JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
46, 1, pp. 69-83, Warsaw 2008

ENTROPY GENERATION DUE TO NON-NEWTONIAN FLUID FLOW IN ANNULAR PIPE WITH RELATIVE ROTATION: CONSTANT VISCOSITY CASE

Ali Kahraman

Technical Education Faculty, Selcuk University, Konya, Turkey
e-mail: alikahraman@selcuk.edu.tr

Muhammet Yürüsoy

Technical Education Faculty, Afyon Kocatepe University, Afyon, Turkey
e-mail: yurusoy@aku.edu.tr

Entropy generation due to Non-Newtonian fluid flow in an annular pipe with relative rotation is investigated. A third grade fluid with constant viscosity is accommodated in the analysis. Relative rotational motion is present between inner and outer cylinders, which induces the flow. Analytical solutions for velocity and temperature distributions are presented, and entropy generation number is computed for different dimensionless values of non-Newtonian viscosity, Brinkman's number and velocity ratio. It is found that the increasing of dimensionless non-Newtonian viscosity lowers the number entropy generation. This is more pronounced in the region close to the annular pipe inner wall. The increasing of Brinkman's number enhances the number entropy generation, particularly in the vicinity of the annular pipe inner wall.

Key words: non-Newtonian fluid, third grade fluid, entropy generation number

1. Introductin

Flow through annular pipes with relative rotation finds many significant engineering applications. In addition to heat transfer situations, the resulting flow is particularly applicable to rotating electrical machines, swirl nozzles, rotating disks, standard commercial rheometers, and other chemical and mechanical mixing equipment, see Maron and Cohen (1991). Considerable research studies were carried out to investigate the non-Newtonian fluid flow. The modeling of viscoelastic materials using differential constitutive equations was introduced by Peters and Baaijens (1997). Their approach was based on the concept of a slip tensor and elastic behavior of structure. Dunn and Fosdick (1974) performed a complete thermodynamic analysis for fluids of the second grade, while Fosdick and Rajagopal (1990) carried out thermodynamic analysis for the third grade fluid. Moreover, Szeri and Rajagopal (1985) investigated the flow of a third grade fluid between two heated horizontal plates. They assumed that the shear viscosity was temperature dependent. The effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe were investigated by Massoudi and Christie (1995). They showed that as dimensionless non-Newtonian viscosity increased, the magnitude of velocity and temperature decreased in the pipe. The approximate analytical solutions for the flow of a third grade fluid in a pipe were presented by Yürüsoy and Pakdemirli (2002). They showed that if certain criteria for the fluid flow were met, the approximate analytical solutions agreed well with the previous numerical results of Massoudi and Christie (1995).

Thermodynamic irreversibility occurring in a flow system gives insight into losses associated with the system. In this case, two major losses can be accounted – due to heat transfer and due to frictional losses. Heat transfer losses alter the thermodynamic work potential of the system while frictional losses lower the pressure in it. Moreover, entropy generation within the system quantifies thermodynamic irreversibility. Consequently, investigation into thermodynamic irreversibility associated with the thermal system through entropy analysis is faithful. Entropy generation in rotational flow systems results from fluid friction and heat transfer. Moreover, entropy analysis provides information for quantification of thermodynamic irreversibility in the flow system. Consequently, entropy minimization lowers frictional and heat transfer losses in the system. Considerable research studies were carried out to examine entropy generation in thermal systems. Bejan (1995) examined the entropy generation and minimization in thermal systems. He indicated that entropy minimization could be used as an effective tool for designing thermal systems. Convective heat transfer in an annular packed bed was investigated by Demirel and Kahraman (2000). They indicated that the volumetric entropy generation map could be used to identify the excessive entropy generation due to operating conditions or design parameters for a required task. The Shannon entropy characteristics of two-phase flow systems were examined by Zhang and Shi (1999). They found that the entropy generation of the bubble flow was the smallest, the slug flow entropy generation was the largest, and the entropy generation of the annular flow was between the bubble flow and slug flow. Mahmud and Fraser (2002)] reported the inherent irreversibility of the fluid flow and heat transfer for non-Newtonian fluids in a pipe and a channel flow. They presented the average entropy generation number graphically for both pipe and channel flows. A study of entropy generation in fundamental convective heat transfer was carried out by Bejan (1979). He showed that flow geometric parameters might be selected in order to minimize the irreversibility associated with a specific convective heat transfer process. Another study was done by Sahin (1998) who introduced a second law analysis of viscous fluids in a circular duct under an isothermal boundary condition. In a more recent paper, Sahin (1999) presented the effect of variable viscosity on the entropy generation rate for a constant heat flux boundary condition for a circular duct. The non-Newtonian fluid flow in an annular pipe without rotation was investigated by Yilbas *et al.* (2004). They presented analytical solutions for velocity and temperature fields by considering a third grade fluid with constant viscosity.

