau_{z1} = X$, $\tau_{z2} = Y$ and $\tau_{z3} = N$. The power distribution module processed the signals and gave required hydrodynamic thrusts using formulas (4.6) or (4.10) depending on the state of thrusters.

The inputs, desired and real outputs as well as tracking errors are shown in Fig. 4 and Fig. 5. The first figure depicts proper work of all thrusters, the second one corresponds to a fault of the 3rd thruster. It can be seen that the failure of a single thruster has small influence on the accuracy of the vehicle motion (position and orientation). In both cases, the tracking error is on the same level. The examples demonstrate the ability of the proposed method of power distribution to cope with such a type of unserviceability of the power transmission system.

## 6. Conclusions

The paper presents a method of power distribution for an unmanned underwater vehicle. The proposed solution is based on the affine model of thrusters and decomposition of the thruster configuration matrix. It makes the method simple and useful for practical use.

The nonlinear model of the vehicle "Ukwiał" was applied for computer simulations. The investigations were carried out for fully efficient power trans-

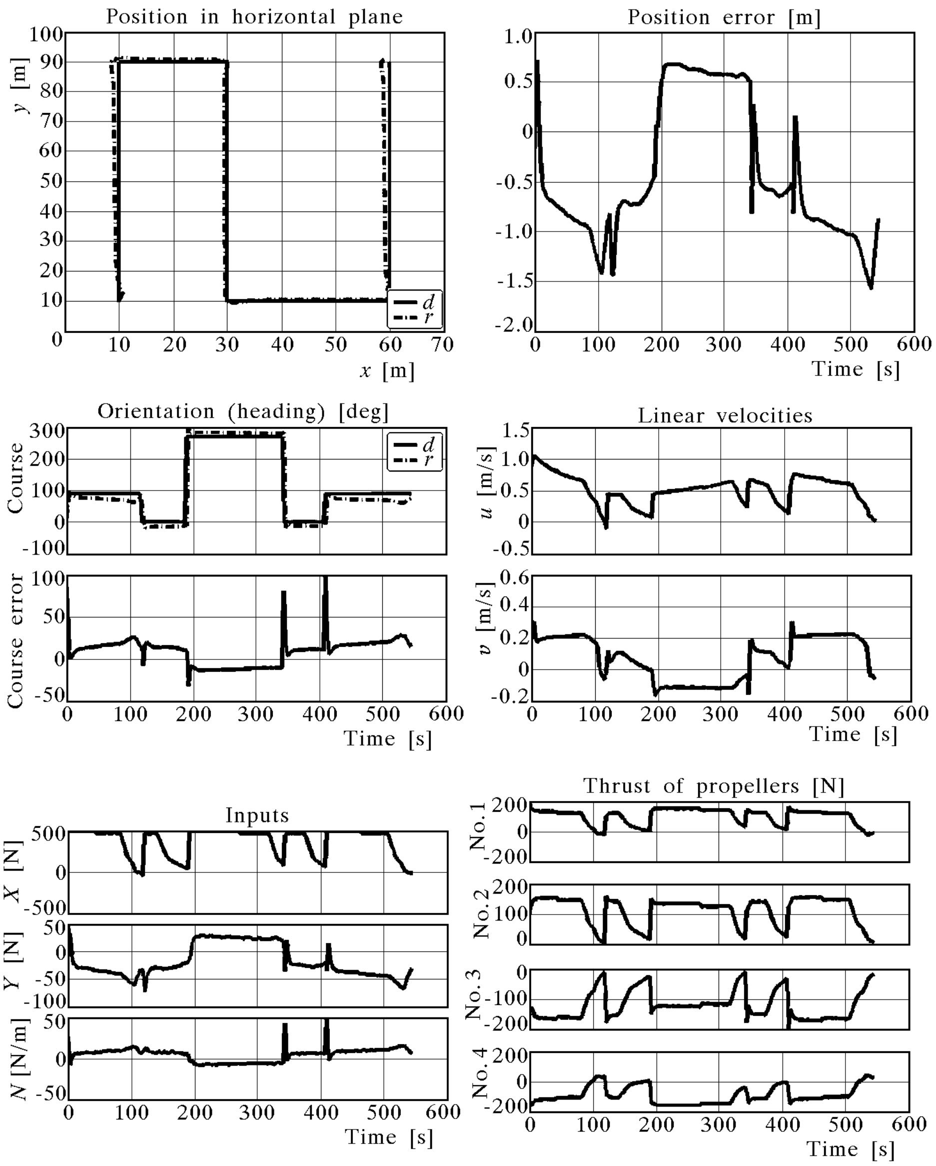


Fig. 4. The vehicle position and orientation ($d$ – desired, $r$ – real), deviation from the position and orientation, inputs and thrusts of propellers for track-keeping. All thrusters operational

