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AN ALTERNATIVE SCHEME FOR DETERMINATION OF JOINT REACTION FORCES IN HUMAN MULTIBODY MODELS

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Multibody models are commonly used in the analysis of human movements. The dynamic formulations often use minimal sets of generalized coordinates, and joint reactions (non-working reactions of model-intrinsic constraints) are excluded from evidence. A separate modeling effort is then required to determine joint reactions, and the arising numerical procedures are computationally arduous. In this paper, a novel efficient approach to the determination of joint reactions is developed, which naturally assists the minimal-form formulations of human body dynamics. The proposed scheme does not involve matrix inversion, and as such it is well suited for both symbolic manipulations and computer implementations. The method is illustrated with a seven-segment planar model of a human body. Some results from the inverse dynamics simulation of somersaults on a trampoline are reported.

Key words: human body modeling, dynamic analysis, joint reaction forces

1. Introduction

Joint reaction forces play an important role in the dynamics of human movements such as walking, jumping and gymnastic exercises; see e.g. Bergman *et al.* (2001) and McNitt-Gray *et al.* (2001). The question of which loads cross the joints during routine or sport activities may be of considerable interest to the clinicians (Wismans *et al.*, 1994). One way to get the information is to build a mathematical model of the human body dynamics and use it for the determination of joint reaction forces by means of numerical simulation. However, dynamical formulations of the human body usually use minimal sets of generalized coordinates, and the joint reactions (non-working reactions of model-intrinsic constraints) are excluded from evidence (Eberhard *et al.*, 1999; Tözeren, 2000; Pandy, 2001; Yamaguchi, 2001). In order to obtain the joint reactions by using classical multibody codes, described e.g. by Langer *et al.* (1987), Nikravesh (1988), García de Jalón and Bayo (1993), Schiehlen (1997) and Blajer (2001), a separate modeling effort is required, and the arising numerical procedures may be computationally arduous.

In this paper, a novel approach to the determination of joint reactions, naturally assisted with the minimal-form formulation of human body dynamics, is described. The idea of the scheme is similar to that of Kane and Levinson (1985), called by them "bringing noncontributing forces into evidence". This extension of Kane's method, in which the joint reaction forces are originally eliminated early in the process of deriving dynamic equations, has then been exploited e.g. by Langer *et al.* (1987), Lesser (1992), Komistek *et al.* (1998), Tisell (2000) and Yamaguchi (2001). While in Kane's approach some auxiliary *fictitious generalized speeds* are used to identify the noncontributing forces, here we introduce *open-constraint coordinates* which express prohibited relative motions in the joints, in addition to the *joint coordinates* that describe relative configurations of adjacent body segments. The followed *augmented joint coordinate method* yields standard formulae that lead to the minimal-form dynamic equations, and simultaneously, a pseudo-inverse matrix to the joint constraint matrix is obtained without much effort. Using the pseudo-inverse, the formulae for the determination of joint reactions are obtained directly in a "resolved" form (no matrix inversion is required). This makes the developed scheme particularly well suited for both symbolic manipulations and computer implementations.

2. Augmented joint coordinate method

For simplicity reasons, the method is introduced by using a seven-segment multibody system shown in Fig. 1a, which models the human body in planar motion with collateral movements of the lower and upper extremities (Blajer and Czaplicki, 2001). The position of the nine-degree-of-freedom system can explicitly be described by $k = 9$ generalized coordinates

$\boldsymbol{q} = [x_H, y_H, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7]^\top$, where x_H and y_H are the hip coordinates, and the angular coordinates φ_i $(i = 1, \ldots, 7)$ that describe the angular onfigurations of the seven segments are all measured from the vertical direction. The six control torques due to muscle forces at the joints are $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6]^\top$.