To investigate the thermodynamic irreversibility in the non-Newtonian annular pipe flow with relative rotation, the present study is carried out. The governing equations of non-Newtonian fluids in cylindrical coordinates are solved using the perturbation method. The velocity and temperature fields are presented analytically after considering the third grade fluid model. The closed form solutions for entropy generation due to fluid friction and heat transfer are obtained, and the entropy number is computed for various values of dimensionless non-Newtonian viscosity and different Brinkman's numbers.

2. Velocity and temperature profiles

Consider the non-Newtonian fluid flow between two concentric cylinders as shown in Fig. 1. A non-dimensional form of equations of motion of a third grade fluid in an annular pipe with relative rotation and heat transfer was derived by Beard and Walters (1964)

$$\begin{aligned} &\mu\Big(v'' + \frac{v'}{r} - \frac{v}{r^2}\Big) + \Lambda\Big(v' - \frac{v}{r}\Big)^2\Big(6v'' - \frac{2v'}{r} + \frac{2v}{r^2}\Big) = 0 \\ &\theta'' + \frac{\theta'}{r} + \Gamma\Big(v' - \frac{v}{r}\Big)^2\Big[\mu + \Lambda\Big(v' - \frac{v}{r}\Big)^2\Big] = 0 \end{aligned} \qquad (2.1)$$

and

$$v(1) = 1 \qquad v(R) = n \qquad \theta(1) = 0 \qquad \theta(R) = 1 \qquad (2.2)$$

where r is the dimensionless radius, v is the dimensionless tangential velocity component, θ is the dimensionless temperature and μ is the dimensionless viscosity. The terms are related to dimensional ones through the following relations

$$\begin{aligned} R &= \frac{\overline{r}_o}{\overline{r}_i} & r &= \frac{\overline{r}}{\overline{r}_i} & v &= \frac{\overline{v}}{\overline{r}_i\overline{\omega}_i} \\ \theta &= \frac{\overline{\theta}-\overline{\theta}_i}{\overline{\theta}_o-\overline{\theta}_i} & \mu &= \frac{\overline{\mu}}{\mu_*} & n &= \frac{\overline{r}_o\overline{\omega}_o}{\overline{r}_i\overline{\omega}_i} \end{aligned} \tag{2.3}$$

where R is the radius ratio, $\overline{r}_i$ is the dimensional radius of the inner cylinder, $\overline{r}_o$ is the dimensional radius of the outer cylinder, dimensional angular velocities are denoted by $\overline{\omega}_i$ and $\overline{\omega}_o$ for the inner and outer cylinders, $\overline{\theta}_i$ and $\overline{\theta}_o$ are the inner outer cylinder dimensional temperatures, μ_* is the reference viscosity, n is the velocity ratio. For positive n, both cylinders rotate in the same direction and for negative n they rotate in the opposite directions.

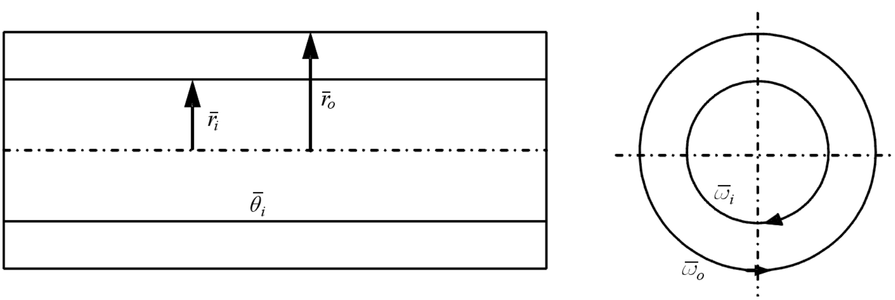

Fig. 1. A schematic view of the annular pipe

The dimensionless parameters involved in equations (2.1) are

$$\varGamma = \frac{\mu_*(\overline{r}_i\overline{\omega}_i)^2}{k(\overline{\theta}_o-\overline{\theta}_i)} \qquad \varLambda = \frac{\beta\overline{\omega}_i^2}{\mu_*} \tag{2.4}$$

where $\varGamma$ is the Brinkman number, $\varLambda$ is the dimensionless parameter related to the non-Newtonian behavior, β is the dimensional material constant for the third grade fluid and k is the thermal conductivity.

