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FREE VIBRATIONS OF A COLUMN LOADED BY A STRETCHED ELEMENT

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The problem of free vibrations of a two-member column loaded by a sretched element is considered in the paper. The influence of rigidity asymmetry on the bending between the stretched element and column rods, and the influence of the rigid mounting of the loading element and its length on the course of natural frequency in relation to the external load are analysed. The regions, for which the tested systems appear to be of divergence or divergence-pseudo-flutter type, are determined for presented physical and geometrical parameters of the column and systems for border values of the coefficient of bending asymmetry. Numerical computations are supported by appropriate results of experimental investigations.

Key words: elastic column, divergence instability, pseudo flutter systems, natural frequency

1. Introduction

In the scientific literature dealing with vibration and stability of slender elastic systems (columns, frames), the following types of systems can be distinguished depending on the course of a curve in the plane: load P – natural frequency ω (characteristic curves):

- divergence systems – loosing their stability due to buckling (conservative systems) (Gajewski and Życzkowski, 1969a,b; Leipholz, 1974; Timoshenko and Gere, 1963; Ziegler, 1968),
- flutter systems – loosing their stability due to growing amplitudes of oscillatory vibrations (non-conservative systems) (Beck, 1953; Bogacz and Janiszewski, 1986; Bolotin, 1963; Langthjem and Sugiyama, 2000),

- hybrid systems – loosing their stability by flutter or divergence due to certain geometrical or physical parameters (non-conservative systems) (Dzhanelidze, 1958; Sundararajan, 1973, 1976; Tomski and Przybylski, 1985; Tomski *et al.*, 1990, 2004),
- divergence-pseudo-flutter systems (Bogacz *et al.*, 1998; Tomski *et al.*, 1994, 1995, 1996, 1998, 1999, 2004) – loosing their stability due to buckling (conservative systems) for which the function in the plane: load P – natural frequency ω (Fig. 1) has the following course:
  - for $P \in< 0, P_c)$ (P_c – the critical load) the angle of the tangent to the characteristic curve can take a positive, zero or a negative value,
  - for $P \approx P_c$ the slope of the characteristic curve is negative,
  - change of the natural vibration form (from the first to the second and inversely) takes place along the characteristic curves, ($M1$, $M2$ denote the first and second form of vibrations, respectively).

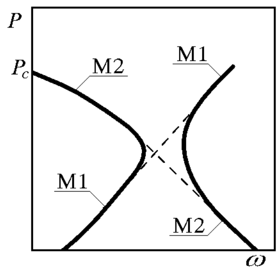

Fig. 1. The course of characteristic curves for a divergence-pseudo-flutter system (cf. Tomski *et al.*, 1995, 1996, 1998, 1999, 2004)

2. Formulation of the problem

Two systems (Fig. 2b,c) are considered in this paper:

- a column loaded by a stretching beam B, Fig. 2b (Tomski *et al.*, 1997, 2004),
- a column loaded by a force through a string C (the force directed towards the positive pole), Fig. 2c (Fieodosjew, 1969; Gajewski, 1970; Gajewski and Życzkowski, 1969).

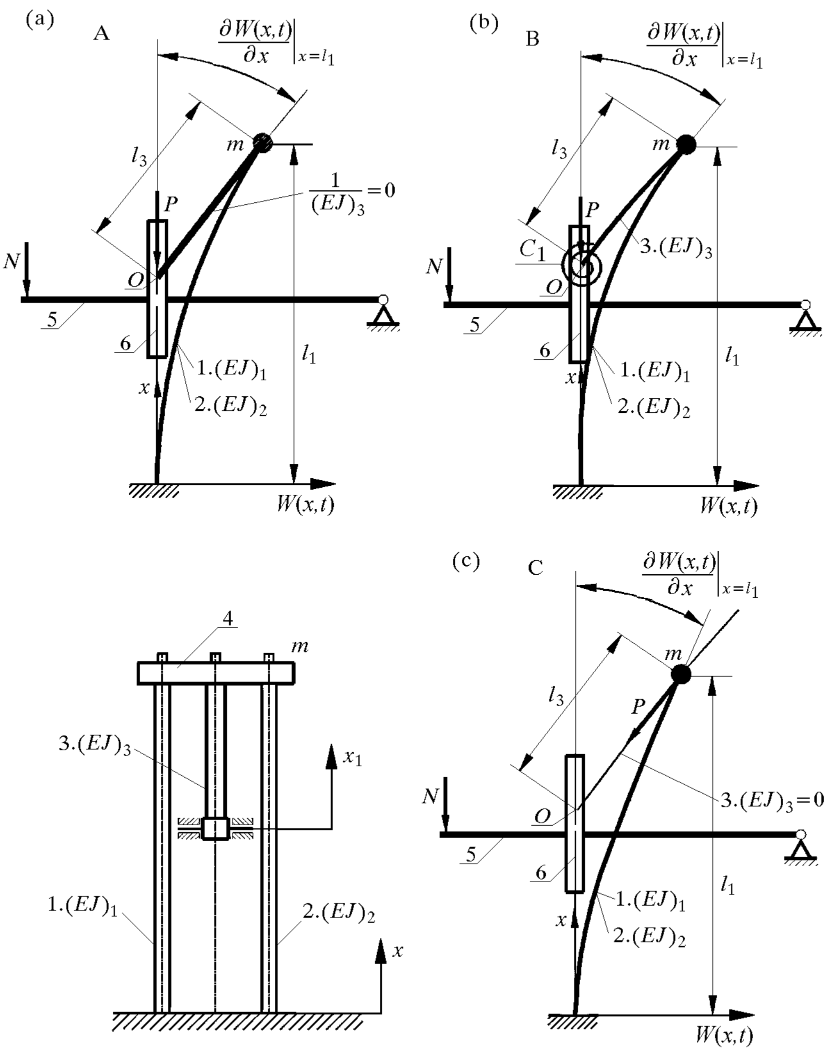

Fig. 2. Physical models of considered systems: (a) column loaded by a follower force applied to the positive pole A, (b) column loaded by a stretching beam B, (c) column loaded by a force directed towards the positive pole C

