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ON HOMOGENIZED MODEL OF PERIODIC STRATIFIED MICROPOLAR ELASTIC COMPOSITES

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The paper deals with modelling problems of periodic stratified composites with micropolar elastic components. By using the linear theory of micropolar elasticity and the homogenization method with microlocal parameters, a homogenized model accounting certain local effects of stresses and coupled stresses is derived. From the obtained model, systems of equations for the "first" and the "second" plane state of strain of the layered composites are presented.

Key words: micropolar elasticity, displacement, rotation, periodically layered composite, homogenized model

1. Introduction

The theory of micropolar elasticity describes elastic bodies as a continuum of oriented particles which may rotate independently of the displacements. The basis of the concept was given by Cosserat E. and F. (1909) and the theory was developed by many other authors (see, for instance Dyszlewicz, 2004; Eringen and Suhubi, 1964; Eringen, 1966, 1968; Nowacki, 1974, 1981). The investigations connected with micropolar bodies had principally a theoretical nature, however important experimental results are given by Gauthier and Jahsman (1975), where methods of determinations of micropolar elastic constants are presented. The theory of micropolar elasticity can be applied to modelling of elastic solids with a microstructure, granular media, multimolecular bodies. Recently, the Cosserat theory has been applied to problems of geomechanics (see, for references Adhikary and Guo, 2002) and geophysics (Teisseyre, 1995).

In the present work, the problem of modelling of periodically layered, micropolar, elastic composites is considered. The basic unit (fundamental layer) is assumed to be composed of $(n+1)$-different micropolar, elastic, isotropic, homogeneous and centrosymmetric layers. Perfect bonding between the layers is assumed. The considerations are based on the linear theory of micropolar elasticity (Eringen and Suhubi, 1964; Eringen, 1966, 1968; Nowacki, 1974, 1981) and the homogenization procedure established by Woźniak (1986, 1987a,b), Matysiak and Woźniak (1987). The approach is based on some concepts of nonstandard analysis combined with some a priori postulated physical assumptions. Application of the homogenization procedure leads to equations given in terms of unknown macrodisplacements, macrorotations as well as some extra unknowns called microlocal parameters. The macrodisplacements, macrorotations describe mean values of deformations, and the microlocal parameters are connected with some local values of deformation gradients, stresses and couple stresses in every component of composites. The homogenization procedure was applied to modelling of periodically layered fluid-saturated porous solids (Kaczyński and Matysiak, 1988; Matysiak, 1992) and diffusion processes in layered composites (Matysiak and Mieszkowski, 2001). The approach is summarized in Matysiak (2001).

Starting from equations of micropolar elasticity, a homogenized model with microlocal parameters for a three dimensional case is derived. The model is described by linear partial differential equations with constant coefficients for macrodisplacements and macrorotations as well as by a system of linear algebraic equations for microlocal parameters.

Equations for plane problems of periodic two-layered composites are derived from three-dimensional models. In the considered case, microlocal parameters are eliminated, and the plane problems are expressed in terms of macrodisplacements and macrorotaions. Finally, derivation of equations of homogeneous micropolar bodies, periodically layered elastic composites and homogeneous elastic solids is presented.

2. Preliminaries

Consider a nonhomogeneous microlocal elastic body which occupies a regular region B in the Euclidean 3-space referred to the fixed Cartesian coordinate system $\boldsymbol{x} = (x_1, x_2, x_3)$. The body in a natural (undeformed) configuration is composed of periodically repeated $(n+1)$-different homogeneous, isotropic, centrosymmetrical layers, see Fig. 1. Let $h_1, \dots, h_{n+1}$ be the thickness of each basic unit of the body, and $h = h_1 + \dots + h_{n+1}$. The axis x_1 is assumed to be normal to the layering. Let $\alpha^{(r)}$, $\beta^{(r)}$, $\lambda^{(r)}$, $\mu^{(r)}$, $\gamma^{(r)}$, $\varepsilon^{(r)}$, $r = 1, \dots, n+1$ be the material constants of the subsequent layers. By $\rho^{(r)}$, $J^{(r)}$, $r = 1,\dots,n+1$, we denote the mass densities and the densities of rotational inertia of the layers. Let $\sigma_{ij}^{(r)}$, $\mu_{ij}^{(r)}$, $i,j = 1,2,3$ be the stress tensors and coupled stress tensors in the layer of the rth kind. Let t denote time, $\boldsymbol{u}(\boldsymbol{x},t) = (u_1,u_2,u_3)(\boldsymbol{x},t)$, and $\boldsymbol{\varphi}(\boldsymbol{x},t) = (\varphi_1,\varphi_2,\varphi_3)(\boldsymbol{x},t)$, denote the displacement and rotation vectors, respectively. Let γ_{ij}, χ_{ij}, $i,j = 1,2,3$ be the components of the strain tensor and the curvature-twist tensor. Perfect bonding between the layers being the components of the composite is assumed. This assumption implies continuity of the displacement and rotation vectors, stress vector and the coupled stress vector on the interfaces (planes between the subsequent layers).

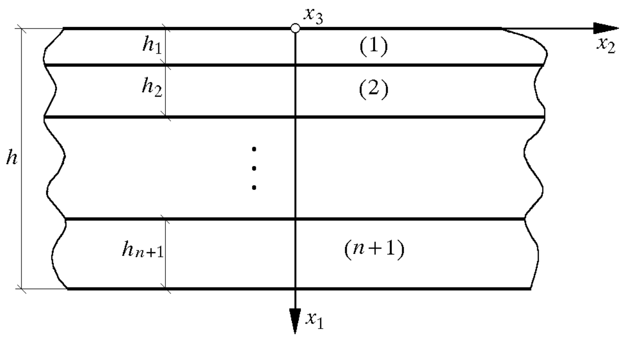

Fig. 1. A scheme of the fundamental layer (basic unit)

The system of equations of motion for the micropolar, isotropic, centrosymmetric, elastic layer of the rth kind takes the following form[1] (Dyszlewicz, 2004; Nowacki, 1974, 1981)

$$\begin{aligned} &\sigma_{ji,j}^{(r)} + \rho^{(r)} X_i - \rho^{(r)} \ddot{u}_i = 0 && i,j,k = 1,2,3 \\ &\varepsilon_{ijk}\sigma_{jk}^{(r)} + \mu_{ji,j}^{(r)} + \rho^{(r)} Y_i - J^{(r)} \ddot{\varphi}_i = 0 && r = 1,\dots,n+1 \end{aligned} \tag{2.1}$$

where ε_{ijk} denotes the permutation symbol (Ricci's alternator).

