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INFLUENCE OF NORMAL MICRO-VIBRATIONS IN CONTACT ON SLIDING MOTION OF SOLID BODY

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The paper shows results of some simulation studies on: (i) normal contact micro-vibrations in a simple model of a friction system and (ii) the influence of such vibrations upon reduction of friction forces in the system. A model of friction contact with two degrees of freedom is adopted, considering non-linear normal contact flexibility of rough surfaces acting on each other. The mechanisms which cause the reduction of the friction force are investigated and described. It is proved that the main cause of the decrease of the friction force, if normal contact micro-vibrations are present, is due to certain dynamic processes taking place within the contact region. This decrease is not due to lower values of the friction coefficient, nor lower mean values of the normal reaction and real contact area, as it is often quoted in many publications.

Key words: mechanical systems, vibration, friction reduction

1. Introduction

According to many to date investigations, there exists powerful mutual interaction between friction and vibrations in mechanical systems (see Tolstoi, 1967; Godfrey, 1967; Budanov *et al.*,1980; Broniec and Lenkiewicz, 1982; Sakamoto, 1987; Martins *et al.*, 1990; Hess and Soom, 1991; Hess and Wagh, 1995; Skåre and Stål, 1992; Kohei and Kyosuke, 2001; Grudziński and Wedman, 1999; Grudziński *et al.*, 2000; Grudziński and Kostek, 2003). Friction may excite vibrations of different kinds, and vibrations in turn may have significant influence on the current value of the friction force and its variation in time. In specific conditions, in dynamic systems, friction may cause nonuniform sliding in the form of stick-slip motion or tangential quasiharmonic oscillations. On the other hand, through excitation and maintaining of properly directed micro-vibrations, it is possible to decrease the friction force as well as the stick-slip effect, thus improving the uniformity of sliding motion. These two aspects of the friction-vibration interaction have long been (Tolstoi, 1967; Godfrey, 1967) and are also currently (Pfeiffer, 1999; Kohei and Kyosuke, 2001; VDI-Berichte, 2002) the subject of interest of investigators who deal with the friction and dynamics of machinery.

Reductions of the friction force observed in investigations, where vibrations between contacting bodies are present, are usually explained in the literature on the subject through decreased values of the friction coefficient (Godfrey, 1967; Broniec and Lenkiewicz, 1982; Skåre and Ståhl, 1992) or the decreased mean value of the normal reaction within the contact area (Tolstoi *et al.*, 1973; Budanov *et al.*, 1980; Hess and Soom, 1991). This is a significant simplification of the considered problem that can lead to false conclusions. Despite the fact that there is a great deal of knowledge on that subject and a wealth of experimental data, there is a lack of appropriate models and theories which would enable reliable understanding of the interaction between friction and vibration in dynamic systems and their qualitative and quantitative investigations with the aid of computer simulation methods.

The goal of this paper is to prove that the main cause of the decrease in the friction force in mechanical systems, where normal contact micro-vibrations are present, are complex non-linear dynamic processes occurring where solid bodies with rough surfaces come into contact, and not a decrease in the friction coefficient or mean normal reaction within the contact area. In the considerations contained herein, the friction coefficient has been assumed to be a constant quantity.

2. Theoretical fundamentals

The investigated object is a system of two bodies in planar friction contact as shown in Fig. 1a. This consists of a slider and a slideway. The surfaces in friction contact are rough (Fig. 1b) and create an elastic interface which is modelled by a system of non-linear springs (Fig. 1c). The slider, with the mass m, is assumed to be a rigid block. The load-deflection characteristic of the interface is presented in Fig. 1d. On the grounds of many investigations (see Levina, 1965, Back *et al.*, 1973; Martins *et al.*, 1990), it may be well described by an involution formula as follows

$$\delta = e_n p^{m_1} \tag{2.1}$$

where: δ denotes the normal deflection of the interface, p – nominal contact pressure, and e_n and m_1 are interface constants depending on the actual type of material and surface treatment of the friction pair, to be determined experimentally.