The method of mounting and loading the considered columns as well as the shape of axes of deflected rods are shown in Fig. 2. The systems are composed of two rods with the flexural rigidities $(EJ)_1$ and $(EJ)_2$, respectively, and the mass per unit length $(\rho_0 A)_1$ and $(\rho_0 A)_2$ (while: $(EJ)_1 = (EJ)_2$, $(\rho_0 A)_1 = (\rho_0 A)_2$, $(EJ)_1 + (EJ)_2 = EJ$, $(\rho_0 A)_1 + (\rho_0 A)_2 = \rho_0 A$). The column rods have the same cross-section and they are made of the same material.

For the column B, see Fig. 2b, the above mentioned elements are loaded by a compressive force P through infinitely rigid element (5) and a stretching beam with the flexural rigidity $(EJ)_3$ and mass per unit length $(\rho_0 A)_3$. The mounting elasticity of the stretched element is determined by the rigidity of a rotational spring C_1. Rods (1,2,3) are connected at the free end by cube (4) with the concentrated mass m. This is done in a rigid way, that is to say that the deflection angles and displacement of the free end are equal for every rod. Changeable length l_3 of the stretched element: bolt, beam or string (position of point O) is realized by mechanical system (6).

The coefficient of asymmetry of the flexural rigidity μ_1 is defined for the considered system

$$\mu_1 = \frac{(EJ)_3}{(EJ)_1 + (EJ)_2} \tag{2.1}$$

For $\mu_1 = 0$ (system C) it was assumed that element (3) is not characterized by flexural rigidity (string). In the case of $1/\mu_1 = 0$ (system A), the flexural rigidity of the considered element is multitudinously higher for the stretching rods of the column (rigid bolt). In this case, the model of the column (Fig. 2a) loaded by the follower force directed towards the positive pole is obtained (Tomski *et al.*, 1998, 2004). System A can be of a divergence or divergence-pseudo-flutter type.

The influence of the mentioned below parameters on the type of the system (divergence, divergence-pseudo-flutter) is analysed for columns with:

- asymmetry of the flexural rigidity between the stretched element and compressive rods of the column (system B)
- mounting elasticity of the stretched element:
  - rigid mounting $(1/C_1 = 0)$ – system B
  - hinged mounting $(C_1 = 0)$ – systems B, C
- length of the stretched element of the column l_3 – systems B, C, and with the constant value of the concentrated mass m at free end of the column.

3. Equations of motion and boundary conditions. Solution to the boundary value problem

The equations of motion for the considered structures have been determined from relationships (3.1), for column B – the three-rod system, and from relationship $(3.1)_1$ for column C) – the two-rod system

$$
\begin{aligned}
&(EJ)_i \frac{\partial^4 W_i(x,t)}{\partial x^4} + S_i \frac{\partial^2 W_i(x,t)}{\partial x^2} + (\rho_0 A)_i \frac{\partial^2 W_i(x,t)}{\partial t^2} = 0 \\
&(EJ)_3 \frac{\partial^4 W_3(x_1,t)}{\partial x_1^4} + S_3 \frac{\partial^2 W_3(x_1,t)}{\partial x_1^2} + (\rho_0 A)_3 \frac{\partial^2 W_3(x_1,t)}{\partial t^2} = 0
\end{aligned}
\tag{3.1}
$$

where i is the ith stretching rod of the system $(i = 1, 2)$ and

$$
S_1 = S_2 = \frac{P}{2} \qquad\qquad S_3 = -P \tag{3.2}
$$

The geometrical boundary conditions for the rigidly restrained point $(x = 0)$ are

$$
W_1(x,t)\Big|_{x=0} = W_2(x,t)\Big|_{x=0} = W_1'(x,t)\Big|_{x=0} = W_2'(x,t)\Big|_{x=0} = 0 \tag{3.3}
$$

The remaining conditions, necessary for solving the boundary value problem, are given in the form

— column B

$$
\begin{aligned}
&\frac{\partial W_1(x,t)}{\partial x}\Big|_{x=l_1} = \frac{\partial W_2(x,t)}{\partial x}\Big|_{x=l_1} = \frac{\partial W_3(x_1,t)}{\partial x_1}\Big|_{x_1=l_3} \\
&W_1(l_1,t) = W_2(l_1,t) = W_3(l_3,t) \qquad\qquad W_3(0,t) = 0 \\
&(EJ)_3 \frac{\partial^2 W_3(x_1,t)}{\partial x_1^2}\Big|_{x_1=0} = C_1 \frac{\partial W_3(x_1,t)}{\partial x_1}\Big|_{x_1=0} \\
&\sum_{i=1}^{2} (EJ)_i \frac{\partial^2 W_i(x,t)}{\partial x^2}\Big|_{x=l_1} + (EJ)_3 \frac{\partial^2 W_3(x_1,t)}{\partial x_1^2}\Big|_{x_1=l_3} = 0 \\
&\sum_{i=1}^{2} (EJ)_i \frac{\partial^3 W_i(x,t)}{\partial x^3}\Big|_{x=l_1} + (EJ)_3 \frac{\partial^3 W_3(x_1,t)}{\partial x_1^3}\Big|_{x_1=l_3} - m \frac{\partial^2 W(l_1,t)}{\partial t^2} = 0
\end{aligned}
\tag{3.4}
$$

