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APPLICATION OF WAVELET TRANSFORM TO IDENTIFICATION OF MODAL PARAMETERS OF NONSTATIONARY SYSTEMS

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In the paper an application of a time-frequency signal analysis technique for modal parameters identification is presented. Procedures of wavelet filtering, which allow estimating mechanical parameters for systems with non-constant parameters have been shown. The wavelets are used to detect natural frequency of the system and transform system time response into the time-scale domain. Presented method has been verified on simulation data for a two degrees of freedom discrete system. Two cases have been considered, with a constant and with a varying damping. The procedure has been applied to a real data recorded during a flight of ISKRA airplane.

Key words: wavelets transform application, modal model parameters identification, nonstationary system analysis

1. Introduction

Many practical engineering systems change dynamic parameters during their operation. One possible reason of the parameter changes is occurrence of damage. The problem of damage detection can be defined as identification of parameter changes in a model of a given system. In literature, that approach is named model based diagnostics, see Batko and Ziółko (2002). The classical approach to the model based damage detection is formulated with an assumption that the system is stationary during an identification experiment. But nonstationary behaviour due to system damage is expected for different experiments. In reality, operational parameters of the system can be changed during a particular experiment, then the system should be treated as nonstationary. Identification and analysis of nonstationary systems are more difficult then in a stationary case.

There are several damage detection techniques based on model parameters identification (Batko and Ziółko, 2002), but very widely used in practical applications is the modal model based method (Uhl, 1997). In original formulation, the modal model can be identified only if a system is linear, stationary with small or proportional damping. These facts seriously limit the application of techniques for modal model parameters identification for damage detection and localization.

In this approach, damage is detected by tracking modal parameters of the structure during its operation (Uhl, 2004). Nowadays, different systems are dedicated for damage detection during operation of mechanical structures; such systems are called the Structural Health Monitoring (SHM). The applicability of modal model for damage detection in SHM has been reported by many authors whose review is available by Uhl (2004). But practical application of the techniques is limited because of lack of efficient methods for the identification of modal models based on operational measurements for nonstationary data.

The paper presents two different approaches for the identification of modal model parameters for a nonstationary mechanical system. As a nonstationary system, a system with varying modal parameters is understood in the paper. The classical methods give incorrect results in presence of system nonstationarity (Klepka and Uhl, 2004). It was proved on simulated data for a two-degrees-of-freedom system with varying stiffness and damping. The stiffness was changed during the simulation and a signal sample with this disturbance due to the system parameters change was used as input data for estimation of the parameters. Time history of the nonstationary system response excited by white noise (stationary) is shown in Fig. 1. The spectrum of the response signal for the signal sample including the time of the parameter change is shown in Fig. 2. As it can be easily seen from the plot, the spectrum has three maxima which suggest three resonances (natural frequences) in the system. But, as predicted from the stationary model responses, the spectrum should have only two maxima. The interpretation can be missed due to system nonstationarity and it should be taken into account if the method of signal analysis is formulated.

To solve the problem of identification inaccuracy due to system nonstationarity, an algorithm for modal parameters identification based on the wavelet transform has been formulated and applied. The accuracy of the method will be proved by simulation results. The method described in the paper has been

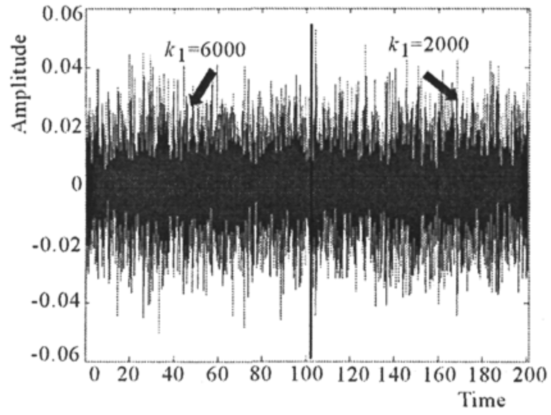

Fig. 1. Time plot of the system response

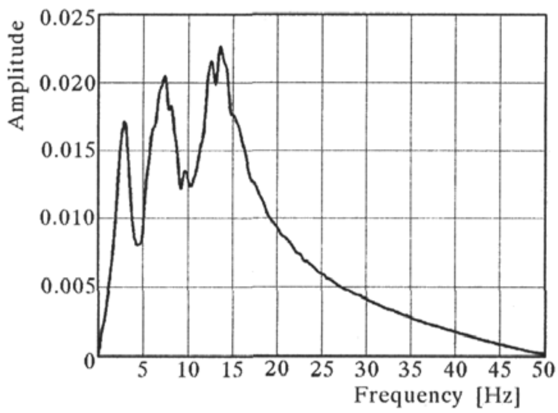

Fig. 2. An identified amplitude frequency characteristic

applied for identification of modal model parameters of an airplane from flight vibration data. The aero-elastic behaviour of the airplane was modiefied due to changes of flight parameters like speed and altitude.

