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DYNAMIC STABILITY OF A METAL FOAM CIRCULAR PLATE

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The study is devoted to a radial compressed metal foam circular plate. Properties of the plate vary across its thickness. The middle plane of the plate is its symmetry plane. First of all, a displacement field of any cross-section of the plate was defined. Afterwards, the components of strain and stress states were found. The Hamilton principle allowed one to formulate a system of differential equations of dynamic stability of the plate. This basic system of equations was approximately solved. The forms of unknown functions were assumed and the system of equations was reduced to a single ordinary differential equation of motion. The equation was then numerically processed that allowed one to determine critical loads for a family of metal foam plates. The results of studies are shown in figures. They show the effect of porosity of the plate on the critical loads. The results obtained for porous plates were compared to homogeneous circular plates.

Key words: circular plate, dynamic stability, porous-cellular metal

1. Introduction

There exist many works on the theory and analysis of plates. Most of them deal with the classical (Kirchhoff) theory, which is not adequate in providing accurate buckling. This is due to the effect of transverse shear strains. Shear deformation theories provide accurate solutions compared to the classical theory. During the last several years, this problem has been developed by many authors. Banhart (2001) provided a comprehensive description of various manufacturing processes of metal foams and porous metallic structures. Structural and functional applications for different industrial sectors have been discussed. Awrejcewicz *et al.* (2001) described regular and chaotic behaviour of flexible plates. Qatu's (2004) book documents some of the latest research in the field of vibration of composite shells and plates and fills certain gaps in this area of research. Malinowski and Magnucki (2005) assumed a non-linear hyphotesis of deformation of a plane cross-section of cylindrical shells. The buckling problem was described for an isotropic porous shell. Magnucki and Stasiewicz (2004) presented a problem of elastic buckling of a porous isotropic beam with varying properties through thickness. They also assumed a non-linear hypothesis. Instead, Szcześniak (2001) first of all described the problem of forced vibration of the plate. Forced vibration dependent on impulsive, harmonic and other loads was analized.

2. Displacements of a porous plate

This work is concerned with two isotropic porous circular plates under radial uniform compression. The first one has a simply supported edge and the other one a clamped edge. It is a continuation of the paper by Magnucka-Blandzi (2008). This kind of material – a metal foam – was described, for example, by Banhart (2001). Mechanical properties of the material vary through thickness of the plate. Minimal value of Young's modulus occurs in the middle surface of the plate and maximal values at its top and bottom surfaces. For such a case, the Kirchhoff and Mindlin plate theories do not correctly determine displacements of the plate cross-section. Wang *et al.* (2000) discussed in details the effect of non-dilatational strain of middle layers on bending of plates subject to various load cases. Magnucka-Blandzi and Magnucki (2005), Magnucka-Blandzi (2006) thoroughly described the non-linear hypothesis of deformation of the plate cross-section. A porous plate (Fig. 1) is a generalized sandwich plate. Its outside surfaces (top and bottom) are smooth, without pores. The material is of continuous mechanical properties. The plate is porous inside, with the degree of porosity varying in the normal direction, assuming the minimal value in the middle surface of the plate. A polar (cylindrical) coordinate system is introduced with the z-axis in the depth direction.

The moduli of elasticity and mass density are defined as follows

$$
\begin{aligned}
E(z) &= E_1[1 - e_0\cos(\pi\zeta)] \\
G(z) &= G_1[1 - e_0\cos(\pi\zeta)] \\
\varrho(z) &= \varrho_1[1 - e_m\cos(\pi\zeta)]
\end{aligned}
\tag{2.1}
$$

