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VIBRATION ENERGY FLOW IN RECTANGULAR PLATES

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The paper presents obtained from literature review formulas on structural intensity calculations. The formulas involve loads (forces and moments) and strains (linear and angular) which enabled the estimation of structural surface intensity for beams, and plates considered here as simple structural elements. The numerical method of intensity evaluation was based on complex modal analysis with the use of the finite element method. The presented calculation results lead to the assessment of distribution of structural intensity vectors on the surface of a steel rectangular plate. The models included the source of vibrations (force excitation) and sink of energy (damper) with known position of application. The changes of the finite elements grid density enabled detailed investigation of the total vibration energy flow in the analysed plates.

Key words: vibration energy flow, structural intensity, intensity vector field

1. Introduction

The quantity of structural intensity was introduced with the purpose of extension of the description of vibration phenomena in structures with the application of vector fields. It has found exceptional application to investigations of vibration energy flow in deformable elastic bodies. The structural intensity represents the averaged in time mechanical energy flow through the unit area perpendicular to the direction of the flow (Gavric *et al.*, 1997). Because of the measurement possibilities, the experimental analysis is conducted in a thin region close to the surface, and is mainly dedicated to thin-walled structures. The analysis in deeper parts of constructional elements is out of practical meaning mainly due to their thin-wall nature. The analysis of the spatial distribution of structural intensity vector fields enables determination and location of transmission paths, sources and sinks of energy of vibrations in application to mechanical systems. It gives the particular information on the streams of energy flow, which is much more advantageous than other methods used earlier to such kind of analysis.

The basic formulations and relationships allowing one to calculate the components of structural intensity vectors for simple constructional elements such as beams, plates, shells and pipes were given in Gavric and Pavic (1993), Gavric *et al.* (1997). Development of the intensity methods in application to analysis of the rigid bodies started in the middle of 70s of the last century. The papers by Noiseux (1970), Pavic (1976) and Verheij (1976) cover mainly the development of measuring methods. As a result, there were two main methods of measurements elaborated: simplified – using the definitional dependencies (Pavic, 1976; Verheij, 1976) and exact based on the wave theory of vibrations (Gavric *et al.*, 1997; Pavic, 1992). The used measurement methods still have disadvantages in their application to practical cases. They are a compromise between the measurement accuracy and simplicity of the measuring system, meaning the number of measuring points. The structural intensity can not be measured directly. Methods for the measurement use deflections (strains) instead of stress components.

Significant development of the structural intensity applications was noted in the second half of 80s and the beginning of 90s, when works showing the possibility of numerical calculation of structural intensity (Gavric, 1991; Gavric and Pavic, 1993) appeared. Fundamental dependencies were based on the assumption of one or two dimensional structures such as beams or plates. The numerical approach was based on the modal model.

The possibility of computational analysis of the structural intensity is very promising due to the prospects of use of already elaborated for other purposes finite element models of mechanical structures. The practical examination on the computational way of the vibration energy flow in complex structures consisting of beams, plates and shells have not been solved satisfactory till now. Main problems were found in the necessity of consideration of complex boundary conditions, complexity of the analysed real structures and applied models of damping.

The method of numerical analysis of structural intensity fields can be used for the modal models obtained from the experimental modal analysis (Gavric, 1997; Gavric *et al.*, 1997). As shown the experiments presented below, the modal model should cover at least first sixty eigenfrequencies. This conclusion is the result of low convergence of the method used, and indicates important contribution of high frequency modes into the vibration energy transfer in the structures.

2. Structural intensity in thin-walled structural elements

For a steady state of vibration the surface structural intensity can be evaluated as a complex quantity (Gavric and Pavic, 1990)

$$\widetilde{S}_{\sigma_{kl}v_l}(\omega) = I_k(\omega) + jJ_k(\omega) \tag{2.1}$$

where $\omega = 2\pi f$ is the angular frequency, f is the frequency of vibrations, $\widetilde{S}_{\sigma_{kl}v_l}(\omega)$ is the cross spectrum function of complex components of the stress and particle velocity.

In practical cases, the real part of the structural intensity, called the active intensity, is only analysed. Only the real part of the intensity is responsible for the energy transfer. The imaginary part represents the energy conservation in the system, and is connected with standing waves. The damping in the structure is necessary for the existence of vibration energy flow. In the case of low damping in the analysed structure, the examination of vibration energy is limited. A curl of the structural intensity is observed instead of the energy flow.

An instantaneous value of the real part of the structural intensity $i_k(t)$ is a time dependent vector equal to the change of energy density in an infinitively small volume (Gavric, 1991). Its kth component is given by the equation

$$i_k(t) = -\sigma_{kl}(t)v_l(t) \qquad\qquad l = 1, 2, 3 \tag{2.2}$$

where $v_l(t)$ is the lth component of the velocity vector, and $\sigma_{kl}(t)$ is the klth component of the stress tensor.