In this Section, velocity profiles will be calculated approximately. Approximate solutions can be obtained by selecting $\varLambda = \epsilon\lambda$, where ϵ is our perturbation parameter, a small quantity, and λ is the ordered non-Newtonian coefficient. The approximate velocity and temperature profiles can then be written as

$$v = v_0 + \epsilon v_1 \qquad \theta = \theta_0 + \epsilon\theta_1 \tag{2.5}$$

Assuming constant viscosity, the non-dimensional viscosity can be taken as one ($\mu = 1$). Substituting all terms into the original equations of motion and separating at each order of ϵ, one has:

- order 1

$$v_0'' + \frac{v_0'}{r} - \frac{v_0}{r^2} = 0 \qquad \theta_0'' + \frac{\theta_0'}{r} + \Gamma\Big(v_0' - \frac{v_0}{r}\Big)^2 = 0 \tag{2.6}$$

and

$$v_0(1) = 1 \qquad v_0(R) = n \qquad \theta_0(1) = 0 \qquad \theta_0(R) = 1 \tag{2.7}$$

- order ϵ

$$\begin{aligned} v_1'' + \frac{v_1'}{r} - \frac{v_1}{r^2} &= -\lambda\Big(v_0' - \frac{v_0}{r}\Big)^2\Big(6v_0'' - \frac{2v_0'}{r} + \frac{2v_0}{r^2}\Big) \\ \theta_1'' + \frac{\theta_1'}{r} &= -\Gamma\Big[2v_0'v_1' + \frac{2}{r^2}v_0v_1 - \frac{2}{r}v_0v_1' - \frac{2}{r}v_0'v_1 + \\ &\quad +\lambda\Big(-\frac{4}{r^3}v_0^3v_0' + {v_0'}^4 + \frac{v_0^4}{r^4} - \frac{4}{r}{v_0'}^3v_0 + \frac{6}{r^2}{v_0'}^2v_0^2\Big)\Big] \end{aligned} \tag{2.8}$$

and

$$v_1(1) = 0 \qquad v_1(R) = 0 \qquad \theta_1(1) = 0 \qquad \theta_1(R) = 0 \tag{2.9}$$

For the first order, solutions satisfying the boundary conditions are

$$\begin{aligned} v_0 &= \frac{(R^2 - r^2) + nR(r^2 - 1)}{r(R^2 - 1)} \\ \theta_0 &= \frac{\ln r}{\ln R} + \frac{\Gamma R^2(R - n)^2}{(R^2 - 1)^2}\Big[1 - \frac{1}{r^2} - \frac{\ln r}{\ln R}\Big(1 - \frac{1}{R^2}\Big)\Big] \end{aligned} \tag{2.10}$$

Substituting these solutions to equations of the order ϵ, one finally obtains

$$\begin{aligned} v_1 &= \frac{8\lambda(R - n)^3}{3(R^2 - 1)^3}\Big(\frac{r^4(R^4 + R^2 + 1) - r^6(R^2 + 1) - R^4}{r^5 R}\Big) \\ \theta_1 &= \frac{A_1}{36}\Big[\frac{\ln r}{\ln R}\Big(1 - \frac{1}{R^6}\Big) + \frac{1}{r^6} - 1\Big] - \frac{A_2}{4}\Big[\frac{\ln r}{\ln R}\Big(1 - \frac{1}{R^2}\Big) + \frac{1}{r^2} - 1\Big] \end{aligned} \tag{2.11}$$

where

$$
\begin{aligned}
A_1 &= \frac{16\Gamma\lambda R^4(R-n)^4}{(R^2-1)^4} \\
A_2 &= \frac{64\Gamma\lambda(R-n)^4(R^4+R^2+1)}{3(R^2-1)^4}
\end{aligned}
\tag{2.12}
$$

