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# THE OFF-DESIGN PERFORMANCE PREDICTION OF AXIAL COMPRESSOR BASED ON A 2D APPROACH

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The two-dimensional compressor flow simulation approach has always been a very valuable tool in compressor preliminary design studies, as well as performance predictions. In this context, a general development of the streamline curvature (SLC) method is elucidated firstly. Then a numerical method based on SLC is developed to simulate the internal flow of the compressor according to the development analysis and conclusion. Two certain transonic axial compressors are calculated by this 2D method. The speed lines and span-wise aerodynamic parameters are compared with the experiment data in order to demonstrate the method presented in this paper.

*Key words:* two-dimensional, compressor, streamline curvature, deviation, loss model

## Nomenclature

$c$ – blade chord
$C, V$ – absolute and relative velocity, respectively
$C_D$ – drag coefficient
$D$ – diffusion factor
$G$ – mass flow
$I$ – relative stagnation enthalpy
$S$ – entropy
$M$ – Mach number
$P, T$ – static pressure and static temperature, respectively
$i, k, \beta, \varphi, \phi, \theta$ – incidence, blade, relative flow, deflection, sweep and camber angle (including circumferential direction), respectively
$m, n$ – direction of meridional and of computation station
$z, r$ – axial and radial (including radius) direction, respectively
$\gamma$ – specific heats ratio
$\sigma$ – solidity
$\rho$ – density
$\omega$ – rotational speed
$\Theta$ – momentum thickness

Subscripts: 1, 2 – upstream and downstream of blade row, respectively.

## 1. Introduction

In the last two decades, with the advances of computer resources and computational fluid dynamics (CFD), 3D viscous based on unsteady Reynolds-average Navier-Stokes (RANS) has been widely applied to simulation and analysis of the compression systems. Denton and Dawes (1999) suggested that "little has changed" on the streamline curvature approach because of the CFD development. The computational precision of SLC heavily depends on prediction models. According to the history of SLC development, incidence and deviation have been changed less than the loss recently. A great deal of effort is used to improve or develop the loss model and shock loss, especially to increase accuracy for a variety of compressor cascades (Aungier, 2003; Boyer, 2003; Pachidis *et al.*, 2006; Swan, 1961).

In the present work, the general development of SLC is elucidated at first. According to analysis of it, one numerical method is set up which considers main factors of SLC in maximum. The deviation is set up based on the reference minimum incidence and considers the main impacts at off-design points for a transonic compressor. The total pressure loss consists of profile loss, secondary loss and shock loss. Every component is calculated respectively at the design and off-design points. A certain transonic axial rotor is calculated and analysised by this 2D method at first. Beside that, one stage compressor is also calculated. The speed lines and span-wise aerodynamic parameters are compared with the experimental data. The result validates that the method presented in this paper is correct and applicable.

## 2. Development of SLC

The SLC method basically solves the discrete equation of full radial equilibrium equation incorporating many models and equations such as deviation and the loss model, state and continuity equations, etc. on computational grids which are constructed in the meridian plane through streamlines and stations (see Fig. 1). The governing equations are derived from the well-known Euler equations

$$\begin{aligned}\frac{\partial V_m}{\partial n} &= \frac{1}{V_m}\Big[\frac{\partial I}{\partial n} - T\frac{\partial S}{\partial n} - \Big(\frac{C_\theta}{r} - \omega\Big)\frac{\partial (rC_\theta)}{\partial n}\Big] + V_m\cos(\phi-\varphi)\frac{\partial \phi}{\partial m} \\ &\quad - \frac{\sin(\phi-\varphi)}{1-M_m^2}V_m\Big[(1-M_\theta^2)\frac{\sin\phi}{r} - \tan(\phi-\varphi)\frac{\partial\phi}{\partial m} + \frac{1}{\cos(\phi-\varphi)}\frac{\partial\phi}{\partial n}\Big] \\ G &= \int\limits_H^T 2\pi r\rho V_m\cos(\phi-\varphi)\,dn\end{aligned} \tag{2.1}$$

