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DYNAMIC COMPENSATION OF DYNAMIC FORCES IN TWO PLANES FOR THE RIGID ROTOR

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The paper presents a method of automatic compensation of dynamic forces acting on a rigid rotors as a result of their unbalance. It is done by free elements located in two planes. The free elements rotate together with the rotor and generate forces that can be opposed to the rotor unbalance. The paper presents physical and mathematical models. It is shown for what positions of the free elements they can compensate the unbalance. Numerical simulations show that the elements status goes to these positions. The vibration forces were defined and it was pointed out that they push the free elements to these positions. The vibration forces take zero values in ball positions for which the system is balanced. They are positions of the equilibrium. The stability of the free elements in these positions is discussed. Ranges of the rotor velocity in which the system balances itself are defined. The paper presents a simulation of the behavior of the system during the balancing for different unbalance situation. The influence of resistance of the free elements on the efficiency of the method is verified. The theoretical results are verified during laboratory experiments.

Key words: rotor, vibration, self-organizing system

1. Introduction

A rotor has static and dynamic unbalances. The first one happens when the center of mass of the rotor is not on the axis of rotation and the second one exists when the axis of rotation is not one of the principal axes of inertia of the rotor. The unbalance of the rotor generates vibration and dynamic forces that act on the rotor and its bearings. They can shorten life of the rotor or they can destroy it. Therefore, all rotors are balanced before they can be used. Special stands are used to balance rotating systems. They measure the rotor unbalance, its position, calculate what counterbalanced masses should be added or remove from the rotor, define the positions where they should be given, and execute the operation (Breal-Kjear, 1973; Harris, 1988). With short rotors, such as grinding wheels or disks, it is enough to balance them in one plane. Longer, rigid rotors have to be balanced at least in two planes (Breal-Kjear, 1973). Elastic rotors have to be balanced in more planes. It all depends on the range of the rotor speed and rotor natural frequencies. The rotor unbalance can change due to its wear or thermal deformation, and therefore the rotor has to be balanced from time to time.

For rotors in which the distribution of mass is changing, for each start or during operation (e.g. washing machine, centrifuge, grinding wheel) this method can not be used. In these situations we can use self-balancing method. This method was proposed by Thearle (1950) for balancing in one plane. The author developed this method (Majewski, 1976, 1978) and checked in what situations it can be used and defined its efficiency. Sokolowska (1981) investigated the possibility of compensating dynamic forces for an object rotating about a fixed point. Sokolowska showed that only a part of the rotor unbalance could be compensated by free elements. In papers Alfriend (1974), Chang and Chou (1991), the authors used a liquid for the stabilization of gyroscopic motion. This kind of balancing is also important for machines that run the risk of failure, e.g. aircraft turbines or compressors when one of the blades brakes. Such break-downs can result in a terrible accident.

If the rotor has static and dynamic unbalance, then there is a combination of a centrifugal force F_o and a moment M_o that rotate together with the rotor. These loadings can also be presented as two oblique forces acting in two arbitrary planes perpendicular to the axis of rotation. So, to dynamically balance the rotor, it is necessary to create two opposing forces. These forces can be generated by free elements, e.g. balls or rollers located in two planes. They should generate a force and a moment that are always opposed to the unbalance. First results of this research were given in Majewski (1980, 1988).

The method has positive and negative features, and it is necessary to define them so that later engineers might decide in what situations this method can be used in practice. The physical and mathematical models will define properties of the method and for which situations it can be used.

The aim of the paper is to determine the possibility of balancing, define the velocity range of the rotor for which the free elements compensate dynamic forces or increase them, and the time of reaction. If the free elements are able to balance the rotor, then it is necessary to define the efficiency of the method and check the influence of each parameter on it. The physical and mathematical model will allow verification of the stability of the dynamic system. The result of research should be verified in laboratory tests.

2. Description of the system

Figure 1 presents a sketch of the system. The mass m_{n11} represents the static unbalance and the two equal masses m_{n21}, m_{n22} represent the dynamic unbalance of the rotor. The bearings D_1 and D_2 of the rigid rotor are supported elastically. The elastic and damping properties of the supports are described by parameters k_{jx}, k_{jy}, and n_{jx}, n_{jy}, ($j = 1, 2$ indicates the number of the support). The positions of the bearings D_1 and D_2 are defined by the coordinates z_j. At the ends of the rotor there are two drums in which free balls

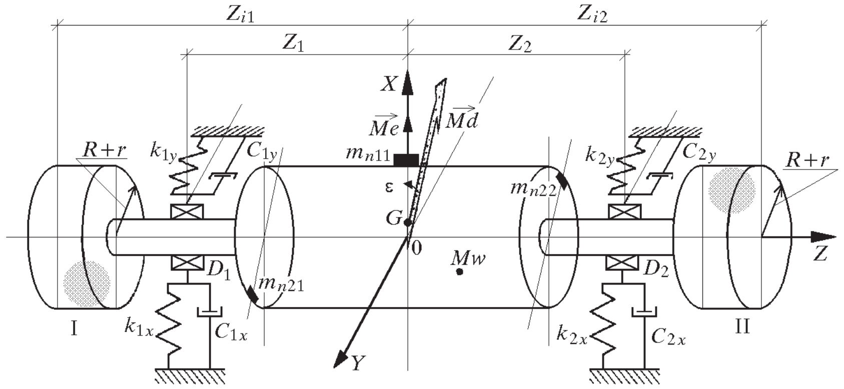

Fig. 1. A system for automatic balancing

or rollers are placed. The position of the drum along the rotor axis is defined by the distance z_i from the mass center of the system. The center of the i-th free element moves along a circular path with the radius R_i. The radius of the ball or roller is r_i. It means that the radius of the drum is $R_i + r_i$. The free elements roll without slipping in the drum. In their motion, with respect to the drums, the free elements have to overcome the viscous and rolling resistance. The static unbalance of the rotor is presented by the principle vector $\overrightarrow{P}_o = \overrightarrow{Me}\omega^2$ and principle moment $\overrightarrow{M}_o = \overrightarrow{Md}\omega^2$. They are perpendicular to the axis of rotation and they spin with the rotor. The coordinate system xyz is fixed to the rotor and spins with it, while the frame XYZ is fixed. It was assumed that the rotor is symmetrical. The mass moment of inertia of the rotor with respect to the axis of rotation is A and the moment with respect to the axis perpendicular to it is B. The displacement of the rotor is defined with respect to the fixed frame XYZ.

3. Equations of motion

Vibrations of the rotor are defined by two components x, y of the displacement of the mass center in the frame XYZ and two angles Θ, Ψ describing the position of the rotor axis with respect to the axis X and Y. The position of i-th free element with respect to the coordinate system fixed to the rotor is described by the angle $\alpha_i(t)$, $i = 1,\dots,N$ (N is the number of the free elements) – Fig. 2.

