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METAL TESTS IN CONDITIONS OF CONTROLLED STRAIN ENERGY DENSITY

Stanisław Mroziński
Dariusz Boroński

University of Technology and Life Sciences, Faculty of Mechanical Engineering, Bydgoszcz, Poland
e-mail: stmpkm@utp.edu.pl; daborpkm@utp.edu.pl

A new concept of the determination of fatigue properties was presented in the paper. Experimental procedures based on strain-life fatigue tests were developed. The total strain energy density parameter calculated during a loading cycle was applied as the control signal for fatigue testing of the specimens. The developed procedure extends the range of fatigue testing methods in the low-cycle fatigue, which should bring a new quality in the case of its use in fatigue life calculations.

Key words: low-cycle properties, fatigue life, plastic strain energy density, elastic strain energy density

Notations

b, c – elastic and plastic exponent, respectively
b_w, c_w – elastic and plasti strain energy exponent, respectively
E – modulus of elasticity, MPa
K_p, K_e – fatigue coefficient in plastic and elastic strain energy criterion, respectively, $\mathrm{MJ/m^3}$
W_t, W_p, W_e – total, plastic and elastic strain energy density, respectively, $\mathrm{MJ/m^3}$
W_e^+, W_e^- – elastic tensile and compression strain energy density, respectively, $\mathrm{MJ/m^3}$
$2N_f$ – reversals to failure
ε_{ac} – amplitude of total strain
$\varDelta\varepsilon_{ac}, \varDelta\varepsilon_{ae}, \varDelta\varepsilon_{ap}$ – total, elastic and plastic strain range, respectively
ε'_f – fatigue ductility coefficient
σ'_f – fatigue strength coefficient, MPa

σ_a, δ_σ – stress amplitude and stress increment, respectively, MPa
δ_ε – strain increment

1. Introduction

With the acceptance of new quantities for fatigue life calculation such as total strain, plastic strain or plastic strain energy density W_p, there appeared the necessity of determination of new fatigue characteristics.

In the case of fatigue damage assessment using a local strain-life approach, the basic characteristic applied in fatigue life calculations is the strain range vs. reversal of loading to the failure curve approximated with the Manson-Coffin relation (Coffin, 1954; Feltner and Morrow, 1961)

$$\frac{\Delta\varepsilon_{ac}}{2} = \frac{\Delta\varepsilon_{ae}}{2} + \frac{\Delta\varepsilon_{ap}}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \tag{1.1}$$

The data needed for determination of the characteristic described by equation (1.1) (total strain or its components) are directly measured during fatigue tests.

Determination of the fatigue characteristic in the energy-life approach demands determination the energy parameter and acceptance of a description model of the fatigue process in the energy approach. Basing on the literature data, it can be stated that presently there are two basic approaches to description of the fatigue process in the energy approach.

In the first case, the total energy cumulated in the fatigue process is taken into account. This energy is comparable with the energy dissipated in static tension tests. Suggestions of such an approach can be found for example in the papers by Feltner and Morrow (1961), Lin (1993), Lin and Haicheng (1998).

In the second approach, the cumulation of dissipated energy in particular cycles of variable loading is taken into account. Three groups of suggestions can be observed in this approach. These take into account:

a) plastic strain energy density W_p solely;

   Suggestions of this kind were presented in the papers by Gołoś (1988), Kaleta (1998), Mroziński and Topoliński (1999). Basing on the literature information, it can be stated that this model of fatigue description proves to be correct in the low-cycle fatigue range.

b) sum of plastic strain energy density W_p and elastic strain energy density W_e;

Suggestions of such an approach can be found in the papers by Ellyin (1989), Gołoś (1989), Gołoś and Ellyin (1988). It proves to be correct for the high-cycle fatigue range in the loading programme.

c) plastic-elastic strain energy density W_t;

   In the above approach separation of plastic-elastic strain energy into components is omitted.

Such descriptions were suggested in the papers by Łagoda (2001), Smith *et al.* (1970).

Fatigue graphs in the energy-life approach with the use of W_p, W_e, or W_t parameters are mostly obtained indirectly basing on tests performed under controlled stress or strain, and adequate energy parameters are the resulting values calculated after realisation of a fatigue test. In the case of cyclically unstable materials it causes difficulties in a unique determination of a fatigue graph. This problem was presented by Mroziński (2006).

