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TWO HEAT CONDUCTION PROBLEMS WITH FRICTIONAL HEATING DURING BRAKING

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Analytical solutions to a boundary-value problem of heat conduction for friction pairs consisting of the half-space sliding (braking at uniform retardation) on a surface of the strip deposited on a semi-infinite foundation and the strip sliding on a surface of the homogenous half-space, are obtained. For materials of the frictional system: cast iron – metal ceramics-steel and metal ceramics-cast iron, evolutions and distributions on depth from the surface of friction for temperatures are studied.

Key words: braking, heat generation, temperature

1. Introduction

Heating problems of friction can be examined through stationary, quasi-stationary and nonstationary statements. We may assume thermal contact as a stationary one in some cases: when the slip velocity is low and the resultant convection have no impact on temperature and heat fluxes. Another case occurs when the process of heat conduction for given external conditions lasts long enough, so that the influence of initial conditions can be ignored. Quasi-stationary thermal contact takes place under the condition of sufficiently long duration of friction between bodies in relative motion, whereas nonstationary thermal contact is either conditioned by a nonstationary distribution of the contact pressure or by time-dependent slip velocity as well as by the fact that the development of heating process is considered from some initial time (Matysiak and Yevtushenko, 2001).

The thermal processes during braking are nonstationary and of short duration. A criterion for evaluation of the frictional thermal strength of materials applied in the contacting pairs, in which the principal role plays the temperature of friction was proposed by Chichinadze (1967).

Ceramic-metal frictional materials are now extensively used in brake systems (Balakin and Sergienko, 1999; Daehn and Breslin, 2006). This is explained by their high thermal stability and wear resistance (Buckman, 1998; Blau, 2001). A friction patch of brakes is designed as a thin cermet strip based either on iron or copper. In the process of braking, this patch is pressed to the counterbody (brake drum, disk, rim of the wheel, etc.). As a result of the friction action on the contact surface, the kinetic energy transforms into heat. Elements of brakes are heated and, hence, the conditions of operation of the friction patches become less favourable: their wear intensifies and the friction coefficient decreases, which may lead to emergency situations (Fazekas, 1953; Ho *et al.*, 1974). Thus, the problem of calculation of temperature is one of the most important problems in the design of brakes (Yun-Bo *et al.*, 2002).

The one-dimensional models correspond to cases when the heat flux can be assumed as normal to the contact surface (Peclet's number must be large). The verification of many analytical solutions with the experimental data which refers to the work of braking devices, shows that the one-dimensional models may be considered as a sufficiently good approximation for computation of brake systems with heat generation taken into account (Fazekas, 1953; Qi and Day, 2007).

Solutions to the problems of heat conduction in a composite solid consisting of two or three infinite slabs between parallel plane boundaries, when the interfaces are subjected to a thermal flux which decreases linearly with time were studied by Newcomb (1959a,b). Analytical and numerical solutions of the one-dimensional thermal problem of friction during braking were obtained for the case of contact between two half-spaces (Yevtushenko *et al.*, 1999). The corresponding solutions for two or three plane-parallel strips with different properties were analysed by Pyryev and Yevtushenko (2000).

In the present paper, we construct solutions to heat conduction problems with frictional heating during braking at uniform retardation for a friction pair consisting of the half-space sliding on a surface of the strip deposited on a semi-infinite foundation and the system consisting of the plane-parallel strip and the half-space. These solutions allow one to determine the temperature of friction elements from the initial time of braking to the moment, when the cooling of the solids is completed. The corresponding problems at uniform sliding were studied by Yevtushenko and Kuciej (2009a,b).