— column C

$$
\begin{aligned}
&\frac{\partial W_1(x,t)}{\partial x}\Big|_{x=l_1} = \frac{\partial W_2(x,t)}{\partial x}\Big|_{x=l_1} \qquad\qquad W_1(l_1,t) = W_2(l_1,t) \\
&\sum_{i=1}^{2} (EJ)_i \frac{\partial^2 W_i(x,t)}{\partial x^2}\Big|_{x=l_1} = 0 \\
&\sum_{i=1}^{2} (EJ)_i \frac{\partial^3 W_i(x,t)}{\partial x^3}\Big|_{x=l_1} + P\Big(\frac{\partial W_1(x,t)}{\partial x}\Big|_{x=l_1} - \frac{W_1(l_1,t)}{l_3} \Big) - m \frac{\partial^2 W(l_1,t)}{\partial t^2}
\end{aligned}
\tag{3.5}
$$

The column undergoes small vibrations, therefore

$$W_i(x,t) = y_i(x)\cos(\omega t) \qquad\qquad W_3(x_1,t) = y_3(x_1)\cos(\omega t) \tag{3.6}$$

The solution to equations of motion (3.1), after previous separation of variables towards time and displacement (3.6), is

$$\begin{aligned} y_i(x) &= C_{1i}\cosh(\alpha_i x) + C_{2i}\sinh(\alpha_i x) + C_{3i}\cos(\beta_i x) + C_{4i}\sin(\beta_i x) \\ y_3(x_1) &= C_{13}\cosh(\beta_3 x_1) + C_{23}\sinh(\beta_3 x_1) + C_{33}\cos(\alpha_3 x_1) + C_{43}\sin(\alpha_3 x_1) \end{aligned} \tag{3.7}$$

where C_{nj} are integration constants $n = 1,2,3,4$, $j = 1,2,3$ and

$$\alpha_j^2 = -\frac{1}{2}k_j^2 + \sqrt{\frac{1}{4}k_j^2 + \Omega_j^2} \qquad\qquad \beta_j^2 = \frac{1}{2}k_j^2 + \sqrt{\frac{1}{4}k_j^2 + \Omega_j^2}$$

while

$$\Omega_j^2 = \frac{(\rho_0 A)_j \omega^2}{(EJ)_j} \qquad\qquad k_i^2 = \frac{S_i}{(EJ)_i} \qquad\qquad k_3^2 = \frac{P}{(EJ)_3}$$

Substitution of solutions (3.7) into the boundary conditions (3.3) and (3.4) for column B or (3.3) and (3.5) for column C (after previous separation of variables towards time and displacement) allows one to receive a system of twelve or eight homogeneous equations. The characteristic equation for the natural frequency of the considered column is obtained when the determinant of the characteristic system of equations equals zero.

4. Characteristic curves in the plane: load-natural frequency

A characteristic curve in the plane: load-natural frequency determines the type of a system. This is why the results of theoretical research are presented below.

Leipholz (1974) introduced a criterion for the loss of stability by divergence. He stated that for conservative columns (divergence systems) described by the boundary conditions

$$\begin{aligned} &y''(x)y(x)\Big|_{x=0}^{x=l_1} = 0 \\ &\Big[y'''(x) + \frac{P}{EJ}y'(x)\Big]y(x)\Big|_{x=0}^{x=l_1} = 0 \end{aligned} \tag{4.1}$$

the course of curves of eigenvalues Ω in relation to the external load λ has the negative slope in the whole range of the load (see curve (a) in Fig. 3)

$$\frac{d\Omega}{d\lambda} = \frac{-\int\limits_0^{l_1} [y'(x)]^2\, dx}{\int\limits_0^{l_1} [y(x)]^2\, dx} < 0 \tag{4.2}$$

where $y(x)$ is the lateral displacement of the column, $\Omega = \Omega_1^2$, $\lambda = P/(EJ)$.

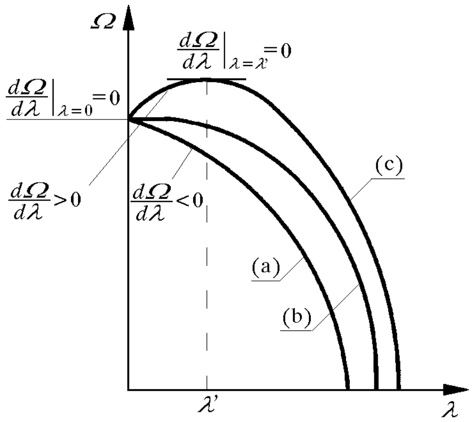

Fig. 3. The course of the basic characteristic curve of the parameter Ω

Leipholz's research, concerning characteristic curves for conservative and non-conservative loads, was generalised in works by Tomski *et al.* (1996, 1997, 1998, 2004). Adequate relationships, representing the eigenvalues in the plane Ω-λ, i.e. the parameter of the natural frequency Ω vs. the parameter of the external load λ, are:

— for a generalised load (conservative systems, cf. Tomski *et al.*, 1996, 2004)

$$\frac{d\Omega}{d\lambda} = \frac{-\int\limits_0^{l_1} [y'(x)]^2\, dx + \rho[y'(l_1)]^2 - \gamma[y(l_1)]^2 + 2\nu y(l_1)y'(l_1)}{\int\limits_0^{l_1} [y(x)]^2\, dx + \frac{m[y(l_1)]^2}{\rho_0 A}} \tag{4.3}$$

where μ, γ, ρ, ν are established coefficients of the generalised load (Tomski *et al.*, 2004),

— for a follower force directed towards the positive pole (A), see Fig. 2a, (conservative systems, cf. Tomski *et al.*, 1998, 200)

$$\frac{d\Omega}{d\lambda} = \frac{-\int\limits_0^{l_1} [y'(x)]^2\ dx + y(l_1)y'(l_1)}{\int\limits_0^{l_1} [y(x)]^2\ dx + \frac{m[y(l_1)]^2}{\rho_0 A}} \tag{4.4}$$

