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GEARS AND GRAPHS

Józef Wojnarowski
Jerzy Kopeć
Stanisław Zawiślak

Faculty of Mechanical Engineering and Computer Science, University of Bielsko-Biala, Poland
e-mail: szawislak@ath.bielsko.pl

The paper presents, among others, a survey of works connected with the problem of the modeling of gears by means of versatile graph theory models. This approach to the problems of gear modeling allows computer based analysis and synthesis. Some recent papers claim that graph representations and derived methods belong to the branch of artificial intelligence due to the possibility of obtaining automatically versatile results, e.g. different constructional design solutions of mechanisms. Some examples are enclosed to explain which class of tasks is solved by means of the graph theory approach, i.e. modeling by means of bond graphs and linear graphs. The following problems have been considered: derivation of systems of equations describing the behaviour of gear subsystems and detection of a redundant wheel. The survey is based on over 60 papers published mainly within last 10 years, some of them in world-wide high level scientific magazines.

Key words: modeling, analysis, design automation, artificial intelligence

1. Introduction

The graph theory is a branch of mathematics which has versatile applications to many fields of engineering, which was summarized in well-known monographs (Deo, 1974; Wojnarowski, 1977a, Murota, 1987). Since the edition of these books, many new results have been achieved, especially in the field of the modeling of mechanisms.

The aim of this paper is to present a selected range of applications of graphs for the modeling of gears. Recently, this area of application of graphs has been extensively developed all over the world. The term "graphs", used in the title of the paper, covers a wide range of objects, i.e. graphs and hypergraphs considered by the graph theory (i.e. linear graphs, digraphs, weighted graphs), as well as bond graphs. The term "gears" is connected with different types of gears: car gear boxes, planetary gears, mixed geared trains as well as parts of machines enclosing geared pairs or geared mechanisms (e.g. geared robot wrists). In fact, as has been just mentioned, bond graphs have strong theoretical connections with graphs in the sense of a pure graph theory, but for their application purpose there is not any need to analyze them. Nevertheless, this direction of investigations is developed as well (Ort and Martens, 1974). The methods described underneath are based not only on graphs themselves, but also on different algebraic objects assigned to the graphs: matrices, structural numbers, vector spaces and systems of equations.

In general, the idea of application of graphs to the modeling of technical objects consists in: automation of analysis, generation of all possible constructional forms or solutions, synthesis of particular mechanical systems, finding particular parts inside technical systems (i.e. a redundant wheel), optimization of particular engineering problems, e.g. the shortest path, the maximum flow or transmission system containing a gear. This approach is recognized as a subarea of the artificial intelligence which should be added to its standard elements like pattern recognition, machine translation or expert systems. This idea arose from the proposal of Shai and Preiss (1999) and it is supported by the present paper. In the opinion of the authors of the present work, the direction of investigation concerning the usage of artificial intelligence methods is worth carrying on and propagating Shai's statement that methods based upon graphs are just such ones.

Another scientist who independently started works in this field is Schmidt *et al.* (2000) who called the methodology developed by her team as "graph grammars". It consists in algorithmization of engineering tasks in a special meta-language (quasi-language). A simple graph G (Bondy and Murty, 1977; Deo, 1974) is a pair (V, E), where V – a set of vertices, E – a set of edges and E is a subset of the cartesian product of the set V. A weighted graph is a triple (V, E, W), where W is a weight function having the domain E and a particular set of values, e.g. R – set of real numbers. The weight can be a real number, e.g. a distance in [km] when we model a road network, but it can be also a vector of two or more components. This will be illustrated in the following sections. A hypergraph H is a pair (V, HE) where HE is a set of hyperedges i.e. the family of subsets of V. A bond graph constitutes the idea of representing relationships between different physical parameters of the system in form of a pictogram or an icon coupled with adequate algebraic formulas and joining rules. The linear graphs and bond graphs are matched with other algebraic objects to create powerful models of engineering systems based on the rules of assignment "system-graph" and derived *intelligent* methodology concerning different problems listed underneath. The graphs can be considered as models of such different technical objects like e.g.: (a) vibrating systems (Wojnarowski, 1977a), (b) trusses (Shai and Preiss, 1999a,b), (c) mechanisms (Chen and Yao, 2000) (d) gears (Wojnarowski, 1977b; Choi and Bryant, 2002), (e) robot wrists (Lin, 1990), (f) networks of particular media (e.g. water, gas, railway, electric current, etc.) (Deo, 1974).

The problems considered by means of these modeling are as follows:

- analysis of vibrating systems (determination of eigen-requencies)
- finding the redundant gear wheel, velocity calculations, dynamical analysis
- enumeration of all possible constructional forms of a gear
- optimization of a particular problem (layout, ratios, weight of the gearbox)
- synthesis of a system with particular properties, e.g. degree of freedom, transmission function.

The question arises how it can be possible to model such different objects and consider such different problems by means of the same class of objects? It is especially characteristic for Polish scientists because interest in this field of knowledge is relatively narrow, just opposite to the situation in western countries, America and The Far East. The answer to such a formulated question consists in modeling process, i.e. what is assigned to a vertex, what is recognized as an edge or hyper-edge, what is a set of weights or how is a bond graph built as well as how to assign more advanced algebraic structures as matrices and matrix-based systems of equations or generalized networks. Moreover, the mystery is veiled in these matrix formulas and systems of quations which can be automatically created in dedicated computer programs where a graph (representing a mechanical system) is the initial data. These standard packages are widely used, e.g. MODELICA, 20-Sim, SIMULINK or others. In general, methods derived from the modeling of graphs are algorithmizable and programmable, which is an essential benefit arising from the effort of learning and applying these graphs.

As has been previously said, different mechanical systems can be modeled by means of graphs, but in this paper we focus our attention on some class of gears.

