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ROBUST ADAPTIVE VIBRATION CONTROL FOR A GENERAL CLASS OF STRUCTURES IN THE PRESENCE OF TIME-VARYING UNCERTAINTIES AND DISTURBANCES

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The problem of active vibration suppression in a wide class of smart structures is addressed. The dynamical model of a structure may be perturbed by uncertain time-varying parameters and external disturbances. A novel adaptive-based control algorithm is presented here to satisfy robustness properties with respect to model uncertainties and environmental disturbances. Reflecting practical situations, the upper bound of perturbations is not required for controller design. The analytical stability of a closed-loop system is presented based on the Lyapunov stability theorem. Furthermore, numerical analysis is also provided to show the effectiveness of the proposed method.

Key words: vibration control, robust adaptive control, piezoelectric materials

1. Introduction

Vibration control of flexible structures is one of the main topics in the field of engineering. In general, vibration suppression algorithms developed by researchers may be passive or active. Passive control, achieved by incorporating mechanical elements into a structure, Skup (2010), is applied when the effects of external disturbances are known in advance. On the contrary, the so-called active control schemes present some self-adaptive mechanisms to reduce vibration of the structure even in the presence of model uncertainties, time-varying loads and unknown disturbances. From a practical viewpoint, implementing active control of flexible structures by piezoelectric materials has been extensively studied during the last decade, Pietrzakowski (2001), Huang and Tseng (2008). This arises from the fact that piezoelectricity is a natural phenomenon which facilitates transforming mechanical energy to electrical energy and vice versa. In addition, piezoelectric materials with low weight and low residual effect exhibit considerable flexibility and can be used in a wide range of temperature. Meanwhile, these materials can be utilized as distributed sensors and actuators incorporated into the structure.

Active vibration suppression has become the focus of attention generally in mechanical and civil structures, see Song *et al.* (2006), Longa *et al.* (2011), and particularly in beams – Trindade *et al.* (2001), Vasques and Rodrigues (2006), Chang (2012), and aircrafts Song and Agrawal (2001), Wachłaczenko (2010). Establishing the controllability concept for dynamical systems by Klamka (1991), and particularly for mechanical systems by Klamka (2005), various reported control schemes for structural systems can be put in the main categories, including: (i) improved conventional control techniques, e.g., proportional and derivative control, Belouettar *et al.* (2008), Fey *et al.* (2010), (ii) optimal control algorithms based on either classical strategies, Stavroulakis *et al.* (2005), Vasques and Rodrigues (2006), or stochastic based optimization techniques, Marinaki *et al.* (2011). Such optimization-based methodologies may ensure the optimal performance in the absence of system uncertainties. Moreover, inverse matrix calculation is needed in the design procedure, whereas the large dimension of such matrices in complex structures takes a considerable time in the implementation process, (iii) intelligent control algorithms based on fuzzy logic, Sharma *et al.* (2005), or neural networks, (iv) robust control schemes, e.g. sliding mode control applied to a simplified dynamics of the structure, Gu *et al.* (2008).

In practice, the system parameters may vary with time due to various circumstances. Among the reported methods, adaptive-based control techniques are powerful tools especially when the variations are slow enough, see Astrom and Wittenmark (1994), Krstic *et al.* (1996). In fact, conventional adaptive methods including adaptive control together with some parameter adjusting mechanism may fail for the case of time-varying perturbations. Investigating into this field, several results have been reported when the variations are periodic, Xu (2004), Ding (2007), or the upper bound of the parameter vector is known in advance, Ge and Wang (2003), Cai *et al.* (2006). On the other hand, two main types of disturbances including time-varying ones and those associated with fixed deformations may also affect the performance of a flexible structure, Irschik (2002). Dealing with the purpose of attenuating or rejecting the influence of disturbances, some robust control methods and also several adaptive approaches have been introduced, especially for the case of periodic disturbances, Bodson and Douglas (1997), Ding (2007).

In this paper, a robust adaptive algorithm is developed to achieve active vibration control of structures. More precisely, an adaptive algorithm is designed to tackle time-varying model uncertainties and incorporated a robust mechanism to deal with external disturbances. In fact, a combination of tools from both robust and adaptive approaches is adopted to achieve the desired performance. Some specific properties of the developed active vibration control algorithm are: (i) it can be applied to a wide class of flexible structures, (ii) there are no conservative assumptions, e.g., on the upper bound, the speed of variations and the periodicity of model uncertainties and external disturbances, (iii) robust stability is ensured by theoretical analysis and verified by various numerical simulations.