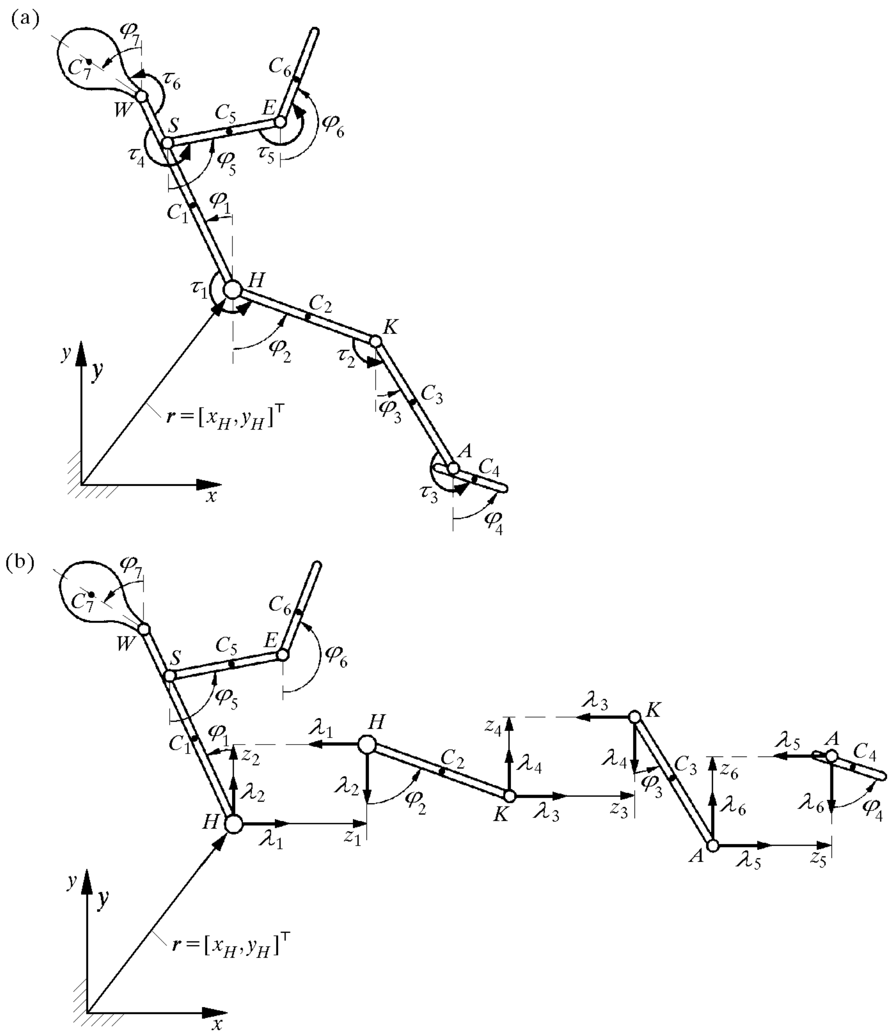

Fig. 1. Seven-segment multibody model (a), and open-constraint coordinates and reaction forces in joints of lower extremities (b)

The derivation of equations of motion in $\boldsymbol{q}$ for the above system belongs to standard multibody codes, well described in the literature; see e.g. Kane and Levinson (1985), Nikravesh (1988), Lesser (1992), García de Jalón and Bayo (1993), Schiehlen (1997), and Blajer (2001). The starting point is the dynamical formulation in $n = 21$ absolute coordinates $\boldsymbol{p} = [x_1, y_1, \theta_1, \dots, x_7, y_7, \theta_7]^\top$, where x_i, y_i and θ_i are the coordinates of the mass center C_i and the orientation angle (here $\theta_i = \varphi_i$) of the ith segment, $i = 1, \dots, 7$. The absolute coordinates $\boldsymbol{p}$ are dependent because of the joints, modeled by $m = 12$ kinematic constraints on the bodies, $k = n - m$, called by Langer *et al.* (1987) the *model-intrinsic* constraints (as distinct from the *model-environment* ones). The constraints express the prohibited relative translations in the joints, and m local *open-constraint coordinates* $\boldsymbol{z} = [z_1, \dots, z_m]^\top$ can be introduced to describe these prohibited relative motions (illustrated in Fig. 1b for the joints of lower extremities). Since $\boldsymbol{z}$ can be expressed in terms of $\boldsymbol{p}$, the constraint equations at the position, velocity and acceleration levels, given in *implicit forms* (Schiehlen, 1997; Blajer, 2001), are

$$
\begin{aligned}
\boldsymbol{z} &= \boldsymbol{\Phi}(\boldsymbol{p}) = \boldsymbol{0} \\
\dot{\boldsymbol{z}} &= \mathbf{C}(\boldsymbol{p})\dot{\boldsymbol{p}} = \boldsymbol{0} \\
\ddot{\boldsymbol{z}} &= \mathbf{C}(\boldsymbol{p})\ddot{\boldsymbol{p}} - \boldsymbol{\xi}(\boldsymbol{p}, \dot{\boldsymbol{p}}) = \boldsymbol{0}
\end{aligned}
\tag{2.1}
$$

where $\mathbf{C} = \partial\boldsymbol{\Phi}/\partial\boldsymbol{p}$ is the $m \times n$ constraint matrix of the maximum row-rank, i.e. constraints $(2.1)_1$ are independent, and the m vector $\boldsymbol{\xi} = -\dot{\mathbf{C}}\dot{\boldsymbol{p}}$ involves the constraint-induced accelerations on the body segments. A particular constraint equation Φ_j $(j = 1, \dots, m)$ depends only on the absolute coordinates of the adjacent bodies in the joint.