The model (Fig. 1c,d) represents, in simplification, a working unit of a machine tool, which is moved along a slideway. The research carried out by Levina (1965), Ostrovski (1965), Dolby and Bell (1971) has shown that for the range of interface pressures and surface finishes used in the joint faces of machine tools, the above form of equation fits the experimental results. Tables with experimental values of the constants e_n and m_1 for several combinations of materials and surfaces finishes can be found in Back *et al.* (1973).

In the adopted model of the interface (Fig. 1) only elastic normal deflections are taken into account. The damping and tangential deflections at the interface are neglected in this work for the sake of simplificity. These factors will been taken into account in farther investigations. As a consequence of the contact flexibility, the slider may micro-vibrate in the normal direction – both freely and as a result of the excitation, see Tolstoi (1967); Budanov *et al.* (1980).

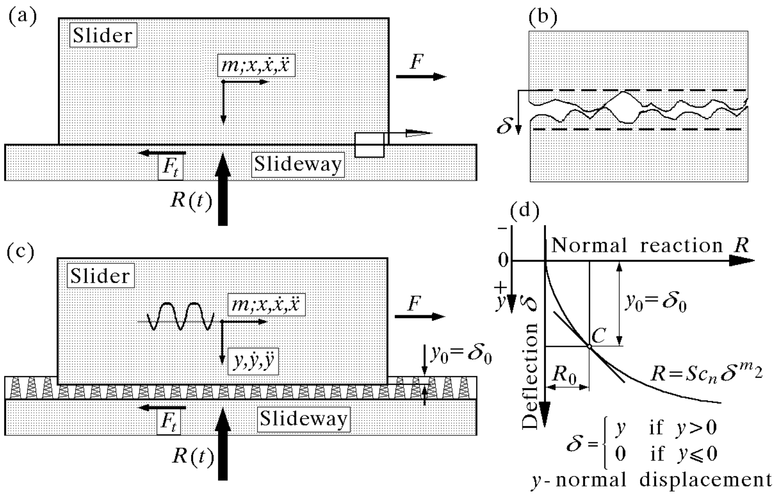

Fig. 1. Model of a simple mechanical system with non-linear contact flexibility and friction

A constant force F is applied to the slider (Fig. 1c) in the direction of the x axis. The force tends to introduce sliding motion. This motion is impaired by the friction force F_t acting in the contact region. The friction force is expressed by the Coulomb law of dry friction as follows

$$\text{if } \dot{x} = 0 \text{ then } |F_t| \leqslant \mu|R| \text{ else } F_t = -\mu|R| \operatorname{sgn} \dot{x} \tag{2.2}$$

where μ is a constant friction coefficient and $R = R[y(t)]$ is the resultant normal reaction in the contact region, which depends on the co-ordinate $y(t)$.This co-ordinate describes the approach and separation of the bodies in contact in the normal direction to the sliding surfaces. The deflection of the interface $\delta = y$ if $y > 0$ and $\delta = 0$ if $y \leqslant 0$; ($y < 0$ corresponds to a loss of contact).

The normal reaction in the contact region is computed from the following formula, see Martins *et al.* (1990)

$$R(y) = \begin{cases} -Sc_n y^{m_2} & \text{for} \quad y = \delta > 0 \\ 0 & \text{for} \quad y \leqslant 0 \end{cases} \tag{2.3}$$

where S means the nominal contact area between the slider and slideway, and c_n, m_2 are constant parameters which characterise the elastic properties of the contact interface

$$m_2 = \frac{1}{m_1} \qquad\qquad c_n = e_n^{-1/m_1}$$

The equation of motion to describe the contact free micro-vibrations of the slider in the normal direction y, is as follows

$$m\ddot{y} - R(y) = mg \tag{2.4}$$

where g denotes the acceleration of gravity.

The equation of sliding motion for the slider (Fig. 1c) may be expressed in the following general form

$$m\ddot{x} - F - F_t = 0 \tag{2.5}$$

As equations (2.4) and (2.5) are non-linear, Runge-Kutta's numerical method of integration of the fourth order was used to calculate the solution.

