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OPTIMIZATION OF THE WORKING CYCLE OF HARBOUR CRANES

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The paper presents one of the possible ways optimization of motion of the harbour crane grab minimization of the working (unloading) cycle, energy consumption and material dissipation during the grab discharging. The optimization procedure of the working cycle is divided into two phases. Firstly, it is optimization of the cargo and grab motion and, secondly, determination of motion of the crane mechanisms upon the obtained optimal parameters of cargo and grab trajectory. The developed mathematical model enables direct application of the optimal control theory methods, i.e. a method of optimization of cargo and grab motion by making use of Pontryagins maximum principle. All relevant expressions are derived analytically.

Key words: optimization, working cycle, harbour crane, maximum principle

1. Introduction

Designing of complex transport systems presents a challenge for the designer, from the point of view of device selection, facilities layout and appropriate software for program-controlled systems. Analysis and selection of a solution for the given design task of complex systems with different levels of links between the elements and mutual influences, which may be deterministic or stochastic, could be found only by application of a modeling process. An approach to the system as a method which gives the best results and a model of the investigating medium which contributes to the observation of the complex reality are used.

The unloading of bulk cargo presents organization of different activities, connected with control and handling of the material flow from a vessel to the transport or storage system, which provides best service conditions of vessels with minimization of costs.

Unloading devices present knot points of unloading terminals, and in the most number of cases, bottle necks, so their functioning is the basic prerequisite for the optimal work of the whole unloading system.

The unloading (working) cycle of an harbour crane grab device consist of: material grabbing from a vessel, grab and cargo transfer from the vessel to the receiving hopper, grab discharging and empty grab return transfer from the receiving hopper to the vessel. Full automation of the unloading process of harbour crane facilities with the grab, is possible but very expensive. On the other hand the crane operator could not repeat the optimal unloading cycle in a longer time period. The only practical feasible solution is to introduce a half-automatic unloading cycle which consists of a manual part, where the crane operator controls grab motion, and of an automatic part in which a computer controls the grab moving according to the given algorithm.

The manual part of the half-automatic unloading cycle consists of lowering of the empty grab to the material surface in the vessel, from one of the three points of the end of the automatic part of the unloading cycle (Fig. 1), material grabbing and grab hoisting with cargo to one of the three points of the beginning of the automatic part of the unloading cycle. The automatic part of the half-automatic unloading cycle consists of grab transfer from one of the three points of the beginning of the automatic part of the unloading cycle to the receiving hopper, grab discharging and empty grab return from the hopper to the one of the three possible points of the end of the automatic part of the unloading cycle. The position of the three points, which presents the beginning/end of the automatic part of the half-automatic unloading cycle is virtual and depends on given geometry of the system, river water level, material level in the vessel, etc. (Oyler, 1977).

2. The mathematical model of an harbour crane with moving grab

Figure 1 shows a simplified harbour crane and cargo moving scheme on which the mathematical model is based. The assumption is that the rope in the initial time is in the vertical position with a defined initial length and the grab position could be one of the three possible. This assumption corresponds to the time immediately before the beginning of the automatic part of the unloading cycle.