In the paper by Summers *et al.* (2001), the authors distinguished two purposes of modeling a system: (a) representing the structure of the system and (b) representing the behavior of the system. It would be best to have one model for these two purposes but, till now, not enough attempts have been made in this context. Both attitudes can be performed by means of graphs: the structure by means of simple graphs and behavior by means of bond graphs. A more detailed explanation of what is understood under these terms is presented afterwords in the present paper.

Besides the explanation of the essence of the modeling process, the goal of this paper is also to provide a wide review of recent papers on these topics published in well known scientific magazines and on a query throughout the internet and private communications obtained via snail-mail from authors or even via personal meetings during conferences. It gives an impression of the state-of-art of this field nowadays. Usually, authors discuss usage of linear graphs and bondgraphs separately, this paper tries to set both these approaches together. The paper encloses also some exemplary considerations as well as a list of approx. 60 references. The reader – who might be then eager to make a detailed study of the described papers – can easily find vast areas for further investigations, which is also the aim of this paper.

2. Graphs as models of gears

Methods of assignment of graphs to mechanical systems are different depending on a task which to be realized and a solution to be found. A comparison of these methods referring to gears was made by Prahasto (1992). The author had taken into account six methods proposed by Freudenstein, Ravisankar, Olson, Hsu, Wojnarowski and himself – basing on his supervisor's (i.e. Prof. Andrews') achievements.

In this paper, we present methods based on graphs (structure analysis) and based on bondgraphs (dynamic analysis). It is worth noting that both presented approaches were generalized to a synthesis task as well, but it is not the subject of the included examples.

In the paper by Summers *et al.* (2001), a definition of a model is formulated as follows:

> "Models are simplified abstract constructs used to predict behavior of a system and to get a quantitative understanding of their operation, improve their performance by variation of parameters,

and find possible critical points before the actual system is built. Models can be catalogued as functional or structural".

It is almost a perfect definition. Aiming at an analysis of a structure, models should represent a system preferably in various ways, which is achieved via graph-based models.

What does artificial intelligence of a graph-based system representation consist in? These models generalize structures of every system, they are assigned or represent the flow of energy, power, rotational speeds, etc. Hence, the analysis can be performed automatically. Moreover, it is possible to solve the inverse problem, i.e. create a family of objects satisfying particular requirements (by some authors the term atlas of solutions" is used).

The assignment of a graph to a mechanical system is a modeling procedure. According to the definition mentioned above, it is a simplified object, but its advantage is that some useful calculations can be made by means of a model which is adequate to the introductory mechanical system.

In the following considerations we focus our attention on the modeling of gears. An exemplary gear is presented schematically in Fig. 1a. In some considerations, we can abstract from the method of supporting a system, therefore, in Fig. 1b, the support is removed from the gear scheme.

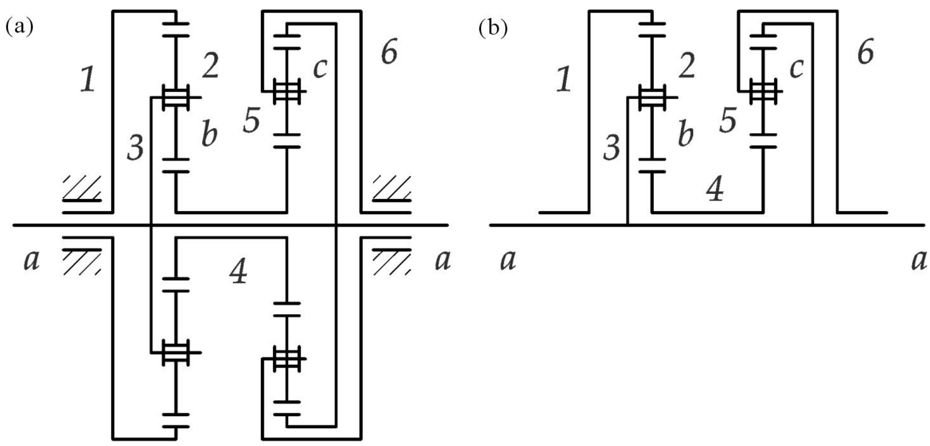

Fig. 1. Gear schemes; (a) general, (b) simplified

Some methods of graph assignment are presented in the following diagrams:

- corresponding to the bearing system (Fig. 2a):

  $G1$ – displacement, Fig. 2a (Hsu and Lin, 1994)