2. The half-space – strip – foundation system

Protective strips such as evaporated coatings and films are used for the improvement of wear-contact characteristics of friction elements. Therefore, the problem of contact interaction of two half-spaces is considered, where one of them is homogeneous and the other is a semi-infinite foundation with the surface covered by a strip of thickness d (Fig. 1). The perfect heat contact and the constant pressures p_0 between the strip and the foundation take place. The homogeneous upper half-space slides with the velocity

$$V(t) = V_0\Big(1 - \frac{t}{t_s}\Big)H(t_s - t) \qquad t \geqslant 0 \tag{2.1}$$

where V_0 is the initial velocity, t – time, t_s – time braking, $H(\cdot)$ – Heaviside's step function) in the direction of the y-axis on the strip surface. Due to friction, heat is generated on the contact plane $z = 0$. It is assumed that the sum of intensities of the frictional heat fluxes directed into each component of the friction pair is equal with the specific friction power

$$q(t) = q_0 q^*(t) \qquad t \geqslant 0 \tag{2.2}$$

where, taking equation (2.1) into account, we have

$$q_0 = fV_0p_0 \qquad q^*(t) = \Big(1 - \frac{t}{t_s}\Big)H(t_s - t) \qquad t \geqslant 0 \tag{2.3}$$

where f is the frictional coefficient, p_0 – pressure.

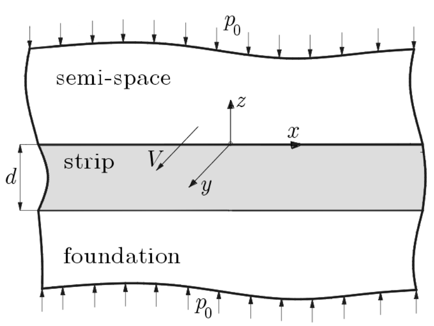

Fig. 1. Scheme of the contact system

On such assumptions, the corresponding boundary-value problem of heat conduction can be written in the dimensionless form

$$\begin{aligned}
&\frac{\partial^2 T_t^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k_t^*}\frac{\partial T_t^*(\zeta,\tau)}{\partial \tau} && 0<\zeta<\infty && \tau>0\\
&\frac{\partial^2 T_s^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T_s^*(\zeta,\tau)}{\partial \tau} && -1<\zeta<0 && \tau>0\\
&\frac{\partial^2 T_f^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k_f^*}\frac{\partial T_f^*(\zeta,\tau)}{\partial \tau} && -\infty<\zeta<-1 && \tau>0
\end{aligned} \tag{2.4}$$

$$\frac{\partial T_s^*}{\partial \zeta}\Big|_{\zeta=0^-} - K_t^*\frac{\partial T_t^*}{\partial \zeta}\Big|_{\zeta=0^+} = q^*(\tau) \tag{2.5}$$

$$T_s^*(0,\tau) = T_t^*(0,\tau) \qquad \frac{\partial T_s^*}{\partial \zeta}\Big|_{\zeta=-1^+} = K_f^*\frac{\partial T_f^*}{\partial \zeta}\Big|_{\zeta=-1^-} \tag{2.6}$$

$$T_s^*(-1,\tau) = T_f^*(-1,\tau) \qquad T_{t,f}^*(\zeta,\tau)\to 0 \qquad |\zeta|\to\infty \tag{2.7}$$

$$\begin{aligned}
&T_t^*(\zeta,0)=0 && 0\leqslant\zeta<\infty\\
&T_s^*(\zeta,0)=0 && -1\leqslant\zeta\leqslant 0\\
&T_f^*(\zeta,0)=0 && -\infty<\zeta\leqslant -1
\end{aligned} \tag{2.8}$$

where

$$q^*(\tau) = \Big(1-\frac{\tau}{\tau_s}\Big)H(\tau_s-\tau) \qquad \tau\geqslant 0 \tag{2.9}$$

and

$$\begin{aligned}
&K_f^* = \frac{K_f}{K_s} && K_t^* = \frac{K_t}{K_s} && k_f^* = \frac{k_f}{k_s}\\
&k_t^* = \frac{k_t}{k_s} && \zeta = \frac{z}{d} && \tau = \frac{k_s t}{d^2}\\
&T_t^* = \frac{T_t}{T_0} && T_s^* = \frac{T_s}{T_0} && T_f^* = \frac{T_f}{T_0} && T_0 = \frac{qd}{K}
\end{aligned} \tag{2.10}$$

where K is the coefficient of heat conduction, k – coefficient of thermal diffusivity, T – temperature, z – spatial coordinate. Moreover, all values and parameters concerned with the top half-space, strip and foundation will have bottom indexes t, s and f, respectively.