2. Properties of wavelet transforms and their application to structural health monitoring

To SHM procedures classical FFT algorithms are commonly applied as a basic signal processing tool. But FFT-based identification methods do not give correct results for nonstationary and nonlinear systems. In practical applications of SHM systems, there are some nonstationarities of signals due to nonstationary environments or faults from a structure itself. There are some methods of structural dynamic testing of nonlinear systems (Uhl, 2001) based on transformation of nonlinear terms to variable parameters. This approach requires methods which help one to process data from nonstationary systems. Time-frequency analysis is the most popular method for analysis of nonstationary signals. In this group of analysis methods, the following are most widely used in practical applications (Batko and Ziółko, 2002; Young, 1993):

- Wigner-Ville distribution (Lin and Qu, 2000)
- Short Time Fourier Transform (Pan and Sas, 1996)
- Choi-Willams Distribution (Peng *et al.*, 2002)
- Cone-Shaped Distribution (Francois and Patrick, 1995)
- Continuous Wavelets Transform (Chancey and Flowers, 2001; Young, 1993)
- Discrete Wavelets Transform (Osipiw *et al.*, 2002).

These methods realise mapping of one-dimensional signal $x(t)$ to a two-dimensional function of time and frequency. The main problem with application of the Wigner-Ville distribution method is the occurrence of an interference term in the time-frequency plane, which misleads the signal analysis. There are no such big interference terms in the Choi-Willams distribution (also Cone-Shaped distribution) method, but computation effort related to its application is enormous, which discards this method in practical solutions (Russel *et al.*, 1998). The Short Time Fourier Transform gives correct results of means constant resolution (interfernce terms are not observed) for all frequencies since it uses the same window for the analysis of the whole signal. But if high frequency resolution is required for a low frequency, it is not possible to obtain good time resolution which is required for a high frequency. The Short Time Fourier Transform is useful for quasi stationary signals but not for real nonstationary signals.

Over the past twenty years, a novel concept of time-frequency transforms has been introduced. It is based on Morlet's wavelets definition given by the formula (Young, 1993)

$$W_x(a,b;\psi) = \frac{1}{\sqrt{a}} \int x(t)\psi^*\Big(\frac{t-b}{a}\Big)\,dt \tag{2.1}$$

where a is the scale factor, b is the time parameter.

The Morlet wavelets theory has been developed to construct an orthogonal wavelet base with excellent time and frequency resolution. There are some different algorithms which are very fast and help one to obtain wavelet signal decomposition from its finest scale approximation. The Morlet wavelet transform can be obtained for a discrete signal using the following formula (Uhl and Bogacz, 2002)

$$W_x(m,n,\psi) = \frac{1}{\sqrt{a_0^m}} \int x(t)\psi^*\Big(\frac{1}{a_0^m}t - nb_0\Big)\, dt \tag{2.2}$$

where m and n are integers.

Different from the STFT, wavelets transforms can be used for multi-scale analysis of a signal through dilation and translation. It means that they can effectively extract time-frequency properties of the signal with good time and frequency resolution. Therefore, the wavelet transforms are a widely applied tool for nonstationary signal processing and nonstationary model parameters identification, and can be very useful for design of SHM systems.

Because of their orthogonal properties, wavelet transforms are applicable for modal parameters identification. It was proved that Morlet's wavelet transform decouple the solution to coupled ordinary differential equations similar to the modal transform (Gouttebroze and Lardies, 2000; Klepka and Uhl, 2003b). This fact allows one to apply Morlet's wavelets for the identification of chosen mode parameters, e.g. modal damping or natural frequencies. For the frequency of a wavelet equal to the natural frequency, representation of only one vibration component is obtained, which allows independent identification modal parameters for each vibration mode.

The modal damping identification procedure and its application will be described in the next section of the paper.

3. Identification of modal damping using Morlet's wavelets

To formulate the method of modal damping identification, a single-degree-of-freedom system is considered (Gouttebroze and Lardies, 2000; Klepka and Uhl, 2003b)

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{3.1}$$

where m, c, k stand for the mass, damping and stiffness, respectively, and f is the excitation force. The solution to (3.1) has the following form

$$x(t) = A(t)\mathrm{e}^{\pm \mathrm{j}\omega_n \sqrt{1-\zeta^2}t} = A(t)\mathrm{e}^{\mathrm{j}\phi(t)} \tag{3.2}$$

For the Morlet function $g(t)$ given by (2.1), the wavelet transform of Eq. (3.2) can be approximated by the equation

$$(W_g x)(a,b) \approx A(b)G^*(a\dot{\phi}(b))\mathrm{e}^{\mathrm{j}\phi(b)} + o(|\dot{A}|, |\ddot{\phi}|) \tag{3.3}$$

where $G^*(\cdot)$ is a complex value adjoint to $G(\cdot)$.