The organization of the paper is as follows. Presenting the mathematical model of flexible structures, the vibration control problem is formulated in Section 2. Section 3 presents the robust adaptive control algorithm and its stability analysis based on the Lyapunov stability theorem. In Section 4, various simulation results are given to illustrate the performance of the proposed vibration suppression method. Finally, the concluding remarks are given in Section 5.

Throughout the paper, $\|\cdot\|$ denotes the Euclidean vector norm and for a $n \times 1$ vector $\mathbf{V}$, the weighted norm is defined as $\|\mathbf{V}\|_Q^2 := \mathbf{V}^{\mathrm{T}}\mathbf{Q}\mathbf{V}$ with a weighting matrix $\mathbf{Q}$. Furthermore, $V \in L_2[0,T]$ if $\int_0^T \|\mathbf{V}(t)\|^2\,dt < \infty$, $T \in [0,\infty)$, and $V \in L_\infty$ if $\|\mathbf{V}(t)\| < \infty$ for all $t \in [0,\infty)$.

2. Mathematical model and problem statement

Mathematical modeling of beams and structures is performed mainly based on linear piezoelectricity, sensor dynamics and equations of motion. Depending on the analysis or control synthesis purposes, each of the aforementioned factors may lead to adopt a suitable method for math formulation of the model. Dealing with the vibration control problem, the finite element method can be used to describe the dynamical equation of motion for a smart mechanical structure as, Trindade *et al.* (2001)

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{D}\dot{\mathbf{X}} + C\mathbf{X} = \mathbf{F}_m + \mathbf{F}_e \tag{2.1}$$

where $\mathbf{X}$ represents the state vector of the system, including travnsersal deflection and rotation variables, $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ denotes the viscous damping matrix and $\mathbf{C}$ stands for the stiffness matrix. The force vector $\mathbf{F}_e$ acts as a control input, produced by electromechanical coupling effects, and $\mathbf{F}_m$ is a mechanical point force vector acting as external disturbance.

From a practical viewpoint, the viscous damping matrix $\mathbf{D}$ and the stiffness matrix $\mathbf{C}$ may not be determined accurately, especially for complex structures, and the existence of uncertainty in the aforementioned matrices is inevitable. The variation of parameters included in such matrices motivates taking a time-varying uncertain dynamical model for controller design. Hence, dynamical equation (2.1) takes the form

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{h}_0(\mathbf{X}, \dot{\mathbf{X}}) + \sum_{i=1}^{p} \boldsymbol{\varphi}_i(\mathbf{X}, \dot{\mathbf{X}})\boldsymbol{\theta}_i(t) = \mathbf{u}(t) + \mathbf{d}(t) \tag{2.2}$$

where

$$\mathbf{h}_0(\mathbf{X}, \dot{\mathbf{X}}) = \mathbf{D}_0\dot{\mathbf{X}} + \mathbf{C}_0\mathbf{X}$$

denotes the nominal part produced by known matrices $\mathbf{C}_0$ and $\mathbf{D}_0$, $\boldsymbol{\varphi}_i(\mathbf{X}, \dot{\mathbf{X}})$ is a dimensionally compatible matrix associated with an unknown time-varying parameter vector $\boldsymbol{\theta}_i(t)$, $i = 1, 2, \ldots, p$, $\mathbf{u}(t)$ stands for the applied control input, and $\mathbf{d}(t)$ denotes the disturbance input.

The following assumptions are made regarding the system.

Assumption 1. The variations of parameters included in $\mathbf{C}$ and $\mathbf{D}$ can be time-varying with unknown bounds, i.e., $\boldsymbol{\theta}_i(t)$ belongs to the compact set $\Omega_i = \{\boldsymbol{\theta}_i(t) : \|\boldsymbol{\theta}_i(t)\| \leqslant \beta_i\}$, $i = 1, 2, \ldots, p$, in which $\beta_i > 0$ is an unknown constant.

Assumption 2. The time-varying external disturbance $d(t)$ is norm bounded with an unknown value, i.e., $\|\mathbf{d}(t)\| \leqslant \delta$, where δ is an uncertain parameter.