Taking the form of function $q^*(\tau)$ (2.9) and the linearity of boundary-value problem (2.4)-(2.8) in to account, the dimensionless temperature at any moment of time τ at the distance $|\zeta|<\infty$ from the surface of friction may be presented as the superposition (Yevtushenko and Matysiak, 2005; Yevtushenko and Kuciej, 2006) $(\tau \geqslant 0)$

$$T^*(\zeta,\tau) = [T^{(0)*}(\zeta,\tau) - T^{(1)*}(\zeta,\tau)]H(\tau) + T^{(1)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s) \tag{2.11}$$

where the upper indexes $k = 0,1$ correspond to solutions to the problem under consideration for the heat flux intensities

$$q^{(k)*}(\tau) = \Big(\frac{\tau}{\tau_s}\Big)^k \qquad \tau > 0, \quad k = 0,1 \tag{2.12}$$

The solutions to the boundary-value problem of heat conduction (2.4)-(2.8) with heat flux intensities (2.12) are obtained by using the integral Laplace transform with respect to the dimensionless time τ in the form ($k = 0,1$, $n = 0,1,2,\ldots,\ \tau \geqslant 0$)

$$\begin{aligned}
T_t^{(k)*}(\zeta,\tau) &= \frac{2\sqrt{\tau}}{1+\varepsilon_t}\Big(\frac{\tau}{\tau_s}\Big)^k \sum_{n=0}^{\infty} \varLambda^n T_{t,n}^{(k)*}(\zeta,\tau) && 0 \leqslant \zeta < \infty\\
T_{t,n}^{(k)*} &= F^{(k)}\Big[\Big(2n + \frac{\zeta}{\sqrt{k_t^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big] + \lambda_f F^{(k)}\Big[\Big(2n+2+\frac{\zeta}{\sqrt{k_t^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big]\\
T_s^{(k)*}(\zeta,\tau) &= q^{(k)*}(\tau)\frac{2\sqrt{\tau}}{1+\varepsilon_t}\sum_{n=0}^{\infty}\varLambda^n T_{s,n}^{(k)*}(\zeta,\tau) && -1 \leqslant \zeta \leqslant 0\\
T_{s,n}^{(k)*}(\zeta,\tau) &= F^{(k)}\Big(\frac{2n-\zeta}{2\sqrt{\tau}}\Big) + \lambda_f F^{(k)}\Big(\frac{2n+2+\zeta}{2\sqrt{\tau}}\Big)\\
T_f^{(k)*}(\zeta,\tau) &= \frac{4\sqrt{\tau}}{(1+\varepsilon_t)(1+\varepsilon_f)}\Big(\frac{\tau}{\tau_s}\Big)^k \sum_{n=0}^{\infty}\varLambda^n T_{f,n}^{(k)*}(\zeta,\tau) && -\infty < \zeta \leqslant -1\\
T_{f,n}^{(k)*}(\zeta,\tau) &= F^{(k)}\Big[\Big(2n+1-\frac{1+\zeta}{\sqrt{k_f^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big]
\end{aligned} \tag{2.13}$$

where

$$\begin{array}{ll}
F^{(0)}(\omega) = \operatorname{ierfc}(\omega) & F^{(1)}(\omega) = 3^{-1}[2(1+\omega^2)F^{(0)}(\omega) - \omega\operatorname{erfc}(\omega)]\\
\varepsilon_t \equiv \dfrac{K_t^*}{\sqrt{k_t^*}} & \varepsilon_f \equiv \dfrac{K_f^*}{\sqrt{k_f^*}}\\
\lambda_t = \dfrac{1-\varepsilon_t}{1+\varepsilon_t} & \lambda_f = \dfrac{1-\varepsilon_f}{1+\varepsilon_f}\\
\lambda = \lambda_t\lambda_f & \varLambda^n = \begin{cases} \lambda^n & 0 \leqslant \lambda < 1\\ (-1)^n|\lambda|^n & -1 < \lambda \leqslant 0\end{cases}
\end{array} \tag{2.14}$$

and where $\operatorname{ierfc}(x) = \pi^{-1/2}\exp(-x^2) - x\operatorname{erfc}(x)$, $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$, $\operatorname{erf}(x)$ – Gauss error function.