— for a column loaded by a stretched beam (B), see Fig. 2b (conservative system, cf. Tomski *et al.*, 1997)

$$\frac{d\Omega}{d\lambda} = \frac{-\int\limits_0^{l_1} [y_1'(x)]^2\ dx - \int\limits_0^{l_2} [y_2'(x)]^2\ dx + \int\limits_0^{l_3} [y_3'(x_1)]^2\ dx_1}{\int\limits_0^{l_1} [y_1(x)]^2\ dx + \int\limits_0^{l_2} [y_2(x)]^2\ dx + \frac{(\rho_0 A)_3}{(\rho_0 A)_1} \int\limits_0^{l_3} [y_3(x_1)]^2\ dx_1 + \frac{m[y_1(l_1)]^2}{(\rho_0 A)_1}} \tag{4.5}$$

— for non-potential systems (non-conservative system, cf. Tomski *et al.*, 2004)

(i) Beck's generalised column

$$\frac{d\Omega}{d\lambda} = \frac{-\int\limits_0^{l_1} [y'(x)]^2\ dx + \eta y(l_1)y'(l_1) - \lambda\eta\Big[y'(l_1)\frac{\partial y(l_1)}{\partial\lambda} - y(l_1)\frac{\partial y'(l_1)}{\partial\lambda}\Big]}{\int\limits_0^{l} [y(x)]^2\ dx + \frac{m[y(l_1)]^2}{\rho_0 A} + \lambda\eta\Big[y'(l_1)\frac{\partial y(l_1)}{\partial\Omega} - y(l_1)\frac{\partial y'(l_1)}{\partial\Omega}\Big]} \tag{4.6}$$

(ii) Reut's generalised column

$$\frac{d\Omega}{d\lambda} = \frac{-\int\limits_0^{l_1} [y'(x)]^2\ dx + \eta y(l_1)y'(l_1) + \lambda\eta\Big[y'(l_1)\frac{\partial y(l_1)}{\partial\lambda} - y(l_1)\frac{\partial y'(l_1)}{\partial\lambda}\Big]}{\int\limits_0^{l} [y(x)]^2\ dx + \frac{m[y(l_1)]^2}{\rho_0 A} - \lambda\eta\Big[y'(l_1)\frac{\partial y(l_1)}{\partial\Omega} - y(l_1)\frac{\partial y'(l_1)}{\partial\Omega}\Big]} \tag{4.7}$$

where η is a coefficient of the follower force (Beck's column) or a coefficient of the follower moment (Reut's column).

In the case of columns A, B, C and generalised loads (conservative systems), the slope of a curve of the natural frequency may be negative (divergence system – curve (a) in Fig. 3), positive (divergence-pseudo-flutter system – curve (c)) or equal to zero (curve (b)) depending on geometrical parameters of the loading and receiving heads.

In the case of the generalised load (cf. Tomski *et al.*, 1996), the parameter of natural frequency $(4.8)_2$ was determined on the basis of Rayleigh's quotient.

Relationship (4.3), conservative condition of load $(4.8)_1$ and boundary conditions at the restrained and free end of the column were used

$$\begin{gathered}\nu+\mu-1=0\\ \Omega=\frac{\int\limits_0^{l_1}[y''(x)]^2\,dx}{\int\limits_0^{l_1}[y(x)]^2\,dx+\frac{m[y(l_1)]^2}{\rho_0 A}}+\lambda\frac{d\Omega}{d\lambda}\end{gathered} \tag{4.8}$$

5. Results of numerical computations

Numerical computations were accomplished on the basis of the solution to the boundary value problem for the considered systems. The influence of length l_3 of the stretched element of the column and the asymmetry of flexural rigidity between the compressed rods and stretched element on the type of the system were determined. The range of μ_1, l_3^* parameters, for which the considered columns were of the divergence type (D) or divergence-pseudo-flutter type (PF), was specified taking into account an educed criterion (systems A, B, C – compare Tomski *et al.*, 1997, 1998, 2994) describing eigenvalue curves in the plane: load P - natural frequency ω, (4.3) and (4.5). The computations were carried out for two extreme cases of the mounting of the stretched element, i.e. hinged mounting $(c_1^*=0)$, see Fig. 4a, and rigid mounting $(1/c_1^*=0)$, see Fig. 4b, with a constant value of the concentrated mass m^* at the free end of the systems, where

$$l_3^*=\frac{l_3}{l_1} \qquad c_1^*=\frac{C_1 l_1}{EJ} \qquad m^*=\frac{m}{l_1\rho_0 A} \tag{5.1}$$

In the case of the hinged mounting (Fig. 4a), the influence of parameter l_3^* on the type of the system for a column loaded by a force directed towards the pole $(C-\mu_1=0)$ and a column loaded by a follower force directed towards the pole $(A-1/\mu_1=0)$ was additionally determined.

Numerical computations were carried out for the considered systems in order to determine the course of changes in natural frequencies in relation to the external load for chosen parameters μ_1, l_3^* (lines 1-4 in Fig. 4). The character of changes of the first two natural frequencies in a dimensionless form Ω_t^* $(t=1,2)$ and additional symmetrical natural frequencies Ω_2^{*s} (Tomski *et al.*,