Assumption 3. Controllability, defined by Klamka (2005), as the possibility to control a dynamical system from an arbitrary initial state to an arbitrary final state using a set of admissible controls, is satisfied here for the presented mechanical systems.

The objective is to design an active control algorithm that ensures vibration suppression in the presence of model uncertainties and external disturbances. As a preliminary step to design such a controller, define the tracking error vector as $\mathbf{e} = \mathbf{X}_d - \mathbf{X}$, where $\mathbf{X}_d$ represents the desired state vector, usually set to zero, for vibration suppression purposes.

3. Robust adaptive controller design

In order to develop the control algorithm and for notational consistency, two error metric functions are defined as $\mathbf{S}(t) = \dot{\mathbf{e}}(t) + \mathbf{e}(t)$ and $\mathbf{S}_r(t) = \ddot{\mathbf{X}}_d(t) + \dot{\mathbf{e}}(t)$. The general structure for the control input is proposed as

$$\mathbf{u} = \mathbf{h}_0 + \mathbf{M}\mathbf{S}_r + \mathbf{K}\mathbf{S} + \mathbf{u}_a + \mathbf{u}_r \tag{3.1}$$

where $\mathbf{K}$ is a positive definite matrix, $\mathbf{u}_a$ presents an adaptive subcontroller, and $\mathbf{u}_r$ is a robust subcontroller to be designed. In fact, $\mathbf{u}_a$ deals with the system parameter uncertainties and $\mathbf{u}_r$ ensures robustness with respect to the environmental disturbances.

In the following, using the Lyapunov stability theorem, the subcontrollers $\mathbf{u}_a$ and $\mathbf{u}_r$ are derived. To this end, choose the Lyapunov function

$$V(\mathbf{e}, \dot{\mathbf{e}}) = \mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} + \frac{1}{2}\mathbf{S}^{\mathrm{T}}\mathbf{M}\mathbf{S} + \frac{1}{2\gamma}\widetilde{\beta}^2 \tag{3.2}$$

where $\widetilde{\beta} = \beta - \widehat{\beta}$ denotes the parameter estimation error and $\gamma > 0$ is the adaptation gain.

The time derivative of V is

$$\dot{V} = 2\mathbf{e}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} + \mathbf{S}^{\mathrm{T}}(\mathbf{M}\ddot{\mathbf{e}} + \mathbf{M}\dot{\mathbf{e}}) + \frac{1}{\gamma}\widetilde{\beta}\dot{\widetilde{\beta}} \tag{3.3}$$

Substituting $\ddot{\mathbf{e}}$ by $\ddot{\mathbf{X}}_d - \ddot{\mathbf{X}}$ in (3.3) and replacing $\mathbf{M}\ddot{\mathbf{X}}$ from (2.2), one can obtain

$$\dot{V} = 2\mathbf{e}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} + \mathbf{S}^{\mathrm{T}}\Big(\mathbf{M}\ddot{\mathbf{X}}_d + \mathbf{h}_0 + \sum_{i=1}^{p}\boldsymbol{\varphi}_i\boldsymbol{\theta}_i(t) - \mathbf{u}(t) - \mathbf{d}(t) + \mathbf{M}\dot{\mathbf{e}}\Big) + \frac{1}{\gamma}\widetilde{\beta}\dot{\widetilde{\beta}} \tag{3.4}$$

Incorporating control law (3.1) into (3.4) yields

$$\dot{V} = 2\mathbf{e}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} - \mathbf{S}^{\mathrm{T}}\mathbf{K}\mathbf{S} + \mathbf{S}^{\mathrm{T}}\Big(\sum_{i=1}^{p}\boldsymbol{\varphi}_i\boldsymbol{\theta}_i(t) - \mathbf{u}_a - \mathbf{d}(t) - \mathbf{u}_r\Big) - \frac{1}{\gamma}\widetilde{\beta}\dot{\widehat{\beta}} \tag{3.5}$$

Taking assumption 1 into account, $\dot{V}$ can be written as

$$\dot{V} \leqslant -\mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} - \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} + \beta\sum_{i=1}^{p}\|\mathbf{S}^{\mathrm{T}}\boldsymbol{\varphi}_i(\mathbf{X},\dot{\mathbf{X}})\| - \mathbf{S}^{\mathrm{T}}\mathbf{u}_a - \mathbf{S}^{\mathrm{T}}\mathbf{d} - \mathbf{S}^{\mathrm{T}}\mathbf{u}_r - \frac{1}{\gamma}\widetilde{\beta}\dot{\widehat{\beta}} \tag{3.6}$$

where $\beta = \max\{\beta_1, \beta_2, \ldots, \beta_p\}$.