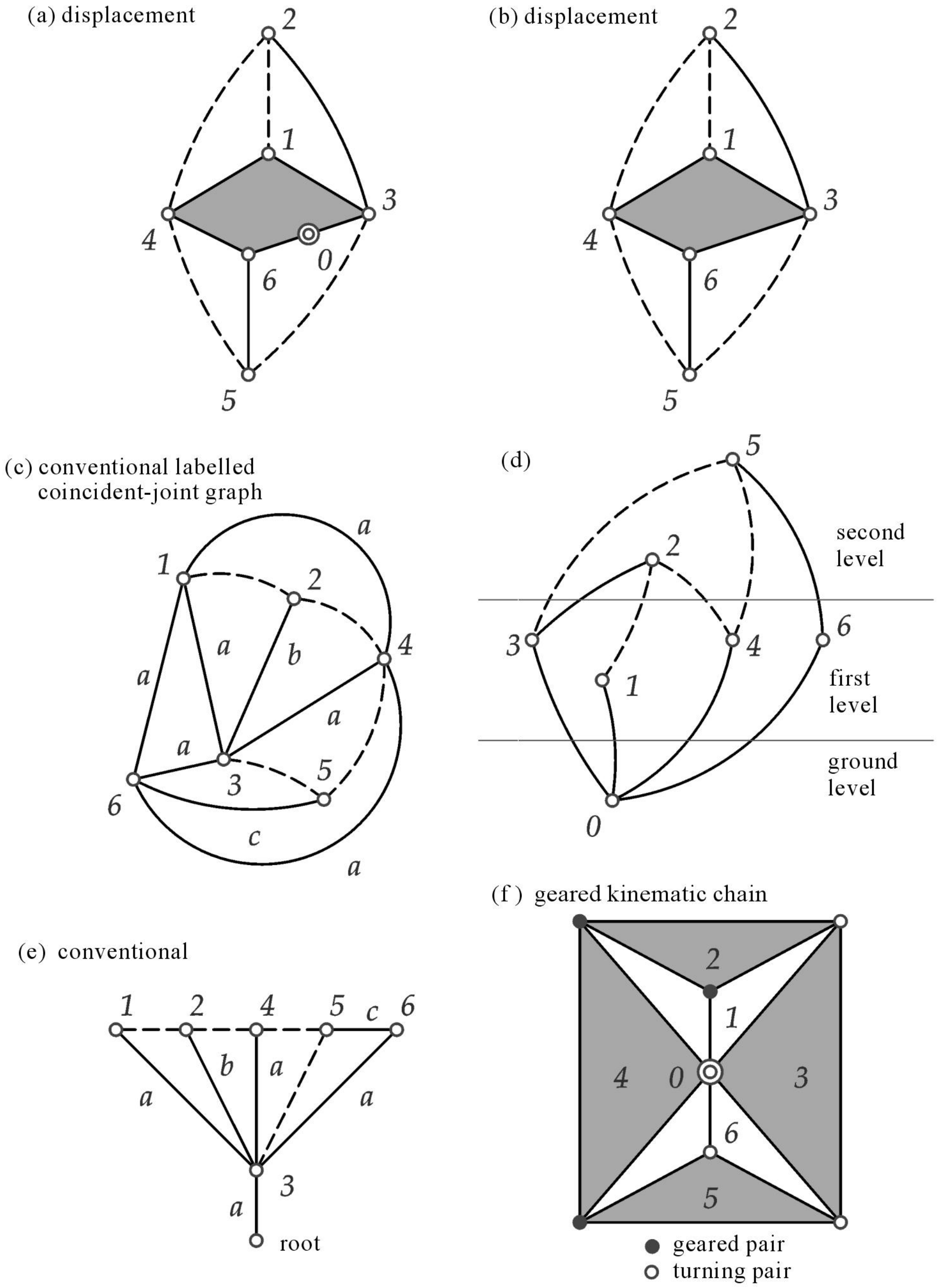

Fig. 2. Graphs assigned to a gear: (a) displacement, (b) displacement simplified, (c) conventional labelled, (d) canonical, (e) conventional, (f) geared kinematic chain

$G2$ – canonical, Fig. 2d (Chatterjee and Tsai, 1996)

$G3$ – conventional, Fig. 2e (Chen and Yao, 2000)

$G4$ – geared kinematic chain, Fig. 2f (Tsai, 2000)

- corresponding to the system without constraints (Fig. 2b):

  $G5$ – displacement, Fig. 2b (Hsu and Lin, 1994)

  $G6$ – conventional labeled or coincident-joint graph, Fig. 2c (Olson *et al.*, 1991).

The elements of the gear are represented by vertices of graphs $G1$-$G3$, $G5$, $G6$, except for case $G4$, where the elements are represented by polygons or edges. In the last mentioned case, the meshings (of engaged elements, geared pairs) are marked by black dots and turning pairs by single circles, respectively. The double concentric circles denote the common axis of several rotational elements fixed on common supports (bearings). The last rule is the same for all the graphs considered here. Relationships between gear elements are coded by means of graph edges. In the case of almost all the graphs, except for $G4$, the geared pairs (in mesh) are represented by dashed lines, and other relationships between the elements by continuous lines which are labeled or are not marked by weights. If they are labeled, the assigned descriptions mean the common axis, $a - a$, see Fig. 1a. Furthermore, an edge can represent the so-called turning pair in the case when one of the elements is mounted on another one. Then the notation of the adequate rotational axis is considered as its label (marked on the general scheme), i.e. b or c, respectively. In the case of graphs $G1$ and $G5$, the coaxial elements are drawn as vertices of a gray filled polygon. The idea of distinguishing the elements of a gear is realized in the case of graph $G2$ by means of the so-called levels: ground (support, bearings), first and second level. Graph $G5$ can be obtained from graph $G6$ by replacing the subgraph generated by vertices $\{1, 3, 4, 6\}$, i.e. clique K_4 and replacing it by a respectable polygon. Rules and derived methods of the analysis of gears are described in detail in the cited references.

In the present paper, a methodology based on graph $G5$ is considered. The methodology of gear synthesis depicted in Fig. 3 has been developed for different representations, but it is beyond the range of this paper and can be studied from the references listed at the end.

The above mentioned graphs are used to analyse the layout of gears, however dynamics can be analyzed by Prahasto's graphs (Prahasto, 1992), Wojnarowski's graphs (Wojnarowski, 1977b) and bondgraphs.

The usage of graphs and bondgraphs of selected gears will be presented in Section 4.