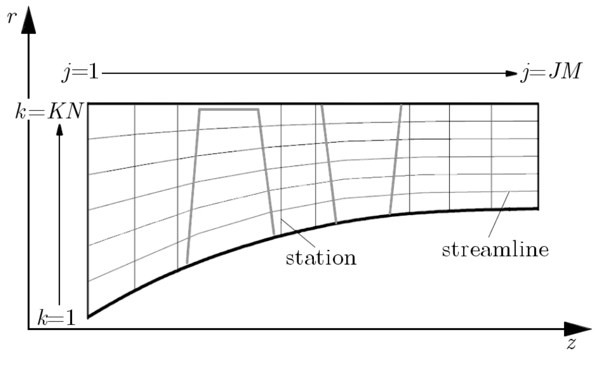


Fig. 1. The computational grids in the meridian plane

The simulation accuracy of SLC heavily depends on prediction models such as minimum loss, incidence angle, deviation angle, total pressure loss and blockage models etc. With the development of experimental equipments and methods, many model modifications or new correlations have appeared to accommodate modern compressors. But most of them are still developed from low-speed correlations of the 1950s.

NASA SP36 low-speed "reference condition" correlations presented by Lieblein (1957, 1959), Lieblein and Roudebush (1956) in the 1950s have been widely used in the deviation and loss prediction (see Fig. 2). Creveling and Carmody (1968), Cetin *et al.* (1987) recommended aseries of modifications for deviation calculation accounting for transonic, 3D effects and off-design condition. Swan (1961), Koch and Smith (1976) developed improved profile loss prediction methods according to Lieblein's work then. Considering additional losses caused by secondary flow, Koch and Smith (1976), Hearsey (1994), Aungier (2003) etc. developed many empirical and semi--empirical models and modifications. Especially, Hearsey's secondary loss model was established on the basis of the profile loss with different distribution along the blade span.

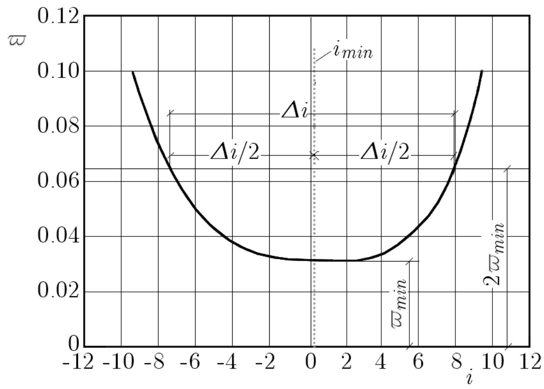


Fig. 2. Illustration of the reference condition

The shock loss is one indispensable part of the total loss at high Mach number in modern compression systems. Miller's normal shock loss (Miller etal, 1961) model has been very popular in shock loss calculation because of its simple algorithm. But the shock loss predicted by it is always larger than in fact at a high Mach number. According to passage shock geometries at different operating conditions in supersonic compressor cascades, recent works by König *et al.* (1994,a,b), Bloch *et al.* (1999) and Boyer (2003) demonstrated that the efficiency characteristic of transonic machines is largely determined by the shock loss. Some shock loss prediction algorithms of arbitrary shape cascades over the entire operating range were then proposed. More recently, Templalexis *et al.* (2006) and Pachidis *et al.* (2006) published several zooming research efforts using 2D SLC component models.

According to the analysis above, the development of SLC is mainly based on its models. The incidence is divided into two forms to a certain extent. One is the reference incidence, another is the minimum incidence. The deviation is mostly set up according to the reference incidence, and develops more slowly than loss. There are two main methods for loss calculation: a) the total loss at the design condition is calculated firstly, and loss at off-design is then calculated based on it, Mach number and incidence, b) the partial losses, such as profile, shock etc. are calculated firstly at every operating condition. The sum of them is the total loss. For calculation of the loss at the off-design condition, there are two reference criterions, which are based on the reference incidence (low-speed minimum incidence) and the minimum incidence. The recent representations are Swan, Pachidis, Boyer, Aungier's works.

## 3. Individual work

### 3.1. Numerical models

#### *3.1.1. Incidence angle*

In general, the SLC approach proceeds around one reference condition. Hence, the reference incidence appears and is used as the reference parameter. The other prediction models are established referring to it. The reference incidence is based on low-speed, two dimensional cascade data from NASA SP36 data or curves. Recently, Milan has introduced one modification which can be used to revise it. The correlations of flow angle and profile geometry parameters can be seen in Fig. 3. The main equations are given by

$$i = \beta_1 - k_1 \qquad i_{ref} = (K_i)_{sh}(K_i)_t(i_0)_{10} + n\theta + C_i \tag{3.1}$$

where $(i_0)_{10}$ represents the zero-camber, minimum loss incidence for 10% thickness NACA 65 series. The constant $(K_i)_{sh}$ and $(K_i)_t$ are thickness and thickness distribution correction factors for particular blade profiles. For example, $(K_i)_{sh}$ is equal to 0.7 for Multiple-circular-Arc type, 1 for NACA 65 series, and 1.1 for C4 series blades. In the present work, $(K_i)_t$ is calculated as a function of cascade thickness, $n$ is the slope of the variation in incidence with the camber angle $\theta$ and $C_i$ is the revision based on Milan *et al.* (2009).