Now, the adaptive and robust terms u_a and u_r are respectively proposed as

$$\mathbf{u}_a = \widehat{\beta}^2\sum_{i=1}^{p}\frac{\boldsymbol{\varphi}_i(\mathbf{X},\dot{\mathbf{X}})\boldsymbol{\varphi}_i^{\mathrm{T}}(\mathbf{X},\dot{\mathbf{X}})\mathbf{S}}{\|\mathbf{S}^{\mathrm{T}}\boldsymbol{\varphi}_i(\mathbf{X},\dot{\mathbf{X}})\|\widehat{\beta} + \sigma\mathrm{e}^{-rt}} \qquad \mathbf{u}_r = \frac{1}{2\rho}\mathbf{S} \tag{3.7}$$

where $\widehat{\beta}$, the estimate of β, is calculated by the adaptation mechanism

$$\dot{\widehat{\beta}} = \gamma\sum_{i=1}^{p}\|\mathbf{S}^{\mathrm{T}}\boldsymbol{\varphi}_i(\mathbf{X},\dot{\mathbf{X}})\| \tag{3.8}$$

Substituting $\mathbf{u}_a$, $\mathbf{u}_r$, and update law (3.8) into inequality (3.6), gives

$$\begin{aligned}\dot{V} &\leqslant -\mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} - \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} + \widetilde{\beta}\sum_{i=1}^{p}\|\mathbf{S}^{\mathrm{T}}\boldsymbol{\varphi}_i(\mathbf{X},\dot{\mathbf{X}})\| + \sigma\mathrm{e}^{-rt} - \frac{1}{\gamma}\widetilde{\beta}\dot{\widehat{\beta}} \\ &\leqslant -\mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} - \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} - \mathbf{S}^{\mathrm{T}}\mathbf{d} - \frac{1}{2\rho}\mathbf{S}^{\mathrm{T}}\mathbf{S} + \sigma\mathrm{e}^{-rt}\end{aligned} \tag{3.9}$$

Using the equivalence

$$-\mathbf{S}^{\mathrm{T}}\mathbf{d} - \frac{1}{2\rho}\mathbf{S}^{\mathrm{T}}\mathbf{S} = -\frac{1}{2\rho}(\mathbf{S} + \rho\mathbf{d})^{\mathrm{T}}(\mathbf{S} + \rho\mathbf{d}) + \frac{1}{2}\rho\|\mathbf{d}\|^2 \tag{3.10}$$

inequality (3.9) can be rewritten as

$$\dot{V} \leqslant -\mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} - \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} - \frac{1}{2\rho}(\mathbf{S} + \rho\mathbf{d})^{\mathrm{T}}(\mathbf{S} + \rho\mathbf{d}) + \frac{1}{2}\rho\|\mathbf{d}\|^2 + \sigma\mathrm{e}^{-rt} \tag{3.11}$$

By omitting some strictly negative terms from the right hand side of inequality (3.11), one can obtain

$$\dot{V} \leqslant -\mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} + \frac{1}{2}\rho\|\mathbf{d}\|^2 + \sigma\mathrm{e}^{-rt} \tag{3.12}$$

and

$$\dot{V} \leqslant -\dot{\mathbf{e}}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} + \frac{1}{2}\rho\|\mathbf{d}\|^2 + \sigma\mathrm{e}^{-rt} \tag{3.13}$$

The following results are then concluded.

(i) By assumption 2, inequality (3.12) implies that $\dot{V}$ is bounded as $\dot{V} \leqslant -\lambda_K\|\mathbf{e}\|^2+\frac{1}{2}\rho\delta^2+\sigma$, where λ_K is the minimum eigenvalue of K. Choosing $\lambda_K > (\rho\delta^2+2\sigma)(2\varepsilon^2)$ for any small $\varepsilon > 0$, there exists a $\kappa > 0$ such that $\dot{V}(\mathbf{e},\dot{\mathbf{e}}) \leqslant -\kappa\|\mathbf{e}\|^2 < 0$ for all $\|\mathbf{e}\| > \varepsilon$. Thus, there is a $T > 0$ such that $\|\mathbf{e}\| \leqslant \varepsilon$ for all $t \geqslant T$. This implies that the error vectors $\mathbf{e}(t)$ are uniformly ultimately bounded (UUB), Krstic *et al.* (1995).