The constitutive relations in the considered case of homogeneous, isotropic, centrosymmetric elastic layers being the composite components can be written as follows (Dyszlewicz, 2004; Nowacki, 1974, 1981)

$$\begin{aligned} \sigma_{ji}^{(r)} &= (\mu^{(r)} + \alpha^{(r)})\gamma_{ji} + (\mu^{(r)} - \alpha^{(r)})\gamma_{ij} + \lambda^{(r)}\gamma_{kk}\delta_{ij} \\ \mu_{ji}^{(r)} &= (\gamma^{(r)} + \varepsilon^{(r)})\chi_{ji} + (\gamma^{(r)} - \varepsilon^{(r)})\chi_{ij} + \beta^{(r)}\chi_{kk}\delta_{ij} \end{aligned} \tag{2.2}$$

[1]Summation convention holds with respect to the repeated indices, and $\varphi_{,i} \equiv \partial\varphi/\partial x_i$, $\dot{\phi} \equiv \partial\phi/\partial t$.

where

$$\gamma_{ji} = u_{i,j} - \varepsilon_{kji}\varphi_k \qquad\qquad \chi_{ji} = \varphi_{i,j} \tag{2.3}$$

and δ_{ij} denotes the Kronecker delta.

By using equations (2.1)-(2.3), the equations of motion can be expressed in the following integral (weak) form

$$\begin{aligned}
&\sum_{r=1}^{n+1}\int_{Br}\Big\{[(\mu^{(r)}+\alpha^{(r)})(u_{i,j}-\varepsilon_{kji}\varphi_k)+(\mu^{(r)}-\alpha^{(r)})(u_{j,i}-\varepsilon_{kij}\varphi_k)+\\
&\qquad + \lambda^{(r)}u_{k,k}\delta_{ij}]\nu_{i,j}-\rho^{(r)}X_i\nu_i+\rho^{(r)}\ddot{u}_i\nu_i\Big\}\,dB=0\\
&\sum_{r=1}^{n+1}\int_{Br}\Big\{\varepsilon_{ijk}[(\mu^{(r)}+\alpha^{(r)})(u_{k,j}-\varepsilon_{mjk}\varphi_m)+(\mu^{(r)}-\alpha^{(r)})(u_{j,k}-\varepsilon_{mkj}\varphi_m)+\\
&\qquad + \lambda^{(r)}u_{m,m}\delta_{kj}]\nu_i+\rho^{(r)}Y_i\nu_i-J^{(r)}\ddot{u}_i\nu_i+\\
&\qquad - [(\gamma^{(r)}+\varepsilon^{(r)})\varphi_{i,j}+(\gamma^{(r)}-\varepsilon^{(r)})\varphi_{j,i}+\beta^{(r)}\varphi_{m,m}\delta_{ij}]\nu_{i,j}\Big\}\,dB=0
\end{aligned} \tag{2.4}$$

for all test functions $\nu_i(\cdot)$ such that $\nu_i(\cdot)|_{\partial B}=0$, and where B_r, $r=1,\dots,n+1$ denotes the part of the region B occupied by the rth material.

Since the body is assumed to be periodic, the material coefficients are h-periodic functions taking constant values in the subsequent layers of the body.

3. Homogenized models of periodic micropolar elastic composites

To obtain a homogenized model of periodic stratified micropolar elastic composites described in Section 2, the homogenization procedure with be applied. This approach, presented in papers by Woźniak (1986, 1987a,b) for thermoelastic composites, is based on some concepts of the nonstandard analysis and some a priori postulated physical assumptions.

In this paper, we shall derive equations of homogenized models omitting the presentation of mathematical assumptions and detailed calculations. Similarly to papers by Matysiak (1992), Woźniak (1987a), the components of the displacement vector $u_i(\cdot)$ and rotation vector $\varphi_i(\cdot)$ are assumed in the form

$$\begin{aligned} u_i(\boldsymbol{x},t) &= U_i(\boldsymbol{x},t) + \underline{f_a(x_1)q_{ai}(\boldsymbol{x},t)} \\ \varphi_i(\boldsymbol{x},t) &= \varPhi_i(\boldsymbol{x},t) + \underline{f_a(x_1)Q_{ai}(\boldsymbol{x},t)} \qquad i=1,2,3 \qquad a=1,\dots,n \end{aligned} \tag{3.1}$$

where $f_a(\cdot):\ R \to R$ are know a priori h-periodic functions, called the shape functions, given in Matysiak and Woźniak (1987)

$$\begin{aligned} &f_a(x_1) = \begin{cases} x_1 - \dfrac{1}{2}\delta_a & \text{for} \quad 0 \leqslant x_1 \leqslant \delta_a \\ \dfrac{\delta_a(x_1-h)}{\delta_a - h} - \dfrac{1}{2}\delta_a & \text{for} \quad \delta_a \leqslant x_1 \leqslant h \end{cases} \\ &f_a(x_1+h) = f_a(x_1) \qquad x_1 \in R \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} \delta_a &= h_1 + \ldots + h_a \qquad a=1,\dots,n \\ h &= h_1 + \ldots + h_{n+1} \end{aligned} \tag{3.3}$$

The functions U_i, $\varPhi_i$ are unknown functions interpreted as the components of macrodispacement, macrorotation. The unknown functions $q_{ai}(\cdot)$, $Q_{ai}(\cdot)$ stand for the kinematical and rotational microlocal parameters, and they are related with the periodic structure of the body.