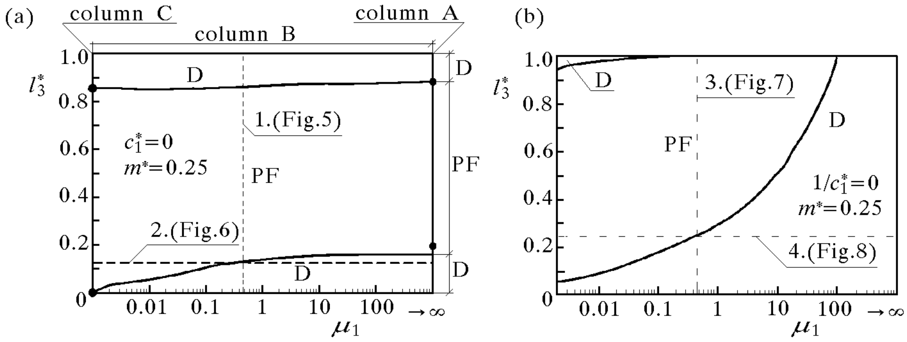

Fig. 4. The effect of μ_1, l_3^* parameters on the type of systems: divergence (D), divergence-pseudo-flutter (PF) for: (a) hinged mounting – columns A, B, C, (b) rigid mounting – column B

1997) in relation to the dimensionless loading parameter λ^* were specified (Fig. 5-Fig. 8). It was assumed that

$$\lambda^* = \lambda l_1^2 = \frac{Pl_1^2}{EJ} \qquad\qquad \Omega^* = \Omega l_1^4 = \frac{\rho_0 A\omega^2 l_1^4}{EJ} \tag{5.2}$$

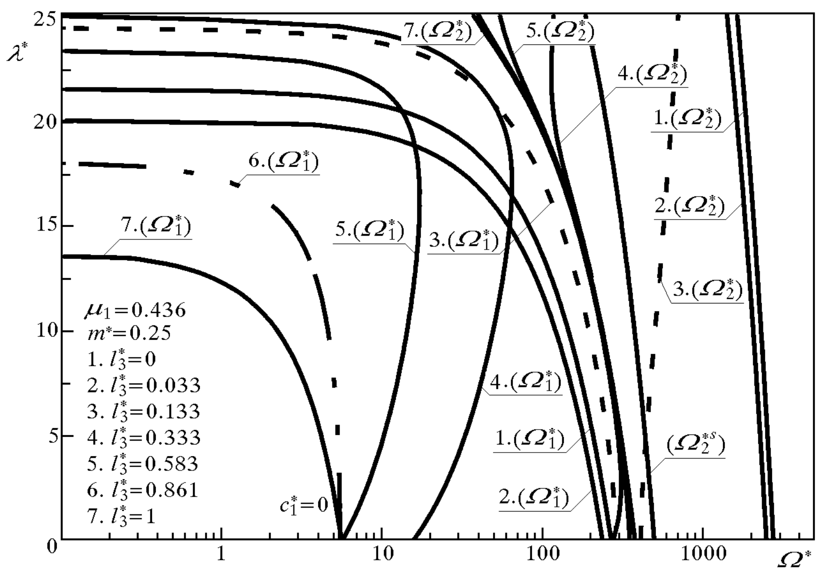

Fig. 5. Characteristic curves for Ω^* parameter in relation to l_3^* for $c_1^* = 0$

Two cases of mounting of the stretched element; i.e. hinged mounting (Fig. 5, Fig. 6) and rigid mounting (Fig. 7, Fig. 8) with the fixed value of the concentrated mass at the free end of the system were considered similarly as in Fig. 4.

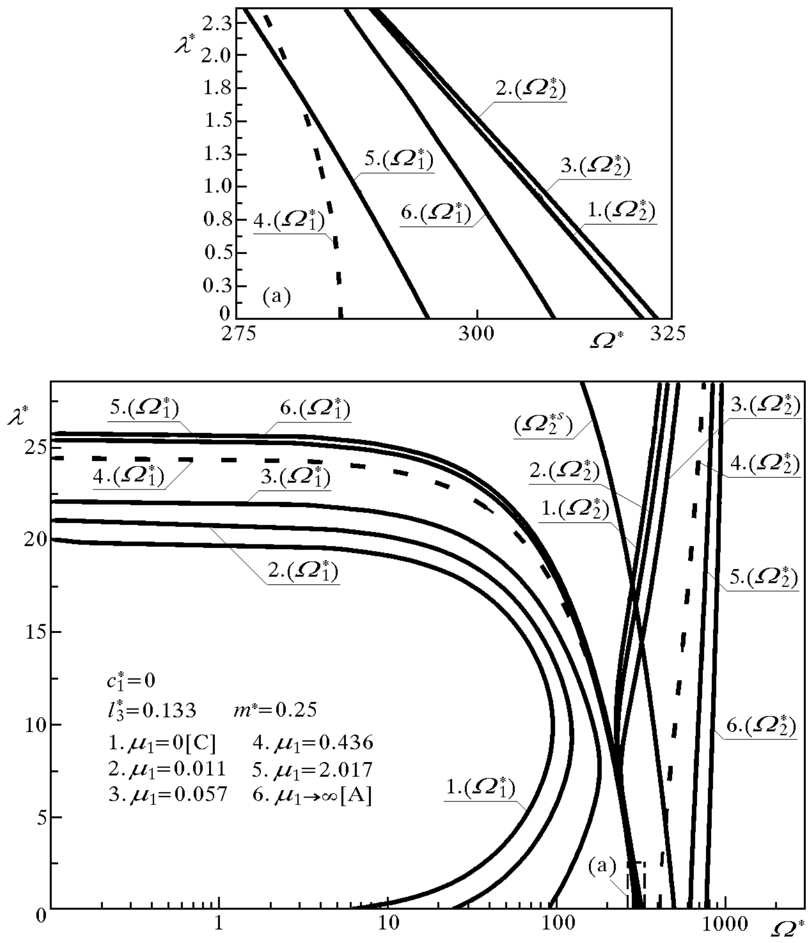

Fig. 6. Characteristic curves for Ω^* parameter in relation to μ_1 for $c_1^* = 0$

The slope of the basic natural frequency Ω^* for $\lambda^* = 0$ may be negative, positive or equal to zero. This is depicted in graphs concerning different parameters μ_1, l_3^* see lines 1-4 in Fig. 4).