where

e_0 – porosity coefficient of elasticity moduli, $e_0 = 1 - E_0/E_1$,

e_m – dimensionless parameter of mass density, $e_m = 1 - \varrho_0/\varrho_1$,

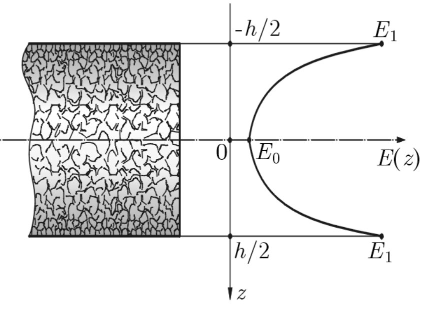

Fig. 1. Scheme of a porous plate

E_0, E_1	–	Young's moduli at $z = 0$ and $z = \pm h/2$, respectively,
G_0, G_1	–	shear moduli for $z = 0$ and $z = \pm h/2$, respectively,
G_j	–	relationship between the moduli of elasticity for $j = 0, 1$, $G_j = E_j/[2(1 + \nu)]$,
ν	–	Poisson's ratio (constant for the plate),
ϱ_0, ϱ_1	–	mass densities for $z = 0$ and $z = \pm h/2$, respectively,
ζ	–	dimensionless coordinate, $\zeta = z/h$,
h	–	thickness of the plate.

Choi and Lakes (1995) presented mechanical properties for porous materials. Taking into account the results of investigations of this paper, the relation between the dimensionless parameter of mass density $e_m = 1 - \varrho_0/\varrho_1$ and dimensionless parameter of the porosity of the metal foam e_0 is defined as follows $e_m = 1 - \sqrt{1 - e_0}$. The field of displacements (geometric model) is shown in Fig. 2. The cross-section, being initially a planar surface, becomes a surface (not a flat surface) after deformation. The surface perpendicularly intersects the top and the bottom surfaces of the plate. Magnucka-Blandzi and Magnucki (2005), Magnucki *et al.* (2006), Magnucki and Stasiewicz (2004) proposed a non-linear hypothesis of cross-section deformation of the structure wall. Applying this hypothesis to a metal foam circular plate, the radial displacement in any cross-section is in the form

$$u(r, z, t) = -h\Big\{\zeta\frac{\partial w}{\partial r} - \frac{1}{\pi}[\psi_1(r, t)\sin(\pi\zeta) + \psi_2(r, t)\sin(2\pi\zeta)\cos^2(\pi\zeta)]\Big\} \tag{2.2}$$

where

$u(r, z, t)$	–	longitudinal displacement along the r-axis,
$w(r, t)$	–	deflection (displacement along the z-axis),
$\psi_1(r, t), \psi_2(r, t)$	–	dimensionless functions of displacements.

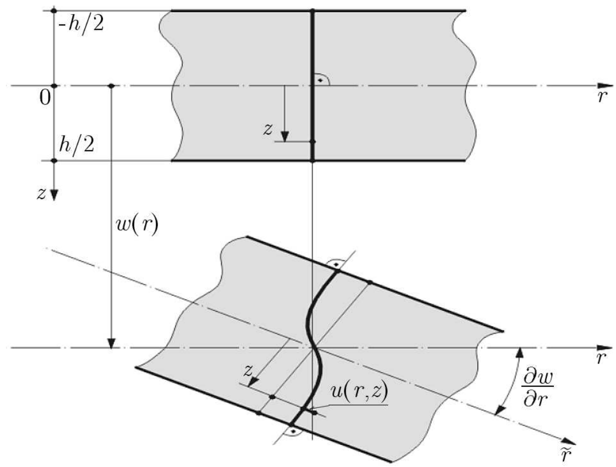

Fig. 2. Scheme of deformation of a plane cross-section of the plate – the nonlinear hypothesies