Since $|f_a(x_1)| < h$ for every $x_1 \in R$, then for small h the underlined terms in equations (3.1) are small and will be neglected (for exact explanation in terms of the nonstandard analysis see papers by Woźniak, 1986, 1987a,b). It is emphasized that $f'_a(\cdot)$ are not small and the terms involving $f'_a(\cdot)$ cannot be neglected. So, we have

$$\begin{aligned} u_{i,1} &\approx U_{i,1} + f'_a(x_1)q_{ai} \qquad & u_{i,\beta} &\approx U_{i,\beta} \\ \varphi_{i,1} &\approx \varPhi_{i,1} + f'_a(x_1)Q_{ai} \qquad & \varphi_{i,\beta} &\approx \varPhi_{i,\beta} \qquad \beta = 2,3 \end{aligned} \tag{3.4}$$

Taking into account the tested functions in the form

$$\nu_i(\boldsymbol{x},t) = V_i(\boldsymbol{x},t) + \underline{f_b(x_1)Z_{bi}(\boldsymbol{x},t)} \qquad \begin{aligned} i &= 1,2,3 \\ b &= 1,\dots,n \end{aligned} \tag{3.5}$$

and substituting equations (3.5) into (2.4) after some calculations similar to those given in Matysiak and Woźniak (1987), Matysiak ((1992) and Woźniak

(1987a), the equations of the homogenized model with microlocal parameters are obtained in the form $(i,j,k,m=1,2,3,\ a=1,\dots,n)$

$$\begin{gathered}
\langle\mu+\alpha\rangle U_{i,jj}+\langle\lambda+\mu-\alpha\rangle U_{j,ij}+\langle(\mu+\alpha)f_a'(x_1)\rangle q_{ai,1}+\\
+\langle(\mu-\alpha)f_a'(x_1)\rangle q_{aj,j}\delta_{1i}+\langle\lambda f_a'(x_1)\rangle q_{a1,i}-\langle\mu+\alpha\rangle\varepsilon_{kji}\varPhi_{k,j}+\\
-\langle\mu-\alpha\rangle\varepsilon_{kij}\varPhi_{k,j}+\langle\rho\rangle X_i-\langle\rho\rangle\ddot{U}_i=0\\
\varepsilon_{ijk}[\langle\mu+\alpha\rangle U_{k,j}+\langle\mu-\alpha\rangle U_{j,k}+\langle(\mu+\alpha)f_a'(x_1)\rangle q_{ak}\delta_{1j}+\\
+\langle(\mu-\alpha)f_a'(x_1)\rangle q_{aj}\delta_{1k}-\langle\mu+\alpha\rangle\varepsilon_{mjk}\varPhi_m-\langle\mu-\alpha\rangle\varepsilon_{mkj}\varPhi_m]+\\
+\langle\gamma+\varepsilon\rangle\varPhi_{i,jj}+\langle\gamma-\varepsilon+\beta\rangle\varPhi_{j,ji}+\langle(\gamma+\varepsilon)f_a'(x_1)\rangle Q_{ai,1}+\\
+\langle(\gamma-\varepsilon)f_a'(x_1)\rangle Q_{aj,j}\delta_{i1}+\langle\beta f_a'(x_1)\rangle Q_{a1,i}+\langle\rho\rangle Y_i-\langle J\rangle\ddot{\varPhi}_i=0
\end{gathered}\tag{3.6}$$

and

$$\begin{gathered}
\langle(\mu+\alpha)f_b'(x_1)\rangle U_{i,1}+\langle(\mu-\alpha)f_b'(x_1)\rangle U_{1,i}+\langle\lambda f_b'(x_1)\rangle U_{k,k}\delta_{i1}+\\
+\langle(\mu+\alpha)f_a'(x_1)f_b'(x_1)\rangle q_{ai}+\langle(\mu-\alpha+\lambda)f_a'(x_1)f_b'(x_1)\rangle q_{a1}\delta_{1i}+\\
-\langle(\mu+\alpha)f_b'(x_1)\rangle\varepsilon_{k1i}\varPhi_k-\langle(\mu-\alpha)f_b'(x_1)\rangle\varepsilon_{ki1}\varPhi_k=0\\
\langle(\gamma+\varepsilon)f_b'(x_1)\rangle\varPhi_{i,1}+\langle(\gamma-\varepsilon)f_b'(x_1)\rangle\varPhi_{1,i}+\langle\beta f_b'(x_1)\rangle\varPhi_{m,m}\delta_{i1}+\\
+\langle(\gamma+\varepsilon)f_a'(x_1)f_b'(x_1)\rangle Q_{ai}+\langle(\gamma-\varepsilon+\beta)f_a'(x_1)f_b'(x_1)\rangle Q_{a1}\delta_{i1}=0
\end{gathered}\tag{3.7}$$

where the symbol $\langle g\rangle$ denotes

$$\langle g\rangle\equiv\frac{1}{h}\int\limits_0^h g(x_1)\,dx_1\tag{3.8}$$

for any h-periodic integrable function $g(\cdot)$.

Equations (3.6) and (3.7) constitute a system of linear algebraic and partial differential equations for $6(n+1)$ unknowns U_i, $\varPhi_i$, q_{ai}, Q_{ai}, $i=1,2,3$, $a=1,\dots,n$. Equations (3.7) stand for a system of linear algebraic equations for microlocal parameters q_{ai}, Q_{ai}. By using equations (3.7), the microlocal parameters can be eliminated from equations (3.6), which leads to 6 linear partial differential equations with constant coefficients for the unknown macrodisplacements U_i and macrorotations $\varPhi_i$.

Using formulae (3.2), (3.3) and (3.8) for an arbitrary h-periodic function $g(\cdot)$ taking a constant value g_r in the layer of the rth kind, $r=1,\dots,n+1$, we have

$$
\begin{aligned}
&\langle g\rangle=\sum_{r=1}^{n+1}\eta_r g_r \qquad\qquad \langle g f_a'(x_1)\rangle=\sum_{r=1}^{a}\eta_r g_r-\omega_a\sum_{r=a+1}^{n+1}\eta_r g_r\\
&\langle g f_a'(x_1) f_b'(x_1)\rangle=\sum_{r=1}^{b}\eta_r g_r-\omega_b\sum_{r=b+1}^{a}\eta_r g_r-\omega_a\omega_b\sum_{r=a+1}^{n+1}\eta_r g_r \qquad b\leqslant a
\end{aligned}
\tag{3.9}
$$

where

$$
\eta_r\equiv\frac{\delta_r}{h} \qquad\qquad \omega_a\equiv\frac{\eta_1+\ldots+\eta_a}{1-(\eta_1+\ldots+\eta_a)} \qquad\qquad \begin{array}{l} r=1,\ldots,n+1\\ a=1,\ldots,n\end{array}
\tag{3.10}
$$

Employing equations (3.9), all material constants in equations (3.6), (3.7) can be calculated by substituting the h-periodic functions α, β, λ, μ, γ, ε, ρ, J for function $g(\cdot)$.