In the case of identical physical properties of the strip and foundation ($K_s = K_f$, $k_s = k_f$), it follows from formulae (2.10) and (2.14) that $K_f^* = 1$, $k_f^* = 1$, $\varepsilon_f = 1$, $\lambda_f = 0$, $\Lambda = 0$. Equations (2.13) at $n = 0$ give a solution to the problem of heat generation at braking with uniform retardation for two half-spaces (Yevtushenko *et al.*, 1999), $k = 0, 1$

$$\begin{aligned} T_t^{(k)*}(\zeta,\tau) &= \frac{2\sqrt{\tau}}{1+\varepsilon_t}\Big(\frac{\tau}{\tau_s}\Big)^k F^{(k)}\Big(\frac{\zeta}{2\sqrt{k_t^*\tau}}\Big) && 0 \leqslant \zeta < \infty \\ T_f^{(k)*}(\zeta,\tau) &= \frac{2\sqrt{\tau}}{1+\varepsilon_t}\Big(\frac{\tau}{\tau_s}\Big)^k F^{(k)}\Big(-\frac{\zeta}{2\sqrt{\tau}}\Big) && -\infty < \zeta \leqslant 0 \end{aligned} \tag{2.15}$$

Substituting solutions (2.15) into the right-hand side of equation (2.11) at $\zeta = 0$, $0 \leqslant \tau \leqslant \tau_s$, we obtain the known formula for calculation of the dimensionless contact temperature (Fazekas, 1953)

$$T_t^*(0,\tau) = T_f^*(0,\tau) = 2\sqrt{\frac{k_t^*\tau}{\pi}}\Big(1 - \frac{2\tau}{3\tau_s}\Big) \qquad 0 \leqslant \tau \leqslant \tau_s \tag{2.16}$$

The numerical results have been obtained for the commercial friction pair ChNMKh cast iron disk (the upper surface) and metal-ceramics FMK-11 frictional element of the patch (strip) on 30KhGSA steel base (foundation), for which (Balakin and Serigienko, 1999):

ChNMKh: $K_t = 51\,\mathrm{Wm^{-1}K^{-1}}$, $k_t = 14\cdot 10^{-6}\,\mathrm{m^2s^{-1}}$,

FMK-11: $K_s = 34.3\,\mathrm{Wm^{-1}K^{-1}}$, $k_s = 15.2\cdot 10^{-6}\,\mathrm{m^2s^{-1}}$,

30KhGSA: $K_f = 37.2\,\mathrm{Wm^{-1}K^{-1}}$, $k_f = 10.3\cdot 10^{-6}\,\mathrm{m^2s^{-1}}$.

The friction conditions are: $p_0 = 1\,\mathrm{MPa}$, $f = 0.7$, $V_0 = 30\,\mathrm{ms^{-1}}$, $t_s = 3.44\,\mathrm{s}$. The initial temperature is equal $20°\mathrm{C}$.

Isolines for the temperature constructed in the coordinate system (z, t) are shown in Fig. 2. The maximum temperature $T_{max} = 740°\mathrm{C}$ is reached on the contact surface $z = 0$ at the moment $t = t_{max} = 1.6\,\mathrm{s}$, which is not much lower than the half value of braking time $t_s = 3.44\,\mathrm{s}$. This result corresponds well with the experimental data $T_{max} = 760°\mathrm{C}$, published in a monograph by Chichinadze *et al.* (1979).