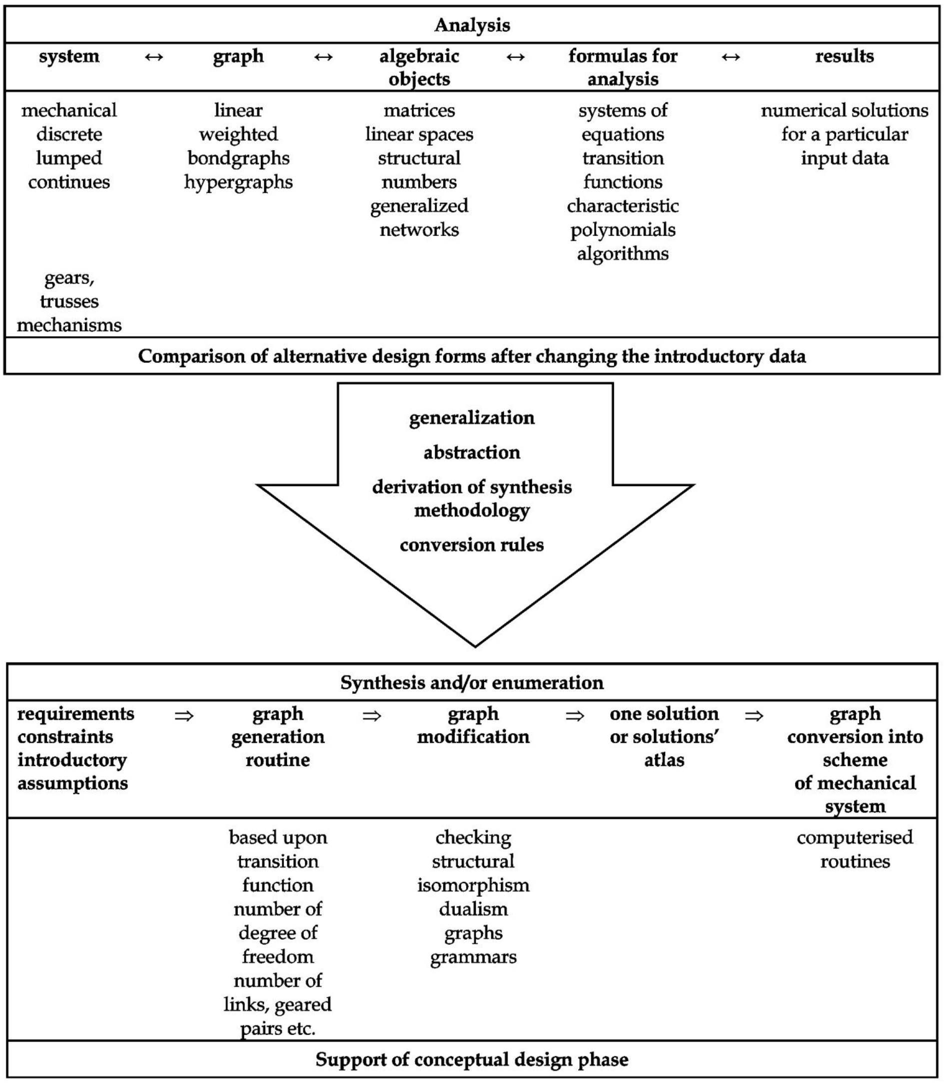

Fig. 3. Graph based methodology of the modeling of mechanical systems

3. Selected gear problems solved by means of graphs

Problems which are considered in references have been mentioned briefly in Section 1. In this paper, some descriptions of contents of the references will be given. One of the highly exploited topics is the **analysis** of gears and geared mechanisms. This problem was considered in papers described hereafter. In Freudenstein and Yang (1972), a kinematic and static analysis of a spur-gear is presented. The authors present the rules on how to assign a graph to a gear and next how to perform the analysis on an exemplary Glover's train. The graph creation rules are also described in Shai's papers (Shai, 1999a,b). Another approach is presented in Lei and Lu [25], where the authors omitted the phase of graph representation assigning a matrix to a gear. They derived a system of equations for kinematic and dynamic analysis of the gear. Topological analysis of gears can also be performed complying with the concept of kinematic fractionation (Liu and Chen, 2000) and kinematic units (Liu and Chen, 2001).

A different graph-based method of gear analysis was presented in the thesis by Prahasto (1992). The author used the vector-network method propagated Andrews (Rogers and Andrews, 1975). The description of the procedure for analyzing planetary gear trains using the vector-network method is presented also in Prahasto and Andrews (1999).

A special branch in the field of gear analysis is the enumeration of all possible design solutions. The solution to this problem allows one, in consequence, to formulate synthesis process rules. The paper dedicated to this approach, see Castillo (2002), focused on the enumeration of 1-dof planetary gear train graphs based on functional constraints.

Another problem is the **optimization** of gears or whole machine subsystems including a gear. In Vetadzokowvska (1999), the author presents an algorithm for synthesis of an optimal 6-stage-gear for assumed transmission ratios for each stage. Graphs obtained by the author and diagrams of gears are included. Another approach to the optimization of gears is presented in Michelena and Papalambros (1995, 1997) – hypergraphs are used for the decomposition of an optimization task of gear trains.

The next problem mentioned here is the **synthesis** of gears and geared mechanisms. Articulated gear mechanisms were synthesized in Chen and Liu (1999). The authors used hierarchical decomposition schemes. They created a database of so-called design primitives and obtained admissible mechanisms. The design primitives have their graph representations. Furthermore, all possible graph representations can be efficiently enumerated. Two 3-dof examples are presented for the sake of illustration. Another method of enumeration of epicyclic gear trains is described in Shina and Krishnamurty (1993) where the so-called standard code technique was used. An efficient methodology for structural synthesis of geared kinematic chains is presented in Hsu and Hsu (1997). The whole catalog of 1-dof geared kinematic chains (or rather their graph representations) is enclosed. Some fundamentals for this methodology were presented by one of the authors in Hsu (1994), where he described mathematical gear models and a special structural code. The idea of the fractionation of gears (Liu and Chen, 2000) was used for topological synthesis of fractionated geared differential mechanisms (Chen and Yao, 2000). The considerations are based upon the 2-dof automotive gear differential. Theoretical grounds for the synthesis of gears are presented in Prasad Raju Pathapati and Rao (2002), where a new technique based on loops for the investigation of displacement isomorphism in planetary gear trains is described. The methodology is based on a graph representation, loop concept and the Hamming number concept. The 6-link PGT was considered. A detailed study of the synthesis of a bevel-gear-type robotic wrist mechanism by means of graphs is presented in Lin (1990). The author created an atlas of wrist mechanisms for a particular task.