The components of the stress tensor $\sigma_{ji}^{(r)}$ and the couple stress tensor $\mu_{ji}^{(r)}$ in the layers of the rth kind can be determined by using equations (2.2), (2.3), (3.3) and (3.4). Thus, we have

$$
\begin{aligned}
\sigma_{ji}^{(r)}&=(\mu^{(r)}+\alpha^{(r)})[U_{i,j}+f_a'(x_1)q_{ai}\delta_{1j}-\varepsilon_{mji}\Phi_m]+\\
&\quad+(\mu^{(r)}-\alpha^{(r)})[U_{j,i}+f_a'(x_1)q_{aj}\delta_{i1}-\varepsilon_{mij}\Phi_m]+\lambda^{(r)}[U_{m,m}+f_a'(x_1)q_{a1}]\delta_{ij}\\
\mu_{ji}^{(r)}&=(\gamma^{(r)}+\varepsilon^{(r)})[\Phi_{i,j}+f_a'(x_1)Q_{ai}\delta_{1j}]+(\gamma^{(r)}-\varepsilon^{(r)})[\Phi_{j,i}+f_a'(x_1)Q_{aj}\delta_{1i}]+\\
&\quad+\beta^{(r)}[\Phi_{k,k}+f_a'(x_1)Q_{ai}]\delta_{ij}
\end{aligned}
\tag{3.11}
$$

$a=1,\ldots,n$; $r=1,\ldots,n+1$; $i,j,k=1,2,3$.

4. Plane problems of two-layered periodic micropolar composites

4.1. The "first" plane state of strain

Consider now a micropolar elastic stratified composite composed of periodically repeated two different layers. Moreover, we confine our attention on plane problems described by displacement and rotation vectors in the form

$$
\begin{aligned}
\boldsymbol{u}(x_1,x_2,t)&=(u_1(x_1,x_2,t),u_2(_1,x_2,t),0)\\
\boldsymbol{\varphi}(x_1,x_2,t)&=(0,0,\varphi_3(x_1,x_2,t))
\end{aligned}
\tag{4.1}
$$

In the considered case $n = 1$, so the set of shape functions is reduced to the following function

$$f_1(x_1) = \begin{cases} x_1 - \dfrac{1}{2}h_1 & \text{for} \quad 0 \leqslant x_1 \leqslant h_1 \\ -\dfrac{\eta x_1}{1-\eta} + \dfrac{h_1}{1-\eta} - \dfrac{1}{2}h_1 & \text{for} \quad h_1 \leqslant x_1 \leqslant h \end{cases} \tag{4.2}$$

where

$$\eta = \frac{h_1}{h} \tag{4.3}$$

Using equations (3.8), (3.10), (3.11) and (4.3), the coefficients in equations (3.6) and (3.7) can be written as follows

$$\begin{aligned}
&(\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\lambda}, \widetilde{\mu}, \widetilde{\gamma}, \widetilde{\varepsilon}, \widetilde{\rho}, \widetilde{J}) \equiv (\langle\alpha\rangle, \langle\beta\rangle, \langle\lambda\rangle, \langle\mu\rangle, \langle\gamma\rangle, \langle\varepsilon\rangle, \langle\rho\rangle, \langle J\rangle) = \\
&\qquad = \eta(\alpha_1, \beta_1, \lambda_1, \mu_1, \gamma_1, \varepsilon_1, \rho_1, J_1) + (1-\eta)(\alpha_2, \beta_2, \lambda_2, \mu_2, \gamma_2, \varepsilon_2, \rho_2, J_2) \\
&([\alpha], [\beta], [\lambda], [\mu], [\gamma], [\varepsilon]) \equiv \\
&\qquad \equiv (\langle\alpha f_1'(x_1)\rangle, \langle\beta f_1'(x_1)\rangle, \langle\lambda f_1'(x_1)\rangle, \langle\mu f_1'(x_1)\rangle, \langle\gamma f_1'(x_1)\rangle, \langle\varepsilon f_1'(x_1)\rangle) = \\
&\qquad = \eta(\alpha_1 - \alpha_2, \beta_1 - \beta_2, \lambda_1 - \lambda_2, \mu_1 - \mu_2, \gamma_1 - \gamma_2, \varepsilon_1 - \varepsilon_2) \\
&(\widehat{\alpha}, \widehat{\beta}, \widehat{\lambda}, \widehat{\mu}, \widehat{\gamma}, \widehat{\varepsilon}) \equiv (\langle\alpha(f_1'(x_1))^2\rangle, \langle\beta(f_1'(x_1))^2\rangle, \langle\lambda(f_1'(x_1))^2\rangle, \\
&\qquad \langle\mu(f_1'(x_1))^2\rangle, \langle\gamma(f_1'(x_1))^2\rangle, \langle\varepsilon(f_1'(x_1))^2\rangle) = \\
&\qquad = \eta(\alpha_1, \beta_1, \lambda_1, \mu_1, \gamma_1, \varepsilon_1) + \frac{\eta^2}{1-\eta}(\alpha_2, \beta_2, \lambda_2, \mu_2, \gamma_2, \varepsilon_2)
\end{aligned} \tag{4.4}$$

Equations (3.6), (3.7) for the plane problems of the periodically two-layered micropolar composite (see, equations (4.1), (3.1), (3.4)) take the following form ($\delta = 1, 2$)

$$\begin{aligned}
&(\widetilde{\mu} + \widetilde{\alpha})U_{1,\delta\delta} + (\widetilde{\lambda} + \widetilde{\mu} - \widetilde{\alpha})U_{\delta,\delta 1} + ([\lambda] + [\mu] - [\alpha])q_{11,1} + ([\mu] - [\alpha])q_{1\delta,\delta} + \\
&\qquad + 2\widetilde{\alpha}\varPhi_{3,2} + \widetilde{\rho}X_1 - \widetilde{\rho}\ddot{U}_1 = 0 \\
&(\widetilde{\mu} + \widetilde{\alpha})U_{2,\delta\delta} + (\widetilde{\lambda} + \widetilde{\mu} - \widetilde{\alpha})U_{\delta,\delta 2} + ([\mu] + [\alpha])q_{12,1} + [\lambda]q_{11,2} - 2\widetilde{\alpha}\varPhi_{3,1} + \\
&\qquad + \widetilde{\rho}X_2 - \widetilde{\rho}\ddot{U}_2 = 0 \\
&(\widetilde{\gamma} + \widetilde{\varepsilon})\varPhi_{3,\delta\delta} + 2\widetilde{\alpha}(U_{2,1} - U_{1,2}) - 4\widetilde{\alpha}\varPhi_3 + 2[\alpha]q_{12} + ([\gamma] + [\varepsilon])Q_{13,1} + \\
&\qquad + \widetilde{\rho}Y_3 - \widetilde{J}\ddot{\varPhi}_3 = 0
\end{aligned} \tag{4.5}$$

and

$$\begin{aligned}
(\widehat{\lambda} + 2\widehat{\mu})q_{11} &= -2[\mu]U_{1,1} - [\lambda]U_{\delta,\delta} \\
(\widehat{\mu} + \widehat{\alpha})q_{12} &= -([\mu] + [\alpha])U_{2,1} - ([\mu] - [\alpha])U_{1,2} + 2[\alpha]\varPhi_3 \\
(\widehat{\gamma} + \widehat{\varepsilon})Q_{13} &= -([\gamma] + [\varepsilon])\varPhi_{3,1}
\end{aligned} \tag{4.6}$$