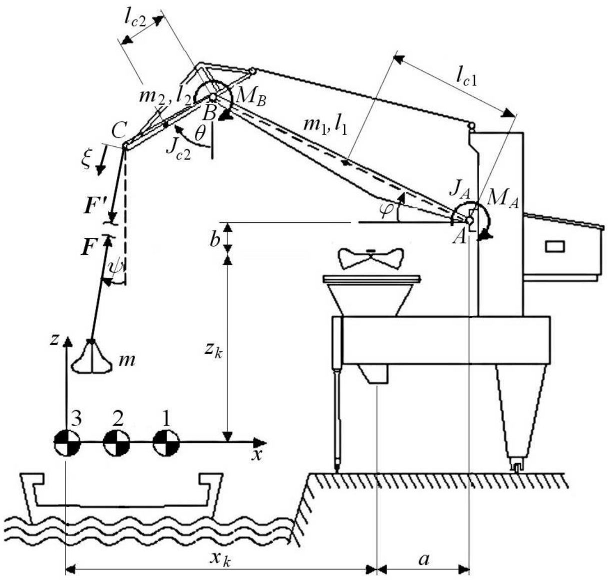

Fig. 1. A simplified scheme of an harbour crane

The generalized coordinates are: φ – angle of the jib, θ – angle of the lever – luffing. The remaining denotations used in the mathematical model are: m_1 – mass of the jib, m_2 – mass of the lever-luffing, m – grab and cargo mass, l_1 – length of the jib, l_2 – length of the lever-luffing, ψ – rope angle, x_k – distance between the vessel and hopper, z_k – height difference between the vessel and hopper, M_A – driving moment acting on the jib, M_B – driving moment acting on the lever-luffing, F – force in the rope, l_{c1} – distance between the point A and the center of gravity of the jib, l_{c2} – distance between the point B and the center of gravity of the lever-luffing, J_A – moment of inertia of the jib with respect to the axis through the point A, J_{c2} – moment of inertia of the lever-luffing with respect to the axis through the center of gravity of the lever-luffing.

It is assumed that the centers of gravity of the jib and lever-luffing lie on straight lines between the points A and B, and B and C, respectively. The driving moments M_A and M_B are reduced to the points A and B. The obtaining of real driving moments requires decomposition of the whole driving structure of the crane which is not the same for all harbour cranes (depends on a manufacturer), and is not a subject of this work. The forces in the rope that connects the lever-luffing and structure of the harbour crane are taken into consideration by reducing the real driving moments to the points A and B.

The optimization procedure of the working cycle will be divided into two phases in the following analysis. Firstly, cargo and grab motion will be optimized and, secondly, the crane driving moments will be determined upon the obtained optimal parameters of the cargo and grab trajectory. According to the thus found motion of the harbour crane, mechanisms and movement of the grab and cargo will be observed separately.

Differential equations which describe motion of the crane mechanisms (Fig. 1) are

$$\begin{aligned}&(J_A + m_2 l_1^2)\ddot{\varphi} - [m_2 l_1 l_{c2}\sin(\varphi-\theta)]\ddot{\theta} + [m_2 l_1 l_{c2}\cos(\varphi-\theta)]\dot{\theta}^2 =\\ &\qquad = M_A - m_1 g l_{c1}\cos\varphi - m_2 g l_1\cos\varphi - F l_1\cos(\psi-\varphi)\\ &-[m_2 l_1 l_{c2}\sin(\varphi-\theta)]\ddot{\varphi} + (J_{c2} + m_2 l_{c2}^2)\ddot{\theta} - [m_2 l_1 l_{c2}\sin(\varphi-\theta)]\dot{\varphi}^2 =\\ &\qquad = M_B + m_2 g l_{c2}\sin\theta + F l_2\sin(\psi-\theta)\end{aligned} \tag{2.1}$$

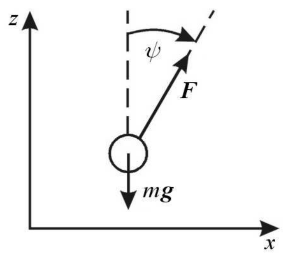

Fig. 2. Forces acting on the grab and cargo

Motion of the grab and cargo will be analyzed in the coordinate system xOz (Fig. 2). At the beginning of motion, the grab and cargo are placed at the point O. In that case, differential equations which describe motion of the grab and cargo are

$$\begin{aligned}&m\ddot{x} = F\sin\psi \qquad && m\ddot{z} = F\cos\psi - mg \qquad && \frac{F}{m} = S\\ &\ddot{x} = S\sin\psi && \ddot{z} = S\cos\psi - g\end{aligned} \tag{2.2}$$

Grab and cargo, for the time interval known in advance $[0, t_c]$:
— from the initial state, $t = 0$

$$\begin{aligned}&x(0) = 0 \qquad && \dot{x}(0) = 0\\ &z(0) = 0 && \dot{z}(0) = 0\end{aligned} \tag{2.3}$$

— should came to the ending state, $t = t_c$

$$\begin{aligned}&x(t_c) = x_k \qquad && \dot{x}(t_c) = 0\\ &z(t_c) = z_k && \dot{z}(t_c) = 0\end{aligned} \tag{2.4}$$

with a limitation that the grab and cargo should pass through the point $(x_k/2, z_k)$ and after that continue to move horizontaly, i.e.

$$x(\tau) = \frac{x_k}{2} \qquad z(\tau) = z_k \qquad z(\tau \leqslant t \leqslant t_c) = z_k \tag{2.5}$$

where the time instant τ is not known in advance.