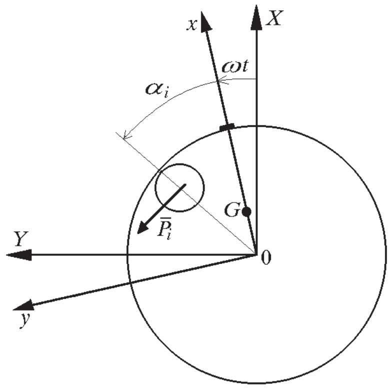

Fig. 2. The coordinate system for the free element

The equation of motion of the rotor and the free elements were obtained from Lagrange's equations. The kinetic and potential energy of the system were defined as a function of generalized coordinates $\boldsymbol{q} = [x, y, \Theta, \Psi]^\top$ describing the vibration of the rotor with respect to the fixed frame XYZ, the angles $\alpha_1,\dots,\alpha_N$ of the free element with respect to the rotor, and their velocities. It was shown in previous papers (Majewski, 1976, 1978) that it is enough to model the rotor unbalance as two generalized forces; the centrifugal force P_o from the static unbalance Me and the moment M_o given by the dynamic unbalance Md. Between the vector of the static and dynamic unbalance there is an angle ε. For any rotor, the magnitude of the unbalance and its location with respect to the rotor are unknowns. The rotor spins with a constant speed ω. Equations of the rotor

$$
\begin{aligned}
&M\ddot{x} + \Big(\sum_{i=1}^{N} m_i z_i\Big)\ddot{\Theta} + \Big(\sum_{j=1}^{2} n_{jx}\Big)\dot{x} + \Big(\sum_{j=1}^{2} n_{jx} z_j\Big)\dot{\Theta} + \Big(\sum_{j=1}^{2} k_{jx}\Big)x + \Big(\sum_{j=1}^{2} k_{jx} z_j\Big)\Theta = \\
&\quad = Me\omega^2 \cos\omega t + \sum_{i=1}^{N} m_i R_i[(\omega + \dot{\alpha}_i)^2 \cos\beta_i + \ddot{\alpha}_i \sin\beta_i]
\end{aligned}
$$

$$
\begin{aligned}
&M\ddot{y} - \Big(\sum_{i=1}^{N} m_i z_i\Big)\ddot{\varPhi} + \Big(\sum_{j=1}^{2} n_{jy}\Big)\dot{y} - \Big(\sum_{j=1}^{2} n_{jy} z_j\Big)\dot{\varPhi} + \Big(\sum_{j=1}^{2} k_{jy}\Big) y - \Big(\sum_{j=1}^{2} k_{jy} z_j\Big)\varPhi = \\
&\quad = Me\omega^2 \sin\omega t + \sum_{i=1}^{N} m_i R_i[(\omega+\dot{\alpha}_i)^2 \sin\beta_i - \ddot{\alpha}_i \cos(\omega t + \alpha_i)] \\[2mm]
&-\Big(\sum_{i=1}^{N} m_i z_i\Big)\ddot{y} + \Big[B + \sum_{i=1}^{N} m_i(z_i^2 + R_i^2\cos^2\beta_i)\Big]\ddot{\varPhi} - \frac{1}{2}\Big(\sum_{i=1}^{N} m_i R_i^2 \sin 2\beta_i\Big)\ddot{\varTheta} + \\
&\quad -\Big(\sum_{j=1}^{2} n_{jx} z_j\Big)\dot{y} + \Big[\sum_{j=1}^{2} n_{jy} z_j^2 + \sum_{i=1}^{N} m_i R_i^2(\omega+\dot{\alpha}_i)\sin 2\beta_i\Big]\dot{\varPhi} + \qquad (3.1) \\
&\quad +\Big[A\omega + 2\sum_{i=1}^{N} m_i R_i^2(\omega+\dot{\alpha}_i)\sin^2\beta_i\Big]\dot{\varTheta} + \Big(\sum_{j=1}^{2} k_{jy} z_j\Big) y + \Big(\sum_{j=1}^{2} k_{jy} z_j^2\Big)\varPhi = \\
&\quad = Md\omega^2 \sin(\omega t - \varepsilon) + \sum_{i=1}^{N} m_i R_i z_j[-(\omega+\dot{\alpha}_i)^2 \sin\beta_i + \ddot{\alpha}_i\cos\beta_i] \\[2mm]
&\Big(\sum_{i=1}^{N} m_i z_i\Big)\ddot{x} - \frac{1}{2}\Big(\sum_{i=1}^{N} m_i R_i^2\sin 2\beta_i\Big)\ddot{\varPhi} + \Big[B + \sum_{i=1}^{N} m_i(z_i^2 + R_i^2\cos^2\beta_i)\Big]\ddot{\varTheta} + \\
&\quad +\Big(\sum_{j=1}^{2} n_{jx} z_j\Big)\dot{x} - \Big[A\omega + 2\sum_{i=1}^{N} m_i R_i^2(\omega+\dot{\alpha}_i)\cos^2\beta_i\Big]\dot{\varPhi} + \\
&\quad +\Big[\sum_{j=1}^{2} n_{jx} z_j^2 - \sum_{i=1}^{N} m_i R_i^2(\omega+\dot{\alpha}_i)\sin 2\beta_i\Big]\dot{\varTheta} + \Big(\sum_{j=1}^{2} k_{jx} z_j\Big)x + \Big(\sum_{j=1}^{2} k_{jx} z_j^2\Big)\varTheta = \\
&\quad = Md\omega^2\sin(\omega t - \varepsilon) + \sum_{i=1}^{N} m_i R_i z_j[(\omega+\dot{\alpha}_i)^2\cos\beta_i + \ddot{\alpha}_i\sin\beta_i]
\end{aligned}
$$

where $\beta_i = \omega t + \alpha_i$.

The mass of the system is $M = M_w + \sum_{i=1}^{N} m_i$, where M_w means the total mass of the rotor and $\sum_{i=1}^{N} m_i$ is the total mass of all free elements.

The differential equation of motion of the free element with respect to the rotor is $(i = 1, \dots, N)$

$$
I_i\ddot{\alpha}_i = m_i R_i[\ddot{x}\sin\beta_i - \ddot{y}\cos\beta_i + \ddot{\varPhi} z_i\cos\beta_i + \ddot{\varTheta} z_i \sin\beta_i] - F_i \qquad (3.2)
$$

where N is the total number of the free elements and I_i is the moment of inertia of the i-th free element reduced to the center of the drum. For the free element that rolls inside the drum without slipping $I_i = m_i R_i^2 + I_{ic}(R_i/r_i)^2$.

The force F_i which is opposed to motion of the free element consists of the viscous F_{iv} and the rolling resistance F_{ir}

$$F_i = F_{iv} + F_{ir} \qquad F_{iv} = n_i R_i^2 \dot{\alpha}_i$$
$$F_{ir} \cong \frac{1}{r_i} m_i (\omega + \dot{\alpha}_i)^2 f_i R_i^2 \tag{3.3}$$

where n_i is the coefficient of viscous resistance, f_i is the coefficient of the rolling resistance for the free element, and r_i is the radius of the ball or roller.

Some of the components in Eqs. (3.1) can be neglected because they are very small with respect to the others. The simulation of the behavior of the system shows that it is possible. Now Eqs. (3.1) can be written in a matrix form

$$\mathbf{M}\ddot{\boldsymbol{q}} + (\mathbf{C}_d + \mathbf{C}_g)\dot{\boldsymbol{q}} + \mathbf{K}\boldsymbol{q} = \boldsymbol{Q}_o(\omega t) + \sum_{i=1}^{N} \boldsymbol{Q}_i(\omega t, \alpha_i, \dot{\alpha}_i) \tag{3.4}$$

where $\boldsymbol{Q}_o(\omega t)$ is the harmonic excitation from the unbalance, $\boldsymbol{Q}_i(\omega t, \alpha_i)$ is the reaction of one free element on the rotor. The components of the first one depend on the speed of the rotor and the static and dynamic unbalance of the rotor. The components of $\boldsymbol{Q}_i(\omega t, \alpha_i)$ are a function of the angle of rotation and the positions of the free elements with respect to the rotor. The matrices of the inertia of the rotor $\mathbf{M}$, damping $\mathbf{C}_d$, and the stiffness $\mathbf{K}$ have the dimension 4×4. The inertial forces generated by the accelerations $\ddot{\alpha}_i$ are very small with respect to the free element centrifugal forces (Gawlak and Majewski, 1991; Majewski, 1976, 1978) and can be neglected in Eqs. (3.1), thus

$$\boldsymbol{Q}_o = \omega^2 [Me\cos\omega t, Me\sin\omega t, Md\cos(\omega t - \varepsilon), Md\sin(\omega t - \varepsilon)]^\top$$
$$\boldsymbol{Q}_i = m_i R_i (\omega + \dot{\alpha}_i)^2 [\cos\beta_i, \sin\beta_i, z_i\sin\beta_i, -z_i\cos\beta_i]^\top \tag{3.5}$$

Equation (3.2) can be written in the form

$$I_i\ddot{\alpha}_i = m_i R_i \boldsymbol{B}_i \ddot{\boldsymbol{q}} - F_i = \overline{P}_i - F_i \qquad i = 1, \dots, N \tag{3.6}$$

where

$$\overline{P}_i = m_i R_i \boldsymbol{B}_i \ddot{\boldsymbol{q}} \qquad \boldsymbol{B}_i = [\boldsymbol{B}_{ic}\cos\omega t - \boldsymbol{B}_{is}\sin\omega t]$$
$$\boldsymbol{B}_{ic} = [\sin\alpha_i, -\cos\alpha_i, z_i\cos\alpha_i, z_i\sin\alpha_i]$$
$$\boldsymbol{B}_{is} = [-\cos\alpha_i, -\sin\alpha_i, z_i\sin\alpha_i, -z_i\cos\alpha_i]$$

Equations (3.4) and (3.6) describe the behavior of the rotor and the free elements during the balancing. They will be used for further analysis of the system dynamics.