By using equations (4.6), the microlocal parameters q_{11}, q_{12}, Q_{13} can be eliminated from equations (4.5). It leads to the following equations for the unknown macrodisplacements U_1, U_2 and macrotation Φ_3

$$
\begin{aligned}
&A_1U_{1,11} + A_2U_{1,22} + A_3U_{2,21} + A_4\Phi_{3,2} + \widetilde{\rho}X_1 - \widetilde{\rho}\ddot{U}_1 = 0\\
&B_1U_{2,11} + B_2U_{2,22} + B_3U_{1,12} + B_4\Phi_{3,1} + \widetilde{\rho}X_2 - \widetilde{\rho}\ddot{U}_2 = 0\\
&C_1\Phi_{3,11} + C_2\Phi_{3,22} + C_3\Phi_3 + C_4U_{2,1} + C_5U_{1,2} + \widetilde{\rho}Y_3 - \widetilde{J}\ddot{\Phi}_3 = 0
\end{aligned}
\tag{4.7}
$$

where

$$
\begin{aligned}
&A_1 = \widetilde{\lambda} + 2\widetilde{\mu} - \frac{([\lambda] + 2[\mu])([\lambda] + 2[\mu] - 2[\alpha])}{\widehat{\lambda} + 2\widehat{\mu}}\\
&A_2 = \widetilde{\mu} + \widetilde{\alpha} - \frac{([\mu] - [\alpha])^2}{\widehat{\mu} + \widehat{\alpha}} \qquad A_4 = 2\Big(\widetilde{\alpha} - \frac{[\alpha]([\mu] - [\alpha])}{\widehat{\mu} + \widehat{\alpha}}\Big)\\
&A_3 = \widetilde{\lambda} + \widetilde{\mu} - \widetilde{\alpha} - \frac{[\lambda]([\lambda] + [\mu] - [\alpha])}{\widehat{\lambda} + 2\widehat{\mu}} - \frac{[\mu]^2 - [\alpha]^2}{\widehat{\mu} + \widehat{\alpha}}\\
&B_1 = \widetilde{\mu} + \widetilde{\alpha} - \frac{([\mu] + [\alpha])^2}{\widehat{\mu} + \widehat{\alpha}} \qquad B_2 = \widehat{\lambda} + 2\widetilde{\mu} - \frac{[\lambda]^2}{\widehat{\lambda} + 2\widehat{\mu}}\\
&B_3 = \widetilde{\lambda} + \widetilde{\mu} - \widetilde{\alpha} - \frac{[\mu]^2 - [\alpha]^2}{\widehat{\mu} + \widehat{\alpha}} - \frac{[\lambda]([\lambda] + 2[\mu])}{\widehat{\lambda} + 2\widehat{\mu}}\\
&B_4 = 2\Big(\frac{[\mu]([\mu] + [\alpha])}{\widehat{\mu} + \widehat{\alpha}} - \widetilde{\alpha}\Big) \qquad C_1 = \widetilde{\gamma} + \widetilde{\varepsilon} - \frac{([\gamma] + [\varepsilon])^2}{\widehat{\gamma} + \widehat{\varepsilon}}\\
&C_2 = \widetilde{\gamma} + \widetilde{\varepsilon} \qquad C_3 = -4\Big(\widetilde{\alpha} - \frac{[\alpha]^2}{\widehat{\mu} + \widehat{\alpha}}\Big)\\
&C_4 = 2\Big(\widetilde{\alpha} - \frac{[\alpha]([\mu] + [\alpha])}{\widehat{\mu} + \widehat{\alpha}}\Big) \qquad C_5 = -2\Big(\widetilde{\alpha} + \frac{[\alpha]([\mu] - [\alpha])}{\widehat{\mu} + \widehat{\alpha}}\Big)
\end{aligned}
\tag{4.8}
$$

The components of stress and couple stress tensors in the layers of the rth kind, $r = 1, 2$, can be obtained by using equations (3.11), (4.1), (4.2) and (4.6).

Thus, we have

$$
\begin{aligned}
&\sigma_{11}^{(r)} = D_1U_{1,1} + D_2U_{2,2} \qquad \sigma_{22}^{(r)} = b_1^{(r)}U_{1,1} + b_2^{(r)}U_{2,2}\\
&\sigma_{12}^{(r)} = E_1U_{1,2} + E_2U_{2,1} + E_3\Phi_3 \qquad \sigma_{33}^{(r)} = c_1^{(r)}U_{1,1} + c_2^{(r)}U_{2,2}\\
&\sigma_{21}^{(r)} = a_1^{(r)}U_{1,2} + a_2^{(r)}U_{2,1} + a_3^{(r)}\Phi_3 \qquad \mu_{13}^{(r)} = F_1\Phi_{3,1}\\
&\mu_{31}^{(r)} = d_1^{(r)}\Phi_{3,1} \qquad \mu_{23}^{(r)} = d_2^{(r)}\Phi_{3,2} \qquad \mu_{32}^{(r)} = d_3^{(r)}\Phi_{3,2}
\end{aligned}
\tag{4.9}
$$