The averaged in time value of (2.2) represents the net energy flow in the mechanical structure (Gavric, 1991)

$$I_k = \langle i_k(t) \rangle \tag{2.3}$$

in the direction of the kth coordinate of a rectangular frame of reference corresponding to the analysed structural element.

The formulations for measurements methods are based on easily measured quantities such as deflections, strains and velocities. The values of partial derivatives are approximated with the finite difference method (Carniel and Pascal, 1985). This causes an instantaneous parallel measurement to be done in thirteen points. The method used for the measurement is inapplicable to calculations.

The components of the structural intensity vector can be calculated for beams, plates and shells as functions of the following variables: bending and twisting moments, shear forces, linear and angular displacements. This approach significantly differs from the measurement methods.

2.1. Structural intensity in rods and beams

A simplest structure, which could provide the energy transportation, is an unbounded thin rod excited to axial vibrations. The structural intensity is related to the axial line of the rod and equal to

$$I = \sqrt{E\rho}\, v^2 \tag{2.4}$$

where E is Young's modulus, ρ – mass density, ν – rod axial velocity.

The only stress occurring in the rod is the axial stress which is in phase with the velocity of vibrations.

In the case of a prismatic beam exposed to a single excitation frequency the vibration velocities are found from the complex representation of the displacements. The stress in the beam is in general a result of bending, shear and twisting. The component along the main axis of the beam is (Gavric and Pavic, 1993)

$$I_x = -\frac{\omega}{2}\mathrm{Im}\,[\widetilde{N}\widetilde{u}_0^* + \widetilde{Q}_y\widetilde{v}_0^* + \widetilde{Q}_z\widetilde{w}_0^* + \widetilde{T}\theta_x^* + \widetilde{M}_y\theta_y^* + \widetilde{M}_z\theta_z^*] \tag{2.5}$$

where $\widetilde{N}$ are tension forces, $\widetilde{Q}_y$ and $\widetilde{Q}_z$ – shear forces, $\widetilde{T}$ – torque, $\widetilde{M}_y$, $\widetilde{M}_z$ – bending moments, $\widetilde{\theta}_x$, $\widetilde{\theta}_y$, $\widetilde{\theta}_z$ – angular displacements, $\widetilde{u}_0$, $\widetilde{v}_0$, $\widetilde{w}_0$ – linear displacements of the beam neutral line according to the x, y, z axis, respectively.

2.2. Structural intensity in plates

In the vibration of thin plates the bending and longitudinal waves are the dominant ones. For the longitudinal waves, the displacements do not vary with depth of the plate. In the case of bending vibrations, the displacements caused by shear forces, bending and twisting moments are added up. Structural intensity vector of a plate subjected to all mentioned kinds of loads is the sum of three components. For bending motion of the plate, the displacements caused by the bending are related to the in-plane displacements as follows

$$u_x = -\frac{h}{2}\frac{\partial u_z}{\partial x} \qquad\qquad u_y = -\frac{h}{2}\frac{\partial u_z}{\partial y} \tag{2.6}$$

In numerical calculations, realised by means of the finite element method, the structural intensity is related to the neutral undeformed middle plane of the plate. In a general case, the plate is subjected to bending, tension and twisting motions. On this assumption, the following relationships on the structural intensity components are derived for thin plates (Gavric and Pavic, 1993; Cieślik, 2002a)

$$
\begin{aligned}
I_x &= -\frac{\omega}{2}\mathrm{Im}\big[\widetilde{N}_x\widetilde{u}_0^* + \widetilde{N}_{xy}\widetilde{v}_0^* + \widetilde{Q}_x\widetilde{w}_0^* + \widetilde{M}_x\theta_y^* - \widetilde{M}_{xy}\theta_x^*\big] \\
I_y &= -\frac{\omega}{2}\mathrm{Im}\big[\widetilde{N}_y\widetilde{v}_0^* + \widetilde{N}_{yx}\widetilde{u}_0^* + \widetilde{Q}_y\widetilde{w}_0^* - \widetilde{M}_y\theta_x^* + \widetilde{M}_{yx}\theta_y^*\big]
\end{aligned}
\tag{2.7}
$$

where: $\widetilde{N}_x$, $\widetilde{N}_y$ – in-plane normal forces, $\widetilde{N}_{xy} = \widetilde{N}_{yx}$ – in-plane tangential forces, $\widetilde{Q}_x$, $\widetilde{Q}_y$ – shear forces, $\widetilde{M}_{xy} = \widetilde{M}_{yx}$ – torques, $\widetilde{M}_x$, $\widetilde{M}_y$ – bending moments.