The modulus of the function G can be obtained on the basis of the following equation

$$|(W_g x)(a,b)| \approx A(b)G^*(a\dot{\phi}(b)) \tag{3.4}$$

For a given dilatation value a_0, with logarithm of equations (3.2) and (3.4) found, the following relation was obtained

$$\ln|(W_g x)(a,b)| \approx -\omega_n \zeta b + \ln[A_0|G^*(\pm a_0 \mathrm{j}\omega_n \sqrt{1-\zeta^2})|] \tag{3.5}$$

Similarly, for a multiple-N-degree-of-freedom system, the wavelet transform of the response has the form (Klepka and Uhl, 2003a; Staszewski, 1997)

$$\ln\Big|\Big(W_g \sum_{i=1}^{N} x_i\Big)(a_i,b)\Big| \approx -\omega_{n_i}\zeta_i b + \ln[A_i|G^*(\pm a_i \mathrm{j}\omega_{n_i}\sqrt{1-\zeta_i^2})|] \tag{3.6}$$

On the basis of equations (3.5) and (3.6), a method for modal damping identification can be formulated. The logarithm of wavelets transform is a combination of straight lines, whose slopes are proportional to the modal damping in the tested system. Based on such a formulation, a identification procedure is determined. The procedure is presented schematically in Fig. 3.

The method explained above was verified numerically.

Example 1

In order to verify the presented method, simulation of a discrete damped system was realised. All calculations were carried out in the MATLAB® environment. For the analysis purposes, the following form of the impulse response signal (Fig. 4a) of a single degree of freedom system is assumed

$$h(t) = \mathrm{e}^{-0.1\cdot 314t}\sin(314t) \tag{3.7}$$

In Fig. 4b, the response signal power spectrum density is shown. In the analysis the following damping and natural frequencies were assumed: $\zeta = 0.1$, $f = 50\,\mathrm{Hz}$.

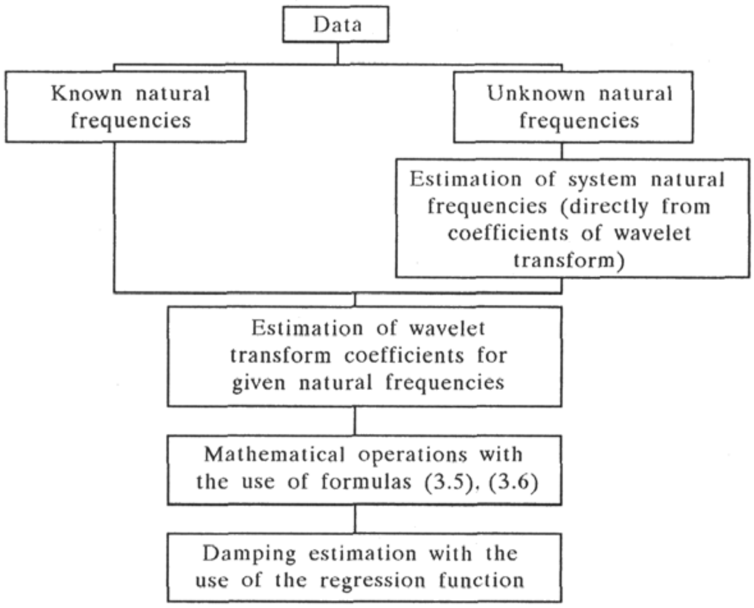

Fig. 3. A scheme of the damping estimation method with the use of the wavelet transform

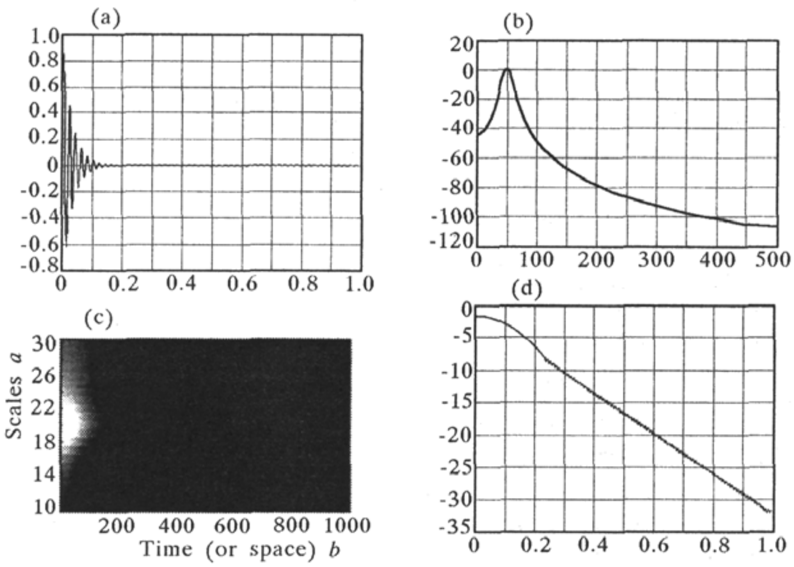

Fig. 4. (a) System impulse response; (b) power spectrum density of the signal $h(t)$; (c) wavelets transform for Morlet's wavelet; (d) values obtained from equation (3.6) for $i = 20$

The signal described by (3.7) is transformed to time-frequency domain using the wavelet transform. The Morlet wavelet employed for the response signal transformation is described by the formula (Robertson *et al.*, 1998)

$$G(t) = \sqrt{\pi f_b}\, \mathrm{e}^{2\mathrm{j}\pi f_c t} \mathrm{e}^{\frac{t^2}{f_b}} \tag{3.8}$$

where f_b denotes the bandwidth parameter, f_c – wavelet central frequency.