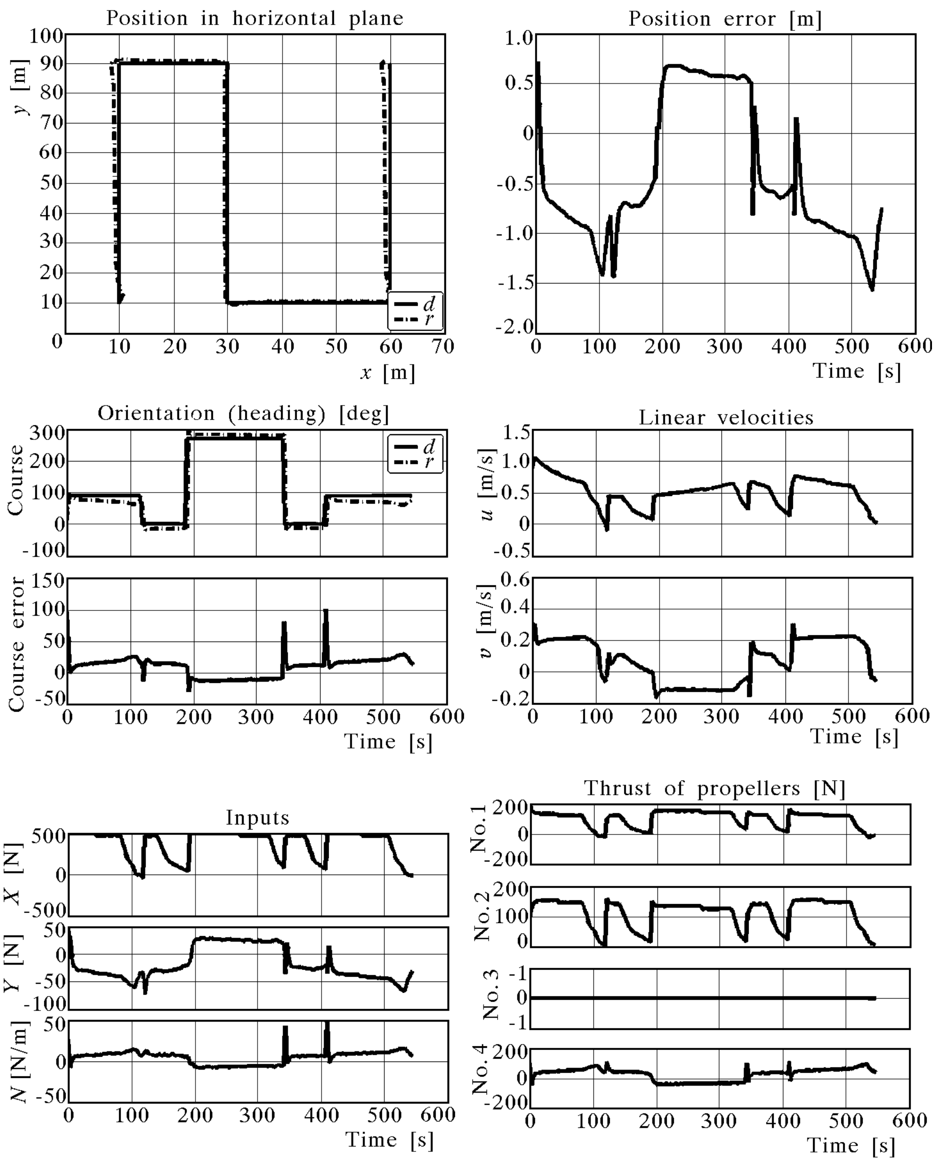


Fig. 5. The vehicle position and orientation ($d$ – desired, $r$ – real), deviation from the position and orientation, inputs and thrusts of propellers for track-keeping. 3rd thruster off

mission system and the system with thrusters being off due to a fault. The obtained results for the tracking control system show that the proposed algorithms enable control of the vehicle in the horizontal plane with high accuracy in both cases.

The main advantage of the approach is its flexibility with regard to the construction of the vehicle power transmission system and number of thrusters. The developed algorithms of power distribution are of a general character and can be successfully applied to all types of the ROVs.

## A. Appendix

The following dynamical model of the ROV has been used in computer simulations

$$\mathbf{M} = \operatorname{diag}\{99.0, 108.5, 126.5, 8.2, 32.9, 29.1\}$$

$$\boldsymbol{D}(\boldsymbol{v}) = \operatorname{diag}\{10.0, 0.0, 0.0, 0.223, 1.918, 1.603\} + \\ + \operatorname{diag}\{227.18|u|, 405.41|v|, 478.03|w|, 3.212|p|, 14.002|q|, 12.937|r|\}$$

$$\mathbf{C}(\boldsymbol{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 26.0w & -28.0v \\ 0 & 0 & 0 & -26.0w & 0 & 18.5u \\ 0 & 0 & 0 & 28.0v & -18.5u & 0 \\ 0 & 26.0w & -28.0v & 0 & 5.9r & -6.8q \\ -26.0w & 0 & 18.5u & -5.9r & 0 & 1.3p \\ 28.0v & -18.5u & 0 & 6.8q & -1.3p & 0 \end{bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{\eta}) = \begin{bmatrix} -17.0\sin\theta \\ 17.0\cos\theta\sin\phi \\ 17.0\cos\theta\cos\phi \\ -279.2\cos\theta\sin\phi \\ -279.2(\sin\theta + \cos\theta\cos\phi) \\ 0 \end{bmatrix}$$

The elements of the thruster configuration matrix $\mathbf{T}$, corresponding to Fig. 3, are as follows

$$\mathbf{T} = \begin{bmatrix} 0.875 & 0.875 & -0.875 & -0.875 \\ 0.485 & -0.485 & 0.485 & -0.485 \\ 0.332 & -0.332 & -0.332 & 0.332 \end{bmatrix}$$

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### Rozdział mocy w układzie napędowym zdalnie sterowanego pojazdu podwodnego

Streszczenie

W pracy przedstawiono sposób rozdziału mocy w układzie napędowym bezzałogowego pojazdu podwodnego. Do rozwiązania zadania dystrybucji naporów na poszczególne pędniki wykorzystano rozkład macierzy konfiguracji pędników względem wartości szczególnych. Zamieszczono algorytmy rozdziału mocy dla sprawnego układu napędowego, jak i dla stanu awarii jednego z pędników. Metoda rozdziału mocy została opracowana z uwzględnieniem jej praktycznego zastosowania w układzie sterowania zbudowanego dla potrzeb polskiej Marynarki Wojennej zdalnie sterowanego pojazdu podwodnego typu "Ukwiał". Zamieszczono wyniki badań symulacyjnych ruchu pojazdu, ze sprawnym i uszkodzonym układem napędowym, przemieszczającego się po zadanej trajektorii w płaszczyźnie poziomej.

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