JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
46, 1, pp. 171-184, Warsaw 2008

ANALYSIS OF LONGITUDINAL TANGENTIAL CONTACT VIBRATION EFFECT ON FRICTION FORCE USING COULOMB AND DAHL MODELS

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The paper presents results of analysis of changes of the friction force in sliding motion affecting a solid body subjected to excited longitudinal tangential contact vibration. The study was conducted using two friction models: firstly, the classical Coulomb model related to rigid body motion on a non-deformable base and secondly, the Dahl model which takes into account tangential contact deformability including the phenomenon of "pre-sliding displacement". It was demonstrated that in the case of vibration motion with a low amplitude (i.e. motion in a micro scale, which is exemplified by longitudinal contact vibration), the Coulomb friction model is not adequate to describe the friction force. It was also shown that the friction force can be reduced in one vibration cycle without instantaneous change of the vector sign of this force, which in literature is often quoted as the main reason for friction force reduction at longitudinal tangential contact vibration.

Key words: friction models, friction force, tangential contact vibration

1. Introduction

Many studies on the influence of contact micro vibration on the friction force carried out over the past several decades suggest that this force can be reduced by this vibration excitation both in the normal (Godfrey, 1967; Tolstoi *et al.*, 1973; Budanov *et al.*, 1980; Hess and Soom, 1991; Grudziński and Kostek, 2005) and tangential direction to the plane of contact (Pohlman and Lehfeldt, 1966; Mitskevich, 1968; Skare and Stahl, 1992; Katoh, 1993; Sase *et al.*, 1995, 1997; Siegert and Ulmer, 1998, 2001; Kutomi 1999; Littmann *et al.*, 2001a,b;

Storck *et al.*, 2002; Kumar and Hutchings, 2004). In the case of tangential vibration, most studies were carried out within ultrasonic vibration range, which is connected with the fact that this vibration plays an important role among methods aiming at improving manufacturing techniques through lowering friction forces between a tool and a workpiece, particularly in plastic forming and machining. Most of the studies on the influence of tangential micro vibration on the friction force mainly concern practical applications of vibration in various technological processes omitting the mathematical description of the phenomenon.

The first theoretical model describing the influence of longitudinal tangential contact vibration on the friction force was presented by Mitskevich (1968). According to this model, it is possible to reduce the average friction force through a cyclic and instantaneous change of the vector sign of this force registered in every vibration period provided that the amplitude v_a of the vibration velocity is higher than the constant component v_c of the sliding motion velocity. This phenomenon described as the *friction vector effect* is commonly regarded as one of the most important mechanisms which can lower the friction force in sliding motion for excited longitudinal tangential contact micro vibration (Eaves *et al.*, 1975; Siegert and Ulmer, 1998, 2001; Skare and Stahl, 1992). On the basis of the same assumption, Littmann *et al.* (2001a,b) as well as Kumar and Hutchings (2004) using classical friction Coulomb's law elaborated successive models allowing them to determine the friction force during macroscopic sliding of one body against the other while longitudinal tangential contact vibration takes place.

The calculation results of the friction force for these models show significant discrepancies in comparison with the results obtained in empirical research, which were carried out by Littmann *et al.* (2001a,b), Storck *et al.* (2002) or Kumar and Hutchings (2004). A much better consistency of the calculation results as compared with the experimental results given in the above quoted articles by Littmann *et al.* (2001a,b) and Storck *et al.* (2002) was obtained by Tsai and Tseng (2006) for the model developed by Dahl (1968, 1976) which takes into account contact deformability in the tangential direction, or with the use of the elasto-plastic friction model proposed by Dupont *et al.* (2000, 2002), which is a development of Dahl's model.

The present study attempts to explain, through numerical analysis, physical reasons accounting for a much better agreement of friction force calculation results obtained using the Dahl model with the experimental data as compared with other models based on classical Coulomb's friction law.

2. The investigated object

In the numerical calculations of the friction force in sliding motion taking place in the presence of excited longitudinal tangential contact vibration, the authors used a model (Fig. 1) in which body A with the mass m moves with the relative velocity v_r with regard to base B.