Knowing the system natural frequency, on the basis of formula (3.5) the dilatation parameter a can be computed. For the determined initial value of a next steps of the formulated procedure will be carried out. Signal decomposition results are shown in Fig. 4c.

On the basis of formula (3.6) and with the use of regression analysis, a straight line slope coefficient has been determined (Fig. 4d). Having transformed equation (3.6), one finds the damping coefficient to be $\zeta = 0.1$. The obtained result is consistent with the assumed one.

A similar simulation procedure has been realised for a more complex system with two degrees of freedom. The system impulse response function had the form

$$h(t) = \mathrm{e}^{-0.1\cdot 314t} \sin(314t) + \mathrm{e}^{-0.04\cdot 785t} \sin(785t) \tag{3.9}$$

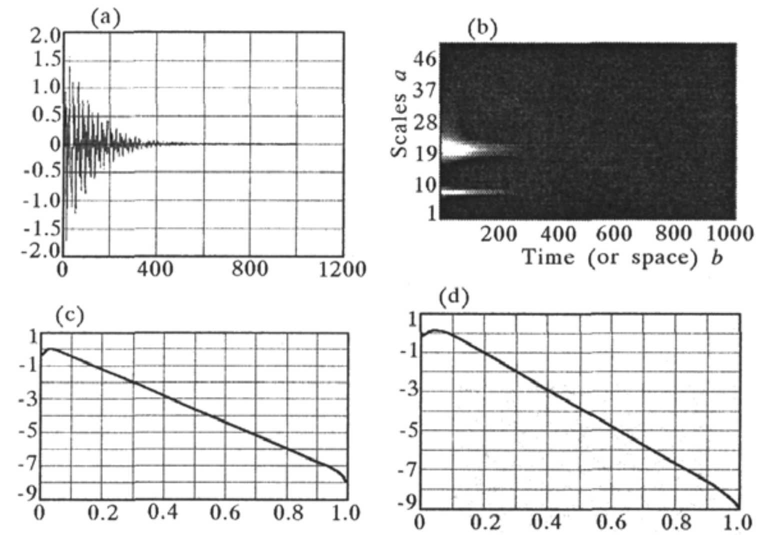

Fig. 5. (a) Impulse response of the simulated system; (b) results of wavelet-based signal decomposition; (c) approximation of the line given by equation (3.6) for $i = 20\%$; (d) approximation of the line given by equation (3.6) for $i = 8$

Natural frequencies were 50 Hz and 125 Hz, while damping factors 0.1 and 0.04. The response time history is shown in Fig. 5a.

As in the case described above, computations were carried out on the basis of relation (3.6). Coefficients charts of the wavelet transform are shown in Fig. 5b.

Two spectral lines corresponding with individual natural frequencies are visible. The following results were obtained: natural frequencies: $f_1 = 50\,\text{Hz}$ and $f_2 = 125\,\text{Hz}$ and corresponding damping values (from the regression analysis): $\zeta_1 = 0.1$ and $\zeta_2 = 0.04$ and (Fig. 5c,d). It is easy to notice that the estimated values of damping parameters and natural frequencies are consistent with the assumed values.

The above described examples show the damping identification procedure for a stationary case. Example 2 shows results obtained for a nonstationary system with a change in parameters.

Example 2

In this case, a single-degree-of-freedom system was been analysed. During simulation of the impulse response the damping parameter was changed significantly. The original value was $\zeta_1 = 0.01$, but after changing had a value of $\zeta = 0.02$. The impulse response time history is shown in Fig. 6a. The time profile of damping changes is shown in Fig. 6b.

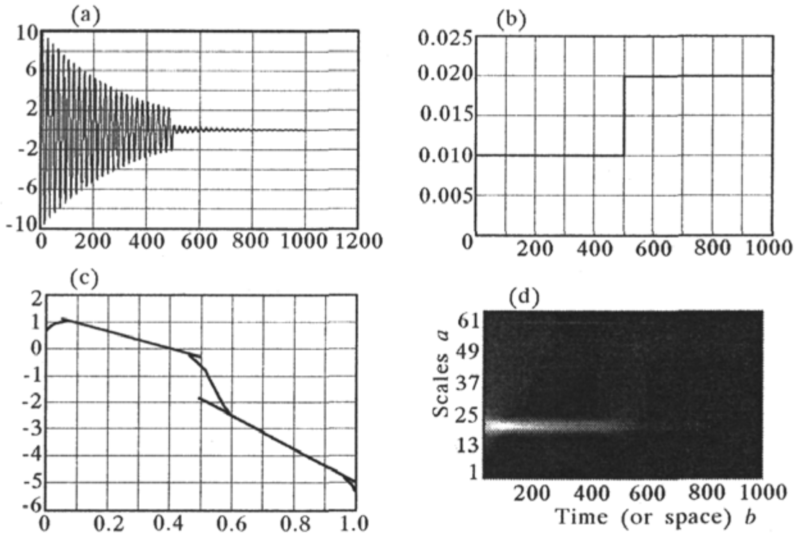

Fig. 6. (a) Time history of the impulse response signal; (b) rofile of damping changes; (c) approximation of formula (3.6) for the nonstationary case; (d) Results of signal decomposition

As a result of transformation of equation (3.6), two straight line intervals of different slopes (Fig. 6c) were obtained. The first segment (ranging from 0.2 to 0.3 second) corresponds to the first damping coefficient, and the second one (ranging from 0.6 do 1 second) corresponds to the second damping coefficient. The following numerical results have been obtained:

- frequency f counted on the basis of formula (3.5): $f = 100\,\mathrm{Hz}$
- damping coefficients for the first and second range: $\zeta_1 = 0.01$, $\zeta_2 = 0.02$, respectively.