Effective realisation of fatigue life calculations with the use of energy parameters demands elaboration of new procedures for finding fatigue characteristics. Their determination should include research work performed with preservation of constant parameters of criterial values in following loading cycles.

In the literature, there can be found examinations under controlled energy parameters (plastic strain energy density or total strain energy density) during tests for metal specimens (Kasprzyczak and Macha, 2006; Mroziński and Boroński, 2006; Słowik *et al.*, 2006), and specimens made of composite materials (Boroński *et al.*, 2006).

The basic aim of this paper is experimental verification of determination of fatigue properties for metal materials under controlled total strain energy density.

An additional aim of the paper is comparative analysis of the results obtained using the presented concept with the results obtained with the use of the classical method (under controlled strain).

2. Assumptions to the testing method in conditions of energy control

Development of modern strength machines, associated mainly with the improvement of digital control systems, allows for applying non-standard methods of research in cyclic loading conditions. The rate of the control process in the PID feedback loop scheme and high quality of measuring sensors and transducers gives the possibility of controlling fatigue machines with the use of new control signal values including total strain energy density or its components.

Conducting a research in conditions of controlled values of strain energy density demands accepting a suitable model of its division into individual components. In the presently used energy descriptions of the fatigue process, it is accepted that the main role in fatigue plays plastic strain energy and (to a lesser degree) elastic strain energy (Gołoś, 1988, 1989; Gołoś and Ellyin, 1988). One can write down the following equation

$$W_t = W_e + W_p \tag{2.1}$$

where: W_t is the total strain energy density, W_p – plastic strain energy density, W_e – elastic strain energy density $(W_e = W_e^+ + W_e^-)$.

Graphical interpretation of the individual components of the energy is presented in Fig. 1a. Basing on the assumptions, to the description of the fatigue graph in the strain-life approach described with equation (1.1), analogous equations were formulated for the energy description. A relation between the energy of plastic strain W_p and the number of loading reversals to failure is as follows

$$W_p = K_p(2N_f)^{c_w} \tag{2.2}$$

where: K_p denotes the regression line constant, c_w – exponent of the regression line.

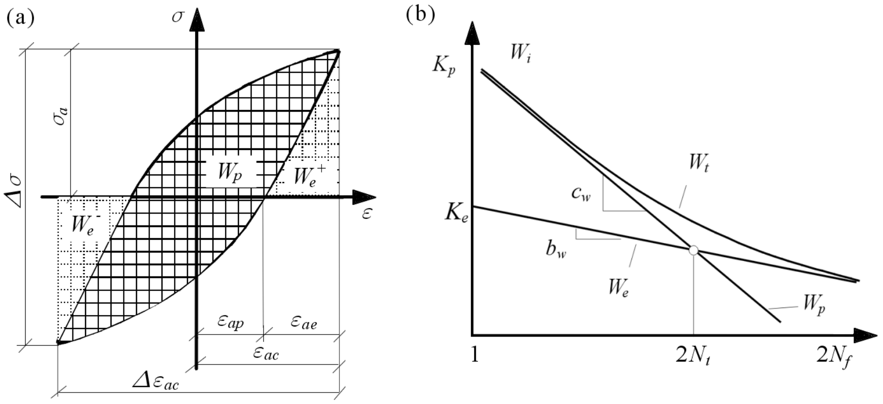


Fig. 1. Components of the total strain energy (a), fatigue chart in the energy approach (b)

In a similar way, the relation between the energy W_e and the loading reversals to failure $2N_f$ is described with an equation

$$W_e = K_e(2N_f)^{b_w} \tag{2.3}$$

where: K_e is the regression line constant, b_w – exponent of the regression line.

The sum of equations (2.2) and (2.3) enables one to define the total energy W_t. Resultant equation (2.4) takes form similar to equation (1.1)

$$W_t = W_e + W_p = K_e(2N_f)^{b_w} + K_p(2N_f)^{c_w} \tag{2.4}$$

In a bilogarythmic co-ordinate system, the W_p and W_e components of the total strain energy W_t described with equations (2.2) and (2.3) are straight lines. The fatigue graph described in the above way is schematically presented in Fig. 1b.

The values of constant coefficients and exponents occurring in the equations are determined in tests conducted in conditions of a constant total strain energy W_t or plastic strain energy W_p. The testing conditions in this case are similar to the fatigue tests performed in conditions of controlled total or plastic strain, described in the ASTM E606-04 [1] standard.

