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PROBLEMS OF NUMERICAL BIFURCATION REPRODUCING IN POST-CRITICAL DEFORMATION STATES OF AIRCRAFT STRUCTURES

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The study presents results of research on the problem of obtaining credible results of nonlinear FEM analyses of thin-walled load-bearing structures subjected to post-critical loads. The similarity of numerical simulation results and actual stress distribution states depends on the correct numerical reproduction of bifurcations that occur during an advanced deformation process.

Key words: bifurcation, experiment, finite elements

1. Introduction

Modern aviation structures are characterised by widespread application of thin-shell load-bearing systems. The strict requirements with regard to the levels of transferred loads and the need to minimise structure mass often become causes for accepting physical phenomena that in the case of other structures are considered as inadmissible. An example of such a phenomenon is the loss of stability of shells that are parts of load-bearing structures within the range of admissible loads (Kopecki, 2010; Niu, 1988).

Thus, an important stage in the design work on an aircraft load-bearing structure is to determine stress distribution in the post-critical deformation state. One of the tools used to achieve this aim is the nonlinear finite elements method. The assessment of the reliability of the results thus obtained is based on the solution uniqueness rule, according to which a specific deformation form can correspond to one and only one stress state. In order to apply this rule, it is required to obtain numerical displacements distribution in the model fully corresponding to actual deformations of the analysed structure (Arborcz, 1985).

An element deciding about of the deformation state structure is the effect of rapid change of the structure shape occurring when the critical load levels are crossed. From the numerical point of view, this phenomenon is interpreted as a change of the relation between the state parameters corresponding to particular degrees of freedom of the system and the control parameter related to the load. This relation, defined as the equilibrium path, in the case of occurrence of the mentioned phenomenon, has an alternative character, defined as bifurcation. Therefore, the fact of taking a new deformation form by the structure corresponds to a sudden change to the alternative branch of the equilibrium path (Mohri *et al.*, 2002).

Therefore, a prerequisite condition for obtaining a proper form of the numerical model deformation is to retain the conformity between numerical bifurcations and bifurcations in the actual structure. In order to determine such conformity, it is required to verify the results obtained by an appropriate model experiment or by using the data obtained during tests on the actual object. It is often troublesome to obtain reliable results of nonlinear numerical analyses, and it requires an appropriate choice of numerical methods dependent upon the type of the analysed structure and precise determination of parameters controlling the course of procedures.

Due to the number of state parameters, the full equilibrium path should be interpreted as a hyper-surface in the state hyperspace, satisfying the matrix equation for residual forces (Felippa, 1976)

$$\mathbf{r}(\mathbf{u}, \mathbf{\Lambda}) = \mathbf{0} \tag{1.1}$$

where $\mathbf{u}$ is the state vector containing displacement components of the structure nodes corresponding to current geometrical configuration, $\mathbf{\Lambda}$ is a matrix composed of the control parameters corresponding to the current load state, and $\mathbf{r}$ is the residual vector containing uncompensated components of forces related to the current system deformation state. The set of control parameters may be expressed by a single parameter being a function of the load. Equation (1.1) takes then the following form

$$\mathbf{r}(\mathbf{u}, \lambda) = \mathbf{0} \tag{1.2}$$

called the monoparametric equation of residual forces.

The prediction-correction methods of determining the consecutive points of the equilibrium path used in modern programs contain also a correction phase based on the satisfaction of an additional equation by the system, called the increment control equation or constraints equation

$$c(\varDelta\mathbf{u}_n, \varDelta\lambda_n) = 0 \tag{1.3}$$

where the increments

$$\varDelta\mathbf{u}_n = \mathbf{u}_{n+1} - \mathbf{u}_n \qquad\qquad \varDelta\lambda_n = \lambda_{n+1} - \lambda_n \tag{1.4}$$

correspond to the transition from the n-th state to $(n+1)$-th state (Ramm, 1987).

In order to find out whether there is full conformity between the character of actual deformations and their numerical representation, it would be required to compare the combinations of the relevant state parameters in all phases of the course of the phenomenon considered herein. Because of the complication of such a comparative system, the deformation processes are represented in practice by applying substitute characteristics called representative equilibrium paths (Bathe, 1996). They define the relations between a control parameter related to load and a selected, characteristic geometric value related to deformation of the structure, an increment of which corresponds to a change in the value of all or some state parameters (Lynch *et al.*, 2004).