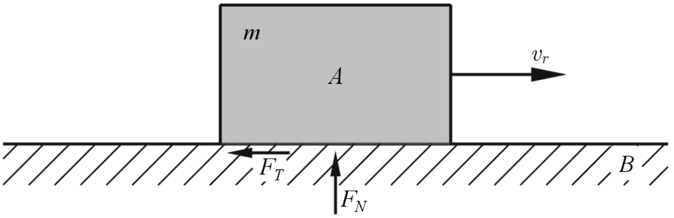

Fig. 1. The model used for calculations

The velocity v_r is the superposition of two component velocities v_c and v_v

$$v_r = v_c + v_v \tag{2.1}$$

where

v_c – a constant component, $v_c =$ const, connected with motion in the macroscopic scale (without vibration)

v_v – a variable component, $v_v = v_v(t)$, connected with excited contact vibration (motion in the microscopic scale).

When harmonic motion is assumed, the variable component can be given in the form

$$v_v(t) = x_a\omega\cos(\omega t) \tag{2.2}$$

where

x_a – amplitude of excited vibration

ω – circular frequency of excited vibration

t – time.

The product $x_a\omega$ is the amplitude v_a of the velocity of excited vibration

$$v_a = x_a\omega \tag{2.3}$$

Hence, equation (2.1) can be written as

$$v_r(t) = v_c + v_a\cos(\omega t) \tag{2.4}$$

In the Coulomb friction model, it is assumed that the interacting surfaces of contact are ideally rigid. The friction force F_T can be given by

$$F_T = F_C \operatorname{sgn}(v_r) \tag{2.5}$$

where

$$F_C = \mu F_N \tag{2.6}$$

F_C is the Coulomb friction force, μ – coefficient of kinetic friction and F_N – normal contact reaction of the contact area. From relation (2.5), it can be seen that in this model the friction force is constant in its absolute value and only its plus/minus sign can change, depending on the sign of the relative velocity v_r of sliding.

In fact, machined surfaces which compose the contact area are not ideally smooth. The process of machining causes deflections from the ideal state. These include roughness, waviness and shape errors. As a result, these surfaces do not adhere one to an other ideally throughout all the nominal area of contact, but instead they only adhere in some micro-areas (Fig. 2a) distributed in a random fashion on this surface.

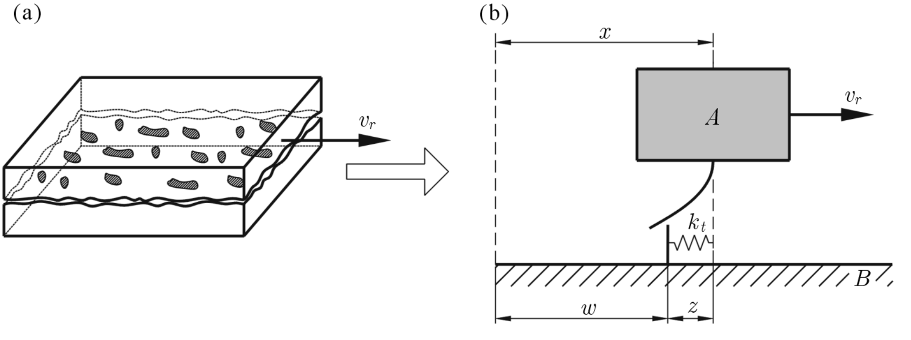

Fig. 2. Real contact of two bodies and its model: (a) a scheme of contact of two machined surfaces, (b) the Dahl friction model showing the distribution of the total displacement x of a rigid body on the elastic z and plastic w components

In the 1970s, the so called dynamic friction models were developed, including the model elaborated by Dahl (1968, 1976). In this model the asperities are modelled by means of micro-springs (Fig. 2b), which when a tangential load is applied deflect in the direction of frictional resistance force. If the deflection is big enough (i.e. when the friction force reaches its maximum value), the contact is broken-away and then sliding takes place.