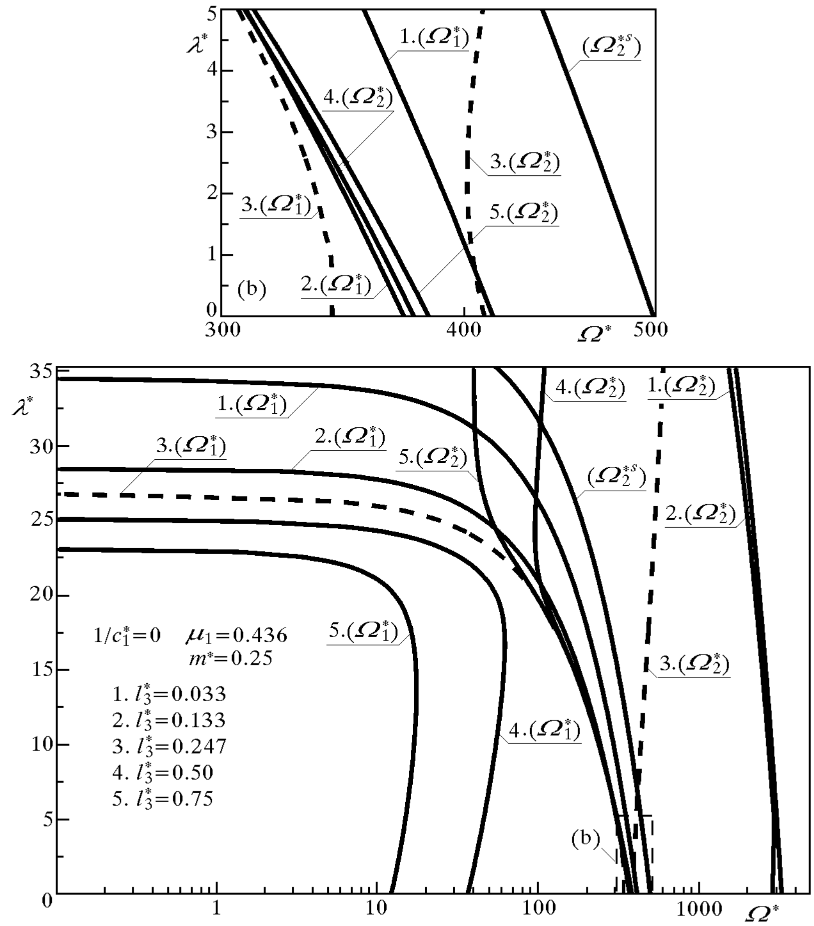

Fig. 7. Characteristic curves for Ω^* parameter in relation to l_3^* for $1/c_1^* = 0$

Changes in the slope of the considered natural frequency curve allows one to rank the considered systems among one of the two types: divergence (D) – $(\partial\Omega^*/\partial\lambda^*)|_{\lambda^*=0} < 0$ or divergence-pseudo-flutter (PF) – $(\partial\Omega^*/\partial\lambda^*)|_{\lambda^*=0} > 0$. The course of eigenvalues was distinguished by broken lines (Fig. 5-Fig. 8). The quality $(\partial\Omega^*/\partial\lambda^*)|_{\lambda^*=0} = 0$ is received for eigenvalues at the determined parameters m^*, μ_1, l_3^*, c_1^*. The course of the natural frequency Ω_2^{*s} corresponding to the symmetrical form of vibrations is identical for every presented graph due to the constant total rigidity EJ and length l_1 of the stretching rods of the system assumed in calculations. The value of the critical load is determined for $\Omega^* = 0$ for the presented curves of changes in the natural frequencies (Fig. 5-Fig. 8).

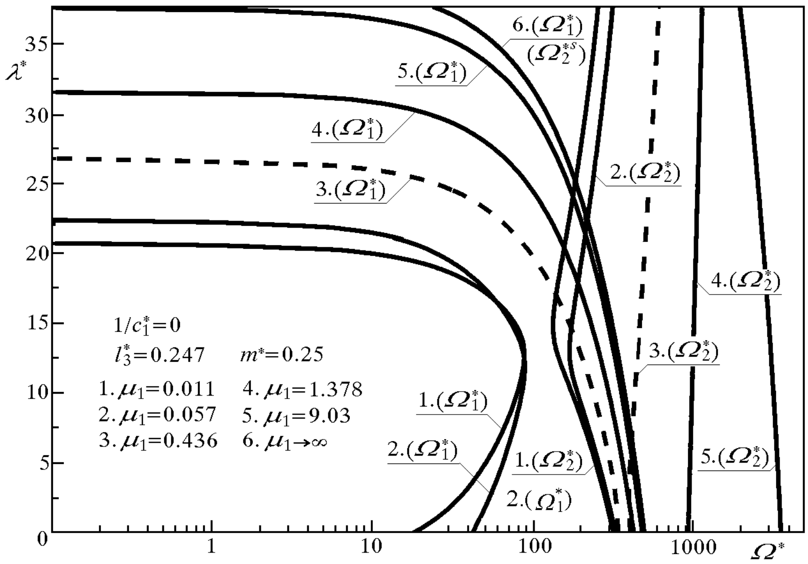

Fig. 8. Characteristic curves for Ω^* parameter in relation to μ_1 for $1/c_1^* = 0$

6. Results of experimental research

Numerical computations were carried out for the systems considered in the paper. The course of natural frequencies in relation to the external load for the column loaded by the stretching rod B and for the column loaded by the force directed towards the pole C, was verified on experimental set-ups (Tomski *et al.*, 1996, 1998, 2004). Physical and geometrical parameters are given in Tables 1 and 2.