In the case of a large number of state parameters, it is not possible at all to represent the character of bifurcation by applying a representative equilibrium path. Sometimes, changes of state parameters resulting from local bifurcation may show the lack of perceptible influence on the representative value, which results in non-occurrence of any characteristic points on the representative path. In general, however, these changes cause a temporary drop in the control parameter value (Fig. 1).

So both the experiment itself and nonlinear numerical analyses may result only in a representative equilibrium path (Rakowski and Kacprzyk, 2005). In that case, the problem of the numerical representation of bifurcation comes down to the preservation of conformity of the representative equilibrium path obtained by a numerical method with the one obtained experimentally, where a sine qua non for the application of the solution uniqueness rule is to recognise the similarity of the post-critical deformation forms of the experimental and numerical models as sufficient (Doyle, 2001).

An additional problem that occurrs during the experimental determination of the equilibrium path, results from the lack of ability of recording the said temporary, little drops in the load arising from local bifurcations, which changes the values of some state parameters. In the

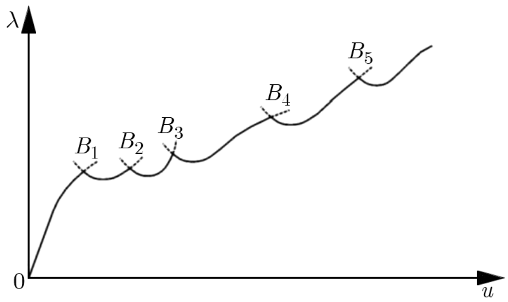

Fig. 1. Bifurcation points on a representative equilibrium path (u – representative geometric value, λ – control parameter related to load)

majority of experiments, the load of the tested model is achieved by force control, e.g. using a gravitational system, or displacement, by means of various types of load-applying devices.

However, even in the case of devices with high a level of technical advancement, in general it is not possible to register precisely short-lasting force changes occurring from the beginning of a bifurcation phenomenon to the moment of reaching the consecutive deformation form by the model. Therefore, the representative equilibrium path obtained as a result of the experiment is of smooth characteristics and its formation is based on measuring points corresponding to the consecutive deformation states determined.

In the case of nonlinear numerical analysis in the finite element approach, the accuracy of the obtaining of the representative equilibrium path may be much better. The existing commercial programs usually offer the results of all the increment steps, followed during the calculation process, and thus they also allow observing slight fluctuations of the control parameter. The only limitation here is exclusively the value of the incremental step itself. In spite of this, due to the lack of possibilities of relating the results obtained to the relevant detailed changes of the experimental characteristics, it seems appropriate to determine the numerical representative equilibrium path of the same level of simplification as in the case of the experiment.

2. Analysis of exemplary structures

The comparative analysis of such representative equilibrium paths is not, however, a method that allows a complete enough verification of the reliability of the results of numerical calculations. An example of a problem in which the calculated results have been deemed incorrect despite the seeming full conformity of the representative equilibrium paths is a thin-shell open cylindrical structure with edges strengthened by stringers, working in conditions of constrained torsion (Fig. 2). This type of systems is quite often used in aviation structures. They form areas of cockpits and large cut-outs, e.g. in cargo airplanes and, they are usually adjacent to much stiffer fragments of the structures.

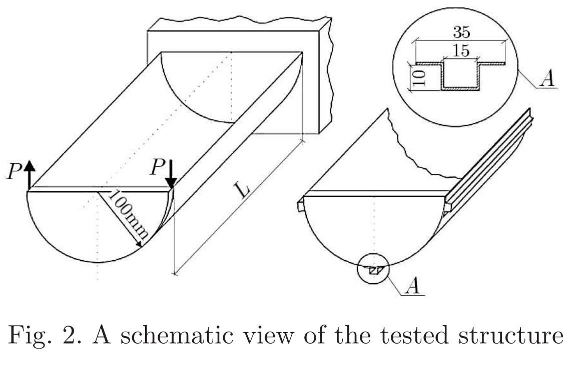

Fig. 2. A schematic view of the tested structure

The area adjacent directly to the closing frame turned out to be crucial in the problem under consideration. The stringer strengthening the edge of the structure was buckled, and the experimental model sustained plastic deformation. The relation between the total angle of torsion of the examined structure and the torque constituting the load was adopted as the representative equilibrium path. In spite of the seeming conformity of the obtained characteristics, a different character of deformation was observed in the critical area (Fig. 3). The divergence seems to result here from the symmetric character the bifurcation phenomenon initiating the deformation. Different deformations correspond to two possible variants of the actual equilibrium path the differentiation of which is not possible in the case of using a simplified representative path. So, in spite of the conformity of the characteristics, the effective stress distributions obtained numerically cannot be considered as reliable.