There are assumed small deformations which enabled the superposition of independent displacements for flat finite element of plate or shell type.

3. Calculation of structural intensity components

3.1. Modal approach to structural intensity calculations

Complex displacements and stresses needed for structural intensity evaluation can be obtained by the modal approach. The software used in computation with the finite element method uses in most cases the real stiffness and mass matrices. As a result of calculations the real displacements and stresses are obtained. The procedure of structural intensity calculation is based on the complex response of the structure with a modal representation of the structure without dissipation (Gavric, 1990). The damping is considered in two forms. The structural internal damping of the structure is taken into consideration as modal damping. The additional damping (energy absorber, sink), placed in the known location, is treated as a external loading

$$
\widetilde{\boldsymbol{R}} = -\widetilde{\mathbf{S}}\widetilde{\boldsymbol{X}} \tag{3.1}
$$

where $\tilde{\mathbf{S}}$ is the additional stiffness matrix, $\widetilde{\boldsymbol{X}}$ – displacement vector.

An excitation by a sinusoidal force in the form $\widetilde{\boldsymbol{F}}\exp(j\omega t)$ was assumed. The equation of motion of the structure including two models of damping is (Gavric and Pavic, 1993)

$$
-\omega^2\mathbf{M}\widetilde{\boldsymbol{X}} + \mathbf{K}\widetilde{\boldsymbol{X}} = \widetilde{\boldsymbol{F}} + \widetilde{\boldsymbol{R}} \tag{3.2}
$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices.

The complex response of the structure is obtained from the calculations of eigenvalues. The internal proportional damping is considered by use of complex eigenfrequencies. The real values of the eigenfrequencies matrix $\boldsymbol{\omega}_0^2$ are replaced with the values including the modal loss factor η_i for the ith mode. The complex eigenfrequencies are then equal to $\widetilde{\omega}_{0i}^2 = (1 + j\eta_i)\omega_{0i}^2$.

The complex response of the structure is obtained from the calculations done with a post-processor program operating on the values obtained from the classical FEM model in order to compute the complex modal response in modal coordinates. For the purpose of structural intensity calculations an own program written in the MatLab environment was elaborated (Cieślik, 2001, 2002b). Program realises calculations as well as presents the results in a graphical form.

3.2. Calculations of structural intensity based on the finite element method

The structural intensity, defined by (2.2), depends on particle velocity and stress. As a result of the calculations, one can get displacements in the nodes and stresses in a point inside a finite element. In the process of calculations these values are need to be found in the same points. Because the accuracy of stress calculations is less than that for displacements, the values of the structure response are taken in points of highest accuracy for the stress. The displacements in these points are found with the help of a shape functions for the finite element from relative displacements of the element.

The shape function matrix is, in a general case, related to the local system of coordinates for the finite element. The vector of modal displacements have to be defined in the same coordinate system. The displacements inside the finite element are also related to the local system of coordinates. Application of the procedure allows one to find the vector containing proper displacements of element centroides for a given eigenvalue. Complex displacements and stresses, both related to the element centroide and necessary for evaluation of the structural intensity, are calculated by modal superposition from definitions (2.5) and (2.7). The obtained structural intensity is complex and expressed in the local coordinate system connected with the finite element. It should be transformed into the general system of coordinates connected with the system of finite elements, the analysed structure.

4. Case study – rectangular plate

For the purpose of verification of the structural intensity applicability to the identification of discontinuities, a numerical experiment was performed. The model chosen for the analysis was a three-dimensional structure of a rectangular plate convenient for the numerical modelling of the structural intensity. Two general cases were discussed: the change of width and change of thickness in the middle part of the plate. The results were compared with the results obtained for a rectangular homogenous plate.

In the calculations, the same own program for calculations of complex modal model allowing the consideration of additional localised damping in the system, was used.

4.1. Analysed model

Homogeneous rectangular plate

The model chosen for the calculations was a homogeneous rectangular flat plate. The plate had mechanical properties of a structural steel and dimensions of 1.5 m in width, 2.5 m in length and thickness of 10^{-2} m. The four finite element method models were arranged using the NASTRAN software. The models differed in number of finite elements. The plate was divided into 60, 240, 960 and 3840 elements. In each case, the model consisted of the same square shell elements of the QUAD4 type. The excitation and damping force were introduced to the model. The harmonic excitation force was applied to the plate at the place indicated in figures by the star. The damping force proportional to the velocity of vibration was applied to the plate at the place indicated in figures by the triangle. The magnitude of the damping force was set to 10^3 N. The direction of its action was chosen perpendicularly to the plane of the plate. The locations of the damping and excitation force were the same for all models.