If such functions $\psi(t), S(t) > 0$ can be found, together with the following conditions

$$\begin{array}{lll} \psi(0) = 0 & \dot{\psi}(0) = 0 & S(0) = g \\ \psi(t_c) = 0 & \dot{\psi}(t_c) = 0 & S(t_c) = g \end{array} \tag{2.6}$$

so that appropriate solutions to equations (2.2) fulfill conditions (2.3), (2.4) and (2.5), the whole system can be controlled.

By increasing the order of differential equations (2.2), those equations can be written as

$$\dddot{x} = \dot{S}\sin\psi + S\dot{\psi}\cos\psi \qquad \dddot{z} = \dot{S}\cos\psi - S\dot{\psi}\sin\psi \tag{2.7}$$

and conditions (2.6) can be written as

$$\begin{array}{lll} \ddot{x}(0) = 0 & \dddot{x}(0) = 0 & \ddot{z}(0) = 0 \\ \ddot{x}(t_c) = 0 & \dddot{x}(t_c) = 0 & \ddot{z}(t_c) = 0 \end{array} \tag{2.8}$$

In that way, the task of motion control of the grab and cargo can be stated in a following form

$$x^{IV} = u_x \qquad z^{IV} = u_z \tag{2.9}$$

and

$$\begin{array}{llll} x(0) = 0 & \dot{x}(0) = 0 & \ddot{x}(0) = 0 & \dddot{x}(0) = 0 \\ z(0) = 0 & \dot{z}(0) = 0 & \ddot{z}(0) = 0 & \\ x(t_c) = x_k & \dot{x}(t_c) = 0 & \ddot{x}(t_c) = 0 & \dddot{x}(t_c) = 0 \\ z(t_c) = z_k & \dot{z}(t_c) = 0 & \ddot{z}(t_c) = 0 & \\ x(\tau) = \dfrac{x_k}{2} & z(\tau) = z_k & z(\tau \leqslant t \leqslant t_c) = z_k & \end{array} \tag{2.10}$$

where u_x and u_z are allowed values of control which belong to an open set.

The beginning condition for $\dddot{z}$ is not set in order to ensure movement in the z direction at the beginning of the movement, while the ending condition for $\dddot{z}$ is automatically fulfilled due to the transverse condition.

According to (2.2) and (2.7), equations (2.9) and conditions (2.10) are equivalent to equations (2.2) and conditions (2.3)-(2.6).

3. Optimal motion of the grab and cargo

The main objectives of the optimization process are the minimal working (unloading) cycle, minimal rope inclination angle, minimal dissipation of the material and, therefore, minimal expense of energy needed for motion of an harbour crane.

By introducing new variables y_i $(i = 1, 2, \dots, 8)$, system (2.9) and conditions (2.10) can be written in a following form

$$
\begin{array}{llll}
\dot{y}_1 = y_2 & \dot{y}_2 = y_3 & \dot{y}_3 = y_4 & \dot{y}_4 = u_x \\
\dot{y}_5 = y_6 & \dot{y}_6 = y_7 & \dot{y}_7 = y_8 & \dot{y}_8 = u_z
\end{array}
\tag{3.1}
$$

and

$$
\begin{array}{llll}
y_1(0) = 0 & y_2(0) = 0 & y_3(0) = 0 & y_4(0) = 0 \\
y_5(0) = 0 & y_6(0) = 0 & y_7(0) = 0 & \\
y_1(t_c) = x_k & y_2(t_c) = 0 & y_3(t_c) = 0 & y_4(t_c) = 0 \\
y_5(t_c) = z_k & y_6(t_c) = 0 & y_7(t_c) = 0 & \\
y_1(\tau) = \dfrac{x_k}{2} & y_5(\tau) = z_k & y_5(\tau \leqslant t \leqslant t_c) = z_k &
\end{array}
\tag{3.2}
$$

which allows direct application of Pontryagins maximum principle. The values u_x and u_z are control values in the x and z direction (Bugaric *et al.*, 2004; Sage and White, 1977; Zrnic *et al.*, 1995).