(ii) Taking inequality (3.13) into account and following a procedure similar to that given in (i), the boundedness of $\dot{\mathbf{e}}(t)$ is concluded.

(iii) In many practical situations, the disturbance inputs, e.g. a constant load for a specific time duration, Stavroulakis *et al.* (2005), a sinusoidal periodic wind-type pressure, Baniotopoulos and Plalis (2002), and zero mean white Gaussian noise, are energy bounded, i.e., $\mathbf{d} \in L_2[0,T]$. Hence, integrating inequality (3.10) from $t=0$ to $t=T$ yields

$$\int\limits_0^T \|\mathbf{e}(t)\|_K^2\,dt + V\big(\mathbf{e}(T),\dot{\mathbf{e}}(T)\big) \leqslant V\big(\mathbf{e}(0),\dot{\mathbf{e}}(0)\big) + \frac{\sigma}{r}(1-\delta\mathrm{e}^{-rT}) + \frac{1}{2}\rho\int\limits_0^T \|\mathbf{d}(t)\|^2\,dt$$

for all $0 \leqslant T < \infty$. This implies that $\mathbf{e}(t)$ is square-integrable, i.e. $\mathbf{e}(t) \in L_2[0,T]$, which together with the boundedness property of $\mathbf{e}(t)$ and $\dot{\mathbf{e}}(t)$, Barbalat's lemma (see the appendix) Krstic *et al.* (1995), ensures the convergence of $\mathbf{e}(t)$ and the closed-loop stability, despite the system uncertainties and external disturbances.

Remark 1. Choosing a smaller $\rho > 0$ provides the system with a faster response. This may be obtained at the expense of larger control effort. In fact, there exists a trade-off between the value of subcontroller gain ρ and the magnitude of control input $\mathbf{u}$.

Remark 2. From a practical viewpoint, the exponential term in $\mathbf{u}_a$ formed by $\sigma > 0$ and $r > 0$ provides the smoothness of the control law without violation the convergence property of the tracking error.

Remark 3. Unlike some previous works, Trindade *et al.* (2001), Stavroulakis *et al.* (2005), Marinaki *et al.* (2011), the inverse calculation of $\mathbf{M}$, whose large dimension in complex structures takes a considerable time in the implementation process, is not required here. Moreover, the effects of model uncertainties and disturbances even with unknown bounds, are well suppressed.

Remark 4. In order to alleviate the increase in the estimation value $\widehat{\beta}$ without bound occurring in the case of imperfect implementation of adaptation mechanism (3.8), an effective modification is adopted here. To this end, substitute update law (3.8) with

$$\dot{\widehat{\beta}} = \mu\sum_{i=1}^{p}\|\mathbf{S}^{\mathrm{T}}\boldsymbol{\varphi}_i(\mathbf{X},\dot{\mathbf{X}})\| \tag{3.14}$$

where

$$\mu = \begin{cases} \gamma & \text{if} \quad \|\mathbf{e}\| > \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

which ensures that all the signals and states of the closed loop system are bounded and $\|\mathbf{e}\|$ is robustly converged to a (small) prescribed bound $\varepsilon > 0$. In fact, this modification acts as a projection algorithm and, therefore, the stability analysis can be followed similar to that of conventional projection methods in the literature, Astrom and Wittenmark (1994), Khalil (1996). Briefly discussing, adaptation mechanism (3.14) is activated whenever the norm of the tracking error $\mathbf{e}$ exceeds the prescribed bound $\varepsilon > 0$ and, consequently, the instability due to the increase in $\widehat{\beta}$ is alleviated.

The aforementioned analysis shows the capability of the proposed control algorithm for vibration suppression in a wide class of flexible structures in the presence of model uncertainties and environmental disturbances.

4. Results and discussion

In order to verify the effectiveness of the proposed robust adaptive vibration control algorithm, the controller is applied to a flexible beam instrumented with a piezoelectric sensor and actuator, as schematically shown in Fig. 1.