The model of a simply supported plate was chosen. The only feasible motion was rotation around the edges of the plate. Translatory motions of edges in any directions were neglected. The model was most characteristic for the most technical cases of the plate mounting in practice.

Rectangular plate with various thickness and width

The next models chosen for the calculations were rectangular flat plates with various thickness in the middle. The plates had mechanical properties of a structural steel and dimensions of 1.5 m in width and 2.5 m in length with thickness of 10^{-2} m. Four different models made up with the finite element method were prepared using the NASTRAN software. Three models differed in the thickness of the middle part of the plate. The fourth model had two symmetrical rectangular cut-outs in the middle part.

In the square region $(1 \times 1\,\mathrm{m})$ in the middle of the plate the thickness was changed to $0.5 \cdot 0^{-2}$ m and $2 \cdot 10^{-2}$ m, respectively. The model consisted of 450 square shell elements of the QUAD4 type. The plate with the cut-out had a constant thickness for all its area. The model consisted of 362 square shell elements of the same QUAD4 type. The dimensions and shape of square elements were the same for all four cases. Similarly, as in the analysis of the homogenous plate, the excitation and damping forces were introduced to the model. The harmonic excitation force was applied to the plate at the place indicated in figures by the star. The damping force proportional to the velocity of vibration was applied to the plate at the place indicated in figures by the triangle. The magnitude of the damping force was set to 10^3 N. The direction of its action was chosen perpendicularly to the plane of the plate. The model of a simply supported plate was chosen. The only feasible motion was rotation around the edges of the plate.

4.2. Results of calculations

Homogeneous rectangular plate

The main target of the analysis was testing the convergence of numerical solution and determination of the number of mode shapes necessary to obtain the solution with a specified accuracy. Calculations were carried out for chosen numbers of the first mode shapes, accordingly: 10, 13, 16, 20, 25, 32, 40, 50, 63, 80 and 100. In cases of low numbers of mode shapes, significant changes in the distribution of vectors for the same density of the mesh of elements were observed. The positions of the excitation and damping forces were not clearly shown by the structural intensity vectors distribution. As the number of mode shapes increased, the more and more regular and ordered vector fields of the structural intensity were observed, Fig. 1-Fig. 4.

For the purpose of the assessment of the necessary number of mode shapes required for the achievement of the correct and convergent numerical solution a relative measure of convergence was chosen. The measure was the ratio of the difference of the vector magnitude in a chosen observation point, calculated for the two consecutive cases of calculations for the nth and $(n-1)$th number of the mode shape, to its value calculated for the $(n-1)$th number of the mode shape. In each of four cases of the finite element mesh, the measure of convergence was less than 0.01 for the number of 63 mode shapes taken into

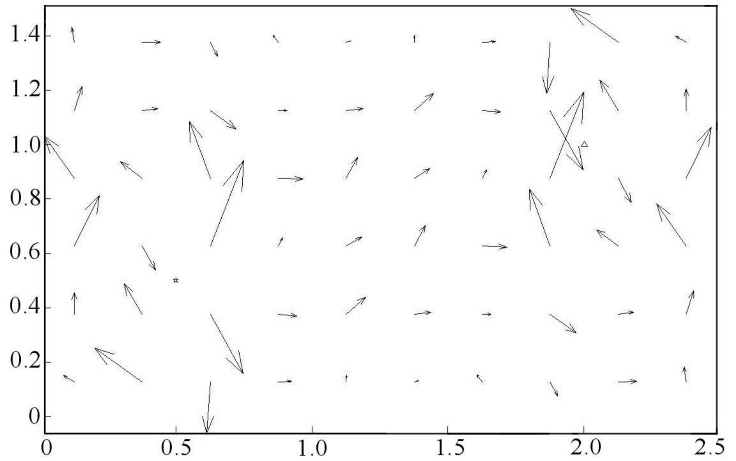

Fig. 1. Distribution of the structural intensity vectors for a simply supported rectangular plate. Model consists of 60 elements. 100 mode shapes calculated

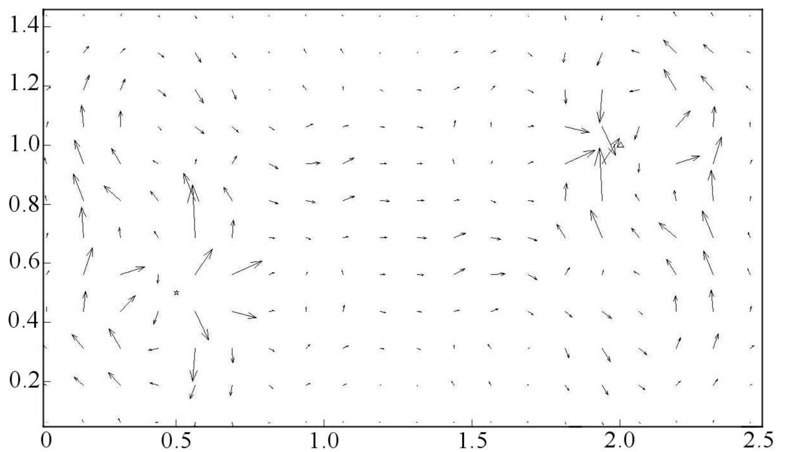

Fig. 2. Distribution of the structural intensity vectors for a simply supported rectangular plate. Model consists of 240 elements. 100 mode shapes calculated