Gears are also analyzed by means of **bondgraphs** (Orlikowski, 2003; Choi and Bryant, 2002; Wojnarowski and Kopeć, 2003; Cichy, 2001). The last of the listed publications is a book (i.e. lecture notes), where methods of the transmission system modeling are described. Gears are just subsystems of such devices. In Choi and Bryant (2002), the bond graph method was coupled with a finite element model of shafts of a gear, which is an unique approach. Orlikowski (2003) analyzes a more complex transmission system including geared pairs. In general, it can be stated that it is not easy to find a publication concerning strict application of bond graphs to the gear modeling. In Wojnarowski and Kopeć (2003), the authors intensively developed this topic.

The considerations presented above can be summarized by the scheme presented in Fig. 3. After a vast exploration of problems via generalization and abstraction, gear synthesis methods have been developed. The advantage of the synthesis methodology is comprehensively and fully proved in Castillo (2002), Lin (1990), Olson *et al.* (1987).

4. Exemplary problems

In the present section, some selected methods are roughly described to give an impression what are the approaches to the gears modeling by means of graphs. The making use of these chosen methods is realised on different objects to underline the versatile possibilities and the powerfulness of the graph based methodology. As was mentioned previously, the essence of every method consists in the graph assigning rules. Three problems will be considered:

- calculations of a gear ratio (see Section 4.1)
- detection of redundant wheel (see Sectio 4.2)
- determination of eigen-vibration of gears by means of bond graphs (see Section 4.3).

4.1. Representation of kinematic gear structure by means of graphs

The goal of this section is to present a method of modeling planetary gears by means of graphs and analyzing the existence of redundant gear wheels.

Planetary gears have many advantages, e.g. they are compact, light and permit an essential reduction of rotational speed; therefore they are frequently used in vehicles, machining tools and heavy-duty machines. The graph theory is a useful tool for structural analysis and synthesis of kinematic chains containing gear wheels (Tsai, 2000).

A redundant gear wheel in a planetary gear is such a wheel which can be removed without changing kinematic characteristics of the gear. The recognition of redundant gear wheels is important for making proper schemes of power transmission in automatic gear boxes. It has also a fundamental meaning for the automatic synthesis of gears. The theory has a further application which is not the subject of this paper, i.e. it allows one to enumerate gears having similar structure schemes, the so-called atlases of all possible solutions. This permit one to choose the most suitable solution and can help a designer in his conceptual work on a particular project, see e.g. Olson *et al.* (1991).

In Hsu and Lin (1994), the method of assigning a graph to a gear has been proposed. The graph was called the displacement or kinematic graph of a gear. Detection of redundant gear wheels requires special matrix representation associated with this graph. Methods of assigning a kinematic graph to an exemplary gear, building a matrix (which is not used here) and searching for the redundant wheel are briefly presented below. Other graph representations of gears are also known.

A layout of shafts and wheels of a planetary gear is usually presented by means of a scheme like shown in Fig. 1. Due to the extent of the present work, only some elements of the graph-based methodology are enclosed. A full description can be found in the references listed at the end. The number of degrees of freedom can be calculated by means of Grubbler's formula

$$
\begin{aligned}
&F = 3(N-1) - 2P_o - P_z \\
&P_z = N - F - 1
\end{aligned}
\tag{4.1}
$$

where

F – number of the degree of freedom
N – number of elements
P_o – number of rotational pairs
P_z – number of meshed pairs.

Elements of the gear are labeled by numbers 0 – 6, furthermore positions of axes are marked by letters a, b and c. The kinematic chain, in which the support system is neglected, is presented in Fig. 2b. In our case, the values of parameters are as follows: $N = 7$, $P_z = 4$ and therefore $F = 2$. One-dof planetary gear trains are considered in Olson *et al.* (1991), and the atlases of graphs assigned to 1-, 2- and 3-dof gears are listed in Tsai (2000).

Based on Hsu and Lin (1994), the kinematic chain can be transformed into a displacement graph according to the rules roughly described and graphically illustrated in Fig. 2a. The displacement graph for a particular kinematic chain can be used for the determination of the equation of motion automatically (Freudenstein and Yang, 1972). The methodology is based upon the well known idea of fundamental cycles of a graph. The sketch of the routine can be formulated as follows.

A fundamental cycle (the so-called f-cycle), which corresponds to two engaged wheels i and j together with the coupled carrier k, consists of three vertices, one meshing edge $i - j$ and two turning edges or/and edges of a polygon $i-k$ and $j-k$. The vertex k, which is not incident to the meshing edge $i - j$, is a coupled transition vertex. For the sake of simplicity, the f-cycle is marked as $(i,j)k$. In this code, the first two numbers determine two meshing wheels (planet and geared wheel, respectively). The third one is the transmission/displacement vertex representing a carrier. The number of f-cycles is equal to the number of meshing edges in the kinematic chain. It can be seen that there are four f-cycles: $(5,3)6$, $(5,4)6$, $(2,1)3$ and $(2,4)3$. The basic equation connected with the f-cycle $(i,j)k$ describing relationships between angular velocities is as follows (Tsai, 2000)

$$\omega_i - \omega_j = \mp N_{ji}(\omega_j - \omega_k) \tag{4.2}$$

where $N_{ji} = Z_j/Z_i$ and Z_j, Z_i – numbers of teeth on the geared wheels i and j, the sign $(\mp)$ depends on the type of engagement; it is positive if a positive rotation of the gear wheel i relative to the carrier k causes a positive rotation of the gear wheel j, otherwise it is negative.