3. Numerical investigations of normal contact free micro-vibrations

Normal contact free vibrations of the slider (Fig. 1c) are determined from equation (2.5), considering formula (2.4). For numerical computations, the following data were adopted: m=1 kg, $c_n = 4.52 \cdot 10^{16}\,\mathrm{N/m^4}$, $m_2 = 2$, $S = 42.735 \cdot 10^{-4}\,\mathrm{m^2}$, $g = 9.81\,\mathrm{m/s^2}$. The computations were carried out assuming for $t = 0$ three different initial conditions:

$$
\begin{array}{lll}
(1) & y = 0 & \dot{y} = 0 \\
(2) & y = \frac{1}{3}y_0 & \dot{y} = 0 \\
(3) & y = \frac{2}{3}y_0 & \dot{y} = 0
\end{array}
$$

where $y_0 = \delta_0 = 2.25 \cdot 10^{-7}$ m, and δ_0 is the deflection of the interface corresponding to the static equilibrium position of the slider. The results of computations are presented on graphs in Fig. 2 and Fig. 3.

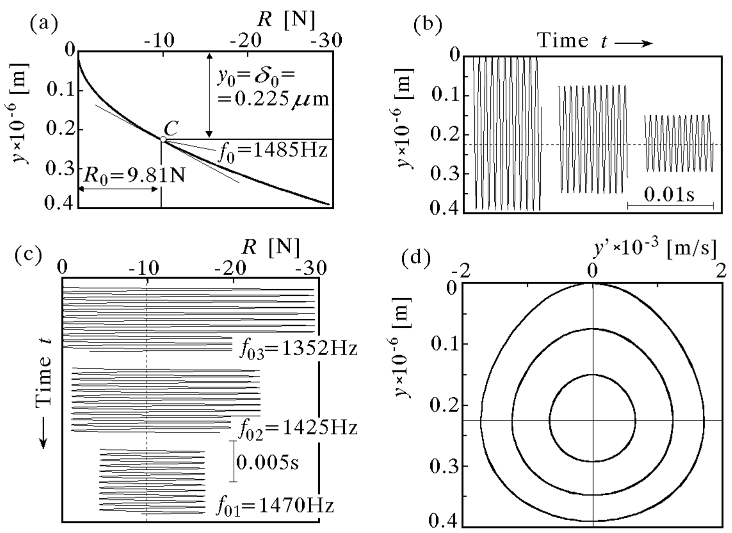

Fig. 2. A load-deflection characteristic of the interface (a) and results of computer simulations illustrating free (non-damped) normal contact micro-vibrations versus time (b), (c) and on the phase plane (d) for three different amplitudes of vibration

Figure 2a shows the load-deflection characteristic of the considered contact interface, where the working point C is marked up, which corresponds to the static equilibrium position of the slider. The frequency of small normal contact vibrations is close to the frequency of linear contact vibrations at that point and may be calculated from equation $f_0 = (1/2\pi)\sqrt{K_n/m}$, where $K_n = 2\sqrt{mgSc_n}$ is the contact stiffness at the point C of the characteristic, and m is the mass of the slider. Substituting the above mentioned data to that formula, the frequency of natural normal contact micro-vibrations was determined to be $f_0 = 1485$ Hz.

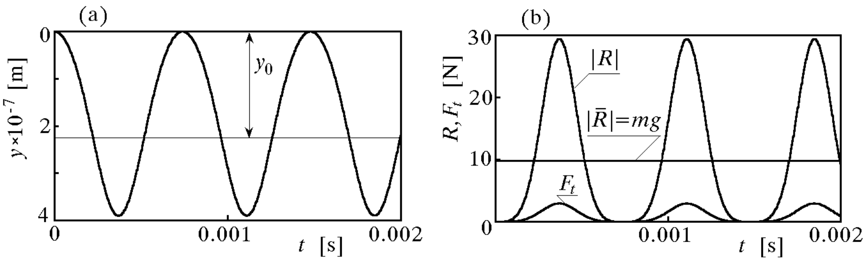

Fig. 3. Normal contact vibrations (a) and contact forces (b) as functions of time