Combining the solutions at each order of approximation and returning back to the original dimensionless parameters, one finally has

$$
\begin{aligned}
v &= \frac{(R^2-r^2)+nR(r^2-1)}{r(R^2-1)} + \\
&\quad + \frac{8\Lambda(R-n)^3}{3(R^2-1)^3}\Big(\frac{r^4(R^4+R^2+1)-r^6(R^2+1)-R^4}{r^5R}\Big) \\
\theta &= \frac{\ln r}{\ln R} + \frac{\Gamma R^2(R-n)^2}{(R^2-1)^2}\Big[1-\frac{1}{r^2}-\frac{\ln r}{\ln R}\Big(1-\frac{1}{R^2}\Big)\Big] + \\
&\quad + \frac{\epsilon A_1}{36}\Big[\frac{\ln r}{\ln R}\Big(1-\frac{1}{R^6}\Big)+\frac{1}{r^6}-1\Big] - \frac{\epsilon A_2}{4}\Big[\frac{\ln r}{\ln R}\Big(1-\frac{1}{R^2}\Big)+\frac{1}{r^2}-1\Big]
\end{aligned}
\tag{2.13}
$$

The perturbation solution is valid if the correction terms are much smaller than the leading terms. Since there are many physical parameters involved, analytical formulas for validity criteria cannot be accomplished. For a simpler case of the normal pipe flow, such criteria have already been presented by Yürüsoy and Pakdemirli (2002). In our case, as well as in the simpler case of Yürüsoy and Pakdemirli (2002), validity does not depend on one parameter, but on a combination of parameters. In all numerical computations, the validity is ensured by making numerical values of correction terms much smaller than the leading terms.

3. Viscous dissipation and entropy generation

The dimensional viscous dissipation term $(\overline{\phi})$ can be obtained from the equations of motion, i.e.

$$
\overline{\phi} = \overline{\mu}\Big(\frac{d\overline{v}}{d\overline{r}} - \frac{\overline{v}}{\overline{r}}\Big)^2 + 2\beta\Big(\frac{d\overline{v}}{d\overline{r}} - \frac{\overline{v}}{\overline{r}}\Big)^4 \tag{3.1}
$$

or ehen inserting the dimensionless quantities

$$
\overline{\phi} = \mu_*\overline{\omega}_i^2\Big(\frac{dv}{dr} - \frac{v}{r}\Big)^2\Big[\mu + 2\Lambda\Big(\frac{dv}{dr} - \frac{v}{r}\Big)^2\Big] \tag{3.2}
$$

The dimensional volumetric entropy generation is defined as Bejan (1995)

$$S'''_{gen} = \frac{k}{\overline{T}_0^2}\Big(\frac{d\overline{\theta}}{d\overline{r}}\Big)^2 + \frac{\overline{\phi}}{\overline{T}_0} \tag{3.3}$$

where $\overline{T}_0$ is the reference temperature. The first term in equation (3.3) is the volumetric entropy generation due to heat transfer and the second term is the entropy generation due to viscous dissipation. Substituting equation (3.2) into (3.3), expressing the terms in dimensionless forms, one finally obtains

$$N_S = \Big(\frac{d\theta}{dr}\Big)^2 + T_0\Gamma\Big(\frac{dv}{dr} - \frac{v}{r}\Big)^2\Big[\mu + 2\Lambda\Big(\frac{dv}{dr} - \frac{v}{r}\Big)^2\Big] \tag{3.4}$$

where N_S is the entropy generation number. It is defined by dividing the dimensional volumetric entropy generation to the reference volumetric entropy generation S'''_G. The relevant definitions are

$$N_S = \frac{S'''_{gen}}{S'''_G} \qquad S'''_G = \frac{k(\overline{\theta}_o - \overline{\theta}_i)^2}{\overline{T}_0^2\overline{r}_i^2} \qquad T_0 = \frac{\overline{T}_0}{\overline{\theta}_0 - \overline{\theta}_i} \tag{3.5}$$

In equation (3.4), the first term due to heat generation can be assigned as N_{S_1} and the second term due to viscous dissipation as N_{S_2}, i.e.

$$N_{S_1} = \Big(\frac{d\theta}{dr}\Big)^2 \qquad N_{S_2} = T_0\Gamma\Big(\frac{dv}{dr} - \frac{v}{r}\Big)^2\Big[\mu + \Lambda\Big(\frac{dv}{dr} - \frac{v}{r}\Big)^2\Big] \tag{3.6}$$

Assuming constant viscosity, the non-dimensional viscosity can be taken as one ($\mu = 1$). Since the temperature and velocity profiles are known functions, they can be taken from equations (2.13) and then inserted into equations (3.4) and (3.6) for final evaluations of the entropy generation numbers.

4. Results and discussions

The non-Newtonian fluid flow in an annular pipe with relative rotation is considered in the paper. Entropy generation in the flow field due to fluid friction and heat transfer is formulated. The influence of Brinkman's number and dimensionless non-Newtonian viscosity on the entropy generation number is examined. In the analysis, the constant viscosity case is assumed.