Since the coordinates $\boldsymbol{z}$ describe the prohibited relative translations of the body segments in the joints, the associated constraint reactions $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_m]^\top$ represent physical joint reaction forces related to $\boldsymbol{z}$ (Fig. 1b). The constrained Newton-Euler equations of motion are then

$$
\mathbf{M}\ddot{\boldsymbol{p}} = \boldsymbol{h} - \mathbf{C}^\top\boldsymbol{\lambda} \tag{2.2}
$$

where $\mathbf{M} = \operatorname{diag}(\mathbf{M}^{(1)}, \dots, \mathbf{M}^{(7)})$ and $\boldsymbol{h} = [(\boldsymbol{h}^{(1)})^\top, \dots, (\boldsymbol{h}^{(7)})^\top]^\top$ are the constant generalized mass matrix and the generalized applied forces related to $\boldsymbol{p}$, $\mathbf{M}^{(i)} = \operatorname{diag}(m_i, m_i, J_{C_i})$, m_i and J_{C_i} are the mass of the ith segment and its mass moment of inertia with respect to C_1, and $\boldsymbol{h}^{(i)} = [F_{xi}, F_{yi}, M_{Ci}]^\top$ are components of the total force applied to the ith segment and the total torque applied with respect to the mass center C_i.

In the case at hand, $\boldsymbol{h}$ consists of gravitational forces, interactions from the environment (reactions of the model-environment constraints) and muscle forces (control torques).

The standard *joint coordinate formulation* of multibody system dynamics (Nikravesh, 1988) is based on relations between the absolute coordinates $\boldsymbol{p}$ and the joint coordinates $\boldsymbol{q}$, $\boldsymbol{p} = \boldsymbol{g}(\boldsymbol{q})$, which are the model-intrinsic constraints given *explicitly* (Schiehlen, 1997; Blajer, 2001). Implicit constraint equations $(2.1)_1$ are satisfied identically when expressed in term of $\boldsymbol{q}$, $\boldsymbol{\Phi}(\boldsymbol{g}(\boldsymbol{q})) \equiv \mathbf{0}$.

In the present *augmented joint coordinate method*, the explicit constraint equations are extended by incorporating the open-constraint coordinates $\boldsymbol{z}$

$$\boldsymbol{p} = \boldsymbol{g}(\boldsymbol{q}, \boldsymbol{z}) \tag{2.3}$$

Since $\boldsymbol{z} = \mathbf{0}$, the above relations are equivalent to the traditional form $\boldsymbol{p} = \boldsymbol{g}(\boldsymbol{q})$, and are usually not much more difficult to formulate. In fact, the dependence on $\boldsymbol{z}$ in (2.3) is needed only to grasp the prohibited motion directions related to $\dot{\boldsymbol{z}}$, called *auxiliary fictitious generalized speeds* in Kane's method (Kane and Levinson, 1985). Namely, by differentiating (2.3) with respect to time and then setting $\boldsymbol{z} = \mathbf{0}$, we obtain

$$\dot{\boldsymbol{p}} = \Big(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{q}}\Big)\Big|_{\boldsymbol{z}=\mathbf{0}} \dot{\boldsymbol{q}} + \Big(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}}\Big)\Big|_{\boldsymbol{z}=\mathbf{0}} \dot{\boldsymbol{z}} \equiv \mathbf{D}(\boldsymbol{q})\dot{\boldsymbol{q}} + \mathbf{E}(\boldsymbol{q})\dot{\boldsymbol{z}} \tag{2.4}$$

while the standard formulation $\boldsymbol{p} = \boldsymbol{g}(\boldsymbol{q})$ yields simply $\dot{\boldsymbol{p}} = \mathbf{D}(\boldsymbol{q})\dot{\boldsymbol{q}}$. Again, since the maintenance of joint constraints assures $\dot{\boldsymbol{z}} = \mathbf{0}$, both relations are genuinely equivalent. The explicit constraint equations at the acceleration level are then considered in the standard form

$$\ddot{\boldsymbol{p}} = \mathbf{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{\gamma}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{2.5}$$

where $\boldsymbol{\gamma} = \dot{\mathbf{D}}\dot{\boldsymbol{q}}$ is an n-dimensional vector.