4. Possibility of balancing

As a result of rotor vibrations there are inertial forces $P_1, \dots, P_N$ that push the free elements to new positions

$$\lim_{t \to \infty} \alpha(t) = \alpha_{if} \tag{4.1}$$

The system will be completely balanced with the free elements in these new positions when for any moment of time the rotor excitation is zero. In these positions, the free elements have to fulfill the following conditions

$$\boldsymbol{Q}_o(t) + \sum_{i=1}^{N} \boldsymbol{Q}_i(t, \alpha_{if}) \equiv \boldsymbol{0} \tag{4.2}$$

Conditions (4.2) represent the resultant force acting on the rotor. It can be written as

$$\Big[\boldsymbol{Q}_{co} + \sum_{i=1}^{N} \boldsymbol{Q}_{ci}(\alpha_{if})\Big] \cos \omega t - \Big[\boldsymbol{Q}_{so} + \sum_{i=1}^{N} \boldsymbol{Q}_{si}(\alpha i_{if})\Big] \sin \omega t \equiv \boldsymbol{0} \tag{4.3}$$

or

$$\boldsymbol{Q}_c \cos \omega t - \boldsymbol{Q}_s \sin \omega t \equiv \boldsymbol{0}$$

When the coefficients in front of the time functions are zero

$$\boldsymbol{Q}_{co} + \sum_{i=1}^{N} \boldsymbol{Q}_{ci}(\alpha_{if}) = \boldsymbol{0} \qquad \boldsymbol{Q}_{so} + \sum_{i=1}^{N} \boldsymbol{Q}_{si}(\alpha_{if}) = \boldsymbol{0} \tag{4.4}$$

then condition (4.2) occurs.

From this condition, we obtain

$$\begin{aligned} &Me + \sum_{i=1}^{N} m_i R_i \cos \alpha_{if} = 0 && \sum_{i=1}^{N} m_i R_i \sin \alpha_{if} = 0 \\ &Md - \sum_{i=1}^{N} m_i R_i z_i \sin(\alpha_{if} - \varepsilon) = 0 && \sum_{i=1}^{N} m_i R_i z_i \cos(\alpha_{if} - \varepsilon) = 0 \end{aligned} \tag{4.5}$$

For the first two conditions the mass center of the system is on the axes of rotation and for the next two conditions the axes of rotation become one of the principle axes of the inertia of the rotor.

Now the rotor excitation, which is presented by the right side of Eq. (3.4), is zero. When there is no excitation, the vibrations of the rotor disappear

$$\boldsymbol{q}(t, \alpha_{1f}, \dots, \alpha_{Nf}) \equiv \boldsymbol{0} \tag{4.6}$$

For the free elements distribution $\alpha_{1f}, \ldots, \alpha_{ft}$, the system is balanced and there is no vibration.

If we use, for example, two balls or rollers in each drum then from conditions (4.5) we can define their final positions $\alpha_{1f}, \ldots, \alpha_{4f}$. The total unbalance of the rotor can also be presented as two oblique forces in the planes of drums in which the free elements are located, Fig. 3. The resultant force from compensating elements in the drum compensate one of the oblique forces. If there are only two balls or rollers (one in each drum) then four conditions (4.5) can be fulfilled only if the masses of the balls are selected according to the rotor unbalance, Fig. 3. For one roller in each drum their masses should be selected according to the existing unbalance that is unknown. It is a rather theoretical problem.

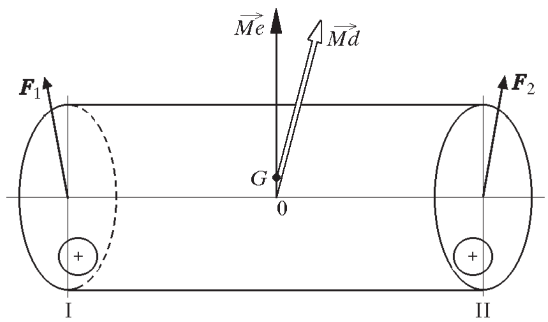

Fig. 3. Forces acting on the rotor

If the system has more than four free elements, then conditions (4.5) can be satisfied not only for one configuration of the balls but also for some other positions. In this situation the final positions of the free elements are not defined exactly. If all free elements are located in one drum, they are not able to compensate the rotor unbalance. The free elements should generate forces that are opposite to the unbalance. If the rotor has no unbalance, then the balls take positions for which they compensate each other. If the drum has two balls, then they occupy positions opposed to each other on one diameter. The static unbalance indicates the location of balls to be symmetrical with respect to the vector $\overrightarrow{Me}$. For the dynamic unbalance, the free elements occupy antisymmetric positions with respect to the vector $\overrightarrow{Md}$. The distance between the drums can not be too small because the moment generated by free elements would be too small to compensate the dynamic unbalance Md of the rotor.

Conditions (4.5) are necessary to obtain the balanced state of the system which consists of the rotor and free elements. It is necessary to prove that the free elements really tend to these positions. Solutions to differential equations (3.1) and (3.2) should give an answer as to which situations the free elements compensate or increase the rotor unbalance.

5. Numerical simulation

It is not possible to determine the exact solutions $\boldsymbol{q}(t)$, $\alpha_i(t)$ to equations (3.1) and (3.2). First results can be obtained from numerical solutions. A system with defined parameters was assumed, a software for simulation prepared and many simulations for different sets of parameters provided. The examples presented in the paper are presented for the following parameters: rotor mass – 4 kg, mass moment of inertia – $A \approx 0.0018\,\mathrm{kg\,m}^2$, $B \approx 0.014\,\mathrm{kg\,m}^2$, $R_1 = R_2 = 0.03\,\mathrm{m}$, $m_1 = m_2 = 0.00166\,\mathrm{kg}$, $z_1 = -z_2 = 80\,\mathrm{mm}$. The most important parameter was the ratio of the rotor velocity to its natural frequencies. The rotor velocity was taken from the range 20-100 rad/s. The diagrams of the coordinates describing the positions of the rotor and the free elements show in which way the free elements move with respect to the rotor. We can point out that the free elements compensate or increase the unbalance, we can also define the time that is necessary for the free elements to reach the final positions, and find put in which way the rotor vibrations change in time. The computer simulation also gives the answer in which way the free elements behave when they are close to the final positions.

Figure 4 shows the results for the rotor with only static unbalance and one ball in each of two drums. The masses of the balls were selected in such a way that they can compensate the rotor unbalance – the conditions (4.5) can be fulfilled. Velocity of the rotor is bigger than its natural frequencies. It is seen that the balls are going to the positions for which the system is dynamically compensated and the vibrations of the rotor vanish. The balls oscillate around the positions α_{1f}, α_{2f} and the time of decay of vibrations depends on the viscous damping in the drum.

In normal conditions, the rotor unbalance is not known and therefore in each drum there should be at least two free elements with masses that can compensate the biggest unbalance that can happen in the system. In Fig. 5, we can observe vibration of the rotor and motion of four balls for the same parameters as in the rotor shown in Fig. 4. The free elements need about $50/\omega$ - $100/\omega$ seconds to reach their final positions. When the balls are close to theirs final positions, the vibration of the rotor vanishes.