The obtained damping coefficients were identical with the assumed values.

The application of the verified identification procedure require one to assume the model structure, mainly assumption of the number of degrees of freedom before the estimation of parameters. But the identification of model structures in a nonstationary case is not an easy task as it was presented, and requires a special procedure. The wavelet transform can be very helpful in solving this problem.

4. Application of wavelet filter for modal analysis of nonstationary systems

To formulate the method of modal parameters estimation from the response signal of a nonstationary system, properties of the Morlet wavelet transform are applied. The idea of the method is based on recovering the time history of a signal from its time-frequency representation. To detect the modal model structure (model order), a stabilisation diagram (Uhl, 1997) is synthesized and analyzed. To obtain the stabilisation diagram, the Frequency Response Functions (FRF) are necessary to be known. Based on the FRF estimators, the modal model parameters can be estimated using the classical formula (Uhl, 1997)

$$H(\mathrm{j}\omega) = \sum_{r=1}^{n} \Big(\frac{Q_r \Psi_r \Psi_r^\top}{\mathrm{j}\omega - \lambda_r} + \frac{Q_r^* \Psi_r^* \Psi_r^\top}{\mathrm{j}\omega - \lambda_r^*} \Big) \tag{4.1}$$

where $A_r = Q_r \Psi_r \Psi_r^\top$ is the vector of modal rest, λ_r – pole of the system.

In the proposed procedure, the FRFs are obtained from the time history of input-output signals after their filtering with filters designed on the base of the wavelet transform. The method is presented schematically in Fig. 7.

Using the classical wavelet transform (Staszewski and Giacomin, 1997), a time-frequency representation of a signal can be obtained. The wavelet transformation can be done by computation of a sequence of wavelet coefficients for

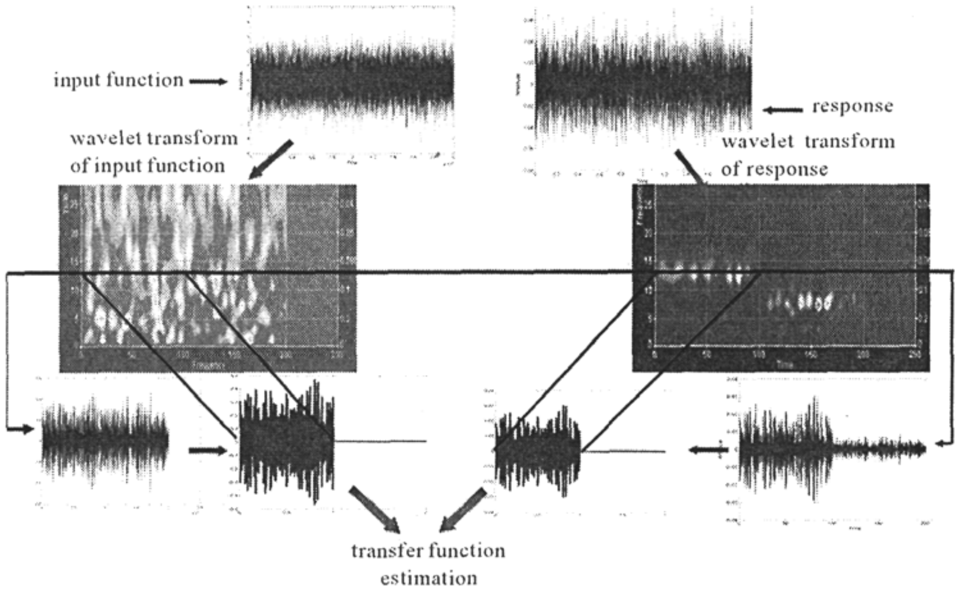

Fig. 7. Scheme of the proposed method

a given parameter a. If the signal under transformation contains a frequency component which corresponds to the parameter a, then the wavelet coefficient will achieve a high value. Repeating this operation for the next value of a, a matrix of wavelet coefficients can be obtained. Each row of this matrix is the time profile for the given frequency, but each column describes the signal spectrum for the given time. High values of wavelet coefficients help one to find the natural frequency of the system. Based on the wavelet coefficients, the signal filter is defined.