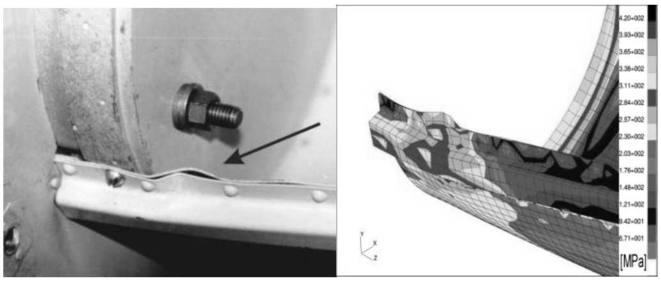

Fig. 3. A comparison of the deformation forms obtained experimentally and numerically

The need for analysis of stress distributions in the case of structures similar to the one presented above occurs quite rarely due to the commonly adopted principle, pursuant to which beam structures after buckling are considered as damaged. (Andrianov *et al.*, 2006). The considerations relating to shells used in the aviation industry, e.g. semi-monocoque structure elements are of much greater practical significance.

Examples of such a system are open cylindrical shells, which were subjected to a cycle of tests during which it was assumed that stringers were characterised by a sufficient margin of stiffness and they do not lose stability (Fig. 4).

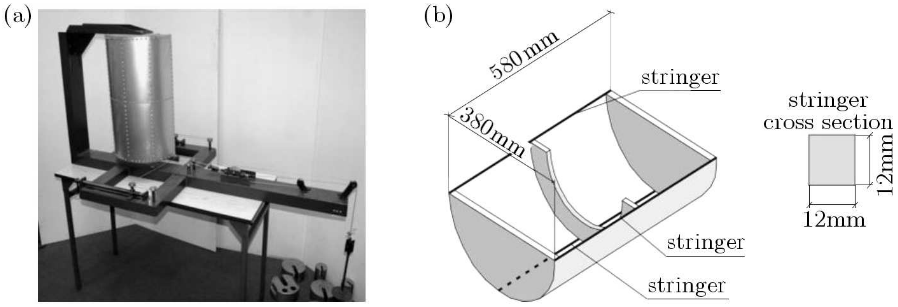

Fig. 4. Experimental stand (a), geometry of the model (b)

So the models made of polycarbonate were strengthened with longitudinal members with a large rectangular cross-section and relatively high values of geometric moments of inertia. Each examined system was subjected to constrained torsion using the test stand presented in Fig. 4. The tests aimed at developing the methodology of determination of stress distributions in the shell structure in post-critical deformation states.

In the condition of torsion, the stress state created in the shell of such a system may be interpreted as an incomplete tension field. As a result, even if there are no geometrical imperfections, the shell loses its stability. On the other hand, the post-critical deformation increment causes significant stress redistribution. The experiments repeated a number of times showed that the final form of post-critical deformations of such systems, occurring at sufficiently high load values, is always the same in spite of the alternative character of the course of the process of changing the structure state.

The fact that the local bifurcations following the increase in loads occur with some scatter of locations and stress levels makes the nonlinear numerical analysis particularly troublesome in this case. It is practically impossible to develop a FEM model allowing one to reproduce accurately the entire process of changes of the structure state, using commercial software, due to the nature of functioning of algorithms for choosing the variants of the equilibrium path at the bifurcation points and the impossibility of user's interference in the form of those algorithms. In this situation, it seems appropriate to focus only on obtaining a numerical solution consistent with the experimental results at the assumed load values.

The selection of an appropriate combination of numerical methods and parameters controlling the course of the analysis seems particularly vital in this case, likewise the proper representation of the model stiffness. Even small mistakes in this respect result in the occurrence of incorrect forms of deformation (Fig. 5).

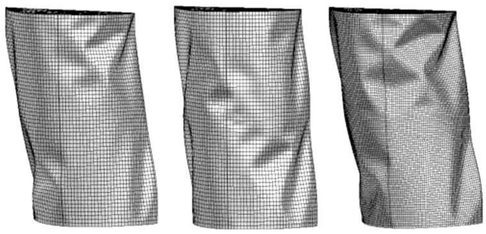

Fig. 5. Incorrect forms of shell stability loss, showing member buckling not revealed in the experiment

It should be emphasised that it seems very risky to rely in the design process on the results of nonlinear numerical analysis of similar structures without appropriate verification by an experiment, if only a relatively cheap model experiment. In practice, multiple repetition of the analysis and systematic comparison of its results with the results of the experiment are required to obtain correct results of the numerical representation of the structure state in the conditions of post-critical loads (Fig. 6).