3. Description of verification tests

3.1. Fatigue testing procedures

To realize fatigue tests in conditions of $W_t = \mathrm{const}$, an original software (developed using Borland C++ Builder) for controlling the fatigue testing machine Instron (8000 series) through the digital controller Instron 8500 (8500 plus or 8800 also) was designed.

The algorithm of the computer code, schematically presented in Fig. 2, enables automatic change of loading parameters of the tested specimen in such a way, that the "measured" (Fig. 1a) value W_t would remain during the whole test on the same level, accepted by the operator. The level of strain energy determines the first loading cycle performed for the set value of the total strain amplitude or nominal stress amplitude. The change of loading parameters of the specimen occurs at any number of cycles, set by the operator. The change of the control signal is decided through the comparison of the base value W_{t_z} with the current energy W_{t_m}.

During verifcation tests, as the control signal of the fatigue machine, the total strain measured with the use of an external extensometer was applied.

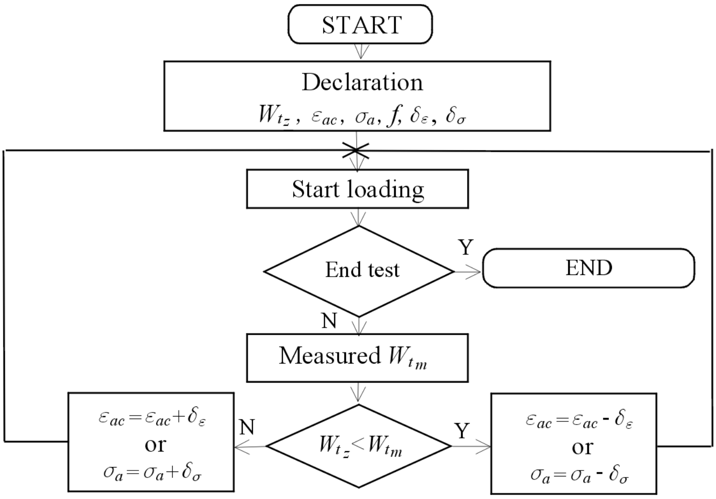


Fig. 2. The algorithm of a computer code used for fatigue tests in conditions of $W_t = \text{const}$

The strain amplitude was corrected in most of the cases every second during the loading cycle by $\delta_\varepsilon = 0.005\%$. The value δ_ε was selected in such a way to make changes of the energy due to variability of the control parameter W_t not higher than 1%. After starting the test and performing the first loading cycle with the amplitude ε_{ac} (or σ_a in the case of selecting a force as the control signal of the machine), there follows determination of the current value of the total strain energy W_t according to the diagram shown in Fig. 1a basing on relation (2.1). The strain energy W_t is calculated as a sum of areas of trapezoids described by the adjacent measurement points of the hysteresis loop (200 points per single loop), similar to numerical procedures described in Szala (1998).

This value is assumed as the basis for the further part of the fatigue test. After obtaining results of measurements and calculations performed in the succeeding loading cycles and after checking the end of the test condition, a comparison of the "base energy" W_{t_z} and the current energy W_{t_m} takes place. According to the comparison results, there follows either increase or decrease of ε_{ac} (or σ_a) by addition or subtraction of the declared value of the strain increment δ_ε (or stress increment δ_σ), with possible change of their value during the test. After realization of the succeeding cycle, the whole sequence of operations is repeated until one of the test criteria is fulfilled.

Tests in conditions of a controlled value of the total strain energy were performed at five levels of W_t presented in Table 1. Their values at particular levels were chosen in such a way that the obtained lives would include the whole low-cycle fatigue range. The loading frequency was 0.2 Hz.

Table 1. Levels of loading applied in fatigue tests

No. of loading level	W_t [MJ/m^3]
1	1.92
2	3.65
3	5.9
4	9.4
5	18.8

3.2. Specimen and material properties

Specimens for the verification test were made of the aluminium alloy PA7 according to the standard (ASTM E 606-04, [1]). Geometry of the specimen is shown in Fig. 3. A chemical constitution of the applied material is presented in Table 2 and mechanical properties – in Table 3.