As shown in Blajer (2001), the $n \times k$ matrix $\mathbf{D}$ defined above is an orthogonal complement matrix to the constraint matrix $\mathbf{C}$ introduced in $(2.1)_2$, i.e.

$$\mathbf{CD} = \mathbf{0} \ \Leftrightarrow\ \mathbf{D}^\top\mathbf{C}^\top = \mathbf{0} \tag{2.6}$$

The said prohibited local motion directions in the joints are then represented as columns in the $n \times m$ matrix $\mathbf{E}$ defined in (2.4). An important characteristic of $\mathbf{E}$ results from the substitution of (2.4) into $(2.1)_2$, which gives $\dot{\boldsymbol{z}} = \mathbf{CD}\dot{\boldsymbol{q}} + \mathbf{CE}\dot{\boldsymbol{z}}$. Since, according to (2.6), $\mathbf{CD} = \mathbf{0}$, it can then be concluded that

$$\mathbf{CE} = \mathbf{I} \ \Leftrightarrow\ \mathbf{E}^\top\mathbf{C}^\top = \mathbf{I} \tag{2.7}$$

where $\mathbf{I}$ denotes the $m \times m$ identity matrix. The $n \times m$ matrix $\mathbf{E}$ produced in (2.4) has thus features of a *pseudo-inverse* (Ben-Israel and Greville, 1980) of the rectangular $m \times n$ constraint matrix $\mathbf{C}$. It is worth to note that $\mathbf{E}$ is obtained here symbolically, and the constraint matrix $\mathbf{C}$ is not involved in the process.

Starting from dynamic formulation (2.2) in the absolute coordinates $\boldsymbol{p}$, the explicit forms of the model-intrinsic constraints, $\boldsymbol{p} = \boldsymbol{g}(\boldsymbol{q}) \Rightarrow \dot{\boldsymbol{p}} = \mathbf{D}(\boldsymbol{q})\dot{\boldsymbol{q}} \Rightarrow \ddot{\boldsymbol{p}} = \mathbf{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{\gamma}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ are enough for converting the constraint reaction-induced dynamic equations to a minimal set of constraint reaction-free dynamic equations in $\boldsymbol{q}$. Then, the pseudoinverse matrix $\mathbf{E}$ enables efficient determination of the 'eliminated' constraint reactions. The transformation formula is

$$\begin{bmatrix} \mathbf{D}^\top \\ \mathbf{E}^\top \end{bmatrix} \Big(\mathbf{M}(\mathbf{D}\ddot{\boldsymbol{q}} + \gamma) - \boldsymbol{h} + \mathbf{C}^\top \boldsymbol{\lambda} \Big) = \mathbf{0} \tag{2.8}$$

while in the standard formulation of the projection method scheme (Blajer, 2001), $\mathbf{E}^\top$ is replaced by $\mathbf{C}\mathbf{M}^{-1}$. The first k equations of (2.8) yield the requested dynamic equations in $\boldsymbol{q}$

$$\overline{\mathbf{M}}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \overline{\boldsymbol{d}}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \overline{\boldsymbol{h}}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) \tag{2.9}$$

where $\overline{\mathbf{M}} = \mathbf{D}^\top\mathbf{M}\mathbf{D}$ is the $k \times k$ generalized mass matrix related to $\boldsymbol{q}$, $\overline{\boldsymbol{d}} = \mathbf{D}^\top\mathbf{M}\gamma$ is the k vector of generalized dynamic forces due to centrifugal and Coriolis accelerations, and $\overline{\boldsymbol{h}} = \mathbf{D}^\top\boldsymbol{h}$ is the k vector of generalized applied forces. From the last m equations of (2.8), one obtains

$$\boldsymbol{\lambda}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, t) = \mathbf{E}^\top \Big(\boldsymbol{h} - \mathbf{M}(\mathbf{D}\ddot{\boldsymbol{q}} + \gamma) \Big) \tag{2.10}$$

which offers a novel formula for the determination of joint reaction forces, distinct from the traditional one obtained after replacing $\mathbf{E}^\top$ with $\mathbf{C}\mathbf{M}^{-1}$ in (2.8), see e.g. Blajer (2001), García de Jalón and Bayo (1993), and Schiehlen (1997) for more details, i.e.

$$\boldsymbol{\lambda}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) = (\mathbf{C}\mathbf{M}^{-1}\mathbf{C}^\top)^{-1}\mathbf{C}(\mathbf{M}^{-1}\boldsymbol{h} - \gamma) \tag{2.11}$$

which is seldom used in biomechanical models.