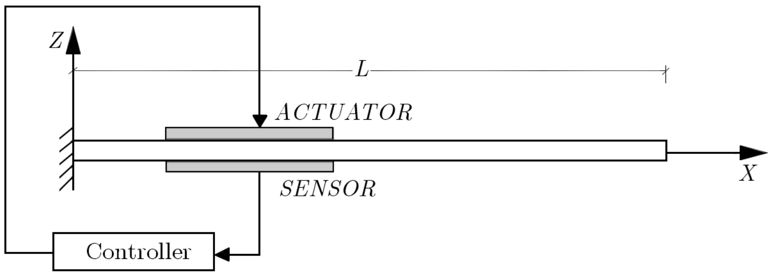

Fig. 1. Configuration of the beam with piezoelectric patches

In simulation studies, the viscous damping and the stiffness matrices are imposed to a sine variation with an amplitude of 30% of the nominal values and period of 0.5. The beam specifications are listed in Table 1.

Table 1. Beam specifications

Parameter	Value
Length [mm]	200
Height [mm]	10
Width [mm]	1
Young's modulus [GPa]	200
Density [kg/m^2]	7800

Three cases are here considered to evaluate the performance of the designed vibration suppression algorithm. At the first step, no disturbances are imposed to the model perturbed by parameter uncertainties.

The control signal is activated at $t = 50\,\mathrm{ms}$ as demonstrated in Fig. 2. The capability of the method in vibration control is illustrated in Fig. 3 and is focused in the steady state for exact analysis. In the second case, a zero mean white noise, as shown in Fig. 4, is imposed to the system which is well suppressed by the proposed robust control algorithm, see Fig. 5. Since the sinusoidal signals can effectively model the effects of wind on structures, Baniotopoulos and Plalis (2002), Stavroulakis *et al.* (2005), a sine disturbance with a period of 0.2 s and an amplitude of 1 is considered in the third case. Figure 6 illustrates a comparison between the time response of vibrations in the absence of control effort and by activating the control input at $t = 50\,\mathrm{ms}$, showing the achievement of the vibration suppression task.

5. Conclusions

The problem of active vibration control is addressed for smart structures. Removing drawbacks of some previous investigations, a novel robust adaptive vibration suppression algorithm is pre-

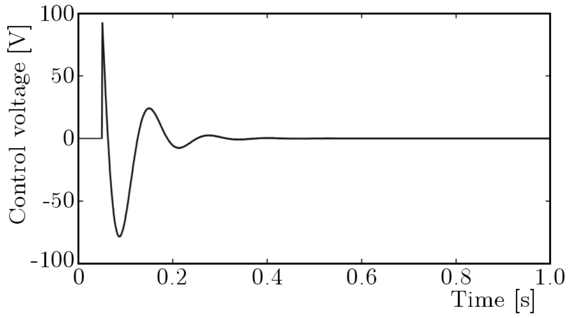

Fig. 2. Control signal for robust adaptive vibration suppression activated at $t = 50\,\mathrm{ms}$

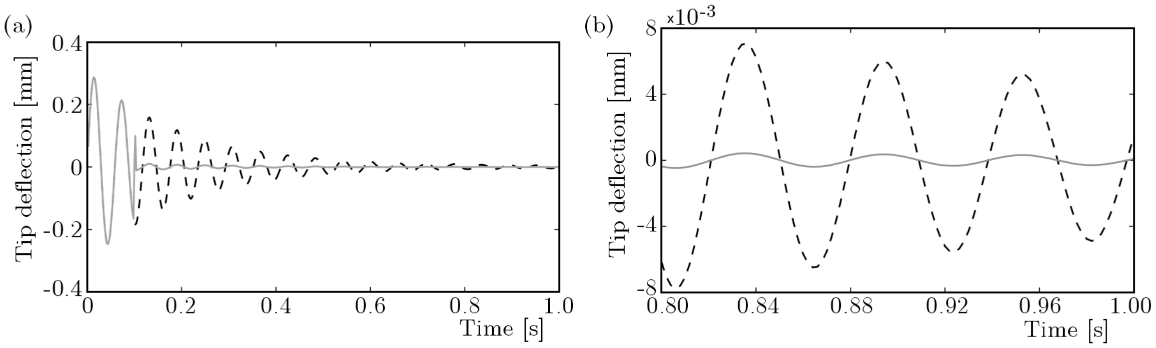

Fig. 3. Time response of vibration with the designed controller (—-), and without control (– –), (a) transient response, (b) steady state response

