

A METHOD OF DETERMINATION OF FEASIBLE
PROPELLER FORCES AND MOMENTS FOR
AN UNDERWATER ROBOT

JERZY GARUS

*Faculty of Mechanical and Electrical Engineering, Naval University
e-mail: j.garus@amw.gdynia.pl*

The paper presents procedure for the optimal allocation of thrust for horizontal motion of underwater robotic vehicles. Computation of propeller thrusts from propelling forces and moments is an optimisation problem based on a model, which the simplest form is unconstrained. In practice, however, where physical limitations must be taken into account, the obtained in such a way solution can be unrealistic. To cope with those difficulties, an algorithm for evaluation of the capacity of a propulsion system to produce required forces and moments and, if necessary, finding their feasible values is proposed. Due to computational simplicity, such an approach is a good solution in real-time applications. A numerical example is provided to demonstrate effectiveness and correctness of the approach.

Key words: underwater robot, hydrodynamic thrust allocation, propulsion system

1. Introduction

Nowadays, it is common to use underwater robotic vehicles (URVs) to accomplish such missions as inspection of coastal and off-shore structures, cable maintenance as well as hydrographical and biological surveys. Motion of URVs in 6 degrees of freedom (DOF) can be described by the following vectors (see e.g. Fossen, 1994; Lisowski, 1981)

$$\begin{aligned}\boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^\top & \boldsymbol{v} &= [u, v, w, p, q, r]^\top \\ \boldsymbol{\tau} &= [X, Y, Z, K, M, N]^\top\end{aligned}\tag{1.1}$$

where:

- $\boldsymbol{\eta}$ – vector of the position and orientation in the earth-fixed frame
- x, y, z – coordinates of the position
- ϕ, θ, ψ – coordinates of the orientation (Euler angles)
- \boldsymbol{v} – vector of linear and angular velocities in the body-fixed frame
- u, v, w – linear velocities along the longitudinal, transversal and vertical axes
- p, q, r – angular velocities about the longitudinal, transversal and vertical axes
- $\boldsymbol{\tau}$ – vector of forces and moments acting on the vehicle in the body-fixed frame
- X, Y, Z – forces along the longitudinal, transversal and vertical axes
- K, M, N – moments about the longitudinal, transversal and vertical axes.

Modern URVs are more and more frequently equipped with automatic control systems in order to execute complex manoeuvres without constant human intervention. Basic modules of a control system are depicted in Fig. 1. An autopilot computes required propelling forces and moments (commands) $\boldsymbol{\tau}_d$ by comparing the desired position and orientation of the robot with their current estimates. The corresponding to them propeller thrusts \boldsymbol{f} are calculated in a thrust distribution module and transmitted to the propulsion system as control inputs.

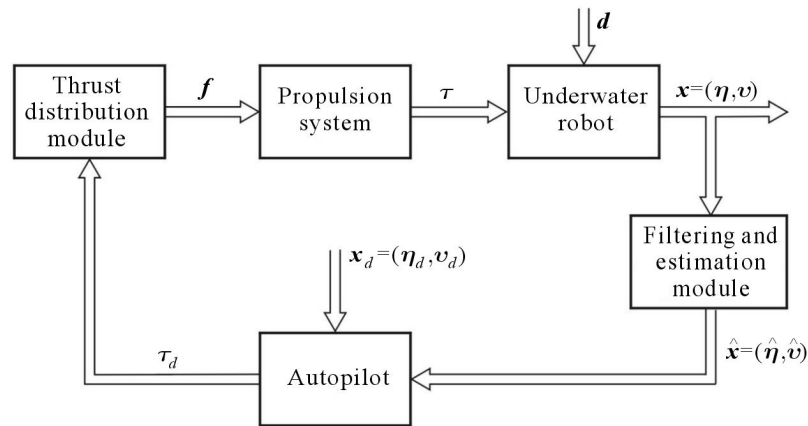


Fig. 1. A structure of the control system (\boldsymbol{d} – vector of environmental disturbances)

Both movement and positioning of an underwater robot is realised only by changing propeller thrusts, which leads to variation of propelling forces and moments. Control laws implemented in the autopilot are of a general character and usually do not take into account constraints put on the maximum and minimum values of thrusts developed by the propellers. It may cause that the desired solution τ_d can not be realised by the propulsion system due to work of one or more thrusters in saturation. Such a situation can contribute to deterioration of the control, and the behaviour of the robot may differ from the required pattern significantly.