(a) (b)

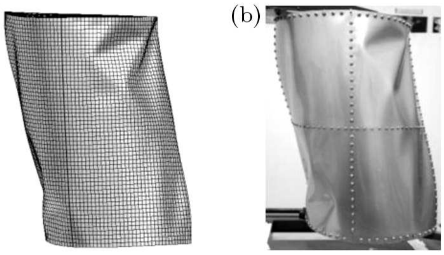

Fig. 6. Accepted as a satisfactory form of post-critical deformation obtained from nonlinear analysis (a) and a deformation form obtained from experimental tests at the identical load (b)

The research results of various load-bearing structures confirm that the difficulty related to carrying out an appropriate nonlinear numerical analysis results from the nature of bifurcation. If the change of the structure form is gentle in nature and it occurs in a small area, then the bifurcations related to it occur gradually, in relatively small subsets of state parameters. The numerical simulation of the process is then easy to perform and it may take place when using prediction-correction methods with a simple correction based on the state control (Marcinowski, 1999). But if the deformation occurs in a larger area, and the change of the form is violent in nature, then the bifurcation corresponds to the simultaneous change of a great number of state parameters, and the determination through a numerical procedure of their appropriate combination, corresponding to the new state of static equilibrium, may be hindered or even impossible. In such a case, it is necessary to match the prediction methods with correction strategies based on arc length control methods, such as the Riks correction or the Crisfield hyperspherical correction (Crisfield, 1997).

Bifurcation changes in form of load-bearing structures, containing shells of considerable curvature, occur more violently if there is a stronger relation of the square of the smaller dimension of the shell segment area limited by the adjacent member frames to the value of the local radius of its curvature (Brzoska, 1965). Thus, semi-monocoque structures of a relatively low number of framing elements are especially troublesome in nonlinear numerical representations.

An example of such a structure is a closed cylindrical shell presented in Fig. 7.

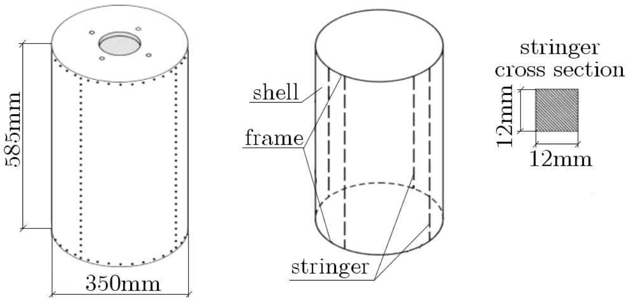

Fig. 7. A schematic view of a complete cylindrical shell reinforced by four members and a schematic view of the structure including dimensions

The structure framing consists of a minimum number of crosswise elements, i.e. two closing frames and four longitudinal members. The type of the structure itself corresponds to solutions commonly used in the aviation technology, e.g. the construction of a fuselage of an aircraft. It should be emphasised, however, the model subjected to examinations constitutes a special instance of a structure of the purposefully minimised number of longitudinal members. The actual solutions are usually based on much more extended framings. The structure described corresponds to an isolated phase of a wider cycle of examinations aiming at determination of direct dependences between the number of framing elements and post-critical deformation distributions.

The examined structure was subjected to constrained torsion using a modified version of the stand presented in Fig. 4.

According to expectations, post-critical deformations occurred in a violent way. Due to the gravitational way of load application, the measurement of the relation between the angle of torsion and the torque, assumed as the representative equilibrium path, corresponded to steady states (Fig. 11).

Using this mode of taking measurements, the representative characteristics do not reflect bifurcation points in an overt way, but attention should be drawn to the occurrence of its horizontal section. It corresponds to this phase of the experiment in which a sudden change occurred in the structure state with the simultaneous constant load level.

With regard to the symmetry, the deformed structure possessed four characteristic grooves in all shell segments (Fig. 8). During the experiment, the surface geometry was registered using the projection Moiré method. Atos scanner manufactured by a German company, GOM Optical Measuring Techniques was used as a registering device.

The problem discussed belongs to one of the most troublesome from the point of view of FEM nonlinear numerical simulation. A number of tests performed using the MSC MARC software revealed the lack of effectiveness of its procedures in the case of this problem, with regard to determination of the appropriate post-buckling state of the structure. The algorithms used in those procedures are characterised by inability to represent the symmetry of the phenomenon.