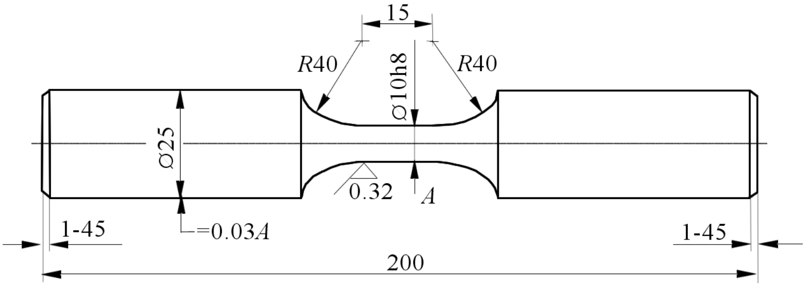


Fig. 3. Dimensions of the PA7 specimen

Table 2. Chemical constitution of the aluminium alloy PA7

Cu [%]	Mg [%]	Mn [%]	Al [%]	Remaining [%]
3.95	1.476	0.54	93.690	0.342

Table 3. Mechanical parameters of the aluminium alloy PA7

σ_y (R_{eH}) [MPa]	σ_u (R_m) [MPa]	R_u [MPa]	E [MPa]	A_5 [%]	Z [%]
321.7	514.7	632.7	72 000	16	21

4. Test results

4.1. The course of the stabilization process

The course of the stabilization process of the PA7 aluminium in conditions of controlled total strain energy density was evaluated basing on analysis of chosen parameters of the hysteresis loop. Examples of changes of some of these parameters for the level $W_t = 5.8\,\mathrm{MJ/m^3}$ are presented in Fig. 4.

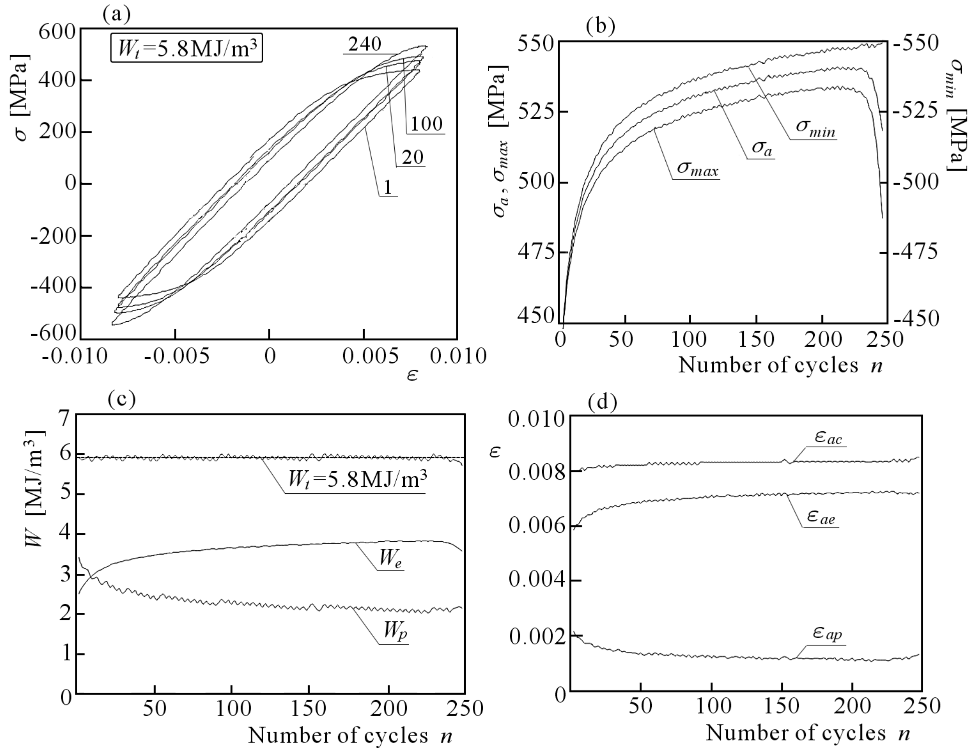
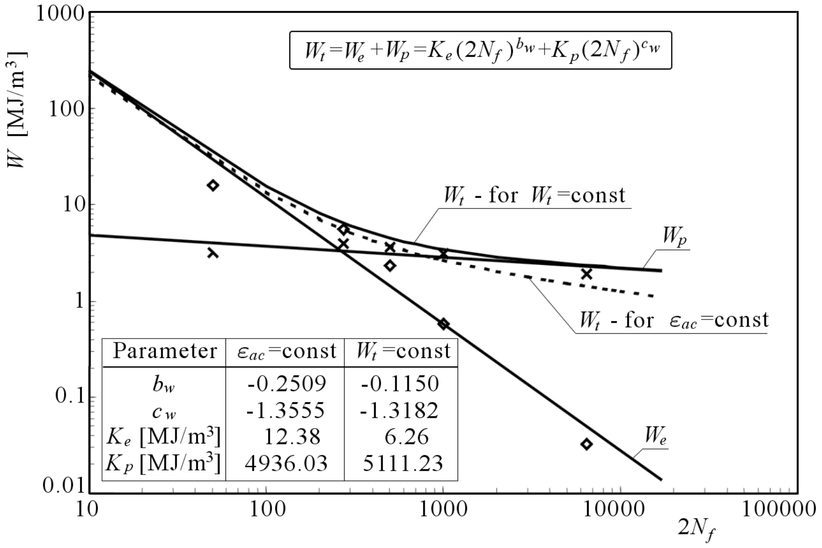