where $(r = 1,2)$

$$
\begin{aligned}
D_1 &= (\lambda^{(1)} + 2\mu^{(1)})\Big(1 - \frac{[\lambda] + 2[\mu]}{\widehat{\lambda} + 2\widehat{\mu}}\Big)\\
D_2 &= \lambda^{(1)} - \frac{[\lambda](\lambda^{(1)} + 2\mu^{(1)})}{\widehat{\lambda} + 2\widehat{\mu}}\\
E_1 &= -\frac{\mu^{(1)} + \alpha^{(1)}}{\widehat{\mu} + \widehat{\alpha}}([\mu] - [\alpha]) + \mu^{(1)} + \alpha^{(1)}\\
E_2 &= (\mu^{(1)} + \alpha^{(1)})\Big(1 - \frac{[\mu] + [\alpha]}{\widehat{\mu} + \widehat{\alpha}}\Big)\\
E_3 &= (\mu^{(1)} + \alpha^{(1)})\Big(\frac{2[\alpha]}{\widehat{\mu} + \widehat{\alpha}} - 1\Big) + \mu^{(1)} - \alpha^{(1)}\\
F_1 &= (\gamma^{(1)} + \varepsilon^{(1)})\Big(1 - \frac{[\gamma] + [\varepsilon]}{\widehat{\gamma} + \widehat{\varepsilon}}\Big)\\
a_1^{(r)} &= \mu^{(r)} + \alpha^{(r)} - \frac{\mu^{(r)} - \alpha^{(r)}}{\widehat{\mu} + \widehat{\alpha}}([\mu] - [\alpha])f^{(r)}\\
a_2^{(r)} &= (\mu^{(r)} - \alpha^{(r)})\Big(1 - \frac{[\mu] + [\alpha]}{\widehat{\mu} + \widehat{\alpha}}\Big)f^{(r)}\\
a_3^{(r)} &= 2\Big(\alpha^{(r)} + \frac{[\alpha]}{\widehat{\mu} + \widehat{\alpha}}(\mu^{(r)} - \alpha^{(r)})f^{(r)}\Big) & c_1^{(r)} &= b_1^{(r)}\\
b_1^{(r)} &= \lambda^{(r)}\Big(1 - \frac{[\lambda] + 2[\mu]}{\widehat{\lambda} + 2\widehat{\mu}}f^{(r)}\Big) & c_2^{(r)} &= \lambda^{(r)}\Big(1 - \frac{[\lambda]}{\widehat{\lambda} + 2\widehat{\mu}}f^{(r)}\Big)\\
b_2^{(r)} &= \lambda^{(r)} + 2\mu^{(r)} - \frac{[\lambda]\lambda^{(r)}}{\widehat{\lambda} + 2\widehat{\mu}}f^{(r)} & d_2^{(r)} &= \gamma^{(r)} + \varepsilon^{(r)}\\
d_1^{(r)} &= (\gamma^{(r)} - \varepsilon^{(r)})\Big(1 - \frac{[\gamma] + [\varepsilon]}{\widehat{\gamma} + \widehat{\varepsilon}}f^{(r)}\Big) & d_3^{(r)} &= \gamma^{(r)} - \varepsilon^{(r)}
\end{aligned}
\tag{4.10}
$$

and

$$
f^{(r)} = \begin{cases} 1 & \text{for} \quad r = 1 \\ -\dfrac{\eta}{1 - \eta} & \text{for} \quad r = 2 \end{cases}
\tag{4.11}
$$

Equations (4.7) and (4.9) with the constant coefficients described by (4.8), (4.10) constitute the governing system of equations for the homogenized model with microlocal parameters of micropolar layered composites in the plane state of strain.

Remark. It should be emphasized that the continuity conditions on interfaces of the stress vector $(\sigma_{11}^{(r)}, \sigma_{12}^{(r)}, 0)$ and the couple stress vector $(0, 0, \mu_{13}^{(r)})$, $r = 1, 2$, are satisfied (see, equations (4.9)).

4.2. The "second" state of strain

Consider now the "second" state of strain described by the displacement and rotation vectors in the form

$$\begin{aligned} \boldsymbol{u}(\boldsymbol{x}, t) &= (0, 0, u_3(x_1, x_2, t)) \\ \boldsymbol{\varphi}(\boldsymbol{x}, t) &= (\varphi_1(x_1, x_2, t), \varphi_2(x_1, x_2, t), 0) \end{aligned} \tag{4.12}$$

Using equations (3.6), (3.7), (4.3), (4.4), (4.12) and (3.1), we obtain the following equations of motion for the considered case of the strain state

$$\begin{aligned} &(\widetilde{\mu} + \widetilde{\alpha})(U_{3,11} + U_{3,22}) + ([\mu] + [\alpha])q_{13,1} + 2\widetilde{\alpha}(\varPhi_{2,1} - \varPhi_{1,2}) + \widetilde{\rho}X_3 - \widetilde{\rho}\ddot{U}_3 = 0 \\ &(2\widetilde{\gamma} + \widetilde{\beta})\varPhi_{1,11} + (\widetilde{\gamma} + \widetilde{\varepsilon})\varPhi_{1,22} + (\widetilde{\gamma} - \widetilde{\varepsilon} + \widetilde{\beta})\varPhi_{2,12} + 2\widetilde{\alpha}U_{3,2} - 4\widetilde{\alpha}\varPhi_1 + \\ &\qquad + (2[\gamma] + [\beta])Q_{11,1} + ([\gamma] - [\varepsilon])Q_{12,2} + \widetilde{\rho}Y_1 - \widetilde{J}\ddot{\varPhi}_1 = 0 \\ &(\widetilde{\gamma} + \widetilde{\varepsilon})\varPhi_{2,11} + (2\widetilde{\gamma} + \widetilde{\beta})\varPhi_{2,22} + (\widetilde{\gamma} - \widetilde{\varepsilon} + \widetilde{\beta})\varPhi_{1,12} - 2\widetilde{\alpha}U_{3,1} - 4\widetilde{\alpha}\varPhi_2 + \\ &\qquad + ([\gamma] + [\varepsilon])Q_{12,1} + [\beta]Q_{11,2} + \widetilde{\rho}Y_2 - \widetilde{J}\ddot{\varPhi}_2 = 0 \end{aligned} \tag{4.13}$$

and

$$\begin{aligned} q_{13} &= \frac{1}{\widehat{\mu} + \widehat{\alpha}}\Big\{-([\mu] + [\alpha])U_{3,1} - 2[\alpha]\varPhi_2\Big\} \\ Q_{11} &= \frac{1}{2\widehat{\gamma} + \widehat{\beta}}\Big\{-(2[\gamma] + [\beta])\varPhi_{1,1} - [\beta]\varPhi_{2,2}\Big\} \\ Q_{12} &= \frac{1}{\widehat{\gamma} + \widehat{\beta}}\Big\{-([\gamma] + [\varepsilon])\varPhi_{2,1} - ([\gamma] - [\varepsilon])\varPhi_{1,2}\Big\} \end{aligned} \tag{4.14}$$