account. Consequently, it was assumed that the number greater than 60 mode shapes corresponds to the exact numerical model of the plate, and properly represents the vibration energy flow in the system.

It was mentioned above that the magnitude of structural intensity vectors decrease with the increase in the number of finite elements. This allows one to draw the conclusion that in the analysis of the energy flow it is not sufficient to observe only the intensity vectors. The better measure of the energy flow seems

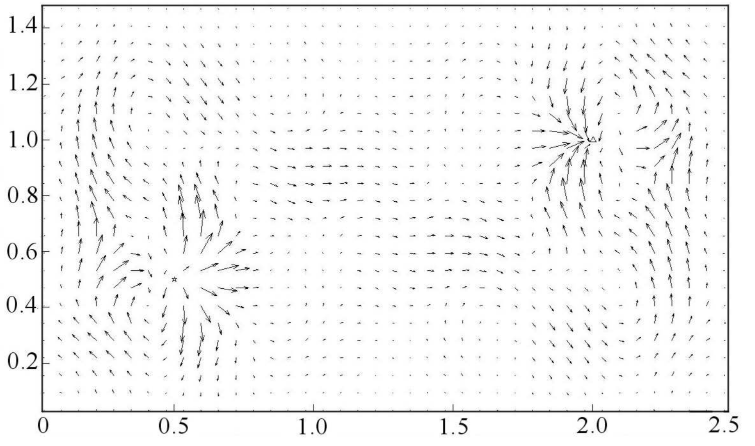

Fig. 3. Distribution of the structural intensity vectors for a simply supported rectangular plate. Model consists of 960 elements. 100 mode shapes calculated

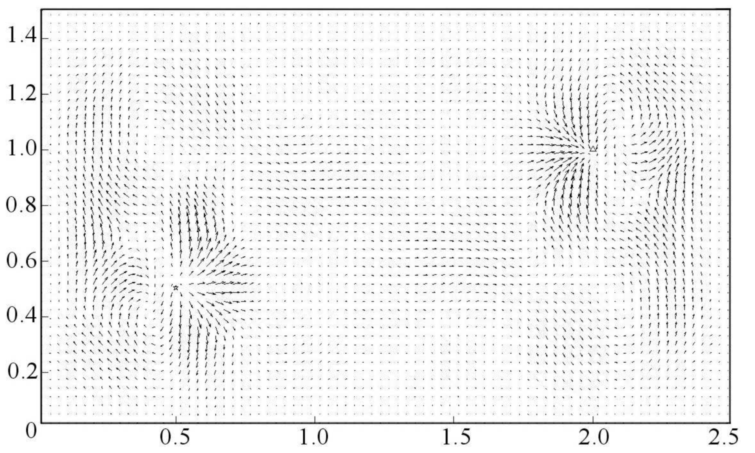

Fig. 4. Distribution of the structural intensity vectors for a simply supported rectangular plate. Model consists of 3840 elements. 100 mode shapes calculated

to be the total energy flow through a closed area around the points of excitation and damping or through the whole width of the plate. For that reason the method of summation of the structural intensity vector magnitude in the closed area was applied. In the general case, the total energy flow should be examined through the cross section perpendicular to the known or assumed direction of energy transfer indicated by an ordered distribution of the structural intensity vectors.