In the particular case $\psi_1(r,t) = \psi_2(r,t) = 0$, the field of displacement u is the linear Kirchhoff-Love hypothesis. Functions $\psi_1(r,t)$, $\psi_2(r,t)$ extend the linear classical hypothesis. In the classical theory, the shear force is equal to 0 (it follows from this linear theory), but in the proposed non-linear hypothesis the shear force does not equal 0, what corresponds with the facts. The geometric relationships, i.e. components of the strain are

$$
\begin{aligned}
\varepsilon_r &= \frac{\partial u}{\partial r} = -h\Big\{\zeta\frac{\partial^2 w}{\partial r^2} - \frac{1}{\pi}\Big[\frac{\partial \psi_1}{\partial r}\sin(\pi\zeta) + \frac{\partial \psi_2}{\partial r}\sin(2\pi\zeta)\cos^2(\pi\zeta)\Big]\Big\} \\
\varepsilon_\varphi &= \frac{u}{r} = -h\Big\{\frac{1}{r}\zeta\frac{\partial w}{\partial r} - \frac{1}{\pi}\Big[\frac{1}{r}\psi_1(r,t)\sin(\pi\zeta) + \frac{1}{r}\psi_2(r,t)\sin(2\pi\zeta)\cos^2(\pi\zeta)\Big]\Big\} \\
\gamma_{rz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = \psi_1(r,t)\cos(\pi\zeta) + \psi_2(r,t)\Big[\cos(2\pi\zeta) + \cos(4\pi\zeta)\Big]
\end{aligned}
\tag{2.3}
$$

where ε_r is the normal strain along the r-axis, ε_φ is the circular strain, and γ_{rz} – shear strain.

The physical relationships, according to Hooke's law, are

$$
\begin{aligned}
\sigma_r &= \frac{E(z)}{1-\nu^2}(\varepsilon_r + \nu\varepsilon_\varphi) \qquad\qquad \sigma_\varphi = \frac{E(z)}{1-\nu^2}(\varepsilon_\varphi + \nu\varepsilon_r) \\
\tau_{rz} &= G(z)\gamma_{rz}
\end{aligned}
\tag{2.4}
$$

Moduli of elasticy (2.1) occuring here are variable and depend on the z-coordinate. The similar porous plate model was presented by Magnucki *et al.* (2006).

3. Equations of stability

The field of displacements in the above defined problem includes three unknown functions: $w(r,t)$, $\psi_1(r,t)$ and $\psi_2(r,t)$. Hence, three equations are necessary for complete description of this problem. They may be formulated basing on the Hamilton principle

$$\delta \int\limits_{t_1}^{t_2} (T - U_\varepsilon + W)\, dt = \delta \int\limits_{t_1}^{t_2} T\, dt - \delta \int\limits_{t_1}^{t_2} (U_\varepsilon - W)\, dt = 0 \tag{3.1}$$

where T denotes the kinetic energy

$$T = \pi h \int\limits_0^R \int\limits_{-1/2}^{1/2} r\varrho(\zeta)\Big(\frac{\partial w}{\partial t}\Big)^2\, d\zeta dr$$

U_ε is the energy of elastic strain

$$U_\varepsilon = \pi h \int\limits_0^R \int\limits_{-1/2}^{1/2} r(\sigma_r\varepsilon_r + \sigma_\varphi\varepsilon_\varphi + \tau_{rz}\gamma_{rz})\, d\zeta dr$$

W is the work which follows from the compressive force

$$W = \pi N(t) \int\limits_0^R r\Big(\frac{\partial w}{\partial r}\Big)^2\, dr$$

where R denotes the radius of the plate, ϱ – mass density of the plate, t_1, t_2 – initial and final times, $N(t)$ – compressive force in the following form

$$N(t) = N_0 + N_a\cos(\theta t)$$

where N_0 is the average value of the load and N_a – amplitude of the load.