The results of experimental research (points) and numerical computations (lines) are presented in Fig. 9 and Fig. 10 (column B) and in Fig. 11 (column C). Systems B1, B2, B3, B5, B6, C are characterised by a hinged mounting of the stretched element $(c_1^* = 0)$ for $x_1 = 0$. The rigid mounting was applied to the remaining cases.

Experimental investigation of the column B was limited to the first three basic natural frequencies (M1, M2, M3) and to two additional frequencies (M2^e, M3^e) corresponding to the symmetrical form of vibrations (Tomski *et al.*, 1997). In the case of column C, numerical computations and experimental investigation were carried out for the first two natural frequencies (M1, M2) for six chosen positions of the pole (length l_3). The results of numerical computations and experimental investigations are in good agreement.

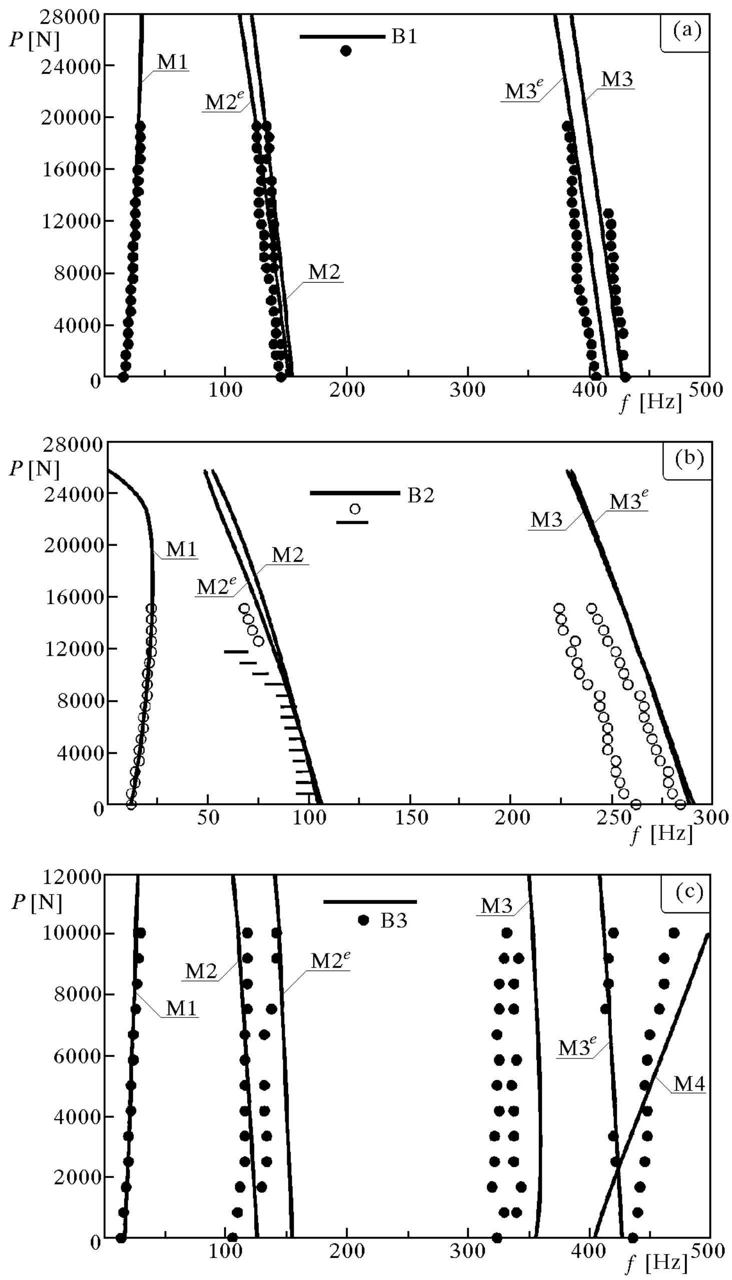

Fig. 9. The course of characteristic curves for column: B1 (a), B2 (b), B3 (c)