During the grab and cargo transfer from a vessel to hopper and vice versa the minimal rope inclination angle as well as no more than one oscillation of the grab and cargo are demarded. Beside that, changes in the rope load as a result of the grab and cargo transfer should be reduced to minimum. In that sense, condition of optimality (3.3) presents good enough measure of behavior of those values

$$
J = \int\limits_0^{t_c} \frac{1}{2}(y_3^2 + y_4^2 + u_x^2 + y_8^2)\, dt \to \inf
\tag{3.3}
$$

Differential equations (3.1) and conditions (3.2) together with condition of optimality (3.3) formulate the task of optimal control.

In other words, on the basis of equations (2.2), it can be concluded that the rope inclination and its angular velocity have greater influence on movement in the x direction, i.e. on values y_3, y_4 and u_x, while a change in the rope load has greater influence on movement in the z direction, i.e. on the value y_8. So, the minimal value of (3.3) fulfills the demands and represents an optimality criterion for the discussed problem. It also provides that the values of control and rope inclination angle do not become too big, ensures minimal number of oscillations, continuity of forces in the rope, uniform work, etc.

The problem defined by relations (3.1)-(3.3) is reduced to a form which makes possible direct application of the maximum principle. For these reasons, considering (3.1) and (3.3), the following function is established

$$H = -\frac{1}{2}(y_3^2 + y_4^2 + u_x^2 + y_8^2) + \sum_{i=1}^{8} \lambda_i y_{i+1} \tag{3.4}$$

where the values λ_i satisfy a system of differential equations

$$\dot{\lambda}_i = -\frac{\partial H}{\partial y_i} \qquad i = 1, \dots, 8 \tag{3.5}$$

and

$$\begin{array}{llll} \dot{\lambda}_1 = 0 & \dot{\lambda}_2 = -\lambda_1 & \dot{\lambda}_3 = y_3 - \lambda_2 & \dot{\lambda}_4 = y_4 - \lambda_3 \\ \dot{\lambda}_5 = 0 & \dot{\lambda}_6 = -\lambda_5 & \dot{\lambda}_7 = -\lambda_6 & \dot{\lambda}_8 = y_8 - \lambda_7 \end{array} \tag{3.6}$$

According to the theorem of the maximum principle, function (3.4) has the maximal value for the optimal solution. According to the condition of extremum

$$\frac{\partial H}{\partial u_x} = 0 \qquad \frac{\partial H}{\partial u_z} = 0 \tag{3.7}$$

the controls in the x and z direction are obtained

$$\begin{array}{l} -u_x + \lambda_4 = 0 \quad \Rightarrow \quad u_x = \lambda_4 \\ \lambda_8 = 0 \quad \Rightarrow \quad \dot{\lambda}_8 = 0 \quad \Rightarrow \quad y_8 = \lambda_7 \end{array} \tag{3.8}$$

The following transverse conditions should be added to conditions (3.2)

$$\lambda_8(0) = 0 \qquad \lambda_8(t_c) = 0$$

what is trivially fulfilled in (3.8).

The structure of systems of differential equations (3.1) and (3.6) shows that the optimization of grab and cargo movement in the x and z directions can be done separately. The system of differential equations for the optimization grab and cargo movement in the x direction has the following form

$$\begin{array}{llll} \dot{y}_1 = y_2 & \dot{y}_2 = y_3 & \dot{y}_3 = y_4 & \dot{y}_4 = \lambda_4 \\ \dot{\lambda}_1 = 0 & \dot{\lambda}_2 = -\lambda_1 & \dot{\lambda}_3 = y_3 - \lambda_2 & \dot{\lambda}_4 = y_4 - \lambda_3 \end{array} \tag{3.9}$$

Boundary conditions are:

— for $t = 0$

$$y_1(0) = y_2(0) = y_3(0) = y_4(0) = 0 \tag{3.10}$$

— for $t = t_c$

$$y_1(t_c) = x_k \qquad y_2(t_c) = y_3(t_c) = y_4(t_c) = 0 \tag{3.11}$$

The system of differential equations for the optimization of grab and cargo movement in the z direction has the following form

$$\begin{array}{llll} \dot{y}_5 = y_6 & \dot{y}_6 = y_7 & \dot{y}_7 = y_8 & \dot{y}_8 = -\lambda_6 \\ \dot{\lambda}_5 = 0 & \dot{\lambda}_6 = -\lambda_5 & \dot{\lambda}_7 = -\lambda_6 & \dot{\lambda}_8 = 0 \end{array} \tag{3.12}$$

Boundary conditions are:
— for $t = 0$

$$y_5(0) = y_6(0) = y_7(0) = \lambda_8(0) = 0 \tag{3.13}$$

— for $t = \tau$

$$y_5(\tau) = z_k \qquad y_6(\tau) = y_7(\tau) = \lambda_8(\tau) = 0 \tag{3.14}$$

— for $\tau \leqslant t \leqslant t_c$

$$y_5(\tau) = z_k \qquad y_6(t) = y_7(t) = \lambda_8(t) = 0 \tag{3.15}$$

Each of differential equations (3.9) and (3.12) defined that way, with conditions (3.10), (3.11) and (3.13)-(3.15) presents a two-point boundary value problem. Due to the configuration of differential equations (3.9) and (3.12), each of them can be solved analytically (Sage and White, 1977).

4. Analytical solutions

According to systems of differential equations (3.1) and (3.9), following relations can be established: (movement in the x direction)

$$\begin{aligned} &u_x = \lambda_4 \qquad \lambda_1 = L_1 \qquad \lambda_2 = -L_1 t + L_2 \\ &\lambda_3 = y_2 + \frac{1}{2}L_1 t^2 - L_2 t + L_3 \\ &\lambda_4 = y_3 - y_1 - \frac{1}{6}L_1 t^3 + \frac{1}{2}L_2 t^2 - L_3 t + L_4 \\ &\dot{y}_4 = \dot{y}_2 - y_1 = -\frac{1}{6}L_1 t^3 + \frac{1}{2}L_2 t^2 - L_3 t + L_4 \end{aligned}$$

Finally, differential equations (3.9) can be reduced to one fourth-order differential equation

$$y_1^{IV} - \ddot{y}_1 + y_1 = -\frac{1}{6}L_1t^3 + \frac{1}{2}L_2t^2 - L_3t + L_4 \tag{4.1}$$

where L_1, L_2, L_3, L_4 are arbitrary constants.

A solution to the previous differential equation has the following form

$$\begin{aligned}y_1 = x = &\Big(A_1\mathrm{e}^{\frac{\sqrt{3}t}{2}} + B_1\mathrm{e}^{-\frac{\sqrt{3}t}{2}}\Big)\cos\frac{t}{2} + \Big(C_1\mathrm{e}^{\frac{\sqrt{3}t}{2}} + D_1\mathrm{e}^{-\frac{\sqrt{3}t}{2}}\Big)\sin\frac{t}{2} + \\ &+E_1t^3 + F_1t^2 + G_1t + H_1\end{aligned}$$

Differentiating the previous expression by t yields expressions for y_2, y_3, y_4

$$\begin{aligned}y_2 = \dot{x} = &\Big[\sqrt{3}\Big(-B_1 + A_1\mathrm{e}^{\sqrt{3}t}\Big)\cos\frac{t}{2} + \Big(D_1 + C_1\mathrm{e}^{\sqrt{3}t}\Big)\cos\frac{t}{2} + \\ &-\Big(B_1 + A_1\mathrm{e}^{\sqrt{3}t}\Big)\sin\frac{t}{2} + \sqrt{3}\Big(-D_1 + C_1\mathrm{e}^{\sqrt{3}t}\Big)\sin\frac{t}{2}\Big]2\mathrm{e}^{-\frac{\sqrt{3}t}{2}} + \\ &+3E_1t^2 + 2F_1t + G_1\end{aligned}$$