The kinetic energy is a function of $\partial w/\partial t$, however the total potential energy $(U_\varepsilon - W)$ does not depend on it. Taking into account principle (3.1), a system of three stability equations for the porous plate under compression is formulated in the following form

$$
\begin{aligned}
&(\delta w) && \frac{\partial}{\partial r}\Big\{r\frac{\partial}{\partial r}\Big\{\frac{1}{r}\frac{\partial}{\partial r}\Big[r\Big(c_0\frac{\partial w}{\partial r}-c_1\psi_1-c_2\psi_2\Big)\Big]\Big\}\Big\}+\\
&&& +4\frac{1-\nu^2}{E_1h^3}\Big[\pi N(t)\frac{\partial}{\partial r}\Big(r\frac{\partial w}{\partial r}\Big)+c_9\varrho_1 rh\frac{\partial^2 w}{\partial t^2}\Big]=0\\
&(\delta\psi_1) && \frac{\partial}{\partial r}\Big\{\frac{1}{r}\frac{\partial}{\partial r}\Big[r\Big(c_1\frac{\partial w}{\partial r}-c_3\psi_1-c_4\psi_2\Big)\Big]\Big\}+\frac{1-\nu}{h^2}\Big(c_5\psi_1+c_6\psi_2\Big)=0\\
&(\delta\psi_2) && \frac{\partial}{\partial r}\Big\{\frac{1}{r}\frac{\partial}{\partial r}\Big[r\Big(c_2\frac{\partial w}{\partial r}-c_4\psi_1-c_7\psi_2\Big)\Big]\Big\}+\frac{1-\nu}{h^2}\Big(c_6\psi_1+c_8\psi_2\Big)=0
\end{aligned}
\tag{3.2}
$$

where

$$
\begin{aligned}
c_0 &= \frac{\pi^3-6e_0(\pi^2-8)}{3\pi^2} & c_1 &= \frac{8-\pi e_0}{\pi^2} & c_2 &= \frac{225\pi-512e_0}{300\pi^2}\\
c_3 &= 2\frac{3\pi-4e_0}{3\pi^2} & c_4 &= \frac{64-15\pi e_0}{30\pi^2} & c_5 &= \frac{3\pi-8e_0}{3}\\
c_6 &= \frac{32-15\pi e_0}{30} & c_7 &= \frac{1575\pi-4096e_0}{2520\pi^2} & c_8 &= 2\frac{315\pi-832e_0}{315}\\
c_9 &= \pi-2e_m
\end{aligned}
$$

Boundary conditions are in the following form:

— for the first case (the plate with a simply supported edge)

$$
\begin{aligned}
&w(R,t)=0 && M_r(R,t)=0\\
&\psi_1(0,t)=\psi_2(0,t)=0 && \frac{\partial w}{\partial r}\Big|_{r=0}=0
\end{aligned}
\tag{3.3}
$$

where the radial bending moment is in the form

$$
\begin{aligned}
M_r &= \frac{E_1h^3}{4(1-\nu^2)}\Big[-c_0\frac{\partial}{\partial r}\mathcal{L}(w)+\frac{c_1}{\pi}\mathcal{L}(\psi_1)+\frac{c_2}{\pi}\mathcal{L}(\psi_2)\Big]\\
\mathcal{L}(f) &= \frac{df}{dr}+\frac{\nu}{r}f
\end{aligned}
$$

— for the second case (the plate with a clamped edge)

$$
\begin{aligned}
&w(R,t)=0 && \psi_1(0,t)=\psi_2(0,t)=0\\
&\frac{\partial w}{\partial r}\Big|_{r=R}=0 && \frac{\partial w}{\partial r}\Big|_{r=0}=0
\end{aligned}
\tag{3.4}
$$

The system of differential equations (3.2) may be approximately solved with the use of Galerkin's method. Hence, three unknown functions are assumed:

— one satisfying boundary conditions (3.3) in the form

$$\begin{aligned}
\psi_1(r,t) &= -6\psi_{a_1}\Big[\frac{2+\nu}{4+\nu}\Big(\frac{r}{R}\Big) - \frac{1+\nu}{4+\nu}\Big(\frac{r}{R}\Big)^2\Big]\\
\psi_2(r,t) &= -6\psi_{a_2}\Big[\frac{2+\nu}{4+\nu}\Big(\frac{r}{R}\Big) - \frac{1+\nu}{4+\nu}\Big(\frac{r}{R}\Big)^2\Big]\\
w(r,t) &= w_a(t)\Big[1 - 3\frac{2+\nu}{4+\nu}\Big(\frac{r}{R}\Big)^2 + 2\frac{1+\nu}{4+\nu}\Big(\frac{r}{R}\Big)^3\Big]
\end{aligned}$$