In the first step of the proposed identification procedure, the wavelet transform of the excitation (or a reference signal if the method for in-operation is applied) and the response are computed. From the scalogram of the response, natural frequences and scale parameters a are found. For each chosen parameter a, time representations of the excitation and response are reconstructed. If a frequency content of the analysed signal is changed in time, then correlation of the wavelet and signal will change as well (Staszewski, 1997). Then, a part of the signal which is correlated is used for estimation modal parameters. The part of the signal where the correlation is changed will be analysed for different parameter a and different frequencies. The filtered signal is used for the estimation of FRF and modal parameters of the structure.

5. Case studies

The proposed method has been applied to the identification of natural frequency and modal damping parameters of an airplane (jet trainer ISKRA) based on in-flight measurements data. Two different identification methods have been applied and results compared. The measurement results contain acceleration signals recorded during flight with varying speed. The first method applied for the estimation of modal parameters was the ARMA model of the measured signal (Cooper, 1995; Peng *et al.*, 2002). The second applied method was the method presented in the paper. The results have been compared in order to verify correctness of the methods. The recorded signal from the accelerometer is shown in Fig. 8.

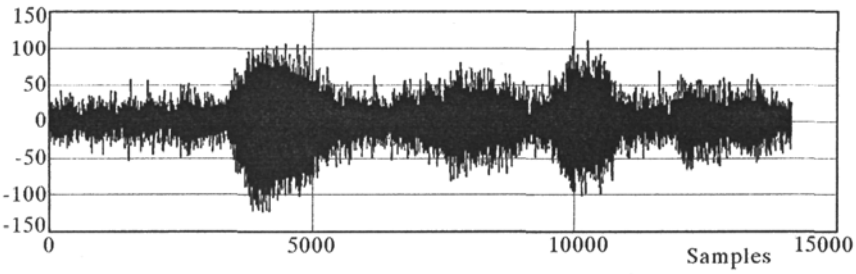

Fig. 8. Time history of vibration acceleration in the ISKRA plane

The signal has been transformed to time-frequency domain using Morlet's wavelet transformation. The results are shown in Fig. 9. As it can be noticed, dynamics of the analysed system is dominated by a frequency about 27 Hz. The natural mode at 27 Hz has been detected and carefully analysed using both methods. The results regarding modal damping coefficients are compared. The comparison is presented in Fig. 10.

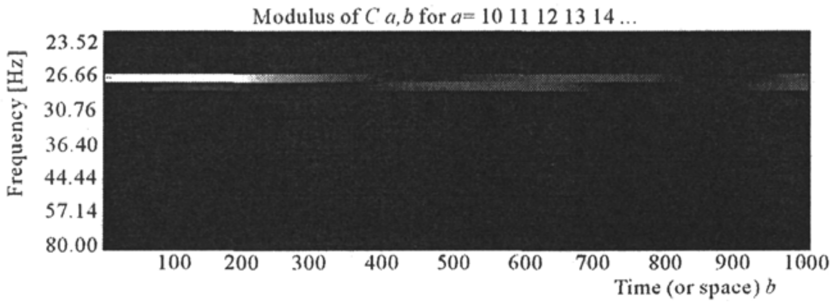

Fig. 9. Example of a scalogram of the recorded signal

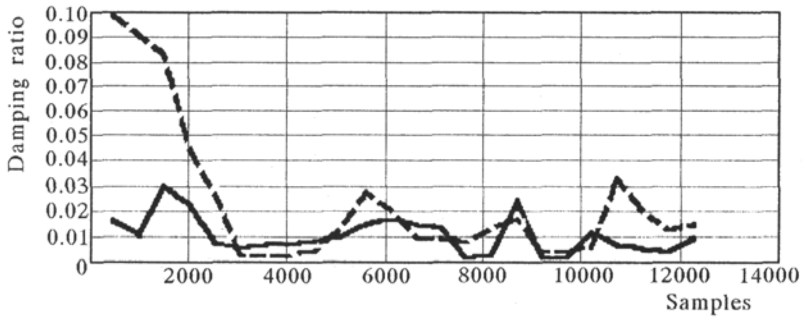

Fig. 10. Comparison of identification results from both methods

As it can be noticed from the presented comparison, the results are similar.

The presented method is applied for the estimation of the modal model structure. In order to perform this estimation based on investigation of the stabilization diagram, cross spectral density (CSD) of measured signals and wavelet transformation have been applied.

The CSD function has been obtained from the formula

$$G_{AB}(f) = A(f)B^*(f) \tag{5.1}$$

where $A(f)$ and $B(f)$ are Fourier transforms of the signals $a(t)$ and $b(t)$, respectively.

But before classical FFT analysis, the signals have been filtered using the presented approach. The results of the first step of the procedure consist of signal scalograms that have been obtained using the VIOMA software (Uhl *et al.*, 2001). The VIOMA software is developed for modal model identification with several different methods. Results in the form of acceleration signal scalograms are shown in Fig. 11. In the scalogram, two dominating frequencies at two separate time intervals have been recognized (inside and outside the rectangular). The modal parameters of the structure in both time intervals have been obtained separately using the BR (Balanced Realization) estimation method, which is implemented in the VIOMA software. The method, in practical applications, gives a very good approximation of modal parameters even for noisy data from an ambient excitation experiment.

To find the model structure, a stabilisation diagram has been synthesised. The results for a signal without wavelet-based filtering are shown in Fig. 12. As it can be noticed, it is very difficult to recognize stabilised modes which indicate model structures and natural frequencies of the system.

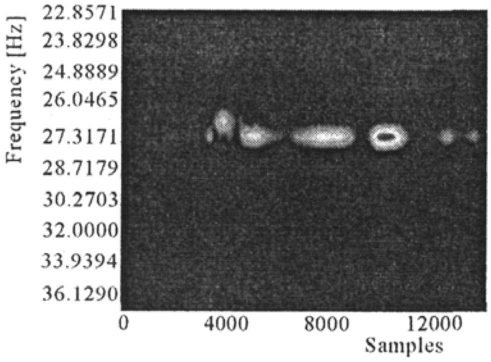

Fig. 11. A scalogram of an acceleration signal

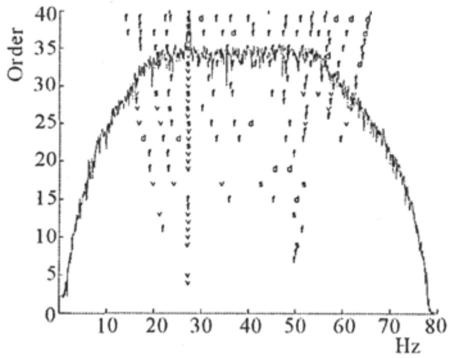

Fig. 12. A stabilisation diagram of a system without wavelet filtering

The signals have been filtered using the formulated wavelet-based filters, and for the filtered data stabilisation diagrams have been built. The filters have very similar characteristics and central frequencies ($a = 41$ and $a = 42$). The results are shown in Fig. 13.

It can be seen in Fig. 13 that the structure (the order of the investigated structure model) of the system is varying and completely different for the interval inside the rectangular (Fig. 11), in which only 4 stabilised modes can be distinguished, and outside the rectangular, where 7 stabilised modes are recognized. The numerical results of modal parameters estimation for both signal intervals are shown in Table 1.

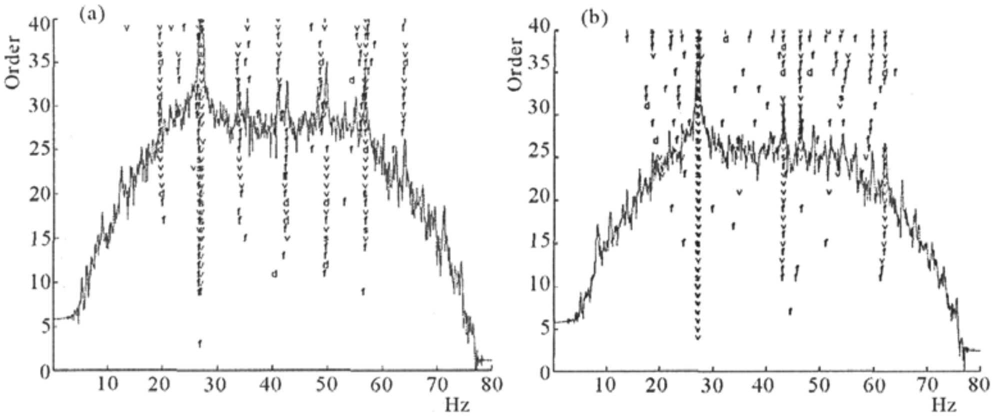

Fig. 13. A stabilisation diagram of a system with wavelet filtering for the scale parameter $a = 42$ (a) and $a = 41$ (b)

Table.1. Results of analysis

Identified mode shapes					
Signal without wavelet filtering		Wavelet filtering for the scale parameter $a = 41$		System with wavelet filtering for the scale parameter $a = 42$	
Frequency [Hz]	Damping [%]	Frequency [Hz]	Damping [%]	Frequency [Hz]	Damping [%]
20.45	1.009	27.26	0.094	26.99	0.182
27.23	0.226	43.17	0.049	41.80	1.197
46.26	2.147	46.29	0.091	49.70	0.310
		54.32	0.788	56.63	0.690
		61.97	0.308	63.91	0.061

As it can be noticed, after wavelet filtering interpretation of the stabilisation diagram is relatively easy and diagrams have a clearer form, which is helpful in recognition of natural modes. Different numbers of natural modes of the system indicate structural nonstationarity of the system, which makes more modes significantly excited when closer to the flutter point.

To confirm the nonstationarity of the system, modal model parameters have been estimated for different signal samples. The wavelet-based filters have been used for signal processing before estimation of the modal parameters. An interval with correlation of a signal with a given wavelet have been chosen for the estimation. The results in a form of a stabilisation diagram for different time samples are shown in Fig. 14 and Fig. 15 and sumarized in Table 2.