$$\begin{aligned}y_3 = \ddot{x} = &\Big[\Big(B_1 + A_1\mathrm{e}^{\sqrt{3}t}\Big)\cos\frac{t}{2} + \sqrt{3}\Big(-D_1 + C_1\mathrm{e}^{\sqrt{3}t}\Big)\cos\frac{t}{2} + \\ &+\sqrt{3}\Big(B_1 - A_1\mathrm{e}^{\sqrt{3}t}\Big)\sin\frac{t}{2} + \Big(D_1 + C_1\mathrm{e}^{\sqrt{3}t}\Big)\sin\frac{t}{2}\Big]2\mathrm{e}^{-\frac{\sqrt{3}t}{2}} + \\ &+6E_1t + 2F_1\end{aligned}$$

$$y_4 = \dddot{x} = \Big[D_1\cos\frac{t}{2} + C_1\mathrm{e}^{\sqrt{3}t}\cos\frac{t}{2} - A_1\mathrm{e}^{\sqrt{3}t}\sin\frac{t}{2} - B_1\sin\frac{t}{2}\Big]\mathrm{e}^{-\frac{\sqrt{3}t}{2}} + 6E_1$$

where $A_1, B_1, C_1, D_1, E_1, F_1, G_1, H_1$ are constants which are to be determined upon boundary conditions (2.3) and (2.4).

For movement in the z direction, according to differential equations (3.1) and (3.12), following the relations can be established

$$\begin{aligned}&u_z = -\lambda_6 &&\lambda_5 = L_5 &&\lambda_6 = -L_5t + L_6\\ &\dot{y}_8 = \dot{\lambda}_7 &&\dot{y}_8 = -\lambda_6 &&\dot{y}_8 = L_5t - L_6\end{aligned}$$

where L_5, L_6 are arbitrary constants.

Substituting $(L_5, -L_6)$ with (A_2, B_2), the required expressions for movement in the z direction are obtained as

$$y_5^{IV} = z^{IV} = A_2t + B_2 \qquad \dddot{y}_5 = \dddot{z} = \frac{1}{2}A_2t^2 + B_2t + C_2$$

$$\ddot{y}_5 = \ddot{z} = \frac{1}{6}A_2t^3 + \frac{1}{2}B_2t^2 + C_2t + D_2$$

$$\dot{y}_5 = \dot{z} = \frac{1}{24}A_2t^4 + \frac{1}{6}B_2t^3 + \frac{1}{2}C_2t^2 + D_2t + E_2$$

$$y_5 = z = \frac{1}{120}A_2t^5 + \frac{1}{24}B_2t^4 + \frac{1}{6}C_2t^3 + \frac{1}{2}D_2t^2 + E_2t + F_2$$

where A_2, B_2, C_2, D_2, E_2, F_2 are constants which are to be determined upon boundary conditions (3.2).

Directly from differential equations (2.2), expressions for ψ and S are obtained as

$$\psi = \arctan \frac{\ddot{x}}{\ddot{z} + g} \qquad\qquad S = \sqrt{\ddot{x}^2 + (\ddot{z}^2 + g)^2}$$

Figure 3 show results of the grab and cargo optimization process in time. Those results are: changes of the grab and cargo velocity in the x direction ($\dot{x}$, Fig. 3a), changes of the acceleration in the x direction ($\ddot{x}$, Fig. 3d), changes of the grab and cargo velocity in the z direction ($\dot{z}$, Fig. 3b), changes of the acceleration in the z direction ($\ddot{z}$, Fig. 3d), changes of the rope inclination angle (ψ, Fig. 3e), changes of the angular velocity of the grab and cargo ($\dot{\psi}$, Fig. 3f), changes of forces in the rope (F/m, i.e. S, Fig. 3g), optimal path of the grab (Fig. 3h), changes of the displacement x (Fig. 3i) and z (Fig. 3j).

Parameters, upon which the results shown in Fig. 3 have been obtained, are: distance between the vessel and hopper in the x direction $x_k = 9\,\mathrm{m}$, distance between the vessel and hopper in the z direction $z_k = 8\,\mathrm{m}$, $t_c = 20\,\mathrm{s}$ – time, known in advance, needed for obtaining one half of the automatic part of the half-automatic unloading cycle, i.e. grab transfer from the vessel to hopper or vice versa (determined upon maximal allowed velocities and accelerations in the x and z directions (Bugaric and Petrovic, 2002)) and $\tau = x^{-1}x_k/2$ – time needed for grab and cargo transfer to one half of the distance between the vessel and hopper, i.e. $z(\tau \leqslant t \leqslant t_c) = z_k$.