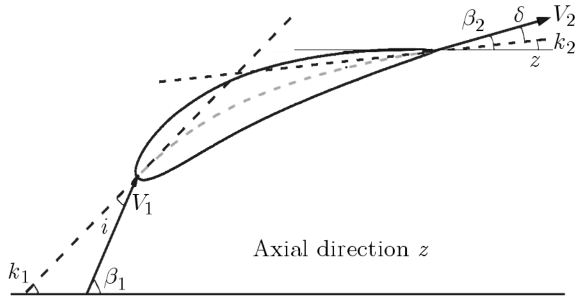


Fig. 3. The correlations between cascade and aerodynamics parameters

*3.1.2. Deviation angle*

It is generally accepted that blade deviation is a crucial parameter for the accurate off--design compressor performance simulation. In the relative frame of referencethe deviation angle is defined by

$$\beta_2 = \delta + k_2 \tag{3.2}$$

where $k_2$ is the outlet blade angle.

For a transonic compressor, there are some factors which should be considered in deviation prediction at off-design conditions. According to the previous works, the deviation is calculated as the following formula in the present work. This approach is based on the reference deviation angle and consideration of the effects of various real-flow phenomena as separating individual deviation sources

$$\delta = \delta_{ref} + \delta_i + \delta_{VA} + \delta_{3D} \tag{3.3}$$

where $\delta_{ref}$ is the reference deviation when the incidence angle equals the reference incidence, which is determined using Carter's rule with the modification recommended by Cetin, $\delta_i$ is the deviation when the actual incidence angle is different from the reference. It is determined from a four-piece curve using NACA-65 series cascade data as its basis (Creveling and Carmody). $\delta_{VA}$ is the expression of the deviation due to axial velocity ratio and $\delta_{3D}$ is the deviation due to the three dimensional effects. It is calculated based on the NASA SP36 curves according to Boyer's method.

*3.1.3. Total pressure loss model*

Modern transonic compressors have a complex three-dimensional flow field, and thus the computation of the total loss coefficient remains difficult. Lieblein (1957), Swan (1961), Koch and Smith (1976) *et al.* proposed many correlations for simulation of the loss. According to their works, the loss model of present work is broken down into three categories as the profile, shock, and secondary, referring to Swan, Boyer, Vassilios Pacthe hidis and Aungie's methods. The following equation identifies the components used in determination of the total pressure loss coefficient

$$\begin{aligned} &\varpi = f_{Re}\varpi_{pro} + \varpi_{sec} + \varpi_M && \varpi_{prof} = 2f_{Re}\sigma\Big(\frac{\Theta}{c}\Big)\frac{\cos^2\beta_1}{\cos^3\beta_2} \\ &\varpi_{sec} = f_{sec}\sigma\Big(\frac{\cos\beta_1}{\cos\beta_2}\Big)^2 C_D && \varpi_M = \frac{1 - \Big[\frac{(\gamma+1)M_B^2}{(\gamma-1)M_B^2+2}\Big]^{\frac{\gamma}{\gamma-1}}\Big[\frac{\gamma+1}{2\gamma M_B^2-(\gamma-1)}\Big]^{\frac{1}{\gamma-1}}}{1 - \Big[1 + \frac{\gamma-1}{2}M_1^2\Big]^{\frac{-k}{k-1}}} \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} &0.02 \leqslant \mathrm{Re}\cdot 10^{-5} \leqslant 0.76 && f_{Re} = -8.735294\mathrm{Re}\cdot 10^{-5} + 8.66888 \\ &0.76 < \mathrm{Re}\cdot 10^{-5} \leqslant 2.00 && f_{Re} = 0.3(\mathrm{Re}\cdot 10^{-5})^2 - 1.5\mathrm{Re}\cdot 10^{-5} + 3.0 \\ &2.00 < \mathrm{Re}\cdot 10^{-5} \leqslant 5.00 && f_{Re} = -0.6666\mathrm{Re}\cdot 10^{-5} + 1.3333 \\ &5.00 < \mathrm{Re}\cdot 10^{-5} && f_{Re} = 5.0(\mathrm{Re}\cdot 10^{-5})^{0.1} \end{aligned} \tag{3.5}$$