The frequency of balls oscillations around their final positions is much smaller than the rotor speed. The vanishing of balls oscillation depends on their resistance. At the final positions, the free elements generate forces that are opposed to the oblique forces in planes of the drums generated by the unbalance.

Other simulations show that for some speeds of the rotor that are close to its natural frequencies the free elements compensate only the static or dynamic

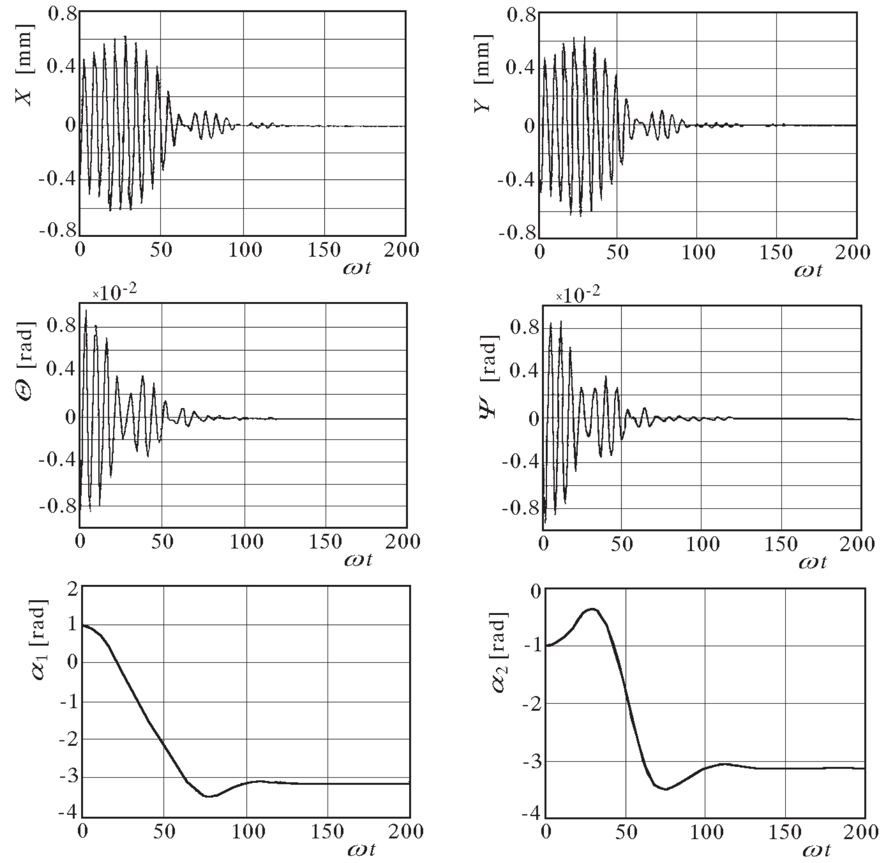

Fig. 4. Balancing with two balls

unbalance. Below natural frequencies, the free elements occupy positions close to the unbalance, and the rotor vibration increases.

6. Vibration forces

The inertial force (Eq. (3.6), Fig. 2) acting on the free element depends on the acceleration of the rotor and the position of the free element. It is defined by the relation

$$\overline{P}_i = m_i R_i \boldsymbol{B}_i \ddot{\boldsymbol{q}} \tag{6.1}$$

The free elements move slowly with respect to the rotor $(\dot{\alpha} \ll \omega)$ and the vibration of the rotor can be written as

$$\boldsymbol{q}(t) \cong \boldsymbol{A}_c \cos \omega t - \boldsymbol{A}_s \sin \omega t \tag{6.2}$$

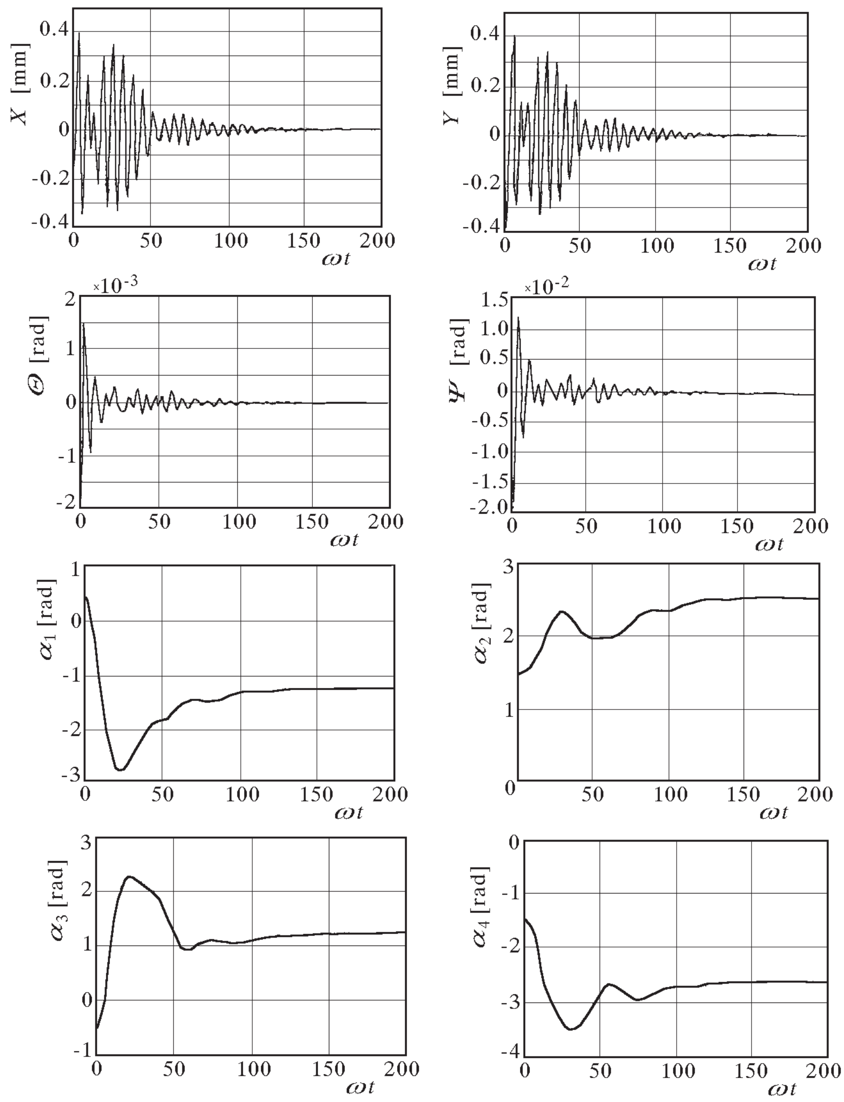

Fig. 5. Balancing with four balls

Accordingly, the acceleration of the vibration is also defined

$$\ddot{\boldsymbol{q}}(t) \cong -\omega^2 \boldsymbol{q}(t) \tag{6.3}$$

Substituting relations (6.3) to (6.1), the inertial force will be defined as a function of time, rotor unbalance and positions of all free elements. We can calculate the average magnitude of this force during one period of vibration $T = 2\pi/\omega$

$$P_i = \frac{1}{T}\int\limits_o^T \overline{P}_i\, dt = -\frac{1}{2} m_i R_i \omega^2 (\boldsymbol{B}_{ci}\boldsymbol{A}_c + \boldsymbol{B}_{si}\boldsymbol{A}_s) \tag{6.4}$$

where

$$\boldsymbol{A}_c = [A_{cx}, A_{xy}, A_{c\varPhi}, A_{c\varTheta}]^\top \qquad \boldsymbol{A}_s = [A_{sx}, A_{sy}, A_{s\varPhi}, A_{s\varTheta}]^\top$$