5. Motion of the crane mechanisms

On the basis of the previous concept of cargo movement, a link between the cargo and crane peak movements can be established as

$$l_1 \cos\varphi + l_2 \sin\theta + \xi \sin\psi + x - x_k - a = 0$$

$$l_1 \sin\varphi - l_2 \cos\theta - \xi \cos\psi - z + z_k + b = 0$$

Realising that this is a redundant system, we can deem that the rope length transition $\xi(t)$, or something else, is a prominent time function, which generally depends on structural characteristics of the crane. Changes of the rope length $\xi(t)$ in time should be determined upon real characteristics of driving mechanisms for a specific type of an harbour crane, depending on a manufacturer. A problem to be resolved now is how the direct task of dynamics and unknown moments M_A and M_B can be determined from differential equations (2.1) on the basis of the obtained parameters of cargo motion.

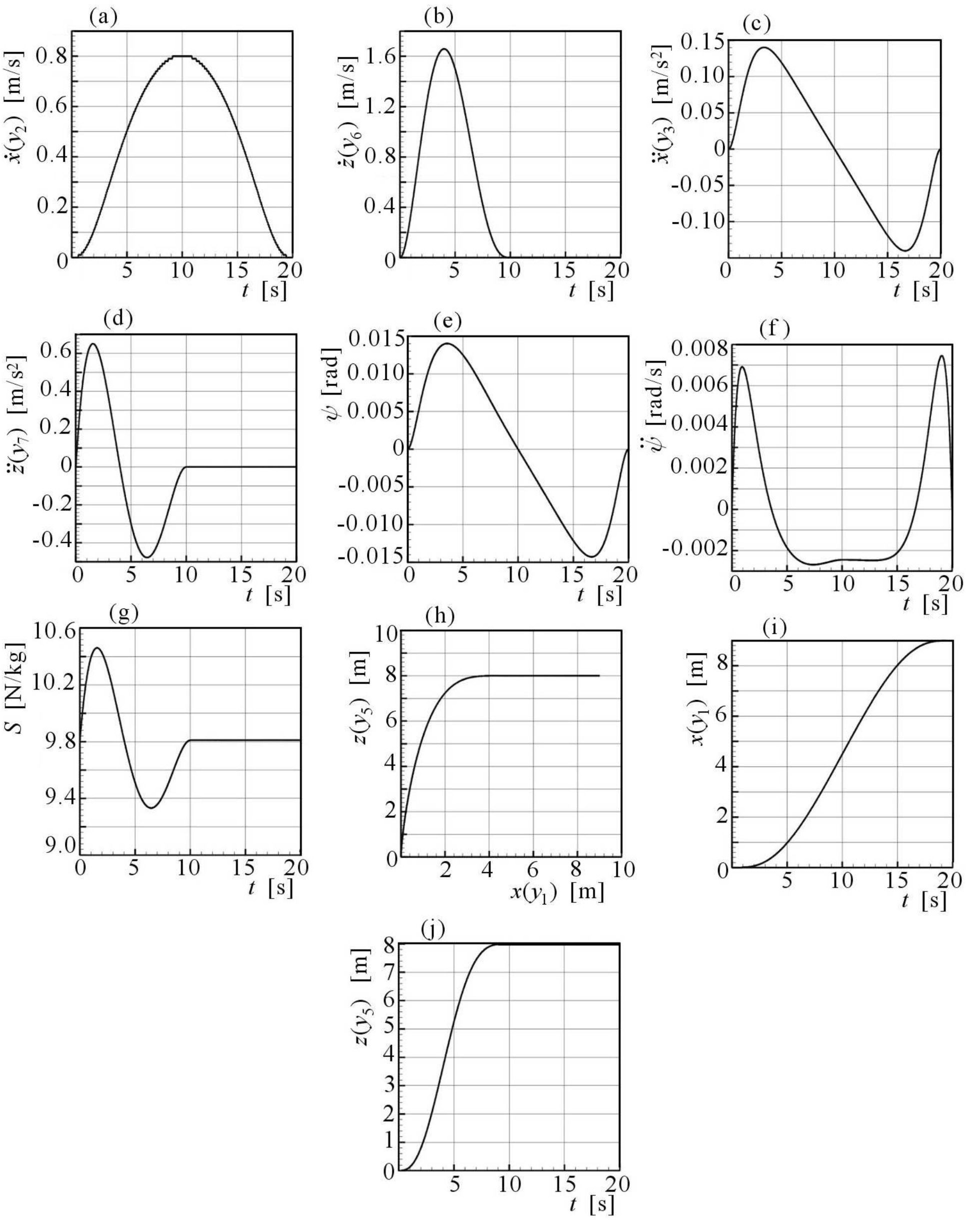

Fig. 3. Functions of the optimized parameters

6. Duration of the working cycle

Duration of the half-automatic working (unloading) cycle consist of the following periods needed for completion of certain operations (Bugaric, 2002):

— automatic part of the half-automatic unloading cycle

$$t_{ac} = 2t_c + t_{gd} = 2 \cdot 20 + 8 = 48\,\mathrm{s}$$

where t_{gd} is the time needed for grab discharging,
— manual part of the half-automatic unloading cycle

$$t_{mc} = t_{gl} + t_{gc} + t_{gh} + t_e = (1.2 \div 7.2) + 15 + (1.2 \div 7.2) + 5 = (22.4 \div 34.4)\,\mathrm{s}$$

where: $t_{gl} = (1.2 \div 7.2)\,\mathrm{s}$ is the time for grab lowering from one of the three possible points of the end of the automatic part of the unloading cycle to the material in the vessel with a velocity of 50 m/min. The lowering distance depends on the water level and varies between 1 and 6 m; $t_{gc} = 15\,\mathrm{s}$ – time needed for closing of the grab; $t_{gl} = (1.2 \div 7.2)\,\mathrm{s}$ – time for grab hoisting from the material in the vessel to one of the three possible points of the beginning of the automatic part of the unloading cycle with a velocity of 50 m/min. The hoisting distance depends on the water level and varies between 1 and 6 m; $t_e = 5\,\mathrm{s}$ – extra time needed for the crane operator to locate the most suitable place for grabbing.