— and one satisfying bounadry conditions (3.4) in the form

$$\begin{aligned}
\psi_1(r,t) &= -6\psi_{a_1}\Big[\Big(\frac{r}{R}\Big) - \Big(\frac{r}{R}\Big)^2\Big]\\
\psi_2(r,t) &= -6\psi_{a_2}\Big[\Big(\frac{r}{R}\Big) - \Big(\frac{r}{R}\Big)^2\Big]\\
w(r,t) &= w_a(t)\Big[1 - 3\Big(\frac{r}{R}\Big)^2 + 2\Big(\frac{r}{R}\Big)^3\Big]
\end{aligned} \tag{3.5}$$

Because of similarity of the solution in both cases, only the second case will be considered hereafter. Substitution of above three functions (3.5) into equations (3.2) and making used Galerkin's method yields a system of three equations in the form

$$\begin{aligned}
&\Big[c_0 - \frac{4\pi(1-\nu^2)R^2}{15E_1h^3}N(t)\Big]w_a(t) - c_1R\psi_{a_1} - c_2R\psi_{a_2} +\\
&\qquad + c_9\varrho_1\frac{4(1-\nu^2)R^4}{105E_1h^2}\frac{d^2w_a}{dt^2} = 0\\
&c_1w_a - c_{13}R\psi_{a_1} - c_{12}R\psi_{a_2} = 0\\
&c_2w_a - c_{12}R\psi_{a_1} - c_{11}R\psi_{a_2} = 0
\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}
c_{10} &= \frac{(1-\nu)R^2}{15h^2} & \qquad c_{11} &= c_7 + c_8c_{10}\\
c_{12} &= c_4 + c_6c_{10} & \qquad c_{13} &= c_3 + c_5c_{10}
\end{aligned}$$

From the second and third equations of system (3.6), the functions ψ_{a_1}, ψ_{a_2} may be calculated, namely

$$\psi_{a_1} = \widetilde{\psi}_{a_1}\frac{w_a}{R} \qquad\qquad \psi_{a_2} = \widetilde{\psi}_{a_2}\frac{w_a}{R} \tag{3.7}$$

where

$$\widetilde{\psi}_{a_1} = \frac{c_1c_{11} - c_2c_{12}}{c_{13}c_{11} - c_{12}^2} \qquad\qquad \widetilde{\psi}_{a_2} = \frac{c_2c_{13} - c_1c_{12}}{c_{13}c_{11} - c_{12}^2}$$

Substitution of functions (3.7) into the first equation of system (3.6) yields the Mathieu equation in the followng form

$$\frac{d^2w_a}{dt^2} + \varOmega^2\Big[1 - 2\mu\cos(\theta t)\Big]w_a = 0 \tag{3.8}$$

where

$$\varOmega^2 = \omega^2\Big(1 - \frac{N_0}{N_{cr}}\Big) \qquad\qquad \mu = \frac{1}{2}\frac{N_a}{N_{cr} - N_0}$$

and

ω – the natural frequencies of the plate

$$\omega = \sqrt{\frac{c_0 - c_1\widetilde{\psi}_{a_1} - c_2\widetilde{\psi}_{a_2}}{c_9}\frac{105E_1h^2}{4R^4(1-\nu^2)\varrho_1}}$$

N_{cr} – the critical force [N/mm]

$$N_{cr} = \frac{15\Big(c_0 - c_1\widetilde{\psi}_{a_1} - c_2\widetilde{\psi}_{a_2}\Big)E_1h^3}{4\pi R^2(1-\nu^2)}$$

4. Numerical calculations of unstable regions

The Mathieu equation is well-known and described in mamy books and papers, for example Doyle (2001), Życzkowski (1988), Gryboś (1980) and others. They concluded that there are separate regions where unbounded solutions exist and regions where all solutions are bounded.