Figure 2 shows velocity profiles along the radial distance in the annular pipe for different dimensionless values of the non-Newtonian viscosity. In the

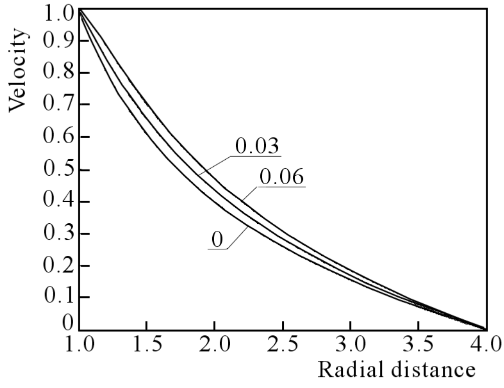

Fig. 2. Velocity profiles along the pipe radius for different dimensionless non-Newtonian viscosities Λ: $n = 0$, $\Gamma = 1$

case of a Newtonian fluid $(\Lambda = 0)$, the velocity profile decays gradually along the radial distance. However, the velocity increases with an increase in the dimensionless non-Newtonian viscosity (Λ).

In Fig. 3 the velocity distribution for different velocity ratios n in plotted. The velocity decreases along the radial direction and exhibits the minimum value at the outer cylinder up to $n \approx 0.5$. For $0.5 < n \leqslant 1$, the minimum velocity occurs inside the annular gap.

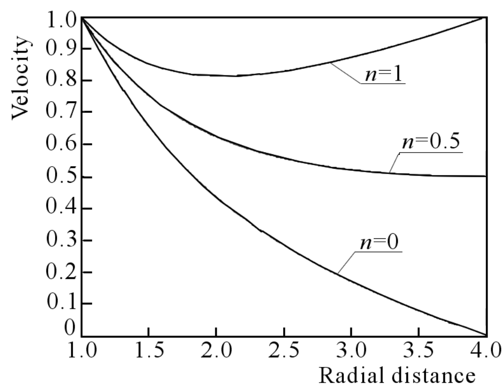

Fig. 3. Velocity profiles along the pipe radius for different velocity ratios n: $\Gamma = 1$, $\Lambda = 0.03$

Figures 4 and 5 show temperature distribution in the annular pipe for different dimensionless non-Newtonian viscosities and Brinkman's numbers Γ, respectively. In Fig. 4, growth of the dimensionless non-Newtonian viscosity reduces the temperature in the flow field. A plot of the radial distance versus temperature is given in Fig. 5 for different values Γ. As Γ increases, the temperature inside the radial direction increases due to the dissipation effect.

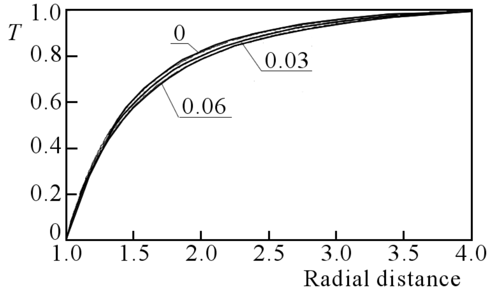

Fig. 4. Temperature profiles along the pipe radius for different dimensionless non-Newtonian viscosities Λ: $n = 0$, $\Gamma = 1$

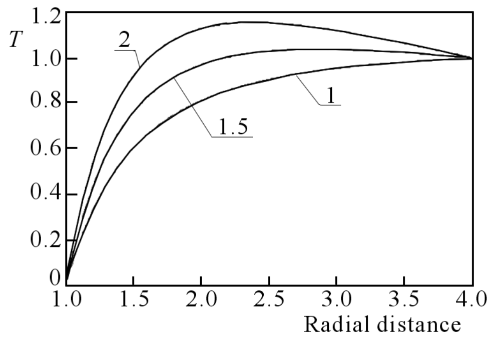

Fig. 5. Temperature profiles along the pipe radius for different Brinkman's numbers Γ: $n = 0$, $\Lambda = 0.01$

Figure 6 shows the entropy generation number due to heat transfer for different dimensionless non-Newtonian viscosities. The entropy generation number attains high values in the vicinity of the annular pipe wall, which is more pronounced in the region close to the inner wall of the pipe (see Fig. 6). Reduction of the dimensionless non-Newtonian viscosity increases the entropy generation rate, in which case the rate of heat transfer enhances.