The free vibrations are asymmetric with respect to the static equilibrium position. Their natural frequency is dependent not only on the position of the working point C, on the curve (Fig. 2a), but on their amplitude as well. As the amplitude rises, the asymmetry of contact vibrations increase and the frequency decreases (Fig. 2b,c). Figure 2d shows phase portraits of the discussed contact vibrations. To be more precise, the normal micro-vibrations and forces acting within the contact region are presented in Fig. 3. Figure 3a shows a graph of free undamaged normal contact vibration and Fig. 3b – time histories of the normal reaction R and friction force F_t that is determined by means of the equation $F_t = \mu|R|$ assuming $\mu = 0.1$.

The contact vibrations make the slider ascend and descend alternately (Fig. 3a). The normal reaction R in the contact region, and thus the friction force F_t, are periodic variables (Fig. 3b). The inertial force originating from the vibrating mass of the slider has large influence upon these forces. It causes the normal reaction in the contact region to fluctuate in a wide range of values. In the case concerned (Fig. 3), the said reaction varies from nearly 0 up to approximately 3 times of its static value. The specified range within which the dynamic load varies is a result of the maximum free vibrations at which the contact between the sliding surfaces has not been lost yet. The range of variation of the normal reaction for smaller vibrations is shown in Fig. 2.

4. Numerical investigations of friction forces and sliding motion

For numerical investigations of friction forces and motion of the slider, the model shown in Fig. 1c loaded with a tangential constant drive force F was adopted. If in the said system normal contact vibrations are not present, then sliding motion of the slider will take place when

$$F \geqslant F_{t0} = \mu mg \tag{4.1}$$

where $F_{t0} = \mu mg$ denotes the nominal static friction force in the absence of contact vibrations.

If there are contact vibrations, then sliding motion of the slider may occur at lower values of the tangential force F where motion will be intermittent, i.e. of the stick-slip type. The condition for motion to occur may be written down as follows

$$F_{t0} > F > -\mu R_{max} = \mu m(g - \ddot{y})_{min} = F_{t\,min} \tag{4.2}$$

where $F_{t\,min}$ denotes the minimum of the static friction force and $\ddot{y}$ is the acceleration of vibrating motion in the direction normal to the friction plane. The latter depends on the current amplitude of contact vibration, whose course is asymmetric relative to the static equilibrium position (Fig. 2 and Fig. 3).

The mean values of the normal reaction and static friction force do not undergo any change here, whereas their instantaneous values do vary within a wide range. In such circumstances, small periodical slip may appear for a driving force F far less than F_{t0} determined by formula (4.1).

Equation (2.6) was used in order to determine motion of the slider due to a constant tangential force F, under consideration of the periodically variable friction force F_t determined from formulae (2.2) and (2.3). The data for computations relevant to the system in question was assumed as in Section 3. The computations were made for the tangential force $F = 0.5\,\mathrm{N}$, which is ca. 50% less than the nominal static friction force $F_{t0} = \mu mg = 0.981\,\mathrm{N}$ for the case when there are no normal contact vibrations in the system. The results of computer simulations are shown in Fig. 4. If there are no contact vibrations (Fig. 4, columns 1 and 1'), then the normal reaction R and friction force F_t assume constant values ($R = R_0 = mg$, $F_t = F_{t0} = \mu mg$), and because $F < F_{t0}$, the slider remains motionless (Fig. 4f,g,h).

The results of computations for small contact vibrations with the amplitude $A = y_0 - y_{min} = y_0/3$ are shown in Fig. 4 in columns 2 and 2'. In this case, the static friction force F_t has periodically a little lower value than the tangential force F. At the points of time, at which the friction force begins to be lower than that of the tangential force F, the slider begins a short-time movement that re-occurs in cycles. Graphs of the displacement x, velocity $\dot{x}$ and acceleration $\ddot{x}$ of that motion are shown in column 2', respectively (Fig. 4).