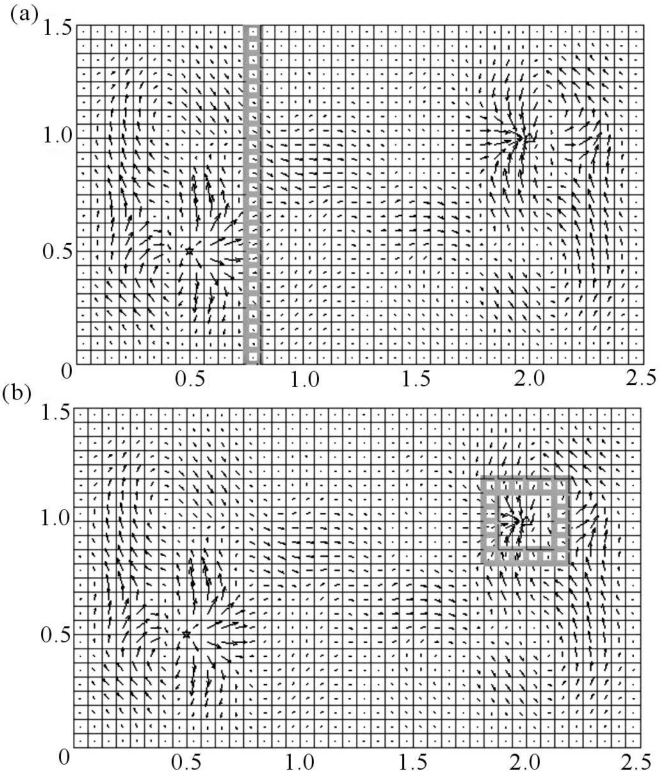

Fig. 5. Regions of summation of the structure intensity vectors: (a) in the whole width of the plate in the middle part, (b) around the point of damping force application

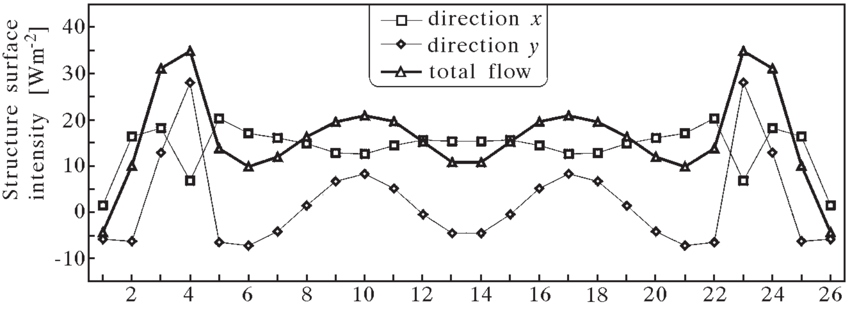

Fig. 6. Results of summation of the structure intensity vectors in chosen cross-sections. The plate model consisted of 960 elements

Examples of the summation are shown in Fig. 5. Figure 6 shows the changes of the energy flow calculated from the vector field in consecutive cross sections of the plate and along the curves surrounding the excitation and sink of the vibration energy. Significant changes are observed for the area around the excitation and sink. This is due to the changes of stress values in the region of force application point. For other cross sections along the whole width of the plate only slight changes in the total energy are observed. The process of structural intensity values summation was done for the horizontal x and vertical y directions parallel to the sides of the plate.

Rectangular plate with various thickness and width

The main target of the analysis was the testing of the effect of plate thickness and shape on the distribution of structural intensity vectors. The obtained results of calculation are shown in Fig. 7-Fig. 10. Particularly in the region of abrupt change in the plate thickness, the distribution of structural intensity vectors changes with respect to the rest of the plate. The positions of the excitation and damping forces are clearly shown by the structural intensity vectors distribution for the case of a homogenous and thin plate.

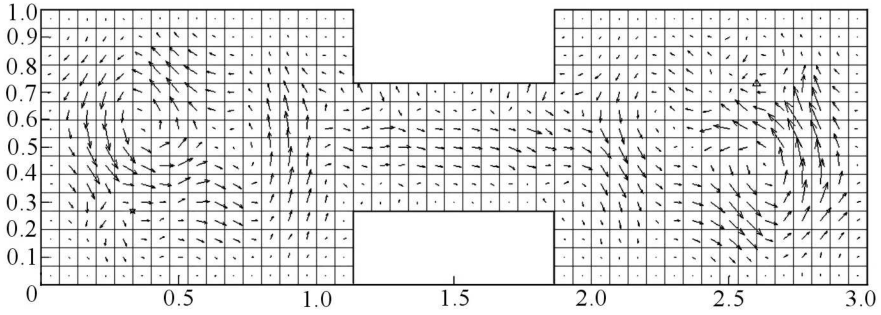

Fig. 7. Distribution of the structural intensity vectors for a simply supported rectangular plate with a cut-out in the middle part

In all cases however, the distribution of intensity vectors in the middle part has a similar shape. The distribution of vectors reveals a distinct direction of the energy flow from the excitation to the sink point. For a plate thicker in the middle part, a vorticity field in the region of the thicker part is observed. The changes of thickness are marked by the abrupt change in the direction and magnitude of the intensity vectors.