For the analyzed gear presented in Fig. 1b, the following system of equations can be written

$$\begin{aligned} \omega_5 - \omega_3 &= +N_{35}(\omega_3 - \omega_6) \\ \omega_5 - \omega_4 &= -N_{45}(\omega_4 - \omega_6) \\ \omega_2 - \omega_3 &= +N_{12}(\omega_1 - \omega_3) \\ \omega_2 - \omega_3 &= -N_{42}(\omega_4 - \omega_3) \end{aligned} \tag{4.3}$$

Assuming the following numbers of teeth on consecutive geared wheels: $Z_1 = 72$, $Z_2 = 16$, $Z_3 = 58$, $Z_4 = 44$, $Z_4' = 32$, $Z_5 = 18$, the transmission ratio R can be calculated. Additionally, we assume that element 6 is stationary (fixed, $\omega_6 = 0$), input element is 1 and output 3. Therefore, the transmission ratio R is equal to

$$R = \frac{\omega_1}{\omega_3} \tag{4.4}$$

Solving the system of equations (4.3), a solution can be found in the following form

$$R = 1 + \frac{N_{42}}{N_{12}}\Big(1 + \frac{N_{35}}{N_{45}}\Big) \tag{4.5}$$

The needed values are as follows: $N_{42} = 2.75$, $N_{12} = 4.5$, $N_{35} = 3.22$, $N_{45} = 1.77$. Finally, for the assumed data, the transmission ratio is equal to $R \approx 2.72$.

The algorithm can be performed in the following steps: assignment of the graph to the system, automatic derivation of the kinematic matrix, automatic generation of f-cycles, creation of equations (4.3), introduction of assumptions for the gear (e.g. the selected element is fixed). Further calculations can also be performed taking into account angular velocities of the carriers, etc.

4.2. Detection of the redundant wheel

In Hsu and Lin (1994) an algorithm was derived, which enabled detection of the redundant gear wheel (in the case of assumed input and output elements). The algorithm consists in the analysis of base cycles in the displacement kinematic) graph. A sketch of the method is presented below in consecutive steps:

(S1) creation of the displacement graph for the analyzed gear

(S2) derivation of all f-cycles

(S3) assumption of the input and output elements

(S4) creation of sets of f-cycles

(S5) for every set consisting of 2 up to $P_z - 1$ cycles, derivation of the associated set of vertices

(S6) checking the criterion for the existence of redundant wheels: $m = F + K$ where: m – number of unknowns, K – number of necessary equations needed for the determination of rotational speeds in the base cycles, F – number of degrees of freedom (dof),

(S7) listing the sets of vertices for which the criterion is satisfied and checking which wheel is redundant in all the sets.

Table 1. Analysis of existence of a redundant wheel

K	f-cycles	Vertex set	Condition $m = F + K$
2	(5,3)6; (5,4)6	{3,4,5,6}	=
	(5,3)6; (2,1)3	{1,2,3,5,6}	>
	(5,3)6; (2,4)6	{2,3,4,5,6}	>
	(5,4)6; (2,1)3	{1,2,3,4,5,6}	>
	(5,4)6; (2,4)3	{2,3,4,5,6}	>
	(2,1)3; (2,4)3	{1,2,3,4}	=

The algorithm can be applied to the considered exemplary gear train (Fig. 1b). The results of analysis for $K = 2$ are depicted in Table l. There are four f-cycles listed above, because there are $P_z = 4$ geared pairs. Therefore, two and three element subsets are considered in the routine. One can extract the following stages of analysis:

1. theoretical assumption of possible input and output elements in the considered gear e.g. $\{1, 3, 4\}$
2. creaating of $F + 1$ (F =dof) combinations of the mentioned elements, $F = 2$ therefore $F + 1 = 3$, and the set is $\{1, 3, 4\}$
3. making a 2-element set of cycles as listed in Table 1
4. checking condition (S6) and drawing conclusions.

In our case: $F = 2$, $K = 2$, and m is equal to the cardinality of the vertex set. For the first row in Table 1 we have $m = 4$, therefore the condition is fulfilled. Analyzing possible inputs or outputs $\{1, 3, 4\}$ for the set $\{1, 2, 3, 4\}$ (i.e. input 1, output 4), elements $(5), 6$ are redundant. Analyzing possible inputs or outputs $\{3, 4, 6\}$ for the set $\{3, 4, 5, 6\}$, it follows that the elements $(2), 1$ are redundant. The planetary gear wheel is highlighted by parentheses.

The conclusion is that the gear has a redundant gear wheel depending on the choice of the input, output and ground. The above mentioned considerations indicate capabilities of the graph-based methodology for gear analysis. The most fruitful is the enumeration of all possible solutions – the so-called atlases of designs, which can be read in Tsai (2000) and will be the subject of further investigations of the authors of this paper.

4.3. Analysis of gear dynamics with bondgraphs

Achievement of design reliability and elimination of failures is easier since a variety of commercial CAD and structural analysis tools became available. However, on the preliminary stage of the design process (the conceptual one), a flexible, compact and ultimate method is needed. One of the valuable tools is the bondgraph method.

Bondgraps describe the power flow in discrete or lump systems, and are cold power flow graphs. The bondgraphs were invented by Paynter, and developed by Karnopp *et al.* (2000) and Thoma (1990).

In this paper, torsional vibration is considered. As a unified pair of variables, the angular velocity (flow variable) and torsional moment (effort variable) are selected. The product of these variables gives the power

$$P = ef = M\omega \tag{4.6}$$

Physical properties of the modeled part, namely inertias and compliances as well as resistances, are regarded in the model. These properties are associated with standard bondgraph elements using common procedures (Karnopp *et al.*, 2000; Thoma, 1990): inertias are described with **I** elements, compliances with **C** elements, energy dissipation with **R** elements, dynamic and kinematic excitations with **Se** and **Sf** elements, respectively. Transformers **TF** describe the corresponding gear ratios. Table 2 contains collected set of standard bondgraph elements.

The lines with half-arrows represent the power flow in the system. The goal is to write down equations from the graph (manually or automatically) and skip differentiation. Comparing the linear graph theory, arcs are cold bonds and vertex elements, respectively. One can obtain the equation of motion automatically by using a computer software. Figure 4shows the way from the existing element or its drawing (Fig. 4a) to the dynamical model in the form of a bondgraph, the simplest model (see Fig. 4b) with no compliance, then extended dynamical models with the compliance of the connecting shaft and friction losses regarded (Fig. 4c), and finally with the wheel compliance (Fig. 4d).