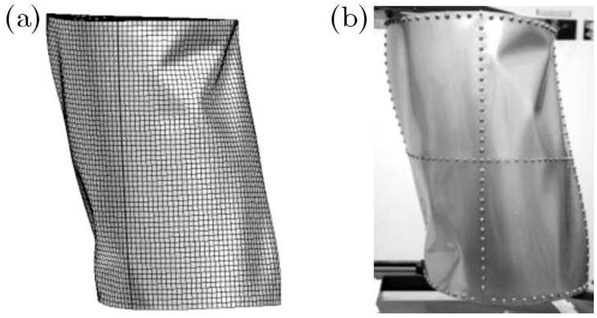

Fig. 8. The advanced post-critical deformation of the examined structure (a) and the distribution of contour lines representing size of the deformation found by making use of the Moiré projection method (b)

With the idealised geometric form of the model, the obtaining of the new form of the structure after crossing the critical load occurs only in one of the segments, in spite of the apparently correct, symmetrical initiation of the stability loss. This proves faultness of the algorithms for choosing the appropriate variants of the equilibrium path in the case of the appearance of changes in the state parameters combination in several of their independent subsets.

The situation was improved when shell imperfections were implemented by applying normal forces to the skin in the central points of a particular skin (Fig. 9).

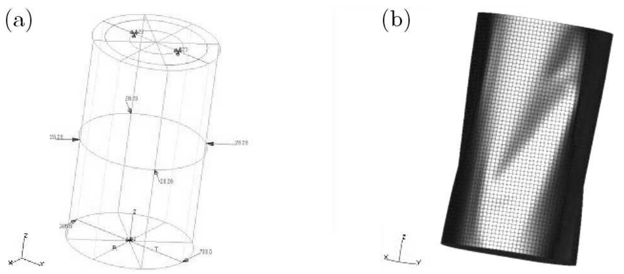

Fig. 9. A geometrical model of a structure made in MSC PATRAN environment with boundary conditions and loading (a). The incorrect form of deformation obtained in the case of too many elements (b)

However, even in the case of applying this type of forcing, it was very difficult to obtain results that would fully correspond to the experimental results. Assuming the use of skin elements with linear shape functions, the appropriate density of the mesh turned out to be the key factor, but its excessive density caused incorrect forms of post-critical deformations (Fig. 9).

A better result, in the case of application of beam elements as a representation of stringers, was obtained with the use of a relatively low density of mesh. This proves the rightness of the thesis, proved a number of times in many studies, pursuant to which the decrease in the general number of degrees of freedom corresponding to the number of state parameters in nonlinear procedures used in the available commercial programs, often brings benefits that considerably exceed the deficiencies of a mathematical description resulting from the decrease in the number of elements.

The best result was obtained only after a fundamental change in the concept of the FEM model, when a different kind of finite elements was applied as the representation of stringers (thick shell element was used instead of the recommended beam element). However this solution, which from the point of view of mathematical description is much less correct, it turned out much more effective in the case of relatively low values of the total angle of torsion of the structure. The results of analysis of this FEM model version obtained using the secant prediction method and strain correction strategy are presented in Fig. 10.

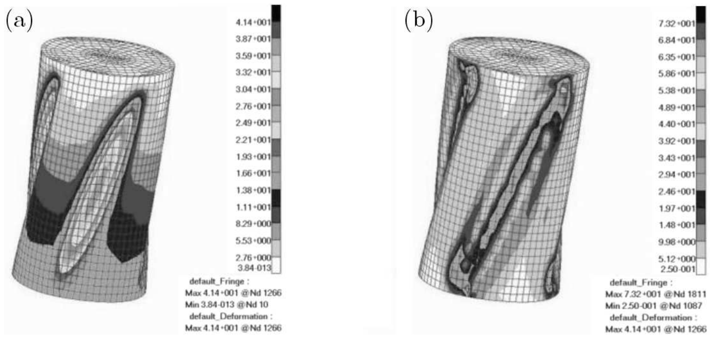

Fig. 10. The deformation distribution (a) and reduced stress distribution according to Huber-Mises hypothesis (b) for 100% of the maximum load (stringers modelled with thick shell elements)

The strain-correction strategy turned out to be the most effective in the case of significant, violent change in the form of deformations, when the representative equilibrium path contains a relatively long "horizontal section".

The relation between the representative equilibrium paths is presented in Fig. 11.