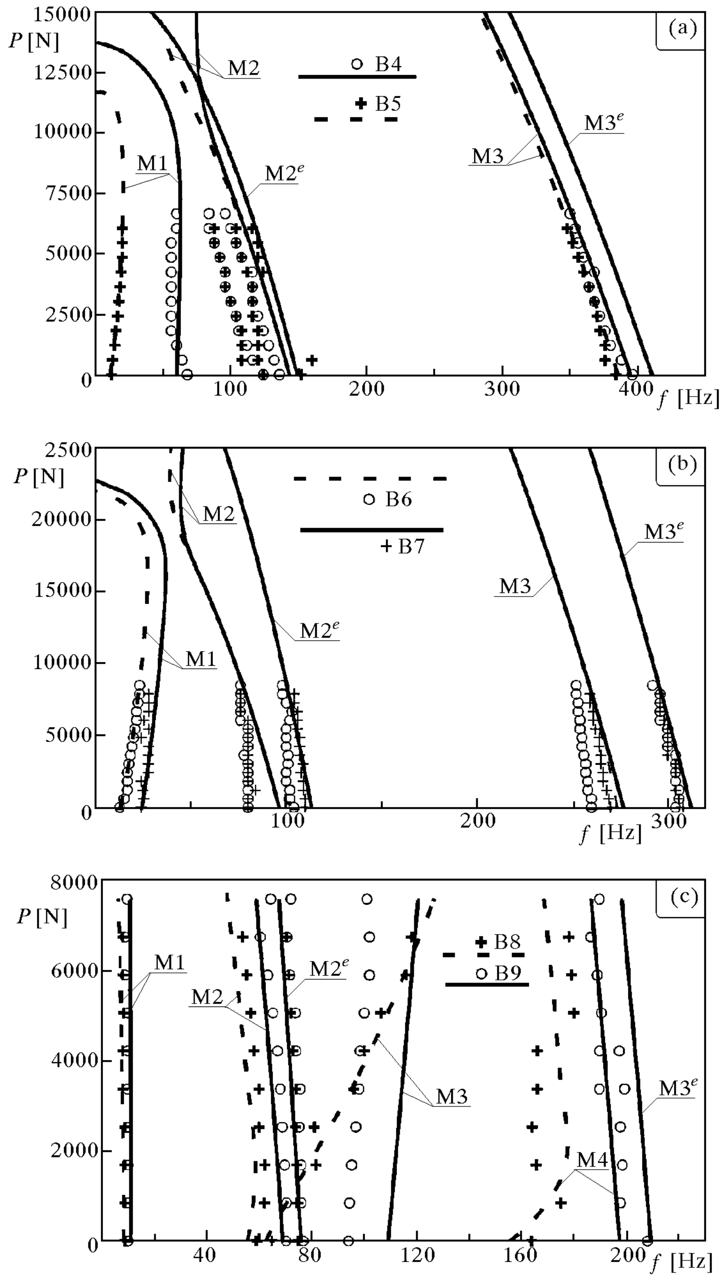

Fig. 10. The course of characteristic curves for column: B4 and B5 (a), B6 andB7 (b), B8 and B9 (c)

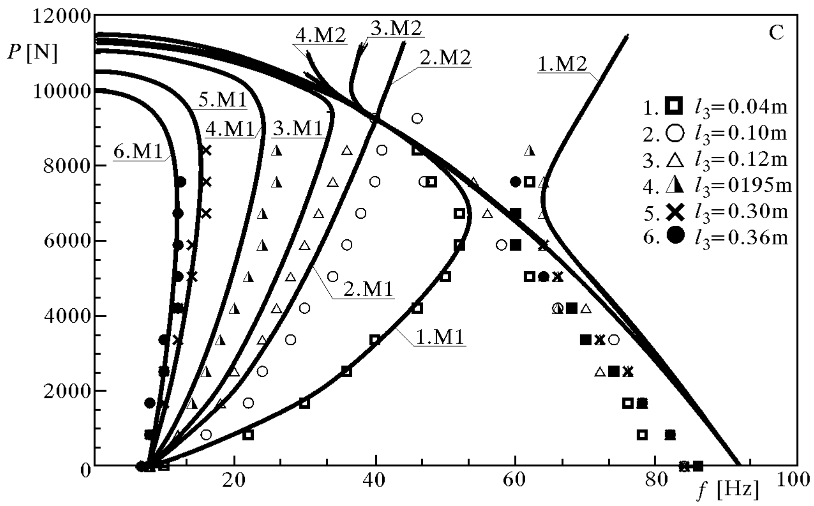

Fig. 11. The course of characteristic curves for column C

Table 1. Physical and geometrical parameters of the column loaded by the stretching rod B

Column	EJ [Nm^2]	$\rho_0 A$ [kg/m]	$(EJ)_3$ [Nm^2]	$(\rho_0 A)_3$ [kg/m]	l_1 [m]	l_3 [m]	m [kg]
B1	716.58	2.401	2132.6	2.918	0.63	0.31	0.39
B2	362.062	2.586	2132.6	2.918	0.63	0.31	0.35
B3	716.58	2.401	76.32	0.315	0.63	0.31	0.43
B4	152.68	0.631	589.04	0.877	0.61	0.305	0.34
B5	152.68	0.631	589.04	0.877	0.61	0.305	0.34
B6	362.062	2.586	76.32	0.315	0.61	0.305	0.35
B7	362.062	2.586	76.32	0.315	0.61	0.305	0.35
B8	716.58	2.401	38.81	0.219	0.9	0.9	0.58
B9	716.58	2.32	831.49	1.041	0.9	0.9	0.58

Table 2. Physical and geometrical parameters of the column loaded by the force directed towards the positive pole C

Column	EJ [Nm^2]	$\rho_0 A$ [kg/m]	$(EJ)_3$ [Nm^2]	$(\rho_0 A)_3$ [kg/m]	l_1 [m]	m [kg]
C	206.17	1.199	–	–	0.6	1.03

The above presented changes of natural frequencies in relation to the external load are typical for divergence-pseudo-flutter systems.

7. Summary

On the basis of experiments and carried out numerical simulations for the presented two-member column loaded by the stretching rod, it can be stated in this paper that:

- Different characteristic cases of column loading, i.e. from Euler's load, through loads by forces directed towards the positive pole to loads by a follower force directed towards the positive pole, can be obtained in relation to assumed values of parameters determining the elasticity of mounting of the stretching rod.
- The considered systems (a column loaded by the stretched element and a column loaded by the force directed towards the positive pole) appear to be ones of the two characteristic types, namely divergence or divergence-pseudo-flutter systems. This can be resolved on the basis of the course of the natural frequency in relation to the external load for the given geometrical and physical parameters.
- The obtained results of numerical computations and experimental investigations regarding the course of natural frequencies in relation to the external load showed good agreement.

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Drgania swobodne kolumn obciążonych poprzez rozciągany element

Streszczenie

W pracy rozważa się zagadnienie drgań swobodnych dwuprętowej kolumny obciążonej poprzez rozciągany element. Analizuje się wpływ asymetrii sztywności na zginanie pomiędzy elementem rozciąganym układu a prętami kolumny, wpływ sztywności zamocowania elementu obciążającego oraz jego długości na przebieg częstości drgań własnych w funkcji obciążenia zewnętrznego. Dla prezentowanych parametrów fizycznych i geometrycznych kolumny oraz układów dla granicznych wartości współczynnika asymetrii na zginanie wyznacza się obszary, w których omawiane układy są typu dywergencyjnego lub dywergencyjnego pseudoflaterowego. Obliczenia numeryczne poparte są odpowiednimi wynikami badań eksperymentalnych.

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