Table 2. Basic elements of bondgraphs

Element	Symbol	Equation
Effort source	**Se**	$e =$ is known
Flow source	**Sf**	$f =$ is known
Inertial element	**I** : m	$f = \frac{1}{m}\int e\ dt + f_0$
Compliance	**C** : c	$e = \frac{1}{c}\int f\ dt + e_0$
Resistance	**R** : r	$e = rf$
1-Nod	**1**	$f =$ idem, $\sum e_i = 0$
0-Nod	**0**	$e =$ idem, $\sum f_i = 0$
Transformer	**TF** : n	$e_1 = ne_2,\ f_2 = nf_1$
Gyrator	**GY** : r	$e_1 = rf_2,\ e_2 = rf_1$

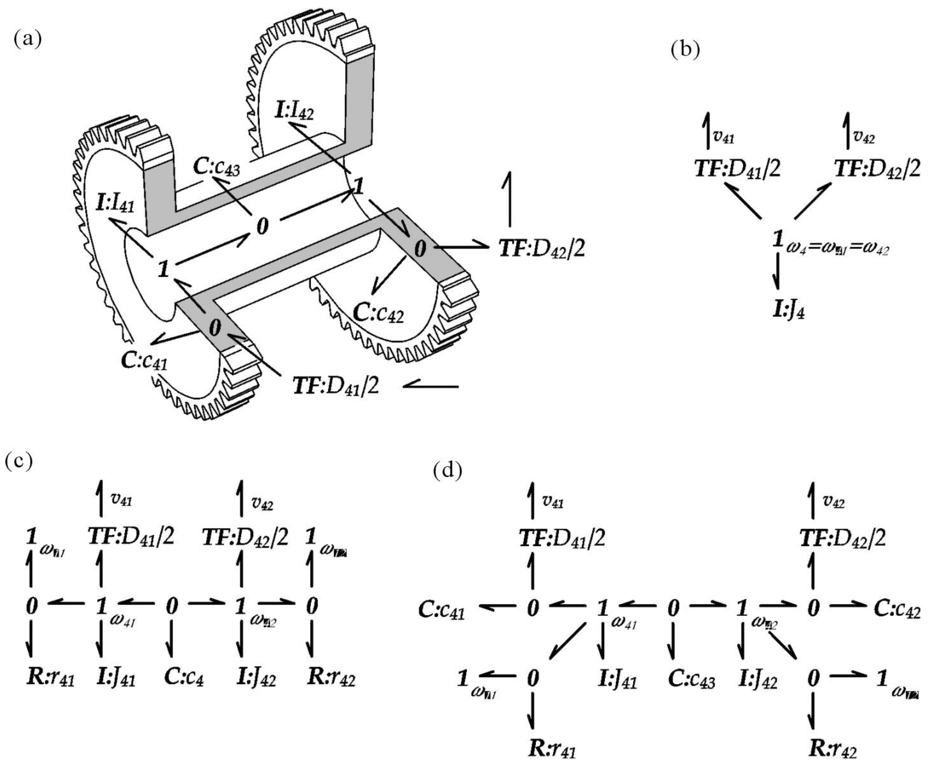

Fig. 4. Development of a bondgraph model of a gear part: from a "rough" graph to an enhanced model (causality analysis marks are not added to the graphs)

The next stage is derivation of equations from the graph. The causality analysis procedure (see Karnopp *et al.*, 2000) is required. It creates a proper order for equations and permits automatic generation of solutions. Short perpendicular lines and numbers under the bonds denote the causality assignment order and equation form (details in Karnopp *et al.*, 2000). The set of equations has a different form depending on the causality analysis. Figures 5a,b,c show possible causality forms and Fig. 5d shows a case when a proper denoting graph (causality assignment) is impossible.

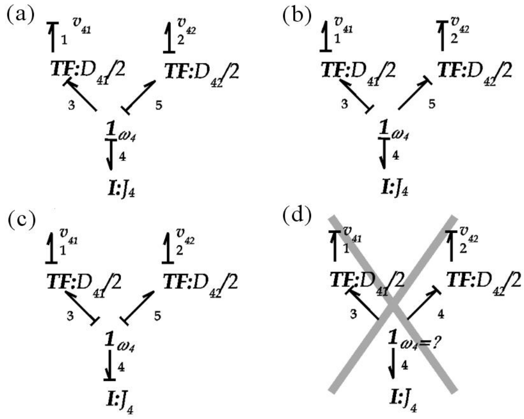

Fig. 5. Bondgraphs for a simple model with different causalities

The full set of equations has the form

$$\begin{cases} f_1 = \text{is known} & f_5 = f_3 \\ e_2 = \text{is known} & e_3 = -e_4 - e_5 \\ f_3 = \dfrac{1}{n_1} f_1 & e_4 = J_4 \dfrac{df_4}{dt} \\ e_1 = \dfrac{1}{n_1} e_3 & f_2 = n_2 f_5 \\ f_4 = f_3 & e_5 = n_2 e_2 \end{cases} \tag{4.7}$$

hence

$$\begin{cases} f_1 = \text{is known} \\ e_2 = \text{is known} \\ e_4 = J_4 \dfrac{1}{n_1} \dfrac{df_1}{dt} \end{cases} \tag{4.8}$$

The arranged sets of equations for bondgraphs from Fig. 5b and 5c have the form

$$\begin{cases} e_1 = \text{is known} \\ f_2 = \text{is known} \\ e_4 = J_4 \dfrac{1}{n_2} \dfrac{df_2}{dt} \end{cases} \qquad \begin{cases} e_1 = \text{is known} \\ e_2 = \text{is known} \\ \dfrac{df_4}{dt} = J_4(-n_1 e_1 - n_2 e_2) \end{cases} \tag{4.9}$$

It is impossible to express a consistent system of equations for the graph from Fig. 5d. This system of equations is contradictory

$$\begin{cases} f_1 = \text{is known} \\ f_2 = \text{is known} \\ e_4 = J_4 \dfrac{1}{n_1} \dfrac{df_1}{dt} \qquad \text{and} \qquad e_4 = J_4 \dfrac{1}{n_2} \dfrac{df_2}{dt} \\ f_4 = f_3 = \dfrac{1}{n_1} f_1 \qquad \text{and} \qquad f_4 = f_5 = \dfrac{1}{n_2} f_2 \end{cases} \tag{4.10}$$

Figure 6 shows bondgraphs of compliant models after performing the causality analysis.