Based on Życzkowski (1988), the first unstable region is determined by

$$2\varOmega\sqrt{1-\mu} < \theta < 2\varOmega\sqrt{1+\mu} \tag{4.1}$$

and the second one by

$$\Omega\sqrt{1-2\mu^2} < \theta < \Omega\sqrt{1+\frac{1}{3}\mu^2} \tag{4.2}$$

Assuming two dimensionless parameters

$$\alpha_1 = \frac{N_0+N_a}{N_{cr}} \qquad\qquad \alpha_2 = \frac{N_0}{N_{cr}}$$

inequality (4.1) for the first unstable region is in the form

$$2\omega\sqrt{1-\frac{1}{2}(\alpha_1+\alpha_2)} < \theta < 2\omega\sqrt{1+\frac{1}{2}(\alpha_1-3\alpha_2)} \tag{4.3}$$

and inequality (4.2) for the second one is in that form

$$\omega\sqrt{\frac{(1-\alpha_2)^2-\frac{1}{2}(\alpha_1-\alpha_2)^2}{1-\alpha_2}} < \theta < \omega\sqrt{\frac{(1-\alpha_2)^2+\frac{1}{12}(\alpha_1-\alpha_2)^2}{1-\alpha_2}} \tag{4.4}$$

Geometric illustration of constraints of these parameters are shown in Fig. 3. I – determines the parameters α_1 and α_2 for which only the first unstable region exists. II – determines the parameters α_1 and α_2 for which only the second unstable region exists. If $\alpha_1 > 1$, then the compressive force reaches the critical value. Only the first unstable region exists if $\alpha_1 > 1$.

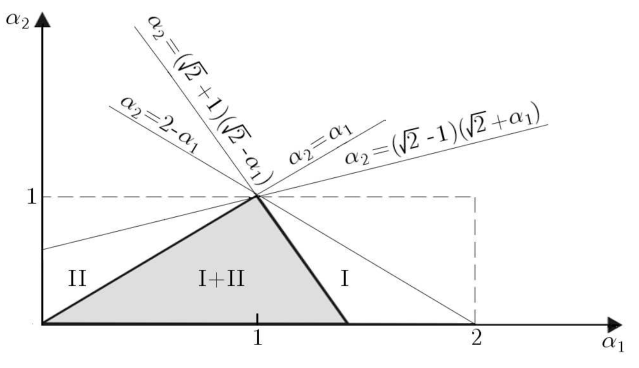

Fig. 3. Constraints of the parameters α_1 and α_2

There are three examples considered below, where the effect of porosity change is shown. A family of plates of height $h = 10$ mm and radius $R = 1500$ mm are taken into account. The material constants are

$$E_1 = 2.05\cdot 10^5\,\text{MPa} \qquad\qquad \varrho_1 = 7.78\cdot 10^{-6}\,\frac{\text{kg}}{\text{mm}^3}$$

In the first instance, $\alpha_1 = 1$, $\alpha_2 = 1/2$ are assumed, which means

$$N(t) = \frac{1}{2}N_{cr}[1 + \cos(\theta t)]$$

So, the first and the second unstable region have the form

$$\omega < \theta < \sqrt{3}\omega \qquad\qquad \frac{1}{2}\omega < \theta < \frac{\sqrt{78}}{12}\omega$$

The plots of these two unstable regions are shown in Fig. 4.