Fig. 4. Chosen parameters of the hysteresis loop in function of the number of loading cycles ($W_t = 5.8\,\mathrm{MJ/m^3}$): (a) hysteresis loop, (b) stress σ_a, $\sigma_{a\,max}$, $\sigma_{a\,min}$, (c) strain energy density W_t, W_p, W_e, (d) strain amplitude ε_{ac}, ε_{ap}, ε_{ae}

As it was expected, the course of changes of the energy W_t and W_p shows little pulsations resulting from the correction of the control signal by $\pm\delta_\varepsilon$. Analysis of the obtained graphs allows one to notice that for the presented level of the total strain energy density, the tested material showed minor hardening. The confirmation of this statement are the courses of changes of W_p (Fig. 4c), total strain amplitudes ε_{ac} and stress amplitudes ε_a.

4.2. Fatigue life graph

The results obtained during fatigue tests performed for a constant value of total strain energy density in the loading cycle allows one to determine a graph of fatigue life according to the description presented in Section 2. According to the accepted assumptions, changes of elastic and plastic strain energy were approximated with straight lines, whereas the total strain amplitude was calculated by summing up its two components. The test results were elaborated with the use of the least squares method and presented graphically in form of fatigue graphs in the W-$2N_f$ co-ordinate system. Values of all determined coefficients and exponents of equation (2.5) were specified in Fig. 5.



Parameter	ε_{ac}=const	W_t=const
b_w	-0.2509	-0.1150
c_w	-1.3555	-1.3182
K_e [MJ/m^3]	12.38	6.26
K_p [MJ/m^3]	4936.03	5111.23

Fig. 5. A fatigue graph of the PA7 alloy in the energy based approach

Comparison analysis of the fatigue life graphs obtained by two described methods (i.e. W_t-$2N_f$ for ε_{ac} = const and W_t-$2N_f$ for W_t = const) allows one to notice their differentiation dependent on the loading amplitude level (e.g. total strain amplitude level). Differences between the W_t energy are small for higher levels and grow together with level decreasing. Higher values of the fatigue life were obtained for the W_t = const approach in the whole range of W_t values. Differences between the fatigue life resulting from the chart presented in Fig. 5 reach very high values for low levels of W_t. It will take a significant effect in the case of fatigue life calculations realised by methods based on the energy approach. Particular analysis of the influence of selected methods for the fatigue properties determination on calculation results of the fatigue life will be carried out in further investigations.

5. Summary

The results of verification tests confirmed the possibility of realization of low-cycle fatigue tests in conditions of controlled total strain energy density. The developed method for determination of fatigue properties in the energy approach, with the procedures described in Mroziński and Boroński (2006) extends the area of presently performed fatigue research in the range of low-cycle fatigue.

The results from fatigue tests carried out in conditions of $W_p = \text{const}$ (Mroziński and Boroński, 2006) and $W_t = \text{const}$ enabled among others, determination of fatigue graphs in the energy approach, which, in consequence, may contribute to the improvement of effectiveness of fatigue life calculations of structural parts.

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Badania metali w warunkach kontrolowanej gęstości energii odkształcenia

Streszczenie

W pracy przedstawiono nową koncepcję określania własności zmęczeniowych. Opracowana metoda badań bazuje na założeniach prowadzenia badań zmęczeniowych sformułowanych dla opisu odkształceniowego. Parametr gęstości energii odkształcenia całkowitego obliczany podczas cyklu obciążenia próbki został zastosowany jako sygnał sterujący próbą zmęczeniową. Opracowana metoda badań poszerza badania w zakresie zmęczenia niskocyklowego, a w przypadku jej zastosowania może przyczynić się do poprawy wyników obliczeń trwałości.

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