According to the Dahl model, the displacement x of a rigid body is elasto-plastic and can be broken down into two components: an elastic z and a plastic w

$$x = z + w \tag{2.7}$$

The elastic component is connected with elastic deflections of asperities in the tangential direction. Its mutual relation with the friction force F_T can be given by

$$F_T = k_t z \tag{2.8}$$

where k_t is the tangential contact stiffness coefficient.

This deflection can be described with a differential equation (Dahl, 1976)

$$\frac{dz}{dt} = v_r\Big(1 - \frac{k_t}{F_C}\operatorname{sgn}(v_r)z\Big)^{\alpha} \tag{2.9}$$

The parameter α in this equation, defines the shape of a curve describing the dependence of the tangential deflections on tangential force. For brittle materials, the value of this parameter is in the range of $0 < \alpha < 1$, and for ductile materials it is $\alpha \geqslant 1$ (Bliman, 1992).

3. Numerical calculations

For numerical calculations of the friction force the following parameters of excited vibration were taken: frequency $f = 60\,\mathrm{kHz}$ and amplitude $x_a = 0.7\,\mu\mathrm{m}$. They are identical to those which were assumed by Littmann *et al.* (2001a) in their experimental investigation of the longitudinal contact vibration influence on the friction force. The following data were also used: mass of the die $m = 0.02\,\mathrm{kg}$ (Littmann, 2006), kinetic friction coefficient $\mu = 0.1$, value of the parameter $\alpha = 1$ and the tangential contact stiffness coefficient $k_t = 0.056\,\mathrm{N}/\mu\mathrm{m}$. The last coefficient was determined on the basis of data provided in the Tsai and Tseng (2006) paper.

For the assumed frequency f and amplitude x_a, taking into account that $\omega = 2\pi f$, the amplitude of vibration velocity $v_a = 0.264\,\mathrm{m/s}$ was determined. It was done using equation (2.3). The friction force calculations were performed for a sequence of the velocity v_r value with a constant component v_c equal to: $v_c = 0.324$, 0.264, 0.230, 0.199, 0.146, 0.095 m/s, for which Littmann *et al.* (2001b) carried out experimental measurements of the friction force. For the Coulomb model, the value of the F_T force was determined from relation (2.5), and for the Dahl model it was determined from relations (2.8) and (2.9). The results of numerical calculations for the assumed values of motion parameters are shown in graphical forms in Figures 3 and 4.

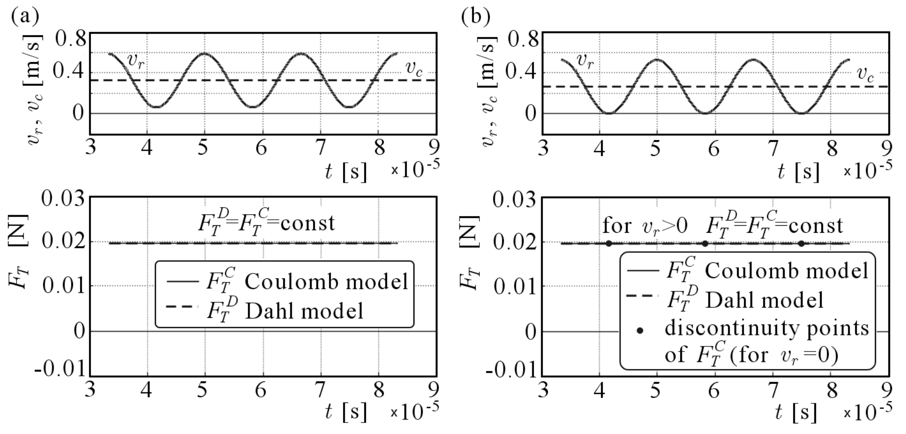

Fig. 3. The friction force for the Coulomb and Dahl models when the amplitude of vibration velocity v_a is less than or equal to the constant component v_c: (a) $v_a = 0.264\,\mathrm{m/s} < v_c = 0.324\,\mathrm{m/s}$, (b) $v_a = v_c = 0.264\,\mathrm{m/s}$

The graphs presented in Fig. 3 demonstrate that for motion in which the amplitude of the excited vibration velocity v_a is smaller or equals to the value of the constant component v_c of the sliding motion velocity, the calculation results using both friction models are identical. The friction force F_T does not change, i.e. tangential contact vibration in this case does not influence the friction force.