The amplitudes of vibration of the rotor are defined by a relation obtained from (3.4)

$$\begin{aligned} (\mathsf{K} - \mathsf{M}\omega^2)\boldsymbol{A}_c - \mathsf{C}\omega\boldsymbol{A}_s &= \omega^2 \boldsymbol{P}_c \\ \mathsf{C}\omega\boldsymbol{A}_c + (\mathsf{K} - \mathsf{M}\omega^2)\boldsymbol{A}_s &= \omega^2 \boldsymbol{P}_s \end{aligned} \tag{6.5}$$

where

$$P_c = [P_{c1}, P_{c2}, P_{c3}, P_{c4}]^\top \qquad P_s = [P_{s1}, P_{s2}, P_{s3}, P_{s4}]^\top$$

and

$$\begin{aligned} P_{c1} &= -P_{s2} = Me + \sum_{i=1}^{N} m_i R_i \cos\alpha_i \\ P_{c2} &= P_{s1} = \sum_{i=1}^{N} m_i R_i \sin\alpha_i \\ P_{c3} &= -P_{s4} = Md\cos\varepsilon - \sum_{i=1}^{N} m_i R_i z_i \sin\alpha_i \\ P_{c4} &= P_{s3} = -Md\sin\varepsilon + \sum_{i=1}^{N} m_i R_i z_i \cos\alpha_i \end{aligned}$$

are component forces acting on the rotor. Each component depends on the static and dynamic unbalance and positions of all free elements with respect to the rotor.

The vibration force of one element can be presented as a sum of four forces generated by four components of the rotor vibration

$$P_i = P_{ix} + P_{iy} + P_{i\varPhi} + P_{i\varTheta} \tag{6.6}$$

and each component is a result of the rotor unbalances Me, Md, and the unbalance given by each free element, e.g.

$$P_{ix} = P_{ixMe}(\alpha_i) + P_{ixMd}(\alpha_i) + \sum_{j=1}^{N} P_{ijx}(\alpha_i - \alpha_j) \tag{6.7}$$

where

$$P_{ixMd} = -\frac{1}{2}m_i R_i \omega^2 a_{xMe} \sin(\alpha_i + \varphi_x)$$
$$P_{ixMd} = -\frac{1}{2}m_i R_i \omega^2 a_{xMd} \sin(\alpha_i + \varepsilon + \varphi_x)$$
$$P_{ijx} = -\frac{1}{2}m_i R_i \omega^2 a_{xj} \sin(\alpha_i - \alpha_j + \varphi_x)$$

a_{xMe}, a_{xMd}, a_{xj} are amplitudes of the rotor vibration in the x direction generated by the static and dynamic unbalance as well as the element j.

Other components of the vibration forces are similar.

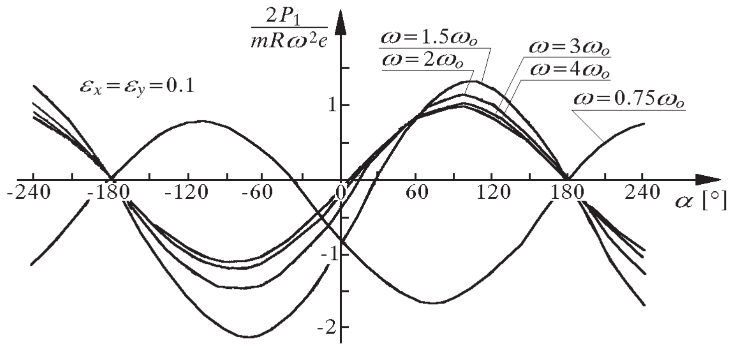

Fig. 6. The vibration force as a function of one free element position

The vibration force P_i acting on each free element is a function of rotor vibrations that are a function of the unbalance of the rotor and the positions of all free elements. Figure 6 shows the force P_{1x} if there is one ball and only static unbalance – balancing in one plane

$$P_{1x} = -\frac{1}{2}m_1 R_1 \omega^2 [a_{xMe} \sin(\alpha_i + \varphi_x) + a_{x1} \sin(\alpha_1 - \alpha_1 + \varphi_x)] \tag{6.8}$$

It is seen that the ball has two positions for which the force P_1 can assume the zero magnitude. One position is exactly at $\alpha_{1f} = \pi$ which fulfills conditions (4.5) and the system is dynamically balanced. For the other position, the free element increases the unbalance of the system. Which of these positions the ball will occupy depends on the velocity of the rotor. Only one of them is a dynamically stable equilibrium. It can be observed in Fig. 6 that for one position of the ball, the derivative of the force with respect to the angular displacement is negative and it is in the position of equilibrium.

For balancing in two planes, there should be at least one ball or roller in each plane, and the forces P_1, P_2 are functions of two variables α_1, α_2. Figure 7 presents the force P_2 changing with the position of both balls. It is seen that there are many positions in which the force acting on one ball disappears. They are the possible positions of equilibrium of the ball. But the force acting on the second ball is not zero. The second ball moves with respect to the rotor which makes the first ball cannot stay in the previous position. The balls can occupy only such positions for which all vibration forces $P_1, \ldots, P_N$ become simultaneously zero.

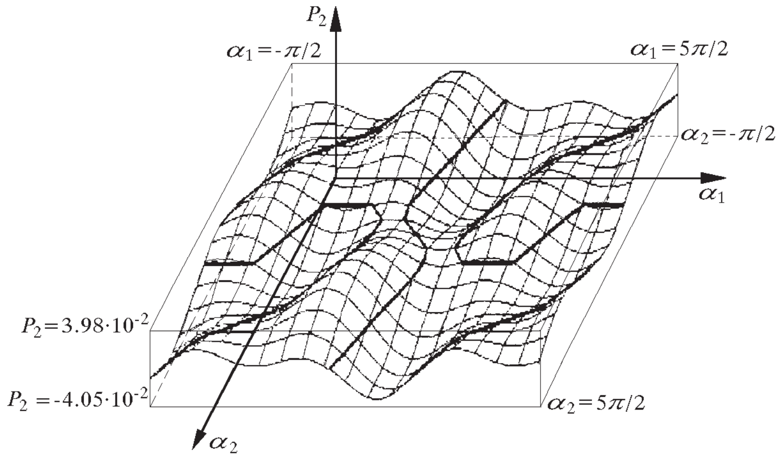

Fig. 7. The vibration force as a function of two free element position

If we know the vibration forces, then we can explain why the free elements change their positions, in what direction they go, and what final positions they occupy. It will allow us to verify the dynamic stability of these positions and define the efficiency of the method.

7. Stability

Sometimes the balls compensate the rotor unbalance and sometimes they increase it. To define properties of the system, it is necessary to analyze the dynamic stability of the balls in their final positions $\alpha_{1f}, \ldots, \alpha_{Nf}$. It can be done if the forces acting on the balls are known (6.4). The final positions of the balls are stable if the roots λ of determinant (7.1) have negative real parts

$$\Big|\frac{\partial P_i}{\partial \alpha_j} - \kappa_{ij}\lambda\Big| = 0 \qquad i,j = 1,\ldots,N \tag{7.1}$$

where κ_{ij} is the Kronecker symbol. The derivative of the vibration force with respect to the position of the ball can be obtained from (6.4)

$$\frac{\partial P_i}{\partial \alpha_j} = -\frac{1}{2} m_i R_i \omega^2 \Big[\Big(\frac{\partial \boldsymbol{B}_{ic}}{\partial \alpha_j} \boldsymbol{A}_c + \frac{\partial \boldsymbol{B}_{is}}{\partial \alpha_j} \boldsymbol{A}_s \Big) + \Big(\boldsymbol{B}_{ic} \frac{\partial \boldsymbol{A}_c}{\partial \alpha_j} + \boldsymbol{B}_{is} \frac{\partial \boldsymbol{A}_s}{\partial \alpha_j} \Big) \Big] \tag{7.2}$$

At the final positions of the free elements $\alpha_{1f}, \dots, \alpha_{Nf}$, when the system is balanced $\boldsymbol{A}_c(\alpha_{1f}, \dots, \alpha_{Nf}) = \boldsymbol{A}_s(\alpha_{1f}, \dots, \alpha_{Nf}) = \boldsymbol{0}$, the above relation takes the form

$$\frac{\partial P_i}{\partial \alpha_j} = -\frac{1}{2} m_i R_i \omega^2 \Big[\boldsymbol{B}_{ic} \frac{\partial \boldsymbol{A}_c}{\partial \alpha_j} + \boldsymbol{B}_{is} \frac{\partial \boldsymbol{A}_s}{\partial \alpha_j} \Big] \tag{7.3}$$

The vibration of the rotor is a composition of vibrations from the rotor unbalance and the action of all free elements. Then

$$\frac{\partial \boldsymbol{A}_c}{\partial \alpha_j} = \frac{\partial \boldsymbol{A}_{cj}}{\partial \alpha_j} \qquad\qquad \frac{\partial \boldsymbol{A}_s}{\partial \alpha_j} = \frac{\partial \boldsymbol{A}_{sj}}{\partial \alpha_j}$$

The derivative of the amplitude, with respect to the position of the free elements, can be defined from relations (6.5)

$$\begin{aligned} [\mathbf{K} - \mathbf{M}\omega^2] \frac{\partial \boldsymbol{A}_{cj}}{\partial \alpha_j} - \omega \mathbf{C} \frac{\partial \boldsymbol{A}_{sj}}{\partial \alpha_j} &= \frac{\partial \boldsymbol{Q}_{cj}}{\partial \alpha_j} \\ \omega \mathbf{C} \frac{\partial \boldsymbol{A}_{cj}}{\partial \alpha_j} + [\mathbf{K} - \mathbf{M}\omega^2] \frac{\partial \boldsymbol{A}_{sj}}{\partial \alpha_j} &= \frac{\partial \boldsymbol{Q}_{sj}}{\partial \alpha_j} \end{aligned} \tag{7.4}$$

Developing determinant (7.1), we obtain the characteristic equation

$$a_n \lambda^n + a_{n-1} \lambda^{n-1} + \ldots + a_1 \lambda + a_o = 0 \tag{7.5}$$

The roots of the above equation have negative real parts if all the coefficients are positive and the principle minors of determinates (7.5) are also positive

$$\varDelta_i > 0 \qquad \text{for} \qquad i = 2, \dots, N-1 \tag{7.6}$$

The coefficients $a_n, \dots, a_o$ of determinant (7.5) are a function of the rotor speed. Depending on the rotor velocity, the roots λ can be positive or negative, and condition (7.1) can be fulfilled or not. Detailed analysis of condition (7.1) allows us to define the ranges in which the state of full balance of the rotor can be obtained.

If we take the simplest situation, e.g. only static unbalance of the rotor, with one free element located in the plane of unbalance $(z_1 = 0)$, then relation (7.1) gives

$$\lambda = -\frac{1}{2} m R \omega^2 \Big[\boldsymbol{B}_{1c} \frac{\partial \boldsymbol{A}_{1c}}{\partial \alpha_1} + \boldsymbol{B}_{1s} \frac{\partial \boldsymbol{A}_{1s}}{\partial \alpha_1} \Big] \tag{7.7}$$

The system can be balanced if the free element has the mass $m = Me/R$ and it should occupy the position opposed to the unbalance $\alpha_{1f} = \pi$. For these parameters the vectors $\boldsymbol{B}_{1c}$, $\boldsymbol{B}_{1s}$ have the form

$$\boldsymbol{B}_{ic} = [0,1,0,0] \qquad \boldsymbol{B}_{is} = [1,0,0,0]$$

To make the problem simpler, we can neglect damping of the rotor, and now the derivatives of the amplitudes take the form

$$\begin{aligned} &\frac{\partial A_{cx1}}{\partial \alpha_1} = c_x \cos\alpha_{1f} = -c_x && \frac{\partial A_{sx1}}{\partial \alpha_1} = c_x \sin\alpha_{1f} = 0 \\ &\frac{\partial A_{cy1}}{\partial \alpha_1} = c_y \sin\alpha_{1f} = 0 && \frac{\partial A_{sy1}}{\partial \alpha_1} = c_y \cos\alpha_{1f} = -c_y \end{aligned} \tag{7.8}$$

where

$$c_x = \frac{s_x^2}{1-s_x^2}\frac{m}{M}R \qquad c_y = \frac{s_y^2}{1-s_y^2}\frac{m}{M}R \qquad s_x = \frac{\omega}{\omega_{ox}} \qquad s_y = \frac{\omega}{\omega_{oy}}$$

The coefficient c_x is negative when the rotor speed is greater than its natural frequency ω_{ox} in the direction x. We have the same situation with the coefficient c_y which is negative for $\omega > \omega_{oy}$, and condition (7.7) takes the form

$$\lambda = \frac{1}{2}mR\omega^2(c_x + c_y) < 0 \tag{7.9}$$

It is seen that when the rotor speed is greater than $\omega > \omega_{ox}$ and $\omega > \omega_{oy}$, the above condition is fulfilled $(c_x < 0, c_y < 0)$ and the position of the free element $\alpha_{1f} = \pi$ is stable. For the rotor speed $\omega < \omega_{ox}$ and $\omega < \omega_{oy}$, the position of the free element opposed to the unbalance is unstable and the free element cannot compensate the rotor unbalance. Using relations (7.8) and (7.9), it is also possible to define the rotor speed from the range between the minimum and the maximum natural frequency of the rotor $\omega_{o\,min} - \omega_{omax}$ in which the position $\alpha_{1f} = \pi$ is also stable.

If there are more free elements placed in different planes $(z_i \neq 0)$, the directions of vibrations are coupled, there exist static and dynamic unbalances of the rotor, then condition (7.1) is more complicated. The exact range of the speed of the rotor in which the positions of the free elements $\alpha_{1f}, \ldots, \alpha_{Nf}$ are stable or unstable can be defined from numerical simulation of the relations (7.1)-(7.3). If they can stay in theses positions, it means that they can compensate the dynamic forces generated by the rotor and the system will be balanced.

The presented system can balance itself automatically if the rotor speed is greater than the natural frequencies. It is also possible in a small range of the rotor speed between the minimum and maximum natural frequencies.

8. Efficiency of the method

If we do not take into consideration the resistance of the free elements in their motion with respect to the drum, then they will occupy the final positions $\alpha_{1f},\dots,\alpha_{Nf}$, and the system that consists of the rotor and free elements can be fully balanced. The viscous resistance does not affect the final positions because it exists only when the free elements move with respect to the rotor. The viscous resistance can only reduce velocity of the free elements while they approach the final positions and suppresses their vibrations about the final positions. Balls or rollers can be taken as the free elements. They roll with respect to the rotor. The normal reaction between the ball or roller and the circular path is a function of the angular velocity of the rotor. This reaction is also a function of rotor vibrations. But when the free elements are close to the final positions the vibrations are very small and the normal reaction can be taken as constant. The rolling resistance depends on the normal reaction and the coefficient of the rolling resistance. The latter one is a function of the rigidity of the element and the rigidity of the circular path. As a result of the resistance, there will be a new position of equilibrium of the compensating element that differences by $\varDelta_i$ with respect to the "ideal" position α_{if}. If the free elements are not in the positions $\alpha_{1f},\dots,\alpha_{Nf}$, then they do not completely compensate the static and dynamic unbalance of the rotor. So, there is a residual unbalance and residual vibrations of the rotor. They are a function of the speed of the rotor and the coefficient of the rolling resistance. If the vibration forces are known, then the errors in the positioning of the free elements can be defined, and in the next step the residual unbalance of the system can be calculated.

The position of the i-th element is the position of equilibrium when

$$\Big|P_i(\alpha_{1f}+\varDelta_1,\dots,\alpha_{nf}+\varDelta_n)\Big| - \Big|F_{ir}\Big| \leqslant 0 \qquad i=1,\dots,N \tag{8.1}$$

where

$$F_{ir} = \frac{m_i R_i \omega^2 f_i}{r_i \operatorname{sgn}(\dot{\alpha}_i)} \qquad\qquad P_i(\alpha_{1f},\dots,\alpha_{Nf}) = 0$$

For a small displacement $\varDelta_i$ of the free element with respect to its theoretical position α_{if}, it can be taken as

$$\begin{aligned} P_i(\alpha_{1f}+\varDelta_1,\dots,\alpha_{Nf}+\varDelta_N) &\cong P_i(\alpha_{1f},\dots,\alpha_{Nf}) + \sum_{j=1}^{N}\varDelta_j\frac{\partial P_i}{\partial\alpha_j}\Big|_{\alpha_{jf}} = \\ &= \sum_{j=1}^{N}\varDelta_j\frac{\partial P_i}{\partial\alpha_j}\Big|_{\alpha_{jf}} \end{aligned} \tag{8.2}$$

The derivatives of the vibration forces with respect to the positions of the free elements are defined by relation (7.3). Relation (8.1) changes into

$$\Big| -\frac{1}{2}m_iR_i\omega^2\sum_{j=1}^{N}\Big(B_{ic}\frac{\partial A_{cj}}{\partial \alpha_j}+B_{is}\frac{\partial A_{sj}}{\partial \alpha_j}\Big)\varDelta_j\Big| - \Big|F_{ir}\Big| \leqslant 0 \tag{8.3}$$

It is a system of algebraic equations from which the displacements of the free elements can be defined $(i = 1, \dots, N)$

$$\begin{aligned}\Big| -\frac{1}{2}m_iR_i\omega^2\Big[\sin\alpha_{if}\sum_{j=1}^{N}\Big(\frac{\partial A_{cxj}}{\partial \varDelta_j}-z_i\frac{\partial A_{c\varPsi j}}{\partial \varDelta_j}-\frac{\partial A_{syj}}{\partial \varDelta_j}+z_i\frac{\partial A_{s\varTheta j}}{\partial \varDelta_j}\Big)\varDelta_j + \\ +\cos\alpha_{if}\sum_{j=1}^{N}\Big(-\frac{\partial A_{cyj}}{\partial \varDelta_j}+z_i\frac{\partial A_{c\varTheta j}}{\partial \varDelta_j}-\frac{\partial A_{sxj}}{\partial \varDelta_j}-z_i\frac{\partial A_{s\varPsi j}}{\partial \varDelta_j}\Big)\varDelta_j\Big]\Big| - |F_{ir}| \leqslant 0\end{aligned} \tag{8.4}$$

When the positioning deviation $\varDelta_i$ is not a small, then equations (8.1) should be used.

Equations (8.3) or (8.4) define the range $\varDelta_{i\ min}$-$\varDelta_{i\ max}$ in which the i-th free element can occupy its position. Because of the rolling resistance, the deviation of the free element is random and it depends on the initial conditions.

For the rotor with the static unbalance only, vibration of the rotor center in the two directions x and y and one free element with the mass $mR = Me$, relation (8.5) gives

$$-\frac{2|F_r|}{mR\omega^2|A_x\cos\varphi_x + A_y\cos\varphi_y|} < \varDelta_1 < \frac{2|F_r|}{mR\omega^2|A_x\cos\varphi_x + A_y\cos\varphi_y|} \tag{8.5}$$

The deviation $\varDelta_1$ is proportional to the resistance F_r. If the resistance of the free element increases, then the error of the positioning increases and residual unbalance of the system increases as well. The errors of positioning are a function of rotor amplitudes of vibrations and the shifts angles φ_x, φ_y that are a function of the rotor speed.

Figure 8 shows the maximum errors in the positioning that can happen if two balls were used to balance the system. They are a function of the rotor speed. It is seen that the errors of positioning are smaller for a speed a little greater than the natural frequencies of the rotor.

The rolling resistance is much smaller than the sliding friction. To obtain the smallest errors of the method, we should use the free elements as balls and rollers that can roll instead of slipping. Using other elements as sand, shots, or liquid involve the slipping and greater friction. The rotor with a liquid cannot be completely balanced. Under vibration forces, the liquid moves to the compensating position. But when the system decreases its unbalance,

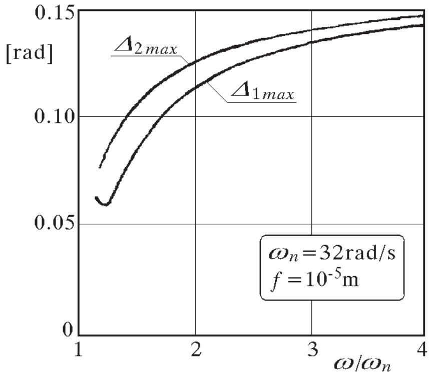

Fig. 8. Maximum errors in the positioning of free elements

then the vibrational forces also vanish and the liquid moves back (Gawlak and Majewski, 1991).

The system with the free elements in positions $\alpha_{1f} + \Delta_1, \dots, \alpha_{Nf} + \Delta_N$ is not completely balanced. If the deviations $\Delta_1, \dots, \Delta_N$ are known, then the final unbalance of the system can be defined. The residual unbalance is a function of the deviations of all free elements. The residual unbalance will be defined in the coordinates $x0y$ fixed with the rotor.

The components of the residual static unbalance have the form

$$\begin{aligned} Me_{Rx} &= Me + \sum_{i=1}^{N} m_i R_i \cos(\alpha_{if} + \Delta_i) \\ Me_{Ry} &= \sum_{i=1}^{N} m_i R_i \sin(\alpha_{if} + \Delta_i) \end{aligned} \tag{8.6}$$

The total static unbalance is

$$Me_R = \sqrt{Me_{Rx}^2 + Me_{Ry}^2} \tag{8.7}$$

When the deviation Δ_i is small then

$$\begin{aligned} Me_{Rx} &\cong -\sum_{i=1}^{N} m_i R_i \Delta_i \sin\alpha_{if} \\ Me_{Ry} &\cong -\sum_{i=1}^{N} m_i R_i \Delta_i \cos\alpha_{if} \end{aligned} \tag{8.8}$$

The residual dynamic unbalance

$$\begin{aligned} Md_{Rx} &= Md\cos\gamma - \sum_{i=1}^{N} m_i R_i z_i \sin(\alpha_{if} + \varDelta_i) \\ Md_{Ry} &= Md\sin\gamma + \sum_{i=1}^{N} m_i R_i z_i \cos(\alpha_{if} + \varDelta_i) \end{aligned} \tag{8.9}$$

For small errors of the positioning of the free elements, the above relations can be written as

$$\begin{aligned} Md_{Rx} &\cong -\sum_{i=1}^{N} m_i R_i z_i \varDelta_i \cos\alpha_{if} \\ Md_{Ry} &\cong -\sum_{i=1}^{N} m_i R_i z_i \varDelta_i \sin\alpha_{if} \end{aligned} \tag{8.10}$$

The total residual dynamic unbalance is

$$Md_R = \sqrt{Md_{Rx}^2 + Md_{Ry}^2} \tag{8.11}$$

The greater the deviations $\varDelta_1, \ldots, \varDelta_N$, the bigger the system unbalance. The total residual unbalance changes with the rotor speed in the same way as the errors in the positioning of the free elements. Because of the resistance and friction, the free elements are shifted with respect to the theoretical positions $\alpha_{1f}, \ldots, \alpha_{Nf}$, and therefore the system cannot be completely balanced. The smallest resistance results in the smallest residual unbalance.

9. Experiments

To verify the results from theoretical investigation, a laboratory stand was built and experiments were carried out – Fig. 9. Rotor 1 ($M = 4.7\,\mathrm{kg}$, $B \approx 72\cdot 10^{-4}\,\mathrm{kg\,m^2}$) is supported on springs 2. At the end of the rotor, there are two drums 3 with two balls 4 in each drum. Each ball has mass of $m = 21.7\cdot 10^{-3}\,\mathrm{kg}$ and radius $r = 8.75\,\mathrm{mm}$. The drum has radius $R+r = 34.75\,\mathrm{mm}$. The distance between the drums is $l = 240\,\mathrm{mm}$. The rotor unbalance could be changed by changing the number and the positions of bolts 5. The distance between the planes of the bolts is 105 mm.

A series of experiments were carried out with different initial positions of the balls. The balls were kept in these positions by a mechanical or electromechanical blocking system. The rotor was driven by an electric motor and we

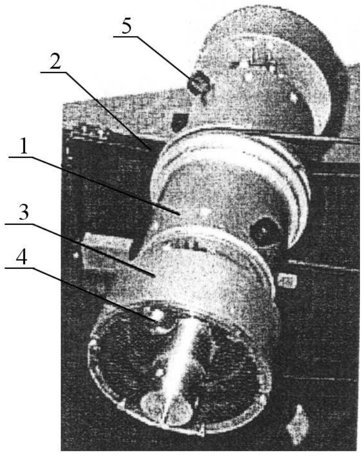

Fig. 9. The laboratory stand for verification of the automatic balancing; 1 – rotor, 2 – springs, 3 – drums, 4 – balls, 5 – bolts

could regulate its speed. When the velocity of the rotor was constant, the motor was disconnected from the rotor, the balls were released and they changed positions with respect to the rotor. The vibrations of the rotor were being recorded and motion of the balls was observed in stroboscopic light. Oscillations in the horizontal direction of the rotor bearings in one of the experiments with the rotor speed greater than its natural frequencies are shown in Fig. 10.

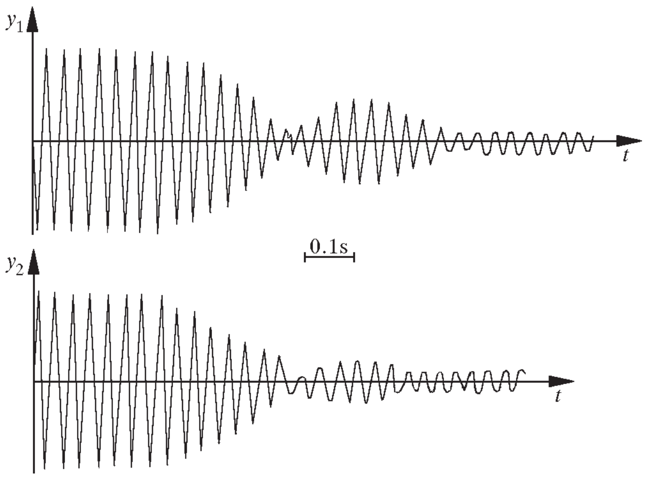

Fig. 10. Vibrations of rotor bearings during automatic balancing

The vibrations vanish when the balls are close to their final theoretical positions. The vibrations do not decay completely because there is a residual unbalance as a result of the errors in the positioning of the balls. In the final positions, the balls were blocked again. For the given rotor unbalance, theoretical final positions of the balls were determined. The initial and final positions of the balls for three experiments with the same dynamic unbalance are given in Table 1. For this particular rotor unbalance, the balls should take the following positions to balance the system; $\alpha_{1f} = 104°$, $\alpha_{2f} = 256°$, $\alpha_{3f} = 76°$, $\alpha_{4f} = 284°$.

Table 1

No.	Initial position [°]				Final position [°]			
	α_1	α_2	α_3	α_4	α_{1t}	α_{2t}	α_{3t}	α_{4t}
1	160	200	20	340	110	240	75	280
2	240	280	250	290	80	232	95	300
3	20	60	60	100	90	230	90	270

In these experiments, the minimum error in the positioning of the balls was $\Delta_{min} = 4°$ and the maximum error $\Delta_{max} = 26°$. When the rotor had no unbalance, then the balls occupied the opposite positions on one diameter in the drum and in this way they compensated each other.

The experiments show that the balls try to balance the rotor but they do not occupy a constant final position. They also vibrate about them. The author explained the reason for this vibration in Majewski (1988). The vibration is caused by the eccentricity of the circular path of the ball.

10. Conclusion

It was shown that the balls can organize themselves in such a way that they compensate the unknown rotor unbalance. It is the greatest advantage of the method. An unbalanced system generates vibration, and because of the inertial forces the free elements move in the direction generating dynamic forces opposed to the rotor unbalance. The changing of the positions of the free elements takes place as long as the unbalance exists. The free elements occupy the final positions for which there are no vibrations. The principle of the method can be presented with the block diagram shown in Fig. 11.

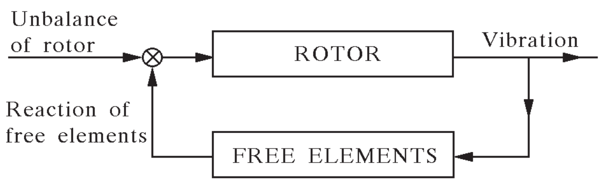

Fig. 11. A block diagram of the method

The behavior of the balls depends on the vibration forces. These forces were defined as a function of the positions of the balls, the rotor unbalance, and the rotor speed. If the vibration forces are known, then the setting of the balls with respect to the rotor can be defined. It is also possible to check if they are dynamically stable or not in these positions, what errors can happen in their positioning, and what is the efficiency of this method.

There are many factors that make the efficiency of the self-balancing smaller than 100%. The most important is friction of the balls or rollers. The resistance should be as small as possible and, therefore, it would be better when the free elements are supported by a magnetic or pneumatic cushion.

The other reason for smaller efficiency is the eccentricity of the drum in which the balls are located. This problem was not discussed here. A related research was given by Majewski (1988).

In this paper, a rigid rotor is analyzed. If the rotor unbalance and compensating elements are in different planes, then the rotor undergoes bending because of the centrifugal forces from the unbalance and the free elements. The bending can deform the rotor which is another reason that the system cannot be balanced completely.

This method can be also used for balancing the system in the case of an accident, e.g. the loss of a blade in a turbine or compressor.

According to conditions (4.5), it is not possible to completely balance the rotor that rotates about a fixed point. There are two angular components of vibration. The vibrations vanish when the resultant moment from the unbalance and free elements about the point of rotation are zero. It does not mean the system is balanced. At the point of the rotor support, there is a dynamic reaction that rotates with the rotor. So, this kind of the system eliminates vibrations but does not eliminate the excitation completely. This feature, however, can be used for elimination of vibrations in general. The idea of a synchronous eliminator was presented by the author in Majewski (1987, 1994, 2000a,b). It is a self-organizing system. The system detects the unbalance and changes the configuration of the free elements to eliminate the vibration.

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Automatyczna kompensacja sił dynamicznych w dwóch płaszczyznach dla sztywnego wirnika

Streszczenie

W pracy przedstawiono metodę automatycznego równoważenia sił dynamicznych działających na sztywny niewyważony wirnik. W tym celu użyto swobodnych elementów (kulek lub rolek) umieszczonych w dwóch płaszczyznach. Elementy swobodne wirujące razem z wirnikiem mogą zmieniać swoje położenie i generują siły, które mogą zrównoważyć niewyważenie wirnika. W pracy przedstawiono model fizyczny i matematyczny. Ustalono położenia elementów swobodnych, w których równoważą niewyważenie wirnika. Przeprowadzono symulację komputerową, która wykazała że elementy swobodne rzeczywiście dążą do tych położeń. Zdefiniowano siły wibracyjne, które wymuszają zmianę położenia elementów swobodnych względem wirnika. Wykazano, że siły wibracyjne zanikają, gdy elementy znajdują się w położeniach równoważących niewyważenie. Zbadano stateczność położeń końcowych. Analizowano wpływ oporów ruchu na końcowy stan wyważenia wirnika. Zbudowano stanowisko laboratoryjne i przeprowadzono eksperymenty, które wykazały, że elementy swobodne rzeczywiście przemieszczają się do położeń, w których równoważą siły dynamiczne działające na wirnik.

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