The results of simulations for the same value of the tangential force F as before, but at higher amplitudes of contact vibrations are shown in columns 3 and 3', 4 and 4', respectively (Fig. 4). An increase in the amplitude of contact vibrations results in an increase in the jerking displacements Δx of the slider in stick-slip motion, and of the instantaneous $(\dot{x})$ and average $(\overline{\dot{x}})$ velocity in sliding motion (Fig. 4f,g).

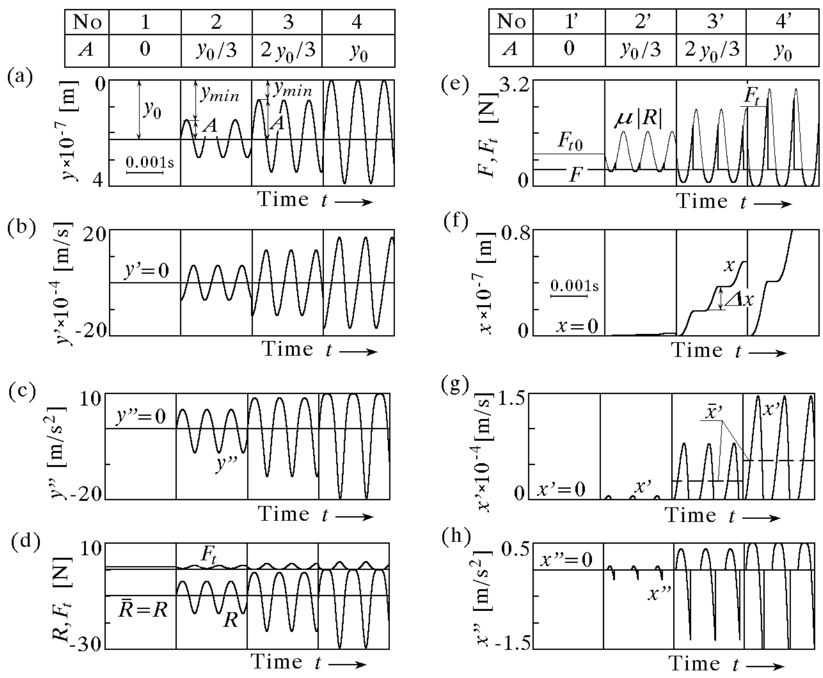

Fig. 4. Results of computer simulations illustrating time histories of contact normal micro-vibrations with different amplitudes $A = y_0 - y_{min}$ and their influence on the normal reaction R, friction force F_t, and kinematic values $x, \dot{x}, \ddot{x}$ characterising motion of the slider

Some more detailed graphs of the friction force F_t and velocity $\dot{x}$ of sliding motion as functions of time are presented in Fig. 5. These are the last two cases from Fig.4 on a greater scale. Figures 5a,b show graphs of the tangential F and friction force F_t in time, and Fig. 5d,e comprise velocity characteristics of sliding motion for two different amplitudes of contact vibrations. The tangential force F is constant, whereas the static friction force is a time variable depending upon the amplitudes of normal contact vibrations. At the point of time when the friction force F_t is taking a value lower than the tangential force F, the phase of sliding motion begins. Primarily, the velocity of this motion rises (when $F_t < F$), then it falls down (when $F_t > F$) and finally, when the velocity $\dot{x}$ drops to zero, the slider stops. Then, typical cyclical stick-slip motion occurs consisting of the sliding and sticking phase (Fig. 5). In the sliding phase, besides the traction F and friction force F_t, there is also a force of inertia of the mass m. The friction force is described by the formula $F_t = F - m\ddot{x}$. During the sticking phase (when $\dot{x} = 0$ and $\ddot{x} = 0$) the real friction force $F_{tr} = F$. This follows from the condition of static equilibrium.

The potential value of the static friction force during this time (t_s) is much higher than F, (Fig. 5a,b, thin lines).