where $f_{Re}$ is a correction due to Reynolds effects, $\varpi_{pro}$ is the profile loss which is calculated mainly based on the relations between wake momentum thickness and diffusion ratio or equivalent diffusion ratio. Here, the equivalent diffusion ratio is used as a parameter to compute the profile loss according to Swan's approach at design and off-design conditions. $\varpi_{sec}$ is the second loss, accounting for the effects of end-wall and secondary flow losses, recently proposed by Templalexis, Pachidis and Aungier. $\varpi_M$ is the shock loss, which is based on the "simple flow model" of Schwenk *et al.* (1957) and Miller *et al.* (1961). $M_B$ is the Mach number before the normal shock.

#### *3.1.4. Blockage model*

Creveling and Carmody (1968) proposed an approach which is based on calculating the available through-flow radius (hub and shroud) via setting blockage coefficients at each axial station. The specific computational equations are as follows:

$$\begin{aligned} &\delta_H = \frac{R_T^2 - R_{He}^2}{R_T^2 - R_H^2} && \delta_T = \frac{R_{Te}^2 - R_H^2}{R_T^2 - R_H^2} \\ &R_{He} = \sqrt{\delta_H R_H^2 + (1-\delta_H)R_T^2} && R_{Te} = \sqrt{\delta_T R_T^2 + (1-\delta_T)R_H^2} \end{aligned} \tag{3.6}$$

where $\delta_H$, $\delta_T$ are blockage coefficients, $R_H$ is the radius at hub of station, $R_T$ is the radius at shroud of station. $R_{He}$ and $R_{Te}$ are the available hub and shroud radii, respectively.

### 3.2. Verification of SLC

A transonic axial rotor (NASA Rotor 37) is simulated firstly to verify the 2D method. The compressor model is constructed according to the detailed geometric data published in NASA Technical Paper No. 1337 by Royce and Lonnie (1980) and ASME paper by Arima (1999). The performance of the SLC is compared with the design and experimental data provided in the report mentioned above. The geometry and overall performance of them is given in these papers.

The rotor was originally designed as an inlet rotor for a core compressor and was tested at NASA Lewis Research Center in the late 1970's. The rotor design pressure ratio is 2.106 at an equivalent mass flow of 20.19 kg/s. The inlet relative Mach number is 1.13 at the hub and 1.48 at the tip at the design speed of 454 m/s (17 188.7 rpm). The rotor has 36 blades with a hub-tip ratio of 0.7, an aspect ratio of 1.19, and a tip solidity of 1.288. The tip clearance is 0.356 mm. The rotor has multiple circular arc (MCA) blade shapes.

For calculational grids shown in Fig. 1, 13 streamlines are displaced from hub to shroud. At the same time, 8 calculation stations are displaced along the axial direction. In experimental conditions, the rotor has a tip clearence of about 0.45 percentage from the shroud. However, in the streamline curvature method, the tip clearence is ignored and replaced with cascades through the 3D modification.

The design operating condition is calculated firstly and compared with the experimental values. Figure 4a shows the spanwise distribution of pressure ratio at 98% of the choke mass flow at the design speed. In this figure we can see that the general tendency of the pressure ratio distribution is close to the experimental data. The prediction data is somewhat smaller than the experimental data except for the hub and tip. The maximum error occurs at the hub and tip. Figure 4b shows the adiabatic efficiency along the blade. The trend of efficiency prediction results is closer to the experimental data than the pressure. The largest error is in the tip region. They all decrease along the span. The operating conditions of 92% (near-stall) has also been simulated. Figures 5a,b shown the comparisons of pressure ratio and efficiency at the near-stall condition. The pressure ratio is basically larger than that at 98% mass flow condition. This phenomenon agrees with the actual situation. The errors are largest near the hub and tip regions. The trend is somewhat different from the experimental data near the tip. The reason is a certain relation with the reference incidence angle. The efficiency significantly decreases at the hub and tip region and larger than the experimental value in the middle region. This might be caused by a cumulative incidence and deviation error which makes the loss smaller than in real conditions in the middle region. At the same time, the comparisons also indicate that the accuracy of SCL reduces more at the near-stall condition. Overall, the trends of pressure ratio and efficiency are basically consistent with the experimental data.