The temperature distribution along the distance $|z|$ from the contact surface at the moment $t = t_{max} = 1.6\,\mathrm{s}$, when temperature reaches its maximum value, at the stop moment $t = t_s = 3.44\,\mathrm{s}$ and at the moment after being stopped $t = 6\,\mathrm{s}$, when the cooling of the tribosystem takes place, is shown in

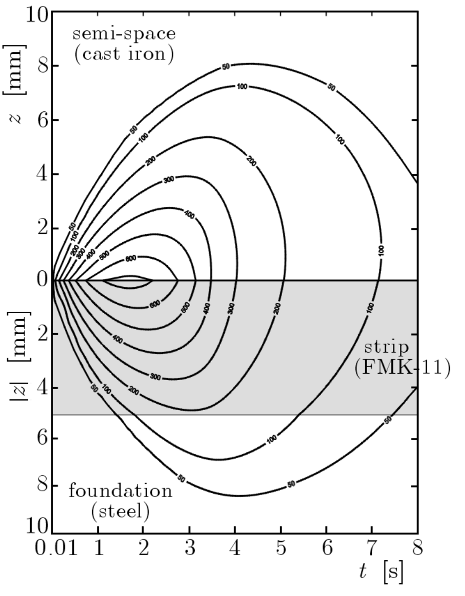

Fig. 2. Isolines of the temperature

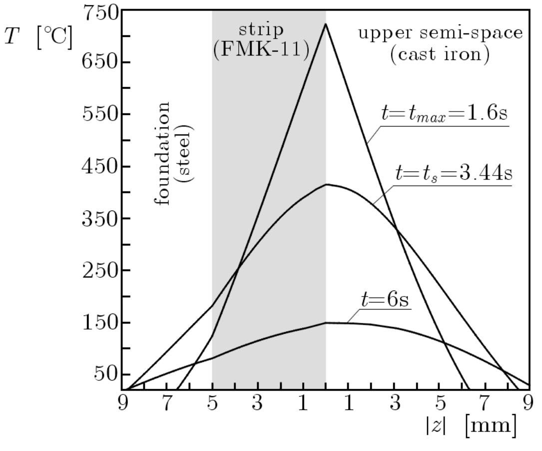

Fig. 3. Temperature distribution along the distance $|z|$ from the contact surface at three time moments

Fig. 3. It can be noticed then that for $t = t_{max}$, the temperature decreases with thickness linearly and could be calculated from the approximate dependence $T(z, t_{max}) \approx 121.01z + 740.62$, $-d \leqslant z \leqslant 0$. The effective heat penetration depth, the depth where temperature decreases to 5% of its maximum value on the contact surface, is equal nearly to 6.5 mm in both directions from the contact surface.

The influence of strip thickness on the maximum temperature is significant for the interval $0.01\,\mathrm{mm} \leqslant d \leqslant 8\,\mathrm{mm}$ (Fig. 4), whereas for values out of this interval, the temperature can be calculated by solving the contact problem with frictional heating during braking for two semi-infinite bodies (2.11) and (2.15).

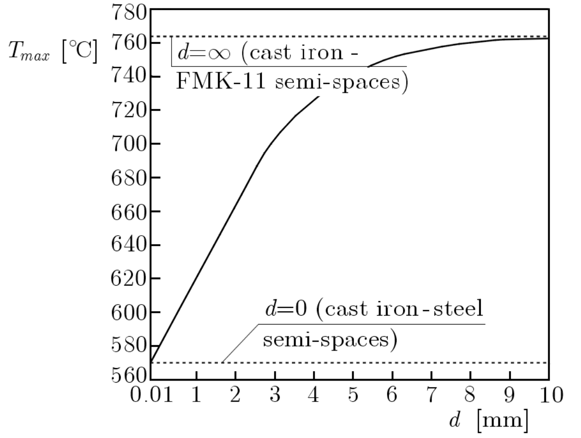

Fig. 4. Dependence of the maximum contact temperature $T_{max} = T(0, t_{max})$ on the strip thickness d

3. The plane-parallel strip – foundation system

The problem of contact interaction of the plane-parallel strip and the half-space is now under consideration. The upper surface of the strip and the foundation in infinity are subjected to the constant pressure p_0. The strip slides over the surface of the half-space along the y-axis of the Cartesian coordinate system $Oxyz$ with the centre at the plane of contact (Fig. 5). The velocity of sliding V (2.1) decreases linearly with time t from the initial value V_0 at $t = 0$ down to zero at the stop time moment t_s (2.1).