In Fig. 7 for $n = 0$, $\Lambda = 0.03$ the entropy generation number versus the radial distance is plotted for different Brinkman's numbers. The entropy generation number is high in magnitude near the inner cylinder due to high gradient of temperature. The entropy generation number falls then exponentially along the radial distance and approaches an asymptote near the outer cylinder. The entropy generation number increases with growth of Brinkman's number.

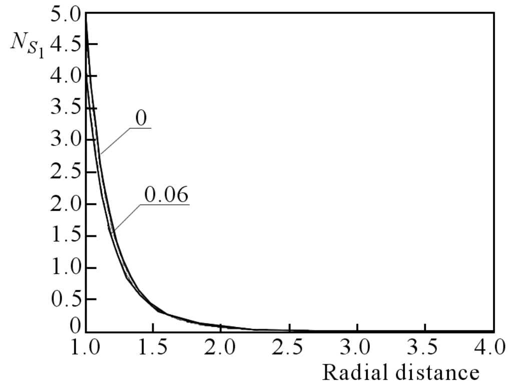

Fig. 6. Entropy generation number due to heat transfer along the pipe radius for different dimensionless non-Newtonian viscosities Λ: $n = 0$, $\Gamma = 1$

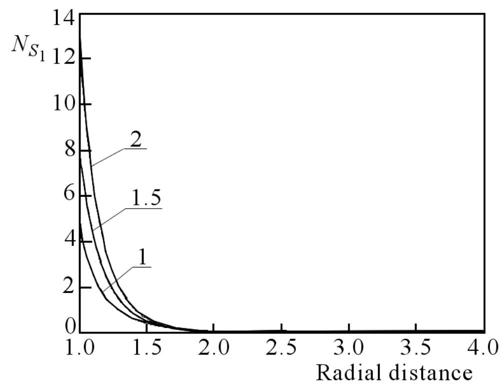

Fig. 7. Entropy generation number due to heat transfer along the pipe radius for different Brinkman's numbers Γ: $n = 0$, $\Lambda = 0.03$

Figures 8 and 9 show the entropy generation number due to fluid friction for different non-Newtonian viscosities and Brinkman's numbers, respectively. The entropy generation number attains high values in the inner pipe wall region. Reduction of the dimensionless non-Newtonian viscosity increases the magnitude of entropy generation number in this region (Fig.8). This is because of the fluid strain, which is high close to the annular pipe wall. Moreover, the maximum entropy generation number moves away from the pipe wall for a dimensionless non-Newtonian viscosity less than 0.03. In the case of Fig, 9, the increasing Brinkman number increases the entropy generation number, particularly in the region close to the annular pipe inner wall.

The effect of the velocity ratio n on the total entropy generation number is plotted in Fig. 10 for $\Lambda = 0.03$, $T_0 = 1.07$, $\Gamma = 1$. The inner wall still

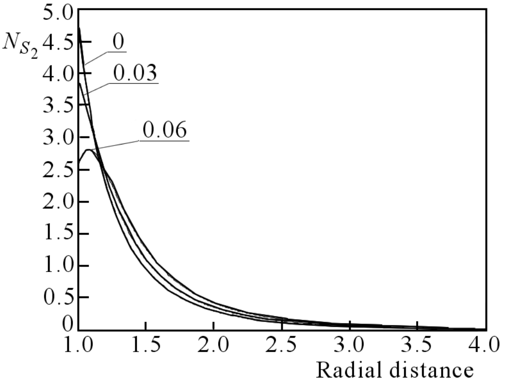

Fig. 8. Entropy generation number due to fluid friction along the pipe radius for different dimensionless non-Newtonian viscosities Λ: $n = 0$, $\Gamma = 1$, $T_0 = 1.07$

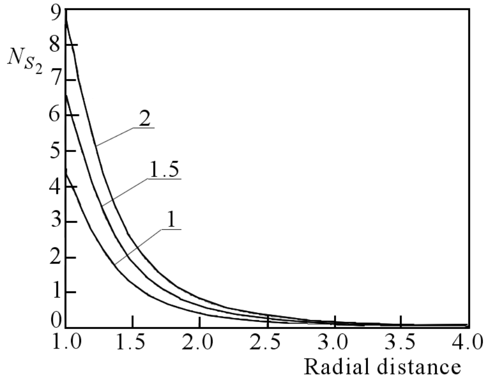

Fig. 9. Entropy generation number due to fluid friction along the pipe radius for different Brinkman's numbers: $n = 0$, $\Lambda = 0.03$, $T_0 = 1.2$

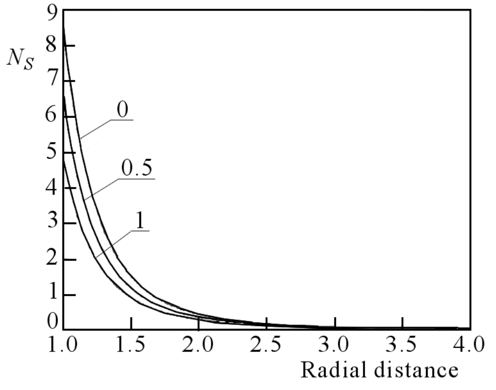

Fig. 10. Entropy generation number due to heat transfer and fluid friction along the pipe radius for different velocity ratios n: $\Lambda = 0.03$, $T_0 = 1.07$, $\Gamma = 1$

acts as a strong concentrator of irreversibility, but now the magnitude of N_S significantly drops at the inner wall for higher n due to lower temperature and velocity gradient.

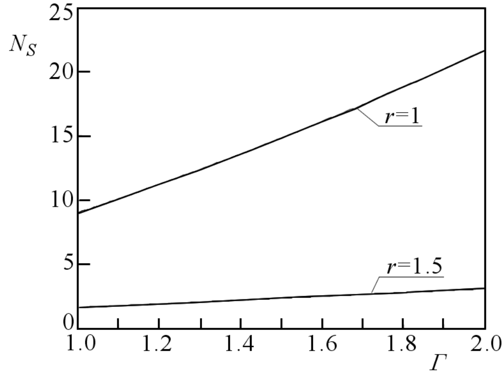

Fig. 11. Entropy generation number due to heat transfer and fluid friction versus Brinkman's number for different radial locations in the pipe: $n = 0$, $\Lambda = 0.03$, $T_0 = 1.2$

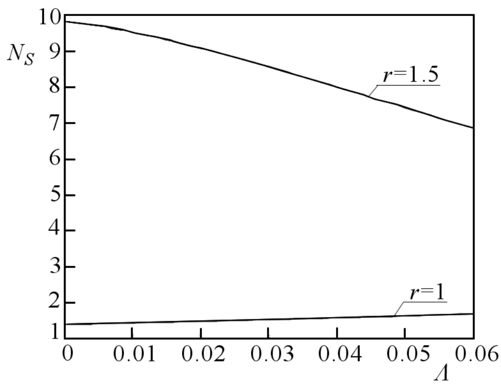

Fig. 12. Entropy generation number due to heat transfer and fluid friction versus dimensionless non-Newtonian viscosity for different radial locations in the pipe: $n = 0$, $\Gamma = 1$, $T_0 = 1.07$

Figures 11 and 12 show the total entropy generation number at different locations in the annular pipe for different Brinkman's numbers and dimensionless non-Newtonian viscosities, respectively. The increasing of Brinkman's number increases the total entropy generation number, particularly in the region close to the annular pipe inner wall. In the case of dimensionless non-Newtonian viscosity (Fig. 12), the total entropy generation number increases with growing dimensionless non-Newtonian viscosity at $r = 1$ in the annular pipe, but at the radial location corresponding to $r = 1.5$, the total entropy generation number decreases with growth of the dimensionless non-Newtonian viscosity. The total entropy generation number is presented against the velocity ratio n in Fig. 13 where n is kept between -1 to 1 for convenience.

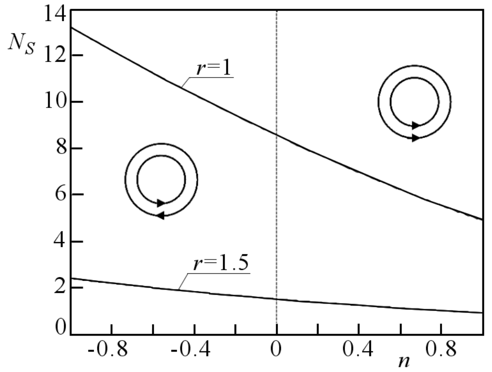

Fig. 13. Entropy generation number due to heat transfer and fluid friction versus velocity ratio for different radial locations in the pipe: $\Lambda = 0.03$, $T_0 = 1.07$, $\Gamma = 1$

5. Conclusions

A non-Newtonian fluid flow in an annular pipe and the influence of dimensionless non-Newtonian viscosity is considered in the paper. Brinkman's number and velocity ratio on the entropy generation number due to fluid friction and heat transfer are examined. The flow inside the annular gap is induced by relative rotation between the inner and outer cylinders. Near the inner cylinder, the entropy generation rate is higher due to higher temperature and velocity gradient. It is found that reduction of the dimensionless non-Newtonian viscosity increases the total entropy generation number in the inner pipe wall region. The profile of the entropy generation number shows asymptotic behavior near the outer cylinder. A growth in Brinkman's number increases the total entropy generation number, particularly in the region close to the annular pipe inner wall.