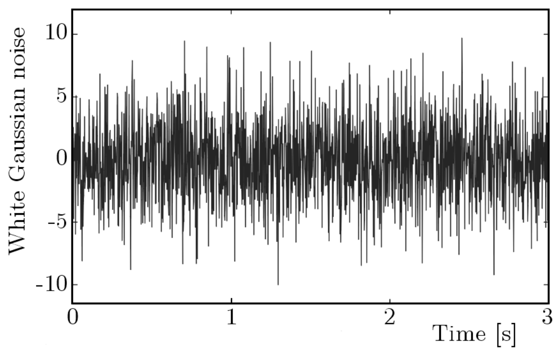

Fig. 4. Zero mean white Gaussian noise

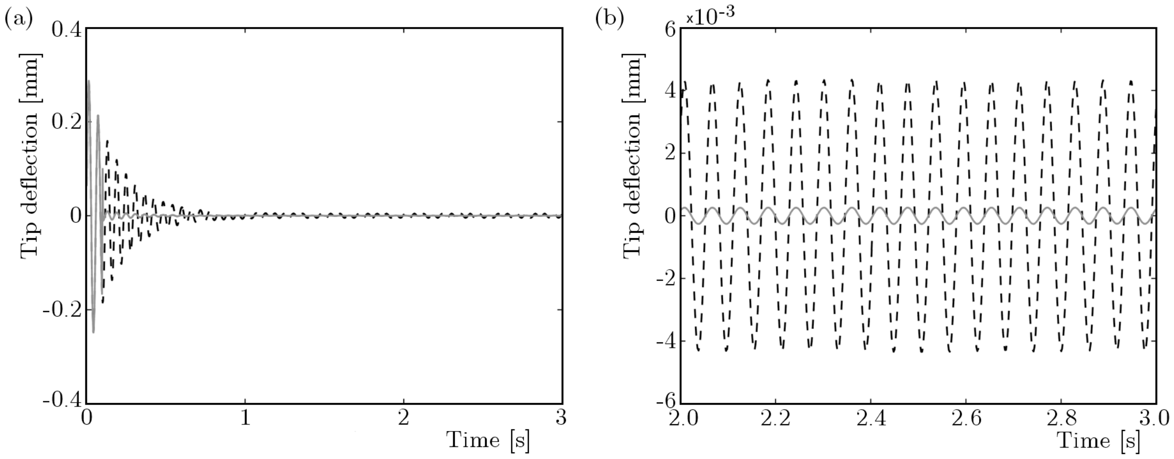

Fig. 5. Time response of vibration in the presence of white noise using the designed controller (—-) and without control (– –), (a) transient response, (b) steady state response

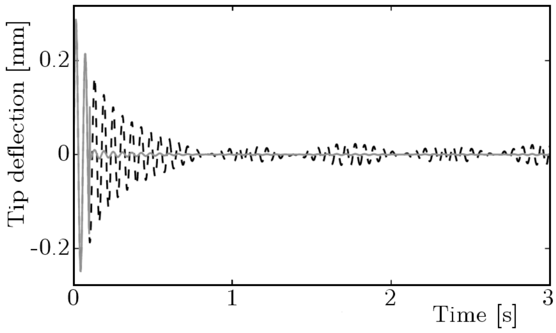

Fig. 6. Time response of vibration in the presence of a sinusoidal disturbance applying the designed robust adaptive controller (—-), and without control (– –)

sented, and the effectiveness of the method is shown by both analytical and simulation analysis. The model uncertainties and disturbances may have unknown bounds. Some kinds of external disturbances, mostly imposed in practical situations, are considered in the analysis. The numerical studies show that the goal of vibration control is achieved by the designed method, despite the model uncertainties and environmental disturbances.

Appendix

One of the results of Barbalat's lemma, used in the stability proof in this paper, is stated by Krstic *et al.* (1995), Ioannou and Sun (1996), as

Lemma: If $\mathbf{e}, \dot{\mathbf{e}} \in L_\infty$ and $\dot{\mathbf{e}} \in L_2$, then $\mathbf{e}(t) \to \mathbf{0}$ as $t \to \infty$.

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