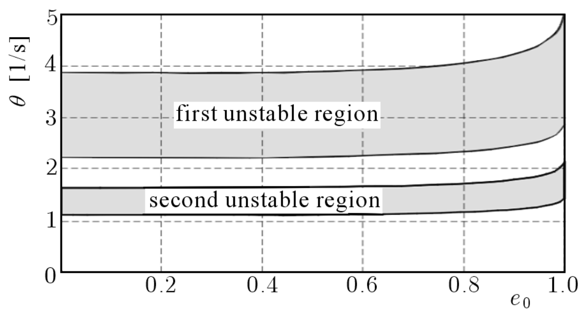

Fig. 4. Stability regions

The next example is for $\alpha_1 = 1/2$ and $\alpha_2 = 1/6$, then

$$N(t) = \frac{1}{3}N_{cr}[2 + \cos(\theta t)]$$

Figure 5 shows two unstable regions. In the above two cases, the compressive force does not reach the critical load, and it can be observed that two unstable regions exist.

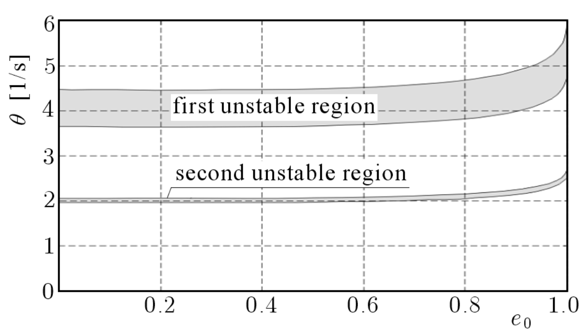

Fig. 5. Stability regions

But in the last example, the compressive force reaches the critical load. This situation was considered for $\alpha_1 = 5/3$ and $\alpha_2 = 1/6$

$$N(t) = \frac{3}{2}N_{cr}\Big[\frac{1}{9} + \cos(\theta t)\Big]$$

Only one unstable region exists there, which is presented in Fig. 6.

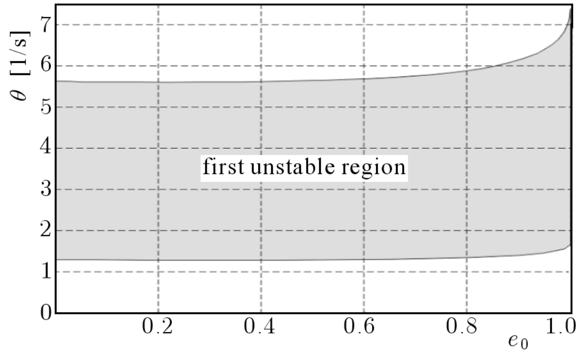

Fig. 6. Stability regions

5. Conclusions

The porous-cellular circular plate is a generalization of sandwich or multilayer plates. Correct hypotheses of plane cross-sections for homogeneous plates are useless in the case of porous-cellular plates as elastic constants vary considerably along their depth. The non-linear hypothesis of deformation of the plane cross-section for porous-cellular plates (structures) includes the linear hypothesis for homogeneous plates and the shear deformable effect. The mathematical model of dynamic stability of the porous-cellular circular plate, under a pulsating compression load, could be reduced to the Mathieu equation. Two unstable regions may determined if the compressive force does not reach the critical load. The influence of the porosity coefficient of elasticity moduli is small for the unstable regions.

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Stateczność dynamiczna płyty kołowej wykonanej z piany metalowej

Streszczenie

Przedmiotem badań jest osiowo ściskana porowata płyta kołowa. Własności płyty są zmienne na jej grubości. Środkowa płaszczyzna płyty jest jej płaszczyzną symetrii. Zdefiniowano pole przemieszczeń dla dowolnego przekroju płyty. Ponadto wyznaczone są składowe stanów odkształceń i naprężeń. Układ równań stateczności dynamicznej płyty wyznaczono z zasady Hamiltona. Następnie równania te rozwiązano metodą Galerkina i otrzymano zwyczajne równanie różniczkowe typu Mathieu. Równanie to rozwiązano metodą Rungego-Kutty i wyznaczono obciążenia krytyczne dla rodziny płyt kołowych. Wyniki przedstawiono na wykresach. Wskazano wpływ porowatości materiału na obciążenia krytyczne płyty. Otrzymane wyniki porównano z odpowiednimi wielkościami dla płyty jednorodnej.

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