Finally, the duration of the working cycle is:

$$t_{uc} = t_{ac} + t_{mc} = 48 + (22.4 \div 34.4) = (70.4 \div 82.4)\,\mathrm{s}$$

7. Conclusions

The characteristic feature of bulk cargo is the great importance of parameters such as fact transport expenses, manipulation and a waiting time. An unloading bulk cargo terminal works 24 hours seven days a week during the sailing period. The presented optimized working cycle of an harbour crane reduces the rope inclination angle, forces in the rope and, therefore, energy required for realisation of such operations.

It is important to underline that the developed procedure for the optimization of grab and cargo motion has universal applications, i.e. results of the optimization process can be applied to any transport device which performs similar tasks (unloading bridges, overhead cranes etc.).

The application of the obtained results lies in the introducing of the half automatic unloading cycle during unloading of the bulk cargo material. In that case, it is possible to achieve the optimal unloading cycle, to minimize material dissipation during grab discharging, to lowering dynamic strains in the crane and also to eliminate the influence of the human factor in realisation of the unloading process (training of operator, weather conditions, night work, etc.).

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Optymalizacja cyklu roboczego dźwigów portowych

Streszczenie

W pracy przedstawiono jeden z możliwych sposobów optymalizacji ruchu chwytaka dźwigu portowego oraz minimalizacji cyklu roboczego pod kątem ograniczania zużycia energii oraz strat materiału podczas rozładunku. Procedurę optymalizacji cyklu roboczego podzielono na dwie fazy. W pierwszej zoptymalizowano ruch przenoszonego obiektu i chwytaka, podczas gdy w drugiej wyznaczono ruch pozostałych elementów dźwigu dla znalezionej optymalnej trajektorii obiektu i chwytaka. Sformułowany model matematyczny układu umożliwił na bezpośrednie zastosowanie metod teorii optymalnego sterowania, w tym optymalizacji z zasady maksimum Pontriagina. Wszystkie związane wyrażenia wyprowadzono analitycznie.

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