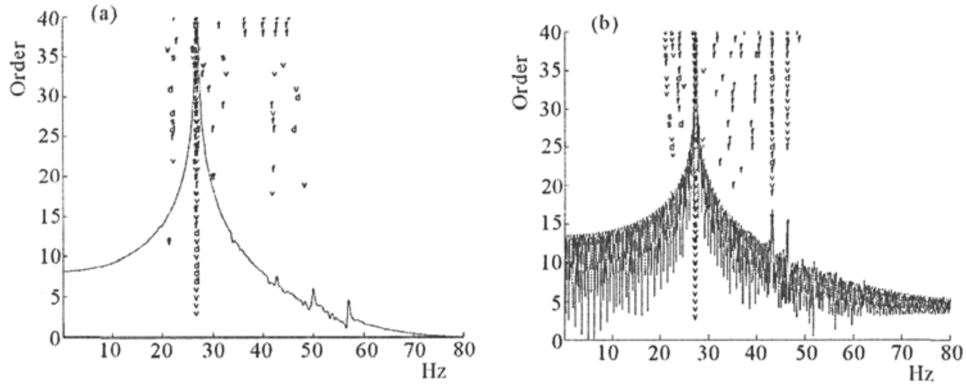

Fig. 14. A stabilisation diagram of a system with wavelet filtering from 3700 to 4200 for the scale parameter $a = 42$ (a) and $a = 41$ (b)

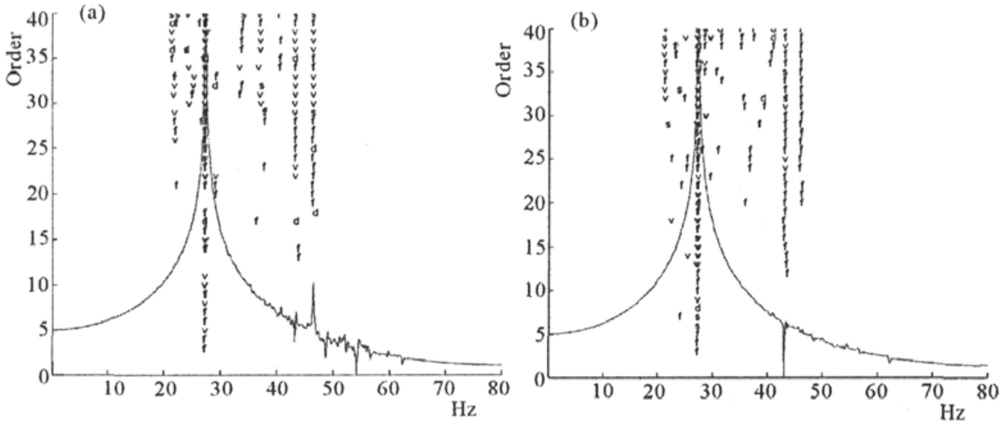

Fig. 15. A stabilisation diagram of a system with wavelet filtering: (a) from 7400 to 8200 for the scale parameter $a = 41$, (b) from 9800 to 10800 for the scale parameter $a = 41$

Table 2. Estimation of natural modes for different signal intervals

Identified natural modes for different time intervals							
Sample No. 3700-4200		Sample No. 4800-5400		Sample No. 7400-8200		Sample No. 9800-10800	
Frequency [Hz]	Damping [%]	Frequency [Hz]	Damping [%]	Frequency [Hz]	Damping [%]	Frequency [Hz]	Damping [%]
26.26	1.02	26.60	2.10	25.79	1.90	28.11	2.18
27.09	0.64	27.25	0.19	27.32	0.25		

The obtained results confirm nonstationary behaviour of the system due to aerodynamic couplings between particular modes. As it can be noticed, the wavelet filtering has significant influence on quality of the obtained results. To obtain a very accurate structure of the system, changes in the filtering based on the wavelet formulation should be repeated for each value of the parameter a, but it is a time consuming task. To avoid this effect, the parameters should be chosen based on visual assessment of a scalogram.

6. Conclusions

Based on the presented theoretical and numerical studies, the following conclusions can be drawn:

- During the process of modal analysis, stationarity of measured signals should be very carefully checked. The nonstationarity can be a reason for significant errors in the identification of parameters and the structure.
- The wavelet transform, particularly Morlet's wavelets, is a very useful tool for modal analysis of a mechanical structure and helps one to identify natural modes of a system even when signals are nonstaionary due to system nonstationarity.
- The wavelet-based filtering should be used as a signal preprocessing procedure if the nonstaionarity of a system is expected. The wavelets help one to recognize nonstionary behaviour of the system and to filter the signal before the estimation procedure.
- A mechanical system with aeroelastic feedback, in which flutter is expected, revealed in its response that more and more dominating vibration modes move to the flutter point in working conditions.

The future research in this area will be focused on modal analysis of nonlinear systems excited by ambient loads.

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Zastosowanie transformaty falkowej do identyfikacji parametrów modalnych układów niestacjonarnych

Streszczenie

W artykule przestawiono możliwość zastosowania transformaty falkowej do identyfikacji parametrów modalnych układów mechanicznych. Pokazano procedury filtracji falkowej pozwalającej na wyznaczanie parametrów dynamicznych układów wykazujących niestacjonarności. Metodę zweryfikowano na danych symulacyjnych dla układu o dwóch stopniach swobody ze zmiennym tłumieniem. Opracowaną metodę zastosowano do badania zmian tłumienia w czasie lotu samolotu ISKRA.

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