Eliminating the microlocal parameters q_{13}, Q_{11}, Q_{12} from equations (4.13) by using (4.14), we obtain a system of equations for the macrodisplacement U_3 and macrorotations $\varPhi_1$, $\varPhi_2$ in the form

$$\begin{aligned} &A_1^*U_{3,11} + A_2^*U_{3,22} + A_3^*\varPhi_{1,2} + A_4^*\varPhi_{2,1} + \widetilde{\rho}X_3 - \rho\ddot{U}_3 = 0 \\ &B_1^*\varPhi_{1,11} + B_2^*\varPhi_{2,12} + B_3^*\varPhi_{1,22} + B_4^*U_{3,2} + B_5^*\varPhi_1 + \widetilde{\rho}Y_1 - \widetilde{J}\ddot{\varPhi}_1 = 0 \\ &C_1^*\varPhi_{2,11} + C_2^*\varPhi_{1,12} + C_3^*\varPhi_{2,22} + C_4^*U_{3,1} + C_5^*\varPhi_2 + \widetilde{\rho}Y_2 - \widetilde{J}\ddot{\varPhi}_2 = 0 \end{aligned} \tag{4.15}$$

where

$$
\begin{aligned}
&A_1^* = \widetilde{\mu} + \widetilde{\alpha} - \frac{([\mu]+[\alpha])^2}{\widehat{\mu}+\widehat{\alpha}} && A_2^* = \widetilde{\mu} + \widetilde{\alpha}\\
&A_4^* = -2\Big(\frac{([\mu]+[\alpha])[\alpha]}{\widehat{\mu}+\widehat{\alpha}} - \widetilde{\alpha}\Big) && A_3^* = -2\widetilde{\alpha}\\
&B_1^* = 2\widetilde{\gamma} + \widetilde{\beta} - \frac{(2[\gamma]+[\beta])^2}{2\widehat{\gamma}+\widehat{\beta}} && B_4^* = 2\widetilde{\alpha}\\
&B_2^* = \widetilde{\gamma} - \widetilde{\varepsilon} + \widetilde{\beta} - \frac{(2[\gamma]+[\beta])[\beta]}{2\widehat{\gamma}+\widehat{\beta}} - \frac{[\gamma]^2-[\varepsilon]^2}{\widehat{\gamma}+\widehat{\varepsilon}} && B_5^* = -4\widetilde{\alpha}\\
&B_3^* = \widetilde{\gamma} + \widetilde{\varepsilon} - \frac{([\gamma]-[\varepsilon])^2}{\widehat{\gamma}+\widehat{\varepsilon}} && C_2^* = B_2^*\\
&C_1^* = \widetilde{\gamma} + \widetilde{\varepsilon} - \frac{([\gamma]+[\varepsilon])^2}{\widehat{\gamma}+\widehat{\varepsilon}} && C_4^* = -2\widetilde{\alpha}\\
&C_3^* = 2\widetilde{\gamma} + \widetilde{\beta} - \frac{[\beta]^2}{2\widehat{\gamma}+\widehat{\beta}} && C_5^* = -4\widetilde{\alpha}
\end{aligned}
\tag{4.16}
$$

The components of stress and couple stress tensors in the layers of the rth kind, $r = 1, 2$, for the "second" plane state of strains, can be obtained by using equations (3.11), (4.3), (4.4), (4.11). Thus we have

$$
\begin{aligned}
&\sigma_{13}^{(r)} = D_1^* U_{3,1} + D_2^* \varPhi_2 && \sigma_{31}^{(r)} = a_1^{*(r)} U_{3,1} + a_2^{*(r)} \varPhi_2\\
&\sigma_{23}^{(r)} = b_1^{*(r)} U_{3,2} + b_2^{*(r)} \varPhi_1 && \sigma_{32}^{(r)} = c_1^{*(r)} U_{3,2} + c_2^{*(r)} \varPhi_1\\
&\mu_{11}^{(r)} = d_1^* \varPhi_{1,1} + d_2^* \varPhi_{2,2} && \mu_{12}^{(r)} = e_1^* \varPhi_{1,2} + e_2^* \varPhi_{2,1}\\
&\mu_{21}^{(r)} = f_1^{*(r)} \varPhi_{1,2} + f_2^{*(r)} \varPhi_{2,1} && \mu_{22}^{(r)} = g_1^{*(r)} \varPhi_{1,1} + g_2^{*(r)} \varPhi_{2,2}
\end{aligned}
\tag{4.17}
$$

where

$$
\begin{aligned}
&D_1^* \equiv (\mu^{(1)} + \alpha^{(1)})\Big(1 - \frac{[\mu]+[\alpha]}{\widehat{\mu}+\widehat{\alpha}}\Big) && b_1^{*(r)} = \mu^{(r)} + \alpha^{(r)}\\
&D_2^* \equiv 2\alpha^{(1)} - \frac{2[\alpha](\mu^{(1)}+\alpha^{(1)})}{\widehat{\mu}+\widehat{\alpha}} && b_2^{*(r)} = -2\alpha^{(r)}\\
&a_1^{*(r)} = (\mu^{(r)} - \alpha^{(r)})\Big(1 - f^{(r)}\frac{[\mu]+[\alpha]}{\widehat{\mu}+\widehat{\alpha}}\Big) && c_1^{*(r)} = \mu^{(r)} - \alpha^{(r)}\\
&a_2^{*(r)} = -2\Big(\mu^{(r)} + \frac{(\mu^{(r)}-\alpha^{(r)})[\alpha]}{\widehat{\mu}+\widehat{\alpha}}\Big) && c_2^{*(r)} = 2\alpha^{(r)}
\end{aligned}
$$