Comparing the obtained results of calculations for rectangular plates presented here as the distribution of structural intensity vectors over the area of

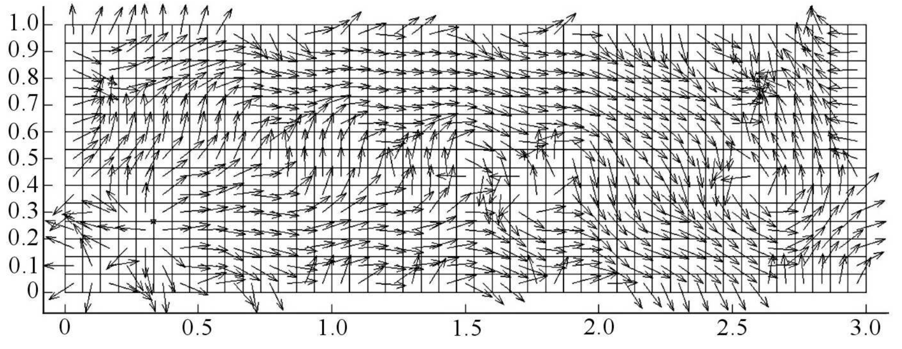

Fig. 8. Distribution of the structural intensity vectors for a simply supported rectangular plate of constant thickness. Vectors normalised to the unit length

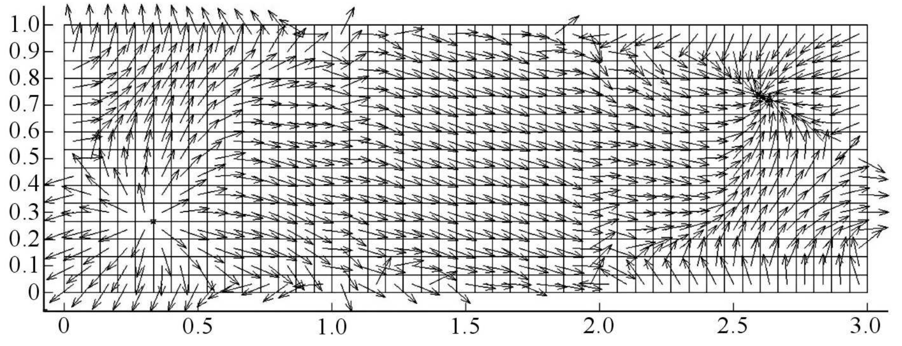

Fig. 9. Distribution of the structural intensity vectors for a simply supported rectangular plate being thinner in the middle. Vectors normalised to the unit length

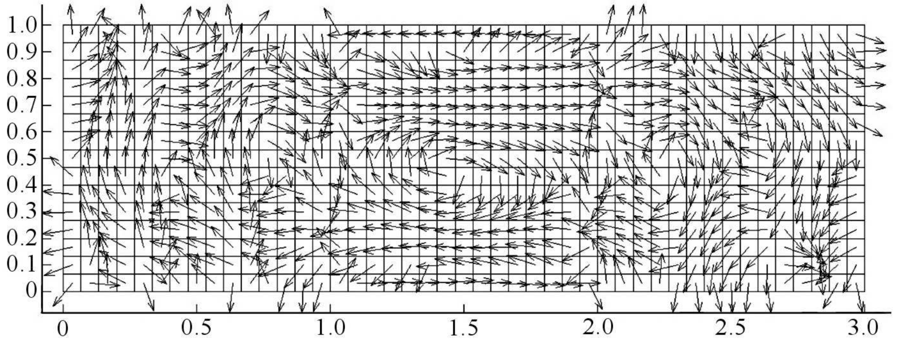

Fig. 10. Distribution of the structural intensity vectors for a simply supported rectangular plate being thicker in the middle. Vectors normalised to the unit length

the plate, one can notice that there is no similarity with the distribution of displacements for each mode shape. This conclusion results from the fact that the mode shapes are connected to the standing waves formed in the plate. In a system with small internal damping the energy flow is not observed or at least it is very low. The effect of the energy flow between the structure elements is observed only for systems with high internal damping caused e.g. by abrupt local changes of properties or a localised damping force.

4.3. Conclusions

The modal analysis based on dense grid of finite elements enabled detailed analysis of vibration energy transportation. The results obtained from calculations are presented in a graphical form in Fig. 1-Fig. 10. The structural intensity vector distributions over the surface of elements of the plate structure are shown. Very specific disturbances of the distribution in points of application of the damping element and application of the exciting force are clearly seen. In the region far from connections as well as points of application of the external damping or excitation, the directions of vectors are ordered parallel and steady regarding the vector magnitude. It means a steady regular flow of the vibration energy in one direction defined by the sense of the vectors. In some places, especially those close to the point of change in the thickness, one can observe the circulation of structural intensity vectors. This effect of vortices, represented by rotational distribution of the intensity vectors, means circulation and conservation of the vibration energy in the system.

The distribution of vectors reveals rotational character of the vector field due to wave reflections and probable places of dissipation of mechanical energy. This effect should be verified by experimental measurement. A particular analysis leads to the conclusion that the intensity vector distribution, by nature, enables localisation of the energy sink in large surface elements. This is a significant advantage as compared to other identification methods.

The distribution of the structural intensity vectors gives a qualitative characteristic of vibration energy transportation in mechanical systems. Introduction of an additional measure in the form of an integral of the structural intensity vector component perpendicular to a certain closed surface, e.g. element cross-section, enables quantitative assessment of energy transfer paths and energy balance in the structure.

The presented method of calculation of the structural intensity vector enables its evaluation for a chosen frequency range and mode shapes (Gavric, 1991). The frequency range of the analysis can not be exactly defined in real numbers. It should be rather related to the frequency range covering approximately the first hundred mode shapes of the analysed structural element or entire structure. This relates the frequency range with mechanical properties and dimensions of the structure.

The derived relationships connect the structural intensity with linear and angular displacements, forces and moments applied to the structures like beams, plates and shells. The method of structural intensity estimation is based on the calculation of displacements in nodes and stresses in inner points, i.e. Gauss points of finite elements. The calculations are carried out with the application of complex modal parameters counted by the use of the numerical modal analysis based on the finite element method.

The damping is considered in two ways: as internal damping represented by modal damping and localised damping by the force proportional to the displacement. Due to complex notation used in the analysis, different models of damping are applicable.

The main disadvantage of the method is its poor convergence. The number of modes which are used in the calculation process should be properly chosen and relatively big. Usually, for calculation of the displacements and their derivatives, the number of modes is taken in such a way so the highest eigenfrequency applied in the calculations is several times higher than the frequency of excitation. In the presented case, the number of mode shapes should be grater than 60. Some authors, however, claim the important contribution of high frequency modes into vibration energy transfer in structures.

The structural intensity is the product of stress and velocity for the given point of the structure. Abrupt changes of stress, which occur in points close to the excitation or discontinuities, can not be well described by a limited number of lower modes. This creates the main problem of accuracy of the calculation method.

The method of analysis of the structural intensity distribution has been applied as a new diagnostic method for the assessment of failure which could occur in structural elements under working loads. It enables the investigation of regions of high concentration of the vibration energy flow, particularly those exposed to the risk of damage or propagating vibration and sound waves to the environment. It can be also considered as the identification of regions in which additional damping could be applied for lowering of the vibration level and resulting noise radiation.

Examples of the examination of the structural intensity distribution in elements of a rectangular plate have confirmed the method usability to the investigation of vibration energy transportation paths and places of its concentration. The method enables a quantitative analysis based on the structural intensity vector distribution. A quantitative analysis based on calculated values of the power of vibrations in particular elements of a structure is possible as well.

The applied modal approach with some limitations enables the making use of the results of experimental modal analysis for the evaluation of structural intensity vectors in real structures.

With respect to the efficiency of analysis of complex structures and limitation of large FEM models, the structure intensity method seems to be a very promising tool for the energy vibration flow investigation in the middle frequency range. The middle frequency range is defined here as the range where dimensions of components of a certain structure are small or comparable to the wave length. In the middle frequency range, the deterministic methods, as e.g. FEM, need a dense grid of elements for the investigation of local changes in the structure. On the other hand, such structures do not fulfil assumptions for statistical methods used in the high frequency range. Such a heuristic definition of the middle frequency range does not specify its real boundary values. Regarding the results of the investigations shown above, it can be stated that the middle frequency range for the purpose of the structural intensity method approximately begins at the frequency corresponding to the 20th mode shape and ends at the 100th. However, the convergence of the structural intensity calculation method is still poor, and mode shapes of a higher order should be taken into account.

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Przepływ energii drganiowej w płytach prostokątnych

Streszczenie

W pracy przedstawiono uzyskane na podstawie danych literaturowych zależności określające natężenie dźwięków strukturalnych. Zależności te uwzględniają odkształcenia (liniowe i kątowe) oraz obciążenia (siły i momenty) pozwalające wyznaczyć wartość natężenia dźwięków strukturalnych dla belek i płyt rozpatrywanych jako elementy konstrukcyjne. Omówiono metodę wyznaczania wartości natężenia dźwięków strukturalnych na podstawie zespolonych parametrów modalnych uzyskiwanych drogą analizy modalnej z użyciem metody elementów skończonych. W pracy podano również przykład obliczeniowy, którego wyniki pozwoliły na ocenę rozkładu wartości wektora natężenia dźwięków strukturalnych na powierzchni prostokątnej płyty stalowej. W modelu przyjęto zadane położenie źródła drgań (siła wymuszająca) i rozpraszania energii (siła tłumiąca). Zmiany gęstości sieci elementów skończonych pozwoliły na szczegółową analizę całkowitego przepływu energii drganiowej w badanej płycie.

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