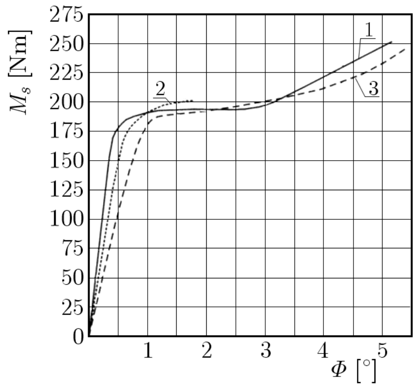

Fig. 11. The presentation of the representative equilibrium paths; 1 – experiment, 2 – numerical analysis – stringers modelled with beam elements, 3 – numerical analysis – stringers modelled with shell elements

3. Conclusion

The presented examples of load-bearing structures represent only some of those widely used in the modern aviation technology. But the criterion applied while selecting them as objects of experimental and numerical analyses was the representativeness for the most commonly met elements of constructions in which the occurrence of local stability loss is acceptable in service load conditions.

The fundamental conclusion that can be drawn from the presented research results is the absolute need for using experimental verifications with regard to FEM nonlinear numerical analyses for this type of structures. The more so that even when the correctness of the obtained results seems unquestionable, they may be in fact burdened with errors resulting from the very limited reliability of the numerical procedures used in commercial programs.

Based on the nonlinear numerical analyses, related to the presented structures and frequently repeated many times, a general recommendation may also be formulated for the maximum possible limitation of the size of a task. Striving for increasing the accuracy of the calculations by increasing the density of finite elements mesh successfully applied in linear analyses, may turn out ineffective in the case of a nonlinear analysis and may lead to incorrect results or the lack of convergence of calculations.

The numerical representation of bifurcation, by virtue of the mere idea of the discrete representation of continuous systems, must be simplified in the finite element method. In such a situation, based on the quoted examples, the need must be emphasised for obtaining the indispensable convergence of the experimental and obtained numerically relations between a selected geometric parameter characterising the essence of the structure deformation and a selected value relating to the load, recognised as the representative equilibrium paths. This convergence, in combination with the accepted as sufficient similarity of post-critical deformation forms, constitutes the grounds for accepting the reliability of stress distributions determined by means of numerical methods.

References

1. Arborcz J., 1985, Post-buckling behavior of structures. Numerical techniques for more complicated structures, *Lecture Notes in Physics*, **228**, USA
2. Andrianov I.V., Verbonol V.M., Awrejcewicz J., 2006, Buckling analysis of discretely stringer-stiffened cylindrical shells, *International Journal of Mechanical Sciences*, **48**, 1505-1515
3. Bathe K.J., 1996, *Finite Element Procedures*, Prentice Hall, USA
4. Brzoska Z., 1965, *Statics and Stability of Bar and Thin-Walled Structures*, PWN, Warszawa, Poland
5. Crisfield M.A., 1997, *Non-Linear Finite Element Analysis of Solid and Structures*, J. Wiley & Sons, New York
6. Doyle J.F., 2001, *Nonlinear Analysis of Thin-Walled Structures*, Springer-Verlag, Berlin, Germany
7. Felippa C. A., 1976, *Procedures for Computer Analysis of Large Nonlinear Structural System in Large Engineering Systems*, Pergamon Press, London, UK
8. Kopecki T., 2010, *Advanced Deformation States in Thin-Walled Load-Bearing Structure Design Work*, Publishing House of Rzeszów University of Technology, Rzeszów, Poland
9. Lynch C., Murphy A., Price M., Gibson A., 2004, The computational post buckling analysis of fuselage stiffened panels loaded in compression, *Thin-Walled Structures*, **42**, 1445-1464
10. Marcinowski J., 1999, *Nonlinear Stability of Elastic Shells*, Publishing House of Technical University of Wrocław, Poland
11. Mohri F., Azrar L., Potier-Ferry M., 2002, Lateral post buckling analysis of thin-walled open section beams, *Thin-Walled Structures*, **40**, 1013-1036
12. Niu M.C., 1988, *Airframe Structural Design*, Conmilit Press Ltd., Hong Kong
13. Rakowski G., Kacprzyk Z., 2005, *Finite Elements Method in Structure Mechanics*, Publishing House of Technical University of Warszawa, Warszawa, Poland
14. Ramm E., 1987, *The Riks/Wempner Approach – An Extension of the Displacement Control Method in Nonlinear Analysis*, Pineridge Press, Swensea, UK

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