There are at least three advantages of the new scheme expressed by (2.10) over the traditional one given by (2.11). Firstly, the joint reactions $\boldsymbol{\lambda}$ are directly obtained in a "resolved" form - no matrix inversion is required as it is necessary in (2.11). The present scheme is thus particularly well suited for both symbolic manipulations and computational implementations. Then, formula (2.10) does not rely on implicit forms (2.1) of model-intrinsic constraint equations, which need not to be introduced at all. Both the derivation of minimal-form dynamic equations (2.11) and the determination of joint reactions are now based on the constraint equations given implicitly in augmented form (2.3). The modification is not usually concerned with much additional modeling effort. Finally, the present scheme can conveniently be used to determine only some joint reactions - only respective entries of $\boldsymbol{z}$ can be introduced, and a number of columns of $\mathbf{E}$ can appropriately be reduced. Such a situation is illustrated in Fig. 1b, where the open-constraint coordinates only in the joints of lower extremities are involved.

3. Illustration

The proposed method was used to calculate reactions in the joints of lower extremities of the human planar multibody model shown in Fig. 1a, used by Blajer and Czaplicki (2001) in analysis of somersaults on a trampoline. The vector $\boldsymbol{h}$ in dynamic equations (2.2) contains gravitational forces, muscle force torques, and, during the support phase, the interaction from the trampoline bed. The way of estimation of this interaction, identification of the model, and inverse dynamics procedure applied to solve the inverse dynamics problem is described in Blajer and Czaplicki (2001).

Since we are interested solely in the joint reactions in lower extremities, the open-constraint coordinates $\boldsymbol{z} = [z_1, \dots, z_6]^\top$ are introduced only to joints H, K and A (Fig. 1b). Consequently, denoted $\boldsymbol{g} = [(\boldsymbol{g}^{(1)})^\top, \dots, (\boldsymbol{g}^{(7)})^\top]^\top$, relation (2.3) will take the traditional form $\boldsymbol{p}^{(i)} = \boldsymbol{g}^{(i)}(\boldsymbol{q})$ for $i = 1, 5, 6, 7$, and the augmented form $\boldsymbol{p}^{(i)} = \boldsymbol{g}^{(i)}(\boldsymbol{q}, \boldsymbol{z})$ for $i = 2, 3, 4$. Omitting for shortness the former relations, the latter ones are

$$
\begin{aligned}
x_2 &= x_H + z_1 + c_2 \sin\varphi_2 \\
y_2 &= y_H + z_2 - c_2 \cos\varphi_2 \\
\theta_2 &= \varphi_2 \\[6pt]
x_3 &= x_H + z_1 + l_2 \sin\varphi_2 + z_3 + c_3 \sin\varphi_3 \\
y_3 &= y_H + z_2 - l_2 \cos\varphi_2 + z_4 - c_3 \cos\varphi_3 \\
\theta_3 &= \varphi_3 \\[6pt]
x_4 &= x_H + z_1 + l_2 \sin\varphi_2 + z_3 + l_3 \sin\varphi_3 + z_5 + c_4 \sin\varphi_4 \\
y_4 &= y_H + z_2 - l_2 \cos\varphi_2 + z_4 - l_3 \cos\varphi_3 + z_6 - c_4 \cos\varphi_4 \\
\theta_4 &= \varphi_4
\end{aligned}
\tag{3.1}
$$

where l_2 and l_3 are the total lengths of segments 2 and 3, and c_2, c_3 and c_4 are the distances HC_2, KC_3 and AC_4, respectively, and C_i $(i = 1, \dots, 7)$ are the mass centers of the seven segments as denoted in Fig. 1.