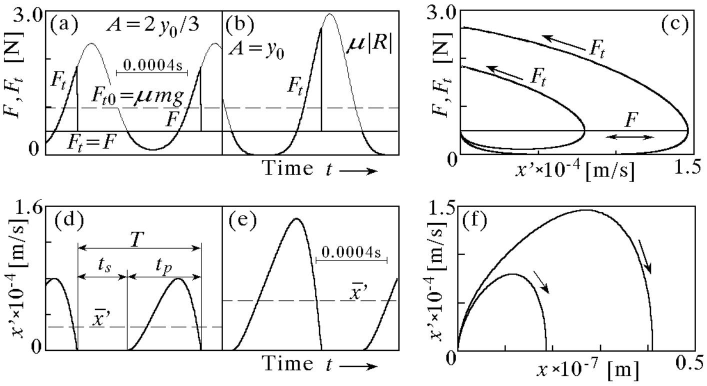

Fig. 5. Detailed curves of the friction force F_t and velocity $\dot{x}$ of the slider in stick-slip motion induced by normal contact micro-vibrations with two greatest amplitudes (from Fig. 4)

Figure 5c shows variations of the friction force F_t in a function of the instantaneous velocity $\dot{x}$ of the slider, and Fig. 5f presents a relation between the displacement x and velocity $\dot{x}$ of sliding motion in the phase plane. Diagrams in Fig. 5c,f concern the sliding phase of intermittent motion. For some determined contact vibrations, stick-slip motion exhibits a stable limit cycle. Times of both individual phases of stick-slip motion are very short and practically unnoticeable. The effect of such micro-slips may not be apparent until some time, e.g. when the nuts on bolts work loose. At greater vibrations, that motion is seen to be continuous with a constant velocity. The presence of such motion is usually explained as a result of reduction in the friction coefficient or an increased distance between mating surfaces, lower mean values of the normal deflection at the interface, and the normal reaction R as well as real contact area (see Tolstoi *et al.*, 1973; Sakamoto, 1987). In the case concerned, none of these factors is present. The decrease of the friction force and the occurrence of motion are both due to rapidly changing normal displacements and dynamic contact forces excited by vibrations.

Figures 4 and 5 show results of simulations for contact vibrations at which the friction surfaces are not yet separated from each other. If the vibrations are of greater amplitudes, there will be a momentary loss of contact, and during this time, the contact forces R and F_t will equal zero. An example of such vibrations and their influence upon the course of sliding motion, where the tangential force $F = 0.5N$ is applied, are illustrated by graphs presented in Fig. 6. Such situations are usually accompanied by specific sound effects (chatter). In theory, any small tangential force F may, in that case, cause motion of the slider in the direction of the x axis. In practice, this may manifest itself, e.g. in the form of vibrating items drifting on a resting tabletop even if the slope of the latter is very small (nearly zero).

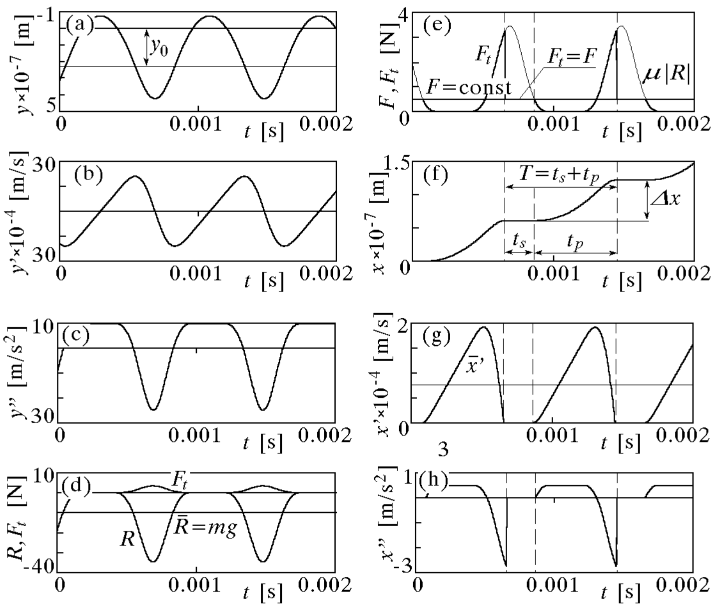

Fig. 6. Time histories of normal contact free micro-vibrations with a momentary loss of contact (a), (b), (c) and their influence upon the normal reaction R, friction force F_t (d), (e) and kinematic values $x, \dot{x}, \ddot{x}$ (f), (g), (h)