The situation changes when the amplitude of the contact vibration velocity v_a is higher than the constant component v_c $(v_a > v_c)$. As it can be seen in graphs presented in Fig. 4, in such a case, the results may differ considerably depending on which friction model is chosen. In the Coulomb friction model, for the whole range of velocities which satisfy the condition that $v_a > v_c$, a change of sign of the relative sliding velocity results in an immediate change of sign of the friction force (Fig. 4). As a result, the calculated values of the average friction force for one period of vibration are much smaller than those determined experimentally, which is seen in the papers by Littmann *et al.* (2001a,b) and Storck *et al.* (2002).

The calculation results using the Dahl model (Fig. 4) demonstrate that a change of an instantaneous friction force in the presence of tangential contact vibration is not abrupt. For small differences between v_a and v_c (Fig. 4a,b), the value of the instantaneous friction force is lowered without the sign change (its

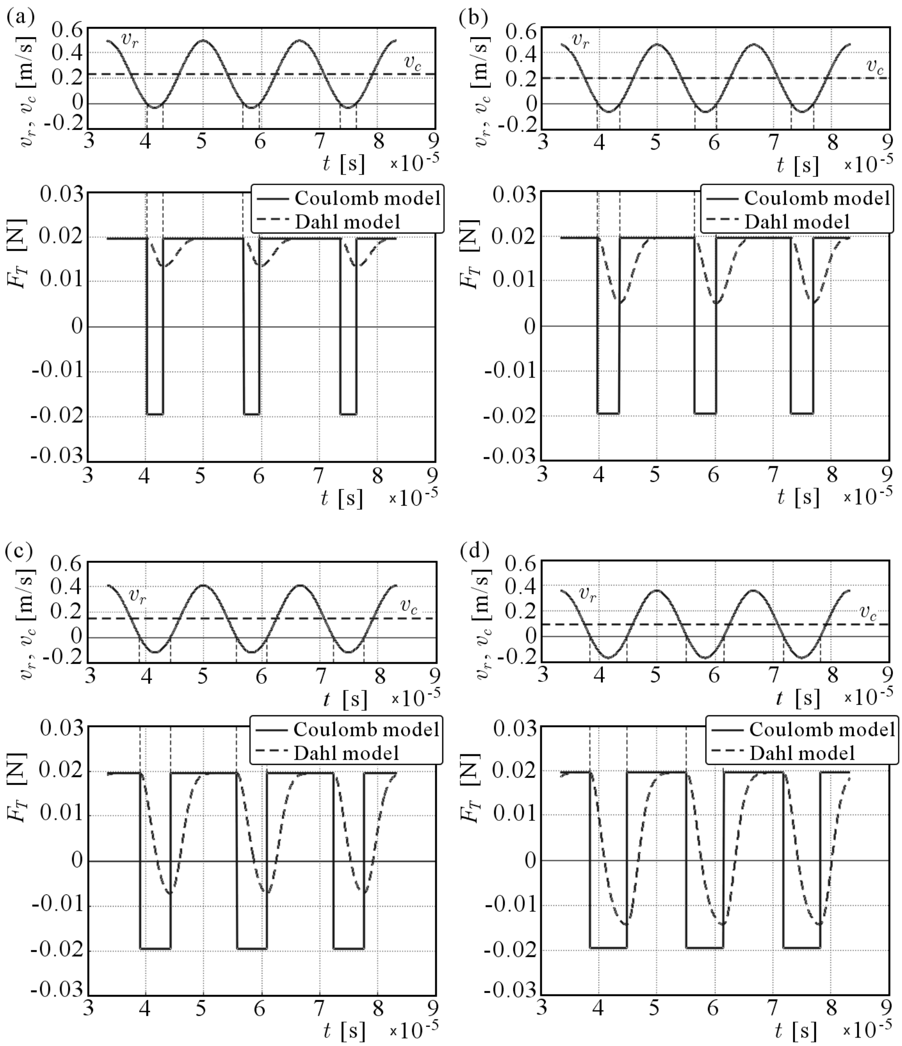

Fig. 4. The friction force for the Coulomb and Dahl models when the amplitude of vibration velocity v_a ($v_a = 0.264\,\mathrm{m/s}$) is greater than the constant component v_c: (a) $v_c = 0.230\,\mathrm{m/s}$, (b) $v_c = 0.199\,\mathrm{m/s}$, (c) $v_c = 0.146\,\mathrm{m/s}$, (d) $v_c = 0.095\,\mathrm{m/s}$

sense is opposite to v_c). For large differences between v_a and v_c (Fig. 4c,d), the instantaneous friction force is gradually reduced to zero and then it changes its sign and starts growing in the opposite direction (in accordance with v_c). It is connected with the direction of elastic deflection of the contact area asperities relative to the motion trajectory of the rigid body, which is illustrated in Fig. 5.

In both analyzed models, for each considered value of the velocity v_c, an

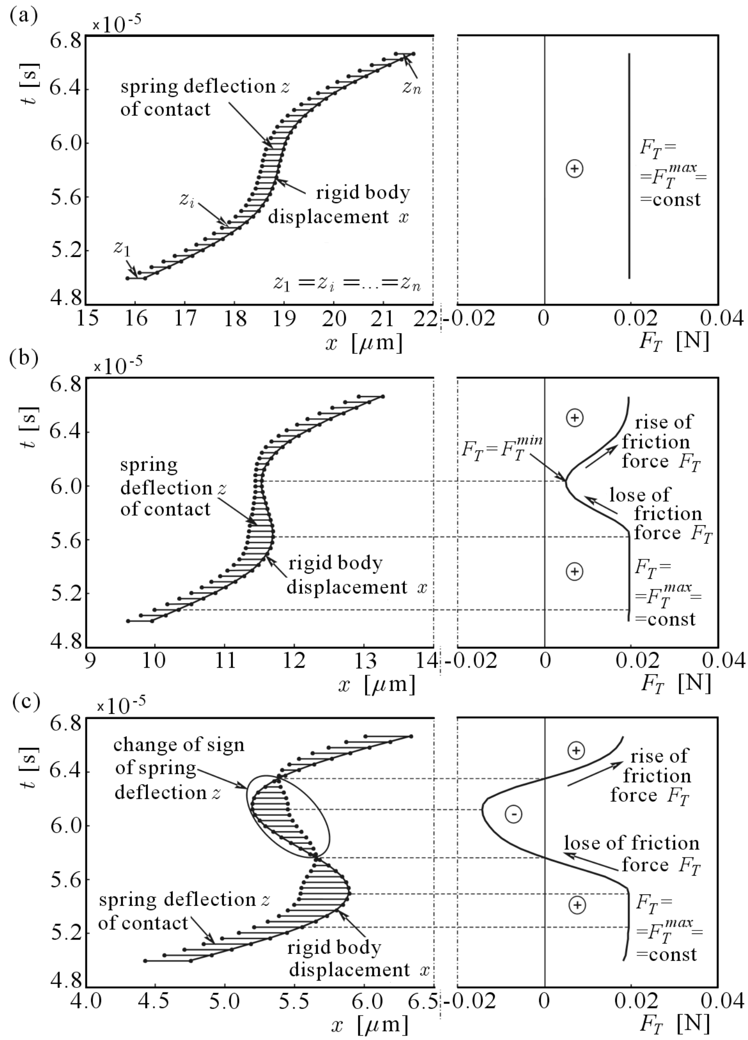

Fig. 5. Diagrams illustrating the relation between the friction force F_T and the elastic deflection of asperities in the contact area;
(a) $v_a = 0.264\,\mathrm{m/s} < v_c = 0.324\,\mathrm{m/s}$, (b) $v_a = 0.264\,\mathrm{m/s} > v_c = 0.199\,\mathrm{m/s}$,
(c) $v_a = 0.264\,\mathrm{m/s} > v_c = 0.095\,\mathrm{m/s}$

average friction force $\overline{F}_T$ in one vibration period $T = 2\pi/\omega$ was determined. This force was evaluated from the following relation

$$\overline{F}_T = \frac{1}{n}\sum_{i=1}^{n} F_T(t + i\Delta t) \tag{3.1}$$

where n is the number of time steps in one vibration period

$$n = \frac{2\pi}{\omega \Delta t} \tag{3.2}$$

The percentage reduction S of the average friction force, calculated from Dahl's and Coulomb's models, which took place as a result of excited tangential contact vibrations, was also evaluated

$$S = \Big(1 - \frac{\overline{F}_T}{F_C}\Big) \cdot 100\% \tag{3.3}$$

The results of $\overline{F}_T$ and S obtained in the calculations are presented in a tabular form in Table 1 where they are compared with the results of experiments carried out by Littmann *et al.* (2001a).

Table 1. A comparison between numerical calculations and experimental results ($v_a = 0.264\,\mathrm{m/s}$)

Velocity v_c [m/s]	Average friction force, $\overline{F}_T$ [N]		
	Coulomb model	Dahl model	Experiment (Littmann *et al.*, 2001a)
0.324	0.0196	0.0196	0.0197
0.264	0.0196	0.0196	0.0187
0.230	0.0131	0.0181	0.0179
0.199	0.0107	0.0156	0.0168
0.146	0.00724	0.0109	0.0112
0.095	0.00454	0.0066	0.0077
Velocity v_c [m/s]	**Reduction of average friction force, S [%]**		
	Coulomb model	Dahl model	Experiment (Littmann *et al.*, 2001a)
0.324	0	0	−0.56
0.264	0	0	4.32
0.230	33.2	7.65	8.45
0.199	45.4	20.4	14.1
0.146	63.1	44.4	42.9
0.095	76.8	66.2	60.6

This comparison clearly shows that the Dahl model provides a far better consistency as compared with the experimental results than other models, including the classical Coulomb friction model which does not take into account tangential contact stiffness.

The friction force value calculated using the Dahl model – equations (2.8) and (2.9) – is a function of contact stiffness in the tangential direction. This is why the quantitative discrepancy between the results obtained using this model and those calculated using the Coulomb model significantly depends on the contact stiffness in the tangential direction.

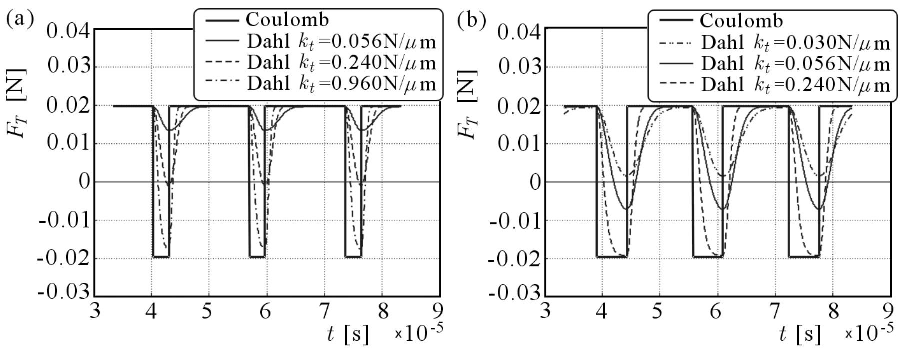

Fig. 6. The friction force for the Coulomb and Dahl models depending on the stiffness of contact in the tangential direction, when $v_a > v_c$; (a) $v_a = 0.264\,\mathrm{m/s}$ and $v_c = 0.230\,\mathrm{m/s}$, (b) $v_a = 0.264\,\mathrm{m/s}$ and $v_c = 0.146\,\mathrm{m/s}$

This problem is graphically illustrated in Fig. 6, which compares the friction force F_T calculated using the Dahl model for contacts with various tangential stiffness with the friction force obtained using the Coulomb model. Figure 6a presents the friction force F_T for velocity $v_c = 0.230\,\mathrm{m/s}$ and amplitude of vibration velocity $v_a = 0.264\,\mathrm{m/s}$, whereas Fig. 6b presents the friction force F_T for $v_c = 0.146\,\mathrm{m/s}$ and $v_a = 0.264\,\mathrm{m/s}$. Tables 2 and 3 present a comparison of the average friction force $\overline{F}_T$ within one period for different contact stiffness parameters calculated using the Dahl model. They are compared with respective $\overline{F}_T$ values calculated using the Coulomb model for the above given values of v_c and v_a.

The graphs presented in Fig. 6 and the comparison shown in Tables 2 and 3 clearly demonstrate that discrepancies in the friction force $\overline{F}_T$ estimated in one period of longitudinal tangential contact vibration using the Coulomb and Dahl models decrease as the contact stiffness increases in the tangential direction.

Table 2. Average friction force $\overline{F}_T$ according to the Coulomb and Dahl models depending on values of k_t, for $v_c = 0.230\,\mathrm{m/s}$ and $v_a = 0.264\,\mathrm{m/s}$

Coefficient of contact stiffness k_t [N/μm]	Average friction force $\overline{F}_T$ [N]	
	Dahl model	Coulomb model
0.030	0.0186	0.0131
0.056	0.0181	
0.120	0.0172	
0.240	0.0160	
0.480	0.0148	
0.960	0.0139	

Table 3. Average friction force $\overline{F}_T$ according to the Coulomb and Dahl models depending on values of k_t, for $v_c = 0.146\,\mathrm{m/s}$ and $v_a = 0.264\,\mathrm{m/s}$

Coefficient of contact stiffness k_t [N/μm]	Average friction force $\overline{F}_T$ [N]	
	Dahl model	Coulomb model
0.030	0.0127	0.00724
0.056	0.0109	
0.120	0.00877	
0.240	0.00783	
0.480	0.00754	
0.960	0.00742	

4. Summary

The presented above analysis demonstrates that depending on the mathematical model used in numerical calculations the results of the influence of longitudinal tangential contact vibration on the friction force vary. A markedly better consistency as compared with the experimental data provided in the literature (Littmann *et al.*, 2001a,b; Storck *et al.*, 2002) is obtained using the Dahl model.

The analysis of numerical calculations conducted using the Dahl model showed that the friction force in sliding motion in the presence of excited longitudinal tangential contact vibration can be reduced without changing the sign of the friction force vector. The view that follows from Coulomb's law, and which is still currently adopted by many authors that the main mechanism of reducing friction force in sliding motion in the presence of tangential contact vibration results in a cyclic, instantaneous sign change of this force, in the light of the presently obtained results has not been confirmed.

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Analiza wpływu drgań kontaktowych stycznych wzdłużnych na siłę tarcia przy wykorzystaniu modelu Coulomba i modelu Dahla

Streszczenie

W pracy przedstawiono wyniki analizy zmian siły tarcia w ruchu ślizgowym ciała sztywnego przy występowaniu wymuszonych drgań kontaktowych stycznych wzdłużnych tego ciała. Badania przeprowadzono przy wykorzystaniu dwóch modeli tarcia: klasycznego modelu Coulomba odniesionego do ruchu ciała sztywnego po nieodkształcalnym podłożu oraz modelu Dahla uwzględniającego podatność kontaktową styczną styku. Wykazano, że w przypadku występowania ruchu drgającego o małej wartości amplitudy (ruch w skali mikro, którego przykładem są drgania kontaktowe styczne) model tarcia Coulomba jest nieadekwatny do opisu siły tarcia. Wykazano również, że siła tarcia może ulec obniżeniu bez występowania zjawiska chwilowej zmiany znaku wektora tej siły, które w literaturze często podawane jest za główną przyczynę obniżenia siły tarcia przy występowaniu kontaktowych drgań wzdłużnych.

Manuscript received March 9, 2007; accepted for print October 8, 2007