Using the relations for $i = 1, \dots, 7$, the 21×9-dimensional matrix $\mathbf{D}$ and the 21-vector $\boldsymbol{\gamma}$ defined in (2.4) and (2.5) can easily be obtained, and dynamic equations (2.9) in $\boldsymbol{q}$ can be derived. The explicit form of the dynamic equations is reported in Blajer and Czaplicki (2001). As far as the joint reactions are concerned, let us denote first $\mathbf{E} = [(\mathbf{E}^{(1)})^\top, \dots, (\mathbf{E}^{(7)})^\top]^\top$, where $\mathbf{E}^{(i)}$ is the 3×6-dimensional matrix related to the ith segment. In the 21×6-dimensional matrix $\mathbf{E}$, we have then $\mathbf{E}^{(1)} = \mathbf{E}^{(5)} = \mathbf{E}^{(6)} = \mathbf{E}^{(7)} = \mathbf{0}$, and the component matrices related to the joints of lower extremities (rows 4 - 12 of $\mathbf{E}$) are

$$\mathbf{E}^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{E}^{(3)} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{E}^{(4)} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.2}$$

The analytical relations for the joint reactions in H, K and A joints are then

$$\begin{aligned}
\lambda_1 &= R_x - m_2(\ddot{x}_H + c_2\ddot{\varphi}_2 \cos\varphi_2 - c_2\dot{\varphi}_2^2 \sin\varphi_2) + \\
&- m_3(\ddot{x}_H + l_2\ddot{\varphi}_2 \cos\varphi_2 + c_3\ddot{\varphi}_3 \cos\varphi_3 - l_2\dot{\varphi}_2^2 \sin\varphi_2 - c_3\dot{\varphi}_3^2 \sin\varphi_3) + \\
&- m_4(\ddot{x}_H + l_2\ddot{\varphi}_2 \cos\varphi_2 + l_3\ddot{\varphi}_3 \cos\varphi_3 + c_4\ddot{\varphi}_4 \cos\varphi_4 - l_2\dot{\varphi}_2^2 \sin\varphi_2 + \\
&- l_3\dot{\varphi}_3^2 \sin\varphi_3 - c_4\dot{\varphi}_4^2 \sin\varphi_4) \\[6pt]
\lambda_2 &= R_y - (m_2 + m_3 + m_4)g - m_2(\ddot{y}_H + c_2\ddot{\varphi}_2 \sin\varphi_2 + c_2\dot{\varphi}_2^2 \cos\varphi_2) + \\
&- m_3(\ddot{y}_H + l_2\ddot{\varphi}_2 \sin\varphi_2 + c_3\ddot{\varphi}_3 \sin\varphi_3 + l_2\dot{\varphi}_2^2 \cos\varphi_2 + c_3\dot{\varphi}_3^2 \cos\varphi_3) + \\
&- m_4(\ddot{y}_H + l_2\ddot{\varphi}_2 \sin\varphi_2 + l_3\ddot{\varphi}_3 \sin\varphi_3 + c_4\ddot{\varphi}_4 \sin\varphi_4 + l_2\dot{\varphi}_2^2 \cos\varphi_2 + \\
&+ l_3\dot{\varphi}_3^2 \cos\varphi_3 + c_4\dot{\varphi}_4^2 \cos\varphi_4) \\[6pt]
\lambda_3 &= R_x - m_3(\ddot{x}_H + l_2\ddot{\varphi}_2 \cos\varphi_2 + c_3\ddot{\varphi}_3 \cos\varphi_3 - l_2\dot{\varphi}_2^2 \sin\varphi_2 - c_3\dot{\varphi}_3^2 \sin\varphi_3) + \\
&- m_4(\ddot{x}_H + l_2\ddot{\varphi}_2 \cos\varphi_2 + l_3\ddot{\varphi}_3 \cos\varphi_3 + c_4\ddot{\varphi}_4 \cos\varphi_4 - l_2\dot{\varphi}_2^2 \sin\varphi_2 + \\
&- l_3\dot{\varphi}_3^2 \sin\varphi_3 - c_4\dot{\varphi}_4^2 \sin\varphi_4) \qquad (3.3) \\[6pt]
\lambda_4 &= R_y - (m_3 + m_4)g - m_3(\ddot{y}_H + l_2\ddot{\varphi}_2 \sin\varphi_2 + c_3\ddot{\varphi}_3 \sin\varphi_3 + l_2\dot{\varphi}_2^2 \cos\varphi_2 + \\
&= c_3\dot{\varphi}_3^2 \cos\varphi_3) - m_4(\ddot{y}_H + l_2\ddot{\varphi}_2 \sin\varphi_2 + l_3\ddot{\varphi}_3 \sin\varphi_3 + c_4\ddot{\varphi}_4 \sin\varphi_4 + \\
&+ l_2\dot{\varphi}_2^2 \cos\varphi_2 + l_3\dot{\varphi}_3^2 \cos\varphi_3 + c_4\dot{\varphi}_4^2 \cos\varphi_4)
\end{aligned}$$

$$\begin{aligned}\lambda_5 &= R_x - m_4(\ddot{x}_H + l_2\ddot{\varphi}_2\cos\varphi_2 + l_3\ddot{\varphi}_3\cos\varphi_3 + c_4\ddot{\varphi}_4\cos\varphi_4 - l_2\dot{\varphi}_2^2\sin\varphi_2 + \\ &- l_3\dot{\varphi}_3^2\sin\varphi_3 - c_4\dot{\varphi}_4^2\sin\varphi_4)\end{aligned}$$

$$\begin{aligned}\lambda_6 &= R_y - m_4g - m_4(\ddot{y}_H + l_2\ddot{\varphi}_2\sin\varphi_2 + l_3\ddot{\varphi}_3\sin\varphi_3 + c_4\ddot{\varphi}_4\sin\varphi_4 + \\ &+ l_2\dot{\varphi}_2^2\cos\varphi_2 + l_3\dot{\varphi}_3^2\cos\varphi_3 + c_4\dot{\varphi}_4^2\cos\varphi_4)\end{aligned}$$

where m_2, m_3 and m_4 are the masses of segments 2, 3 and 4, R_x and R_y are the components of the force acting on the feet (segment 4) from the trampoline, and g is the gravity acceleration. It my be worth noting that the above result is a little different from that possibly obtained by recursive application of the classical dynamic equilibrium D'Alembert's principle, starting from the last (fourth) to the first (second) segment. In the present solution, the joint reaction forces are obtained in a resolved form, while in the other solution, the reactions λ_3 and λ_4 (in K joint) are dependent on λ_5 and λ_6 (in A joint), and the reactions λ_1 and λ_2 (in H joint) are dependent on λ_3 and λ_4 (in K joint).

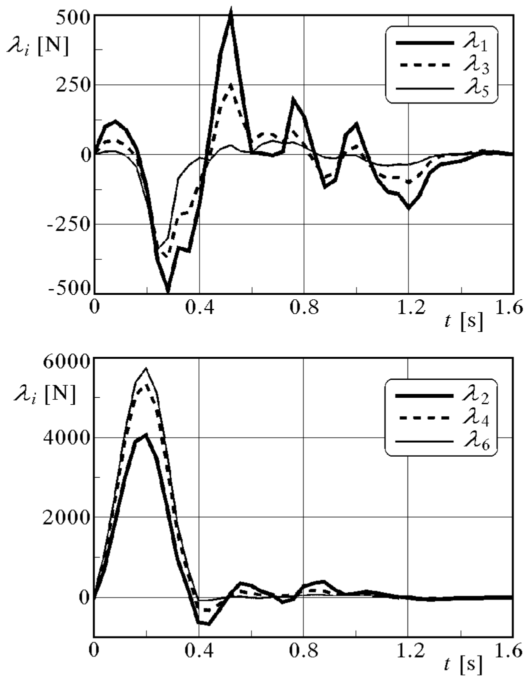

Fig. 2. Reaction forces in joints of lower extremities during back somersault on a trampoline

A back somersault in the pike position was chosen to present the results of calculations. By using $\boldsymbol{q}_d(t)$, $\dot{\boldsymbol{q}}_d(t)$, $\ddot{\boldsymbol{q}}_d(t)$, $R_{xd}(t)$ and $R_{yd}(t)$ from measurements previously reported by Blajer and Czaplicki (2001), the joint reactions were determined from (2.10). Figure 2 shows time variations of the reactions in lower extremities during a single somersault. Obviously, the joints are most loaded at the supporting phase, $0 - 0.4$. One can easily see a significant drop of the vertical reaction from the ankle to hip level, during this phase.

4. Conclusions

The proposed method for the determination of joint reactions is relatively simple and naturally assists minimal-form formulation (2.9) of human body dynamics. Computational scheme (2.10) is particularly efficient - no matrix inversion is required as it is needed in classical code (2.11). As such, scheme (2.10) is well suited for both symbolic manipulations and computer implementations.

The idea of the proposed method is not new, it has much in common with Kane's method of "bringing noncontributing forces into evidence". The novelty of the present formulation lies in its compactness and clarity. The method for the determination of joint reactions is presented in a systematic and practical way.

For simplicity reasons, the proposed method is introduced and illustrated in this paper by means of a very simple planar model of the human body. Nevertheless, the methodology behind is general. The formulation is extendable to a three dimensional case. A more realistic musculoskeletal model can be used as well.