In general, one may conclude that the occurrence of sliding motion of the slider and its behaviour in time depend on the value of the tangential force F (which is lower than the static friction force $F_{t0} = \mu mg$) and amplitude A of normal contact vibrations. Figure 7 presents the influence of these factors on the mean velocity $v = \overline{\dot{x}}$ of sliding motion for $\mu = 0.1$. If $y_{min} \geqslant 0$, the bodies remain in contact all the time (Fig. 4 and Fig. 5), and if $y_{min} < 0$ it leads to a periodical loss of contact between the sliding surfaces (Fig. 6).

The possibility of a momentary loss of contact between the sliding surfaces has been taken into account in the computations. In Figure 7, the area of variations of controllable data (F and A) is noticeable, in which sliding motion does not occur ($v = 0$). This takes place for appropriately small amplitudes of vibrations and values of the tangential force. If the vibration amplitude is $A > 0.225\,\mu$m, which was the case, then some momentary losses of contact will occur, and the slider will move even for very small values of the tangential force F (Fig. 7).

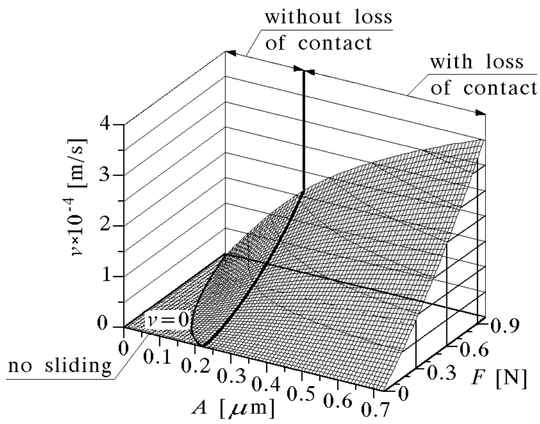

Fig. 7. Influence of the tangential force F and amplitude A of normal micro-vibrations in the contact area on the mean velocity of sliding

Motion of the slider with a constant mean velocity v for $F < T_{t0} = \mu mg$ is possible only if $A > 0$. Should $F > F_{t0}$, then the slider will move with an acceleration dependent, to a certain extent, on the current amplitude of contact vibrations. These cases are of little interest and were not analysed in detail.

5. Conclusions

The adopted model as well as simulation tests carried out in this study both provide theoretical justification and show the mechanism of reduction of the friction force for planar contact of solids if normal micro-vibrations take place. It has been shown that the influence that normal contact micro-vibrations have on the reduction of the friction force, which is often observed and confirmed in many experiments, does not necessarily have to have any connection with the decrease either in the friction coefficient or the mean value of the normal reaction in contact. This influence primarily results from periodic dynamic processes that take place in the frictional contact. In order to properly understand and describe these processes, it is necessary to adopt the frictional contact model with at least two degrees of freedom that would take into account the non-linear normal flexibility of the rough surfaces.

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Wpływ mikrodrgań normalnych w kontakcie na ruch ślizgowy ciała stałego

Streszczenie

W artykule przedstawiono wyniki symulacyjnych badań mikrodrgań w kierunku normalnym w kontakcie ciał stałych o powierzchniach chropowatych oraz ich wpływu na redukcję siły tarcia. Przyjęto model o dwóch stopniach swobody, uwzględniający nieliniową podatność kontaktową normalną oddziaływujących na siebie powierzchni. Zbadano i opisano mechanizm powodujący zmniejszenie oporu tarcia w kontakcie, w przypadku występowania w nim drgań normalnych. Wykazano, że główną przyczyną redukcji oporu tarcia w takim przypadku są pewne procesy dynamiczne występujące w kontakcie, a nie spadki wartości współczynnika tarcia, ani też średniej wartości reakcji normalnej, jak to się podaje w wielu publikacjach.

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