$$
\begin{aligned}
d_1^* &= (2\gamma^{(1)} + \beta^{(1)})\Big(1 - \frac{2[\gamma] + [\beta]}{2\widehat{\gamma} + \widehat{\beta}}\Big)\\
d_2^* &= \beta^{(1)} - \frac{(2\gamma^{(1)} + \beta^{(1)})[\beta]}{2\widehat{\gamma} + \widehat{\beta}}\\
e_1^* &\equiv \gamma^{(1)} - \varepsilon^{(1)} - \frac{[\gamma] - [\varepsilon]}{\widehat{\gamma} + \widehat{\varepsilon}}(\gamma^{(1)} + \varepsilon^{(1)})\\
e_2^* &\equiv (\gamma^{(1)} + \varepsilon^{(1)})\Big(1 - \frac{[\gamma] + [\varepsilon]}{\widehat{\gamma} + \widehat{\varepsilon}}\Big)\\
f_1^{*(r)} &= \gamma^{(r)} + \varepsilon^{(r)} - (\gamma^{(r)} - \varepsilon^{(r)})\frac{[\gamma] - [\varepsilon]}{\widehat{\gamma} + \widehat{\varepsilon}}f^{(r)}\\
f_2^{*(r)} &= (\gamma^{(r)} - \varepsilon^{(r)})\Big(1 - \frac{[\gamma] + [\varepsilon]}{\widehat{\gamma} + \widehat{\varepsilon}}f^{(r)}\Big)\\
g_2^{*(r)} &= 2\gamma^{(r)} + \beta^{(r)} - \frac{(2[\gamma] + [\beta])\beta^{(r)}}{2\widehat{\gamma} + \widehat{\beta}}f^{(r)}\\
g_1^{*(r)} &= \beta^{(r)}\Big(1 - \frac{2[\gamma] + [\beta]}{2\widehat{\gamma} + \widehat{\beta}}f^{(r)}\Big)
\end{aligned}
\tag{4.18}
$$

$r = 1, 2$, and $f^{(r)}$ is in (4.11).

Remark. It should be emphasized that the continuity conditions on interfaces of the stress vector $(0, 0, \sigma_{13}^{(r)})$, and the couple stress vector $(\mu_{11}^{(r)}, \mu_{12}^{(r)}, 0)$, $r = 1, 2$, are satisfied (see, equations (4.16) and (4.17)).

5. Final remarks and conclusions

We have investigated the problem of modelling of periodically layered composites composed of different, homogeneous, isotropic, centrosymmetrical layers. The obtained homogenized model is given in terms of macrodisplacements, macrorotations as well as kinematical and rotational microlocal parameters. The microlocal parameters are determined by a system of linear algebraic equations (3.7), and they can be expressed by the macrodisplacements and macrorotations (for the "first" plane problem we obtained equations (4.6), and equations (4.14) for the "second" plane problem). Thus, the boundary value problems for the considered composites can be determined in terms of the macrodisplacements and macrorotations described by a system of 6 linear partial differential equations with constant coefficients.

From the obtained homogenized model of micropolar composites, we can pass to the following cases of elastic bodies:

Case 1. Homogeneous micropolar bodies

Assuming that the considered body is homogeneous, so

$$\begin{array}{llll} \alpha^{(r)} = \alpha & \beta^{(r)} = \beta & \gamma^{(r)} = \gamma & \lambda^{(r)} = \lambda \\ \mu^{(r)} = \mu & \varepsilon^{(r)} = \varepsilon & \rho^{(r)} = \rho & J^{(r)} = J \end{array} \tag{5.1}$$

$r = 1, \ldots, n+1$ and substituting (5.1) for the functions $g(\cdot)$ in equations (3.9) and (3.10), we obtain

$$\begin{array}{llll} \langle\alpha\rangle = \alpha & \langle\beta\rangle = \beta & \langle\gamma\rangle = \gamma & \langle\lambda\rangle = \lambda \\ \langle\mu\rangle = \mu & \langle\varepsilon\rangle = \varepsilon & \langle\rho\rangle = \rho & \langle J\rangle = J \end{array} \tag{5.2}$$

and $(a = 1, \ldots, n)$

$$\begin{array}{lll} \langle\alpha f'_a(x_1)\rangle = 0 & \langle\beta f'_a(x_1)\rangle = 0 & \langle\gamma f'_a(x_1)\rangle = 0 \\ \langle\mu f'_a(x_1)\rangle = 0 & \langle\lambda f'_a(x_1)\rangle = 0 & \langle\varepsilon f'_a(x_1)\rangle = 0 \end{array} \tag{5.3}$$

Thus, from equations (5.3), (3.7), it follows that

$$q_{ai} = 0 \qquad Q_{ai} = 0 \qquad a = 1, \ldots, n \qquad i = 1, 2, 3 \tag{5.4}$$

and from equations (5.4), (5.2), (3.6) and (3.11) we obtain equations of motion and constitutive relations for homogeneous, micropolar, isotropic and centro-symmetric bodies (Dyszlewicz, 2004; Nowacki, 1974, 1981).

Case 2. Periodically layered elastic bodies

In the case when

$$\begin{array}{lll} \alpha^{(r)} = 0 & \beta^{(r)} = 0 & \gamma^{(r)} = 0 \\ \varepsilon^{(r)} = 0 & J^{(r)} = 0 & r = 1, \ldots, n+1 \end{array} \tag{5.5}$$

from the obtained results given in (3.6), (3.7), (3.11), we obtain a homogenized model of periodically layered, elastic composites composed of $(n+1)$ different isotropic, homogeneous layers (see the result of Matysiak and Woźniak, 1987). Also, from equations (4.7), (4.8) and (5.5) we pass to a system of equations for the plane state of strain for periodically two-layered composites (Kaczyński and Matysiak, 1987, 1988).

Case 3. Homogeneous elastic bodies

If we assume that

$$\lambda^{(r)} = \lambda \qquad \mu^{(r)} = \mu \qquad \rho^{(r)} = \rho \tag{5.6}$$

we obtain from (5.5), (5.6) and (3.6), (3.7), (3.11), equations of the classical theory of elasticity.

The derived homogenized model with microlocal parameters for periodically layered, micropolar composites creates a basis for considerations of boundary value problems for nonhomogeneous bodies.

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O homogenizowanym modelu periodycznych warstwowych sprężystych kompozytów mikropolarnych

Streszczenie

Praca dotyczy zagadnień modelowania periodycznych warstwowych kompozytów o składnikach mikropolarnych. Wykorzystując liniową teorię mikopolarnej sprężystości i metodę homogenizacji z parametrami mikrolokalnymi wyprowadzono model homogenizowany uwzględniający pewne efekty lokalne w naprężeniach i naprężeniach momentowych. Z otrzymanego modelu otrzymano układy równań dla "pierwszego" i "drugiego" płaskiego stanu odkształcenia dla periodycznie warstwowych kompozytów mikropolarnych.

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