References

1. Ben-Israel A., Greville T.N.E., 1980, *Generalized Inverses: Theory and Applications*, Robert E. Krieger Publishing Company, Huntington, New York

2. Bergman G., Deuretzenbacher G., Heller M., Graichen F., Rohlmann A., Strauss J., Duda G.N., 2001, Hip contact forces and gait patterns from routine activities, *Journal of Biomechanics*, **34**, 859-871

3. Blajer W., 2001, A geometrical interpretation and uniform matrix formulation of multibody system dynamics, *ZAMM*, **81**, 247-259

4. Blajer W., Czaplicki A., 2001, Modeling and inverse simulation of somersaults on the trampoline, *Journal of Biomechanics*, **34**, 1619-1629

5. Eberhard P., Spgele T., Gollhofer A., 1999, Investigations for the dynamical analysis of human motion, *Multibody System Dynamics*, **3**, 1-20

6. García de Jalón J., Bayo E., 1993, *Kinematic and Dynamic Simulation of Multibody Systems: The Real-time Challenge*, Springer, New York

7. Kane T.R., Levinson D.A., 1985, *Dynamics: Theory and Applications*, McGraw-Hill, New York

8. Komistek R.D., Stiehl J.B., Dennis D.A., Paxson R.D., Soutas-Little R.W., 1998, Mathematical model of the lower extremity joint reaction forces using Kane's method of dynamics, *Journal of Biomechanics*, **31**, 185-189

9. Langer F.D., Hemami H., Brown D.B., 1987, Constraint forces in holonomic mechanical systems, *Computer Methods in Applied Mechanics and Engineering*, **62**, 255-274

10. Lesser M., 1992, A geometrical interpretation of Kane's equations, *Proceedings of the Royal Society of London*, **A 436**, 69-87

11. McNitt-Gray J.L., Hester D.M.E., Mathiyakom W., Munkasy B.A., 2001, Mechanical demand and multijoint control during landing depend on orientation of the body segments relative to the reaction force, *Journal of Biomechanics*, **34**, 1471-1482

12. Nikravesh P.E., 1988, *Computer-Aided Analysis of Mechanical Systems*, Price-Hall, Enlewood Cliffs

13. Pandy M.G., 2001, Computer modelling and simulation of human movement, *Annual Review of Biomedical Engineering*, **3**, 245-273

14. Schiehlen W., 1997, Multibody system dynamics: roots and perspectives, *Multibody System Dynamics*, **1**, 149-188

15. Tisell C., 2000, *Integration of Database Technology and Multibody System Analysis*, Doctoral Thesis, Department of Machine Design, Royal Institute o Technology, Stockholm, Sweden

16. Tözeren A., 2000, *Human Body Dynamics: Classical Mechanics and Human Movement*, Springer, New York

17. Wismans J.S.H.M., Janssen E.G., Beusenberg M., Koppens W.P., Lupker H.A., 1994, *Injury Biomechanis*, Course notes, Faculty of Mechanical Engineering, Eindhoven University of Technology, Endhoven

18. Yamaguchi G.T., 2001, *Dynamic Modeling of Musculoskeletal Motion*, Kluwer, Norwell, Massachusetts

Alternatywny sposób wyznaczania reakcji w stawach wieloczłonowych modeli ciała człowieka

Streszczenie

Wieloczłonowe modele ciała człowieka są powszechnie wykorzystywane do analizy czynności motorycznych. Dla sformułowań dynamiki tych modeli stosowane są zwykle niezależne współrzędne uogólnione, co powoduje, że reakcje w połączeniach (idealne reakcje więzów wewnętrznych) są eliminowane na wstępnym etapie modelowania. Dla ich określenia wymagane są dodatkowe procedury modelowania matematycznego, a generowane tą drogą zależności charakteryzują się niską efektywnością numeryczną. W niniejszej pracy proponowane jest nieco inne podejście do wyznaczania reakcji w stawach, w sposób naturalny skojarzone z minimalno-wymiarowym formułowaniem dynamiki wieloczłonowych modeli ciała człowieka. Proponowane sformułowania nie wymagają odwracania macierzy, są tym samym efektywne zarówno dla wyprowadzeń symbolicznych, jak i zastosowań numerycznych. Metoda zilustrowana jest za pomocą siedmioczłonowego płaskiego modelu ciała człowieka. Prezentowane są wybrane wyniki obliczeń numerycznych odnoszące się do symulacji dynamicznej odwrotnej sportowca wykonującego salto na trampolinie.

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