Therefore, thrust distribution is one of the tasks of the control system that has essential influence on the quality of control. A procedure of thrust allocation is proposed to be realised in two stages (Fig. 2). In the first stage, the generating capacity of the demanded commands τ_d by the propulsion system is checked and feasible commands τ'_d are determined (i.e. such values of forces and moments which can be produced by the propulsion system). In the second stage, the real allocation of thrusts among the propellers is carried out on the base of τ'_d .

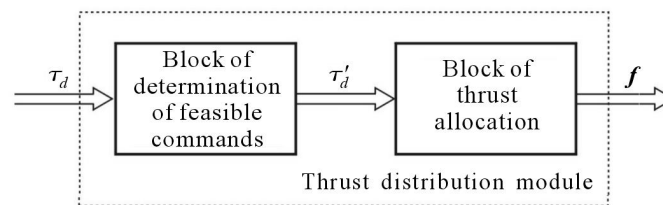


Fig. 2. A block diagram of the thrust distribution module

2. A procedure of thrust allocation for horizontal motion

In conventional URVs, the basic motion is movement in a horizontal plane with some variation due to diving. URVs operate in a crab-wise manner in 4 DOF with small roll and pitch angles that can be neglected during normal operations. Therefore, the spatial motion is regarded as a superposition of two displacements: motion in the horizontal plane and motion in the vertical plane. It allows one to divide the propulsion system into two independent subsystems responsible for motion in the vertical and horizontal planes, respectively. A general structure of such a system is shown in Fig. 3.

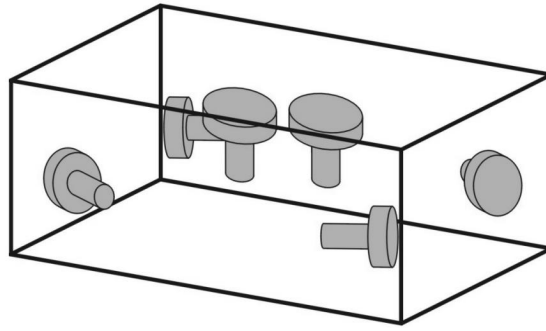


Fig. 3. A structure of a power transmission system with 6 thrusters

The first subsystem enables motion in heave and consists of 2 thrusters. The required force Z_d is equal to the sum of their thrusts.

The second one assures motion in surge, sway and yaw, and composes of 4 thrusters mounted askew with respect to main symmetry axes of the vehicle (see Fig. 4). Hence, the desired forces X_d and Y_d acting in the longitudinal and transversal axes and the moment N_d about the vertical axis are a combination of thrusts produced by the thrusters.

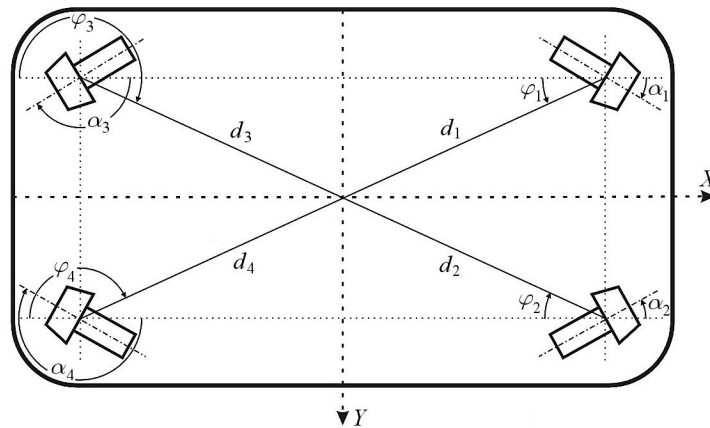


Fig. 4. Layout of thrusters in the subsystem responsible for horizontal motion

Let us denote:

$\boldsymbol{\tau}$ – vector of demanded commands

$$\boldsymbol{\tau}_d = [\tau_{d1}, \tau_{d2}, \tau_{d3}]^\top = [X_d, Y_d, N_d]^\top$$

\mathbf{f} – thrust vector

$$\mathbf{f} = [f_1, f_2, f_3, f_4]^\top$$

and assume that components of both vectors are bounded

$$\begin{aligned} \tau_{di}^2 - (\tau_i^{max})^2 &\leq 0 & \text{for } i = 1, 2, 3 \\ f_j^2 - (f_j^{max})^2 &\leq 0 & \text{for } j = 1, 2, 3, 4 \end{aligned} \quad (2.1)$$

Values of τ_i^{max} and f_j^{max} depend on the design of propellers and configuration of thrusters in the propulsion system.

As shown in Fossen (2002), for horizontal motion, the vector of required propelling forces and moments $\boldsymbol{\tau}_d$ can be described as a function of the thrust vector \mathbf{f} by the following expression

$$\boldsymbol{\tau}_d = \mathbf{T}(\boldsymbol{\alpha})\mathbf{f} \quad (2.2)$$

where

$\mathbf{T}(\boldsymbol{\alpha})$ – thruster configuration matrix

$$\mathbf{T}(\boldsymbol{\alpha}) = \begin{bmatrix} \cos \alpha_1 & \cos \alpha_2 & \cos \alpha_3 & \cos \alpha_4 \\ \sin \alpha_1 & \sin \alpha_2 & \sin \alpha_3 \sin \alpha_4 & \\ d_1 \sin(\alpha_1 - \varphi_1) & d_2 \sin(\alpha_2 - \varphi_2) & d_3 \sin(\alpha_3 - \varphi_3) & d_4 \sin(\alpha_4 - \varphi_4) \end{bmatrix}$$

$\boldsymbol{\alpha}$ – vector of azimuth angles, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]^\top$

α_j – angle between the longitudinal axis and direction of thrust of the j th propeller f_j

d_j – distance of the j th thruster from the centre of gravity

φ_j – angle between the longitudinal axis and the line connecting the centre of gravity with the j th thruster symmetry centre.

The thrust allocation problem, i.e. computation of \mathbf{f} from $\boldsymbol{\tau}_d$, is usually formulated as a least-squares optimisation problem and described in the following form (see e.g. Garus, 2004; Sordelen, 1997)

$$\mathbf{f} = \mathbf{T}^*(\boldsymbol{\alpha})\boldsymbol{\tau}_d \quad (2.3)$$

where the matrix $\mathbf{T}^*(\boldsymbol{\alpha}) = \mathbf{T}^\top(\boldsymbol{\alpha})[\mathbf{T}(\boldsymbol{\alpha})\mathbf{T}^\top(\boldsymbol{\alpha})]^{-1}$ is the generalized inverse.

This method of thrust allocation allows one to find the minimum-norm solution, but it should be noted that (2.3) belongs to unconstrained optimisation problems - i.e., there are no bounds on the elements of the vector \mathbf{f} ,

so the obtained values f_j may not satisfy $(2.1)_2$, and then the generation of the desired vector $\boldsymbol{\tau}_d$ by the propulsion system is not possible. In such a case, a new vector of commands meeting condition $(2.1)_2$ must be determined. A method of evaluation of this vector, called the vector of feasible commands and denoted by $\boldsymbol{\tau}'_d = [\tau'_{d1}, \tau'_{d2}, \tau'_{d3}]^\top$, is presented in the next section.

3. An algorithm for determination of feasible propelling forces and moments

Assume that the propulsion system consists of $n = 4$ identical nonrotational thrusters. It means that the quantities like: d_j , α_j and φ_j are constant for every thruster. Hence, all elements of the configuration matrix $\mathbf{T}(\boldsymbol{\alpha})$ are constant.

Let us denote:

1. τ_1^{max} , τ_2^{max} and τ_3^{max} – maximum values of the propelling forces and moments generated by the propulsion system for horizontal motion

$$\begin{aligned}\tau_1^{max} &= \sum_{j=1}^n |\tau_{1j}^{max}| = \sum_{j=1}^n |f_j^{max} \cos \alpha_j| \\ \tau_2^{max} &= \sum_{j=1}^n |\tau_{2j}^{max}| = \sum_{j=1}^n |f_j^{max} \sin \alpha_j| \\ \tau_3^{max} &= \sum_{j=1}^n |\tau_{3j}^{max}| = \sum_{j=1}^n |f_j^{max} d_j \sin(\alpha_j - \varphi_j)|\end{aligned}$$

2. O – origin of the Cartesian coordinate system,
3. P – point in the 3-dimensional space with coordinates $(\tau_{d1}, \tau_{d2}, \tau_{d3})$,
4. \overrightarrow{OP} – position vector of the point P .

The evaluation of capacity of the propulsion system to generate the desired propelling forces and moment $\boldsymbol{\tau}_d$ requires taking into consideration both limitations (2.1) simultaneously.

The first one indicates that the vector $\boldsymbol{\tau}_d$ is produced only if the position vector \overrightarrow{OP} is entirely contained in a cubicoïd having vertexes in points: $(\tau_1^{max}, \tau_2^{max}, \tau_3^{max})$, $(\tau_1^{max}, \tau_2^{max}, -\tau_3^{max})$, $(\tau_1^{max}, -\tau_2^{max}, \tau_3^{max})$, $(\tau_1^{max}, -\tau_2^{max}, -\tau_3^{max})$, $(-\tau_1^{max}, \tau_2^{max}, \tau_3^{max})$, $(-\tau_1^{max}, \tau_2^{max}, -\tau_3^{max})$,

$(-\tau_1^{max}, -\tau_2^{max}, \tau_3^{max}), (-\tau_1^{max}, -\tau_2^{max}, -\tau_3^{max})$ (see Fig. 5). Since the components of the vector of demanded commands τ_d are a linear combination of thrusts developed by all propellers, then fulfilling only condition $(2.1)_1$ does not guarantee their generation. For example, if to any element of the vector τ_d there corresponds an assignment $\tau_{di} = \tau_i^{max}$, then the full power of the propulsion system is used to its generation and the rest of the components are equal to zero. Therefore, the evaluation of the capacity of the propulsion system to generation of the vector τ_d requires giving consideration to inequality $(2.1)_2$.

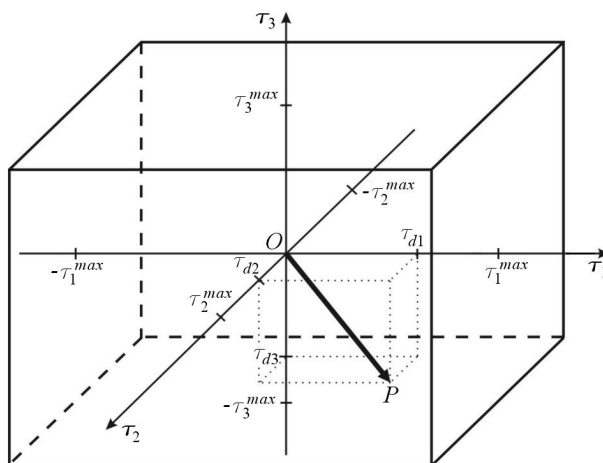


Fig. 5. A view of the cubicoid and the position vector \vec{OP}

The analysis of values that elements of the vector τ_d may take under limitations (2.1) leads to the following conclusion: the quantities τ_{d1}, τ_{d2} and τ_{d3} can be produced by the propulsion system if and only if the position vector \vec{OP} is entirely contained in a trisoctahedron with vertexes in points: $(\tau_1^{max}, 0, 0), (0, \tau_2^{max}, 0), (0, 0, \tau_3^{max}), (-\tau_1^{max}, 0, 0), (0, -\tau_2^{max}, 0), (0, 0, -\tau_3^{max})$ (see Fig. 6). This situation proceeds if the following inequality holds

$$\frac{|\tau_{d1}|}{\tau_1^{max}} + \frac{|\tau_{d2}|}{\tau_2^{max}} + \frac{|\tau_{d3}|}{\tau_3^{max}} \leq 1 \tag{3.1}$$

If (3.1) is false, then the point $P = (\tau_{d1}, \tau_{d2}, \tau_{d3})$ lies outside the octahedron and the vector τ_d can not be generated. It means that the vector of feasible commands $\tau'_d = [\tau'_{d1}, \tau'_{d2}, \tau'_{d3}]^T$ must be determined. Its elements,

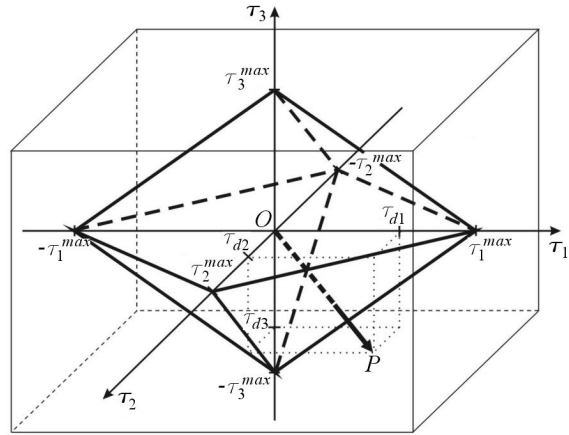


Fig. 6. A view of the trisoctahedron and the position vector \overrightarrow{OP}

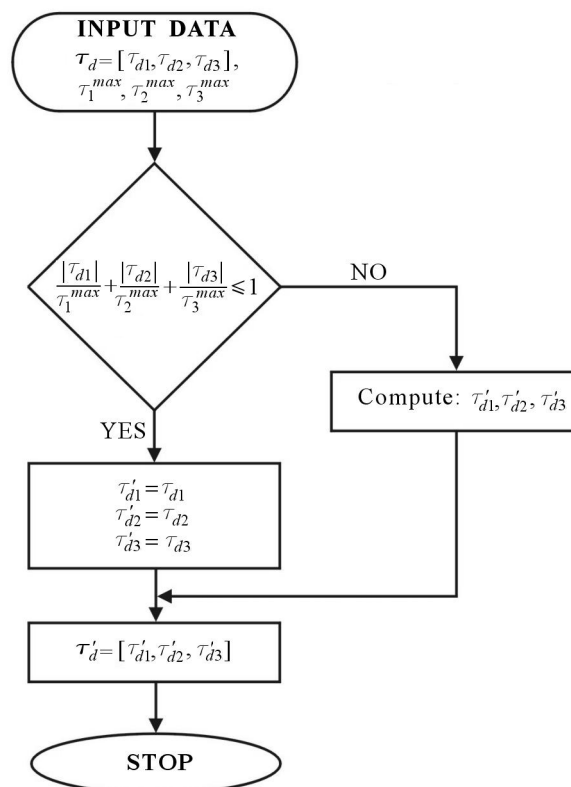


Fig. 7. A block diagram of the algorithm for the determination of feasible commands

on the assumption that reciprocal ratios of corresponding components of the vectors τ_d and τ'_d are preserved

$$\frac{\tau'_{d1}}{\tau'_{d2}} = \frac{\tau_{d1}}{\tau_{d2}} \quad \frac{\tau'_{d1}}{\tau'_{d3}} = \frac{\tau_{d1}}{\tau_{d3}} \quad (3.2)$$

can be computed by means of the following equations

$$\begin{aligned} \tau'_{d1} &= \operatorname{sgn} \tau_{d1} \left(\frac{1}{\tau_1^{max}} + \frac{1}{\tau_2^{max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_3^{max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1} \\ \tau'_{d2} &= \operatorname{sgn} \tau_{d2} \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right| \quad \tau'_{d3} = \operatorname{sgn} \tau_{d3} \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right| \end{aligned} \quad (3.3)$$

Basing on the above considerations, an algorithm for the evaluation of the vector τ_d and determination of τ'_d has been developed (see Fig. 7). Input data to the algorithm are quantities τ_1^{max} , τ_2^{max} , τ_3^{max} and the vector τ_d . The vector of feasible commands τ'_d is computed according to (3.3).

A proof of the dependence (3.3)

Not to decrease of generality of considerations it is assumed that $\tau_{di} \geq 0$ for $i = 1, 2, 3$. It allows to restrict analysis into a subspace limited by positive semi-axes of the coordinate system (see Fig. 8).

Let $A = (\tau_1^{max}, 0, 0)$, $B = (0, \tau_2^{max}, 0)$, $C = (0, 0, \tau_3^{max})$, $P = (\tau_{d1}, \tau_{d2}, \tau_{d3})$ and $P' = (\tau'_{d1}, \tau'_{d2}, \tau'_{d3})$ be points in the 3-dimensional space. An equation of a plane including the points A , B and C has a form

$$\frac{\tau_1}{\tau_1^{max}} + \frac{\tau_2}{\tau_2^{max}} + \frac{\tau_3}{\tau_3^{max}} = 1 \quad (3.4)$$

Let us assume that the point $P' = (\tau'_{d1}, \tau'_{d2}, \tau'_{d3})$ is a common point of a line containing the position vector \overrightarrow{OP} and the plane defined by (3.4). Substituting the coordinates of the point P' into (3.4) and taking into account requirements (3.2), the following set of equations is formulated

$$\begin{aligned} \frac{\tau'_{d1}}{\tau_1^{max}} + \frac{\tau'_{d2}}{\tau_2^{max}} + \frac{\tau'_{d3}}{\tau_3^{max}} &= 1 \\ \frac{\tau'_{d1}}{\tau'_{d2}} &= \frac{\tau_{d1}}{\tau_{d2}} \quad \frac{\tau'_{d1}}{\tau'_{d3}} &= \frac{\tau_{d1}}{\tau_{d3}} \end{aligned} \quad (3.5)$$

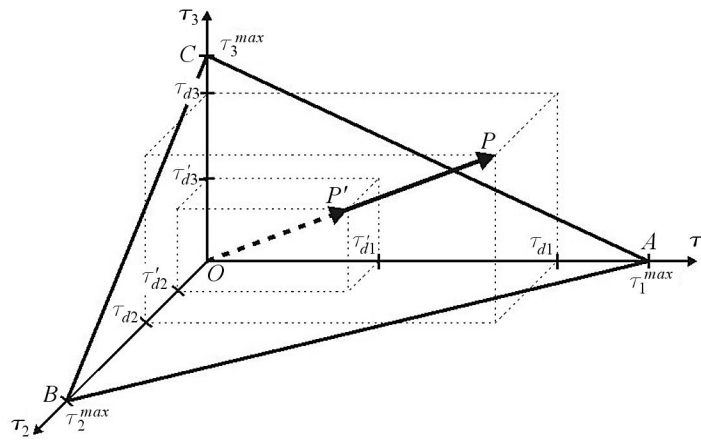


Fig. 8. A view of the position vectors \overrightarrow{OP} and $\overrightarrow{OP'}$ for positive semi-axes of the Cartesian coordinate system

Hence, solving (3.5) the following expressions for calculation of τ'_{d1} , τ'_{d2} and τ'_{d3} are obtained

$$\begin{aligned}\tau'_{d1} &= \left(\frac{1}{\tau_1^{max}} + \frac{1}{\tau_2^{max}} \frac{\tau_{d2}}{\tau_{d1}} + \frac{1}{\tau_3^{max}} \frac{\tau_{d3}}{\tau_{d1}} \right)^{-1} \\ \tau'_{d2} &= \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \quad \tau'_{d3} = \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1}\end{aligned}\quad (3.6)$$

Finally, transformation of the above expressions to the Cartesian coordinate system yields

$$\begin{aligned}\tau'_{d1} &= \operatorname{sgn} \tau_{d1} \left(\frac{1}{\tau_1^{max}} + \frac{1}{\tau_2^{max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_3^{max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1} \\ \tau'_{d2} &= \operatorname{sgn} \tau_{d2} \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right| \quad \tau'_{d3} = \operatorname{sgn} \tau_{d3} \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right|\end{aligned}\quad (3.7)$$

End of the prove

4. Numerical example

The computations are done for the following data of the propulsion system of the underwater robot "Ukwiał" designed and built for the Polish Navy (see Garus, 2004):

$$f_i^{max} = 250 \text{ N} \quad d_i = 0.4 \text{ m} \quad \text{for} \quad i = 1, 2, 3, 4$$

and

$$\boldsymbol{\alpha} = [29.0^\circ, -29.0^\circ, 151.0^\circ, 209.0^\circ]$$

$$\boldsymbol{\varphi} = [-26.5^\circ, 26.5^\circ, 206.5^\circ, 153.5^\circ]$$

Hence

$$\tau_1^{max} = \sum_{i=1}^4 |\tau_{1i}^{max}| = \sum_{i=1}^4 |f_i^{max} \cos \alpha_i| = 875.0 \text{ N}$$

$$\tau_2^{max} = \sum_{i=1}^4 |\tau_{2i}^{max}| = \sum_{i=1}^4 |f_i^{max} \sin \alpha_i| = 485.0 \text{ N}$$

$$\tau_3^{max} = \sum_{i=1}^4 |\tau_{3i}^{max}| = \sum_{i=1}^4 |f_i^{max} d_i \sin(\alpha_i - \varphi_i)| = 332.0 \text{ Nm}$$

Let us assume that $\boldsymbol{\tau}_d = [700.0, -120.0, 30.0]^\top$.

STEP 1

Calculate inequality (3.1) to check the capacity of the propulsion system in order to generate the vector $\boldsymbol{\tau}_d$

$$\frac{|\tau_{d1}|}{\tau_1^{max}} + \frac{|\tau_{d2}|}{\tau_2^{max}} + \frac{|\tau_{d3}|}{\tau_3^{max}} \leq 1$$

$$\frac{|700.0|}{875.0} + \frac{|-120.0|}{485.0} + \frac{|30.0|}{332.0} \leq 1$$

$$1.14 \leq 1.0$$

Since the inequality is false (i.e. the point $P = (700.0, -120.0, 30.0)$ lies outside the trisoctahedron having the vertexes in points: $(875, 0, 0)$, $(0, 485, 0)$, $(0, 0, 332)$, $(-875, 0, 0)$, $(0, -485, 0)$, $(0, 0, -332)$), the vector of feasible commands $\boldsymbol{\tau}'_d$ must be determined.

STEP 2

Calculate the components of the vector $\boldsymbol{\tau}'_d$

$$\tau'_{d1} = \operatorname{sgn} \tau_{d1} \left(\frac{1}{\tau_1^{max}} + \frac{1}{\tau_2^{max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_3^{max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1} =$$

$$= \operatorname{sgn} 700.0 \left(\frac{1}{875} + \frac{1}{485} \left| \frac{-120.0}{700.0} \right| + \frac{1}{332} \left| \frac{30.0}{700.0} \right| \right)^{-1} = 614.6$$

$$\tau'_{d2} = \operatorname{sgn} \tau_{d2} \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right| = \operatorname{sgn} (-120.0) \left| \frac{-120.0}{700.0} 614.6 \right| = -105.4$$

$$\tau'_{d3} = \operatorname{sgn} \tau_{d3} \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right| = \operatorname{sgn} 30.0 \left| \frac{30.0}{700.0} 614.6 \right| = 26.3$$

To check the truth of the calculations, the ratios of the corresponding components of the vectors $\boldsymbol{\tau}_d$ and $\boldsymbol{\tau}'_d$ are computed according to (3.2)

$$\begin{aligned} \frac{\tau_{d1}}{\tau_{d2}} &= \frac{700.0}{-120.0} = -5.8 & \frac{\tau'_{d1}}{\tau'_{d2}} &= \frac{614.6}{-105.43} = -5.8 \\ \frac{\tau_{d1}}{\tau_{d3}} &= \frac{700.0}{30.0} = 22.3 & \frac{\tau'_{d1}}{\tau'_{d3}} &= \frac{614.6}{26.3} = 22.3 \end{aligned}$$

The obtained values indicate that the ratios are preserved. It confirms the correctness of the proposed approach.

5. Conclusions

The paper presents a method of determination of feasible propelling forces and moment for underwater robotic vehicles. For a robot moving in the horizontal plane, it is necessary to distribute the propelling forces and moment $\boldsymbol{\tau}_d \in \mathfrak{R}^3$ among n propellers in terms of the thrust $\boldsymbol{f} \in \mathfrak{R}^n$. Computation of \boldsymbol{f} from $\boldsymbol{\tau}_d$ is an optimisation problem based on a model, which in the simplest form is unconstrained. In real applications, however, due to physical limitations, (e.g. saturations), this task must be solved as a constrained optimisation problem. To cope with those difficulties, a procedure of checking the required forces and moments and the determination of feasible ones has been worked out. It allows one to find such a vector $\boldsymbol{\tau}'_d$ that unconstrained optimisation methods can be used without any restrictions.

The main advantage of the approach is its simplicity and flexibility with regard to the construction of the vehicle power transmission system and number of thrusters. The developed procedure of determination of feasible commands is of a general character and can be successfully applied to all types of URVs.

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Metoda wyznaczania dopuszczalnych sił napędowych i momentu napędowego dla robota podwodnego

Streszczenie

Praca dotyczy zagadnienia rozdziału mocy w układzie napędowym robota podwodnego. Dla ruchu w płaszczyźnie poziomej wektor naporów \mathbf{f} wyznaczany jest na podstawie wektora zadanych sił i momentów napędowych $\boldsymbol{\tau}_d$. Zadanie to rozpatrywane jest najczęściej jako problem optymalizacyjny bez ograniczeń. Otrzymane w ten sposób rozwiązanie zapewnia wygenerowanie żądanych wartości $\boldsymbol{\tau}_d$ tylko wtedy, gdy nie występuje żądanie rozwinięcia przez którykolwiek z pędników naporu przekraczającego wartość graniczną. Jeżeli ma to miejsce, to żądane siły i moment nie mogą być wytworzone. Stąd proponuje się realizację procedury rozdziału mocy dwuetapowo. W etapie pierwszym następuje ocena możliwości wytworzenia przez układ napędowy zadanych sił i momentu $\boldsymbol{\tau}_d$ i wyznaczane są ich wartości dopuszczalne $\boldsymbol{\tau}'_d$, tj. takie, które możliwe są do wygenerowania. W etapie drugim, na podstawie $\boldsymbol{\tau}'_d$, dokonywany jest właściwy przydział naporów na poszczególne pędniki.

Zamieszczony w pracy algorytm wyznaczania dopuszczalnych sił napędowych i momentu napędowego opracowany został z ukierunkowaniem na jego praktyczne zastosowanie w układzie automatycznego sterowania robotem podwodnym „Ukwiał”, eksploatowanym na okrętach Marynarki Wojennej.

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