References

1. Beard D.W., Walters K., 1964, Elastico-viscous boundary layer flows, *Proc. Cambridge Philos. Soc.*, **60**, 667-674

2. Bejan A., 1979, A study of entropy generation in fundamental convective heat transfer, *J. Heat Transfer*, **101**, 719-725

3. Bejan A., 1995, *Entropy Generation Minimization*, CRC Press, New York

4. Demirel Y., Kahraman R., 2000, Thermodynamic analysis of convective heat transfer in an annular packed bed, *International Journal of Heat and Fluid Flow*, **21**, 442-448

5. Dunn J.E., Fosdick R.L., 1974, Thermodynamics, stability, and boundedness of fluids of complexity 2 and fluids of second grade, *Arch. Rat. Mech. Anal.*, **56**, 191

6. Fosdick R.L., Rajagopal K.R., 1980, Thermodynamics and stability of fluids of third grade, *Proc. R Soc. Lond.*, **A 339**, 351-377

7. Mahmud S., Fraser R.A., 2002, Inherent irreversibility of channel and pipe flows for non-Newtonian fluids, *Int. Comm. Heat Mass Transfer*, **29**, 577-587

8. Maron D.M., Cohen S., 1991, Hydrodynamics and heat/mass transfer near rotating surfaces, *Adv. Heat Transfer*, **21**, 141-183

9. Massoudi M., Christie I., 1995, Effect of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe, *Int. J. Non-Linear Mech.*, **30**, 687-699

10. Peters G.W.M., Baaijens F.P.T., 1997, Modeling of non-isothermal viscoelastic flows, *Journal of Non-Newtonian Fluid Mechanics*, **68**, 205-224

11. Sahin A.Z., 1998, Second law analysis of laminar viscous flow through a duct subjected to constant wall temperature, *ASME J. Heat Transfer*, **120**, 76-83

12. Sahin A.Z., 1999, Effect of variable viscosity on the entropy generation and pumping power in a laminar fluid flow through a duct subjected to constant heat flux, *Heat Mass Transfer*, **35**, 499-506

13. Szeri A.Z., Rajagopal K.R., 1985, Flow of a non-Newtonian fluid between heated parallel plates, *Int. J. Non-Linear Mech.*, **20**, 91

14. Yilbas B.S., Yürüsoy M., Pakdemirli M., 2004, Entropy analysis for non-Newtonian fluid flow in annular pipe: Constant viscosity case, *Entropy* 304-315

15. Yürüsoy M., Pakdemirli M., 2002, Approximate analytical solutions for the flow of a third grade fluid in a pipe, *Int. J. Non-Linear Mech.*, **37**, 187-195

16. Zhang Z., Shi L., 1999, Shannon entropy characteristics of two phase flow systems, *Journal of Applied Physics*, **85**, 7544-7551

Generacja entropii przy przepływie indukowanym względnym ruchem obrotowym ścian pierścieniowego przewodu – przypadek przepływu o stałej lepkości

Streszczenie

W pracy zbadano zagadnienie generacji entropii obserwowanej podczas przepływu nieniutonowskiej cieczy przez przewód pierścieniowy, którego ścianki obracają się względem siebie. Do analizy przyjęto płyn trzeciego stopnia o stałej lepkości. Przepływ czynnika jest indukowany względnym ruchem obrotowym zewnętrznego i wewnętrznego cylindra tworzącego ścianki przewodu. Rozwiązania analityczne zaprezentowano dla rozkładu prędkości i temperatury płynu, a liczbę generacyjną entropii wyznaczono dla różnych wartości lepkości nieniutonowskiej cieczy, liczby Birnkmana i stosunku prędkości obwodowej cylindrów. Potwierdzono, że zwiększenie bezwymiarowej lepkości obniża liczbę generacyjną entropii. Ten efekt jest szczególnie wyraźny w obszarze bliskim ściany wewnętrznego cylindra. Wzrost liczby Birnkmana powiększa liczbę generacyjną entropii, także w pobliżu ściany wewnętrznej przewodu.

Manuscript received February 21, 2007; accepted for print July 16, 2007