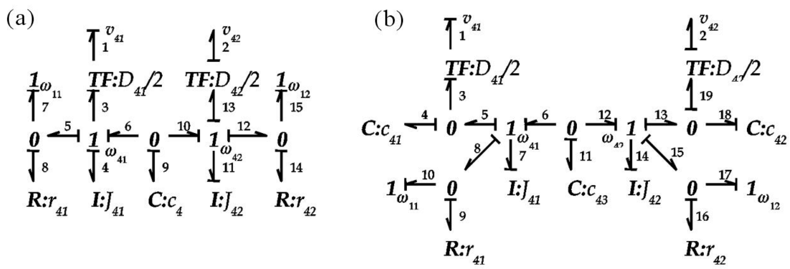

Fig. 6. A bondgraph model for compliant models with causality analysis marks

The equations of dynamics for the model shown in Fig. 4b and Fig. 6a have the form

$$\begin{cases} f_1 = \text{is known} \\ e_2 = \text{is known} \\ e_4 = J_{41} \dfrac{1}{n_1} \dfrac{df_1}{dt} \\ e_9 = \dfrac{1}{c_4} \displaystyle\int \Big(-\dfrac{1}{n_1} f_1 - f_{11}\Big)\, dt \\ \dfrac{df_{11}}{dt} = \dfrac{1}{J_{42}} [e_9 - r_{42}(f_{11} - f_{15}) - n_2 e_2] \end{cases} \tag{4.11}$$

The subscripts correspond to the number of bonds. The equations of dynamics of the model shown in Fig. 4d and Fig. 6b have the form

$$
\begin{cases}
f_1 = \text{is known} \\
e_2 = \text{is known} \\
e_4 = \dfrac{1}{c_{41}} \displaystyle\int \Big(-\dfrac{1}{n_1} f_1 + f_7 \Big)\, dt \\
\dfrac{df_7}{dt} = \dfrac{1}{J_{41}} [-e_4 + e_{11} - r_{41}(f_7 - f_{10})] \\
f_{10} = \text{is known} \\
e_{11} = \dfrac{1}{c_{43}} \displaystyle\int (-f_7 - f_{14})\, dt \\
\dfrac{df_{14}}{dt} = \dfrac{1}{J_{42}} [e_{11} - n_2 e_2 - r_{42}(f_{14} - f_{17})] \\
f_{17} = \text{is known} \\
f_{18} = c_{42} n_2 \dfrac{de_2}{dt}
\end{cases}
\tag{4.12}
$$

5. Final remarks and conclusions

Graphs can be recognized as versatile, concise and powerful models of gears. The assignment of a graph to a particular gear is only the first step of the procedure. The assignment of further algebraic objects is an immanent phase of all described methods. Depending on the procedure of modeling, different tasks can be performed. One can mention here: analysis, synthesis, finding a particular part, optimization or generation of all possible design solutions. Only some of them is roughly described in this work. However, a wide range of publications on this topic is briefly presented. In all cases, the graph-based methodology allows automatization of a particular task and its realization by means of a computer routine. Graph-based methods can be coupled with FEM and evolutionary algorithms, which means – together with all previously mentioned features – that the graph-based methodology can be recognized as an artificial intelligence method. These methods are intensively developed just recently, and enjoy great interest confirmed by numerous investigations which are carried on.

During a design and analysis process, an engineer requires a tool for validation of dynamic properties of a created machine. A bondgraph model is easy to build and modify, regarding or neglecting selected dynamical properties in the model, drawing or erasing adjacent bonds and elements. Some authors (Shai and Preiss, 1999a) claimed that the graph method can be recognized as a new branch of strong artificial intelligence (Kasperski, 2003). The paper supports this point of view due to the fact that the graph-based methods consist in the algorithmization of particular tasks.

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Grafy i przekładnie

Streszczenie

W pracy przedstawiono, między innymi, przegląd prac związanych z zadaniem modelowania przekładni zębatych za pomocą uniwersalnych modeli wykorzystujących teorię grafów. To podejście do zadań modelowania przekładni zębatych umożliwia przeprowadzenie zadania analizy i syntezy za pomocą komputerów. W niektórych najnowszych pracach pojawia się stwierdzenie, że reprezentacja za pomocą grafów i metod pochodnych są działem sztucznej inteligencji, gdyż umożliwiają automatyczne uzyskiwanie pożądanych wyników, np. różnych rozwiązań konstrukcyjnych mechanizmów. Załączono kilka przykładów dla wyjaśnienia, jakie klasy zadań można rozwiązywać za pomocą metod wywodzących się z teorii grafów; tj. modelowanie za pomocą grafów przepływu mocy i grafów liniowych. Omówiono następujące zagadnienia: automatyczne tworzenie układów równań opisujących dynamikę podzespołów przekładni i wykrywanie kół nadmiarowych. Przegląd został oparty o prawie 60 prac, opublikowanych głównie w okresie ostatnich 10. lat; część z nich pochodzi z uznanych czasopism specjalistycznych o ogólnoświatowym zasięgu.

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