The transient dimensionless temperature fields in the strip and in the foundation can be found from the solution to the heat conduction problem of friction during braking $(\tau > 0)$

$$\begin{aligned} \frac{\partial^2 T_s^*(\zeta,\tau)}{\partial \zeta^2} &= \frac{\partial T_s^*(\zeta,\tau)}{\partial \tau} && 0 < \zeta < 1 \\ \frac{\partial^2 T_f^*(\zeta,\tau)}{\partial \zeta^2} &= \frac{1}{k^*}\frac{\partial T_f^*(\zeta,\tau)}{\partial \tau} && -\infty < \zeta < 0 \end{aligned} \tag{3.1}$$

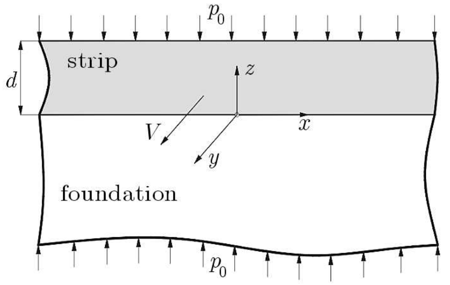

Fig. 5. Illustration of the problem

and $(\tau > 0)$

$$K^* \frac{\partial T_f^*}{\partial \zeta}\Big|_{\zeta=0^-} - \frac{\partial T_s^*}{\partial \zeta}\Big|_{\zeta=0^+} = q^*(\tau) \tag{3.2}$$

$$\begin{array}{ll} T_f^*(0,\tau) = T_s^*(0,\tau) & T_s(1,\tau) = 0 \\ T_f^*(\zeta,\tau) \to 0 & \zeta \to -\infty \\ T_s^*(\zeta,0) = 0 & 0 \leqslant \zeta \leqslant 1 \\ T_f^*(\zeta,0) = 0 & -\infty < \zeta \leqslant 0 \end{array} \tag{3.3}$$

The dimensionless temperatures $T_{s,f}^{(k)*}(\zeta,\tau)$, $k = 0,1$ in the strip and the foundation for the intensities of heat fluxes $q^{(k)*}(\tau)$, $k = 0,1$ (2.12) are found in the form $(\tau \geqslant 0,\ k = 0,1,\ n = 0,1,2,\ldots)$

$$\begin{aligned} T_s^{(k)*}(\zeta,\tau) &= \frac{2\sqrt{\tau}}{1+\varepsilon}\Big(\frac{\tau}{\tau_s}\Big)^k \sum_{n=0}^{\infty} \Lambda^n T_{s,n}^{(k)*}(\zeta,\tau) \qquad 0 \leqslant \zeta \leqslant 1 \\ T_{s,n}^{(k)*}(\zeta,\tau) &= F^{(k)}\Big(\frac{2n+\zeta}{2\sqrt{\tau}}\Big) - F^{(k)}\Big(\frac{2n+2-\zeta}{2\sqrt{\tau}}\Big) \\ T_f^{(k)*}(\zeta,\tau) &= \frac{2\sqrt{\tau}}{1+\varepsilon}\Big(\frac{\tau}{\tau_s}\Big)^k \sum_{n=0}^{\infty} \Lambda^n T_{f,n}^{(k)*}(\zeta,\tau) \qquad -\infty < \zeta \leqslant 0 \\ T_{f,n}^{(k)*}(\zeta,\tau) &= F^{(k)}\Big[\Big(2n - \frac{\zeta}{\sqrt{k^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big] - F^{(k)}\Big[\Big(2n+2 - \frac{\zeta}{\sqrt{k^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big] \end{aligned} \tag{3.4}$$