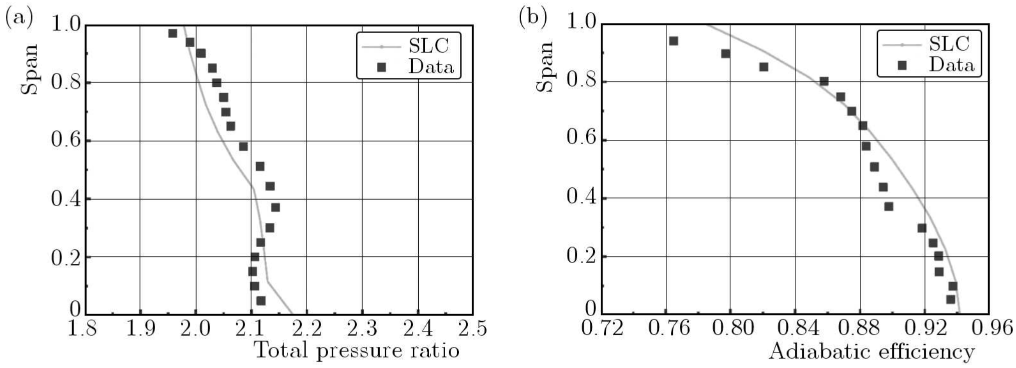


Fig. 4. Total pressure ratio (a) and adiabatic efficiency (b) for 0.98 mass flow rate

The flow field of the rotor is very complex with shock, endwall boundary layers, shock and boundary layer interactions and tip clearance vortex, etc. The deviation and loss prediction are difficult to be captured accurately and have a certain degree of bias, especially at the hub and tip region. However, regardless of the pressure ratio or efficiency, the trends are consistent with the experimental data. The largest errors occur at the annular walls. This indicates that the deviation and loss models take into account inadequate impact of 3D flow effects and the annular walls. Above all, the analysis indicates that the models also need to be further improved to increase the accuracy.

The accuracy of compressor characteristic curves is one of the most important criteria to measure the compressor performance prediction program. For the above reason, in the present

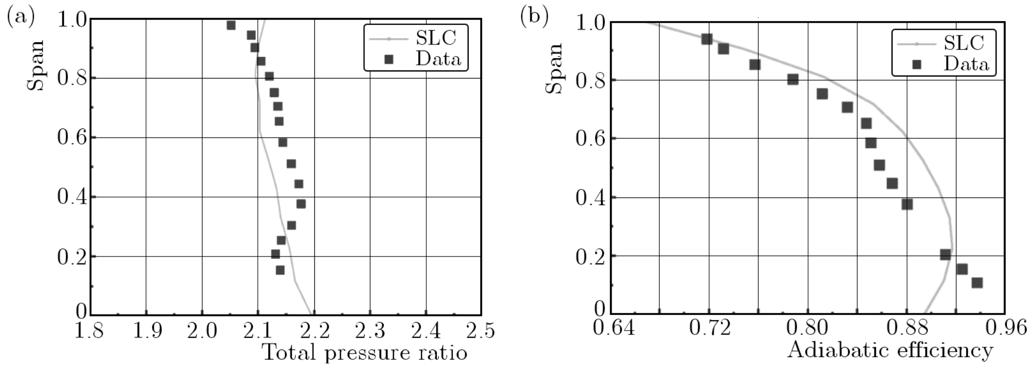


Fig. 5. Total pressure ratio (a) and adiabatic efficiency (b) for 0.92 mass flow rate

work, three conditions (60%, 80% and 100% design speed) are numerically calculated. The results are shown in Figs. 6a,b. From the total pressure ratio comparison chart, it can be seen that all total pressure ratios versus equivalent mass flow speed lines follow the overall shape of the experimental speed lines. And they are in a good qualitative and quantitative agreement with the experimental data away from design point conditions. As for the adiabatic efficiency at various speeds, the prediction values are more severely deviated from the experimental value when the mass flow increases at each rotational speed. The analysis above means that the precision should be further improved.

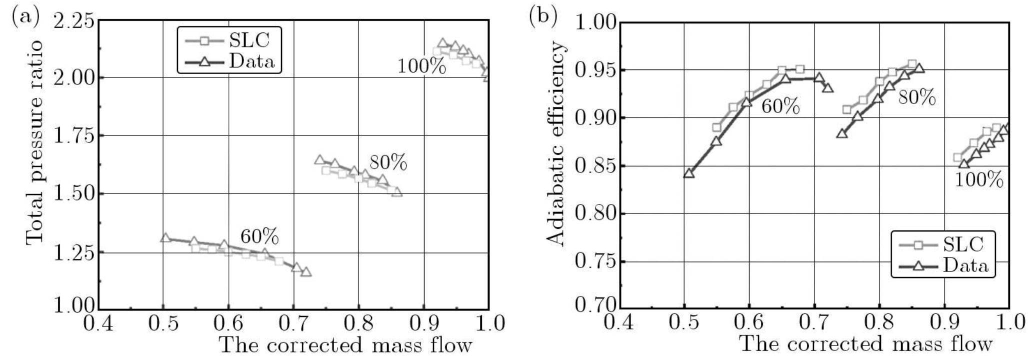


Fig. 6. Performance curve of pressure ratio for the rotor (a) and of adiabatic efficiency for the rotor (b)

Besides NASA Rotor 37 calculations, the code results are compared against design and experimental data provided in NASA Technical Paper 1659 by Royce and Lonnie (1980). This technical paper describes thoroughly the design and performance of a single-stage axial flow compressor. The compressor design pressure ratio is 2.050 and isentropic efficiency is 0.842 at an equivalent mass flow of 20.188 kg/s. The rotor has 36 blades with a hub-tip ratio of 0.7, an aspect ratio of 1.19, and a tip solidity of 1.288. The design speed is 17 188.7 rpm. The stator has 46 blades with multiple circular arc (MCA) blade shapes as the rotor. The tip clearance is 0.356 mm. 13 streamlines are displaced from the hub to shroud, and 13 calculation stations are displaced along the axial direction.

The results in form of comparisons are shown in Figs. 7a,b. From the total pressure ratio comparison chart, it can be seen that all total pressure ratios versus equivalent mass flow speed lines follow the overall shape of the experimental speed lines. And they are in a good qualitative and quantitative agreement with the data away from design point conditions. As the rotation speed decreases, the error becomes smaller in general. This phenomenon indicates that the SLC method used here could be a useful reference for compressor performance optimization at low speeds. As for the adiabatic efficiency at various speeds, at 90% and 100%, the prediction values deviate more severely from the data when the inlet angle increases. The maximum efficiency points are not the same as the data. This may mean the prediction of loss deviates a bit from the real situation at various incidence. The analysis above means that the calculation precision should be further improved.

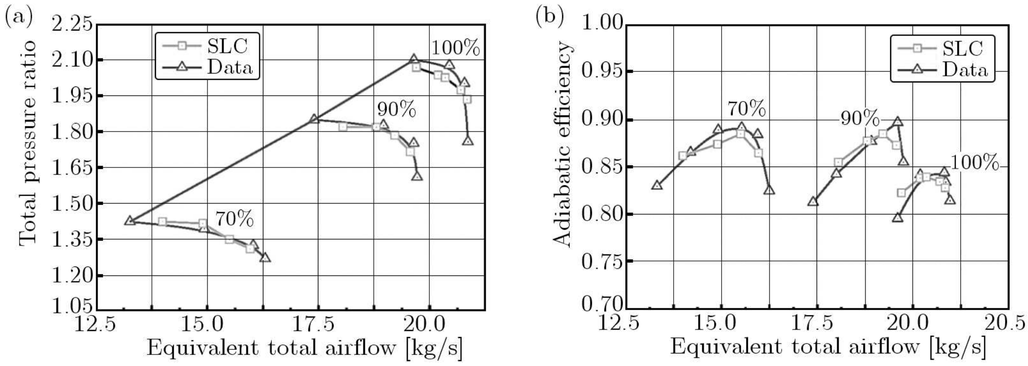


Fig. 7. Performance curve of pressure ratio (a) and of adiabatic efficiency (b) for the compressor

## 4. Conclusions

By using the streamline curvature method based on the deviation and loss models, two compressors are simulated in detail at both design and off-design conditions. By comparing the simulation results with the experiment data, the main conclusions are listed below:

- The SLC method based on the deviation angle and loss models presented in this paper can predict compressor characteristics at both design and off-design points better. Along the rotor span, the prediction trends for the total pressure ratio and efficiency coincide with the experiment data although the prediction error near the hub and tip region is a bit larger.
- The pressure ratio trends for the two compressors follow the overall shape of the equivalent mass flow speed lines. The adiabatic efficiency versus speed lines are in a good qualitative and agreement as well, although the overall shape of the lines do not exactly match the experimental data as shown in Fig. 4. On the whole, the computational models need to be further improved in order to get more accurate solutions.

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