Solutions (3.4) are obtained for zero temperature on the upper surface $\zeta = 1$ of the strip. If this surface is thermally insulated, then the corresponding temperatures are found in form $(3.4)_1$ and $(3.4)_3$, in which $(n = 0,1,2,\ldots)$

$$\begin{aligned} T_{s,n}^{(k)*}(\zeta,\tau) &= F^{(k)}\Big(\frac{2n+\zeta}{2\sqrt{\tau}}\Big) + F^{(k)}\Big(\frac{2n+2-\zeta}{2\sqrt{\tau}}\Big) \\ T_{f,n}^{(k)*}(\zeta,\tau) &= F^{(k)}\Big[\Big(2n - \frac{\zeta}{\sqrt{k^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big] + F^{(k)}\Big[\Big(2n+2 - \frac{\zeta}{\sqrt{k^*}}\Big)\frac{1}{2\sqrt{\tau}}\Big] \end{aligned} \tag{3.5}$$

The numerical results have been obtained for the friction couple ChNMKh cast iron (foundation) and FMK-11 metal-ceramics (the strip, the frictional element of the patch) and the friction conditions of the previous problem.

All the results presented in Figs. 6-8 were obtained for two limiting types of the boundary conditions on the upper surface $z = d$ $(\zeta = 1)$ of the strip: at zero temperature (dashed curves) and for thermal insulation (continuous curves).

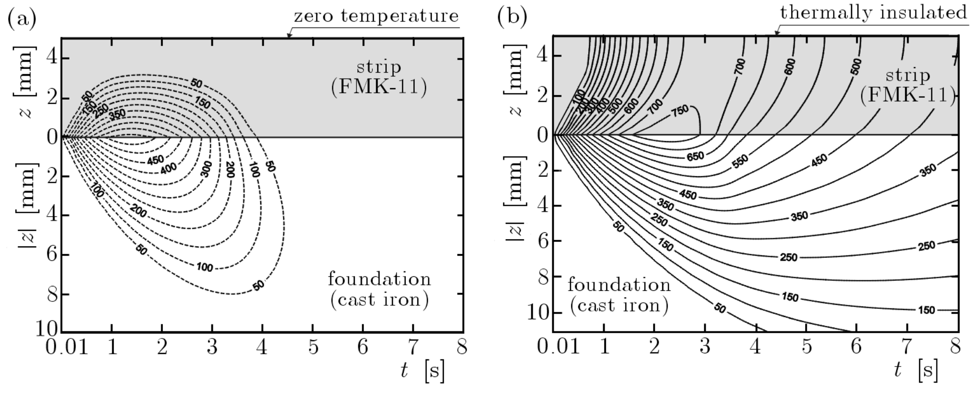

Fig. 6. Isolines of the temperature T [°C] at $t_s = 3.44\,\mathrm{s}$

For the known braking time $t_s = 3.44\,\mathrm{s}$, the maximum temperatures $T_{max} = 593$°C and $T_{max} = 797$°C are reached on the contact surface $z = 0$ at the time moment $t_{max} = 1.6\,\mathrm{s}$ in the case of zero temperature on the upper surface of the strip (Fig. 6a), and $t_{max} = 2.6\,\mathrm{s}$ when this surface is thermally insulated (Fig. 6b). The evolution of temperature on the contact surface $z = 0$ $(\zeta = 0)$ is shown in Fig. 7, provided that the braking starts, the temperature increases quickly, then it reaches its maximum and begins to decrease. The largest value of temperature on this surface is reached in the case of the thermally insulated upper surface of the FMK-11 strip. In the case of maintaing zero temperature on the upper surface of the strip, the temperature on the friction surface, having obtained it is maximum value $T_{max} = 593$°C, decreases quickly and reaches the initial value 20°C after 4.1 s.

Such a decreases in the temperature in the case of the thermally insulated upper surface of the strip is of different nature. Having reached the maximum value $T_{max} = 797$°C, the temperature decreases much more slowly and the time for obtaining the initial temperature value is much longer, and is greater than 20 s.

In the case of thermal insulation of the upper surface of the strip, the increase in thickness of the strip leads to a decrease of the maximum tempe-

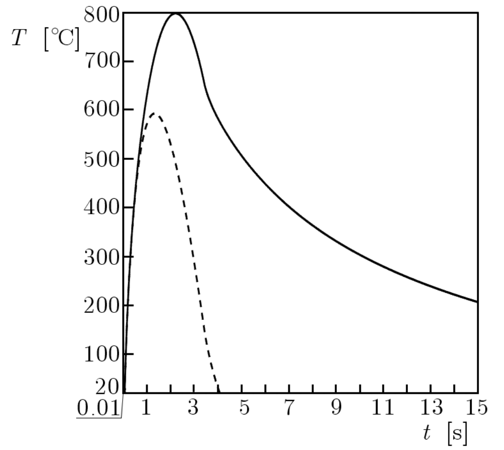

Fig. 7. Evolution of the contact temperature at $t_s = 3.44\,\mathrm{s}$

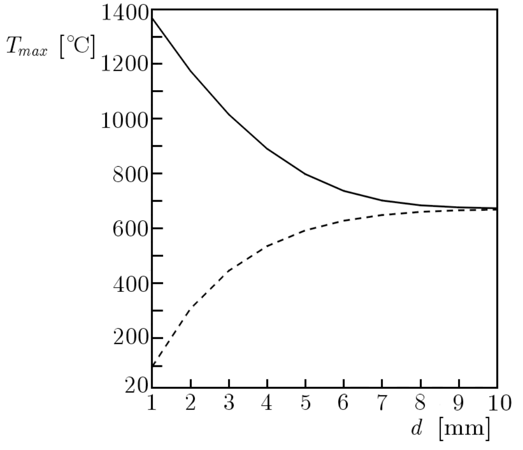

Fig. 8. Dependence of the maximum temperature T_{max} on the strip thickness d

rature (Fig. 8). At the same time, when temperature of the upper surface of the strip is kept at zero, the maximum temperature increases with the growth of thickness of the strip. The boundary conditions on the upper surface of the strip have no influence on the maximum temperature, when the strip is thicker than 10 mm.

4. Conclusions

The presented solution describes a model of the heat generation process during the single-braking mode in a multi-disk brake. As distinct from other solutions, ours determines the temperature in each element of the tribosystem, both in the heating phase at braking and in the cooling phase, when the brake is released. Moreover, the temperature evolution and distribution in relation to thickness of each material of the friction pair: cast iron disc + FMK-11 metal ceramic patch on the steel foundation, were examined. The maximum temperature value $T_{max} = 740°C$, obtained as a result of numerical calculations, corresponds pleasingly with the respective value found in the monograph by Chichinadze *et al.* (1979).

The analytical solution to the transient heat problem of friction during braking for the plane-parallel strip sliding over the semi-infinity foundation has been obtained too. The temperature field for the friction pair FMK-11 (strip) and ChNMKh (foundation) has been studied. The influence of the boundary conditions on the upper surface of the friction element (strip) and the thickness of this element on the distribution of temperature has been investigated. The thickness of the strip at which the solution to the corresponding problem of friction heating during braking for two half-spaces can be applicable is established.

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Dwa zagadnienia przewodnictwa ciepła z uwzględnieniem nagrzewania tarciowego podczas hamowania

Streszczenie

W pracy otrzymano analityczne rozwiązania początkowo-brzegowych zagadnień przewodnictwa ciepła dla dwóch układów par tarciowych: 1) półprzestrzeń – warstwa

– półprzestrzeń, 2) warstwa – półprzestrzeń. Założono, że prędkość ślizgania się elementów tarciowych w rozpatrywanych układach zmniejsza się liniowo z czasem, od wartości początkowej do zera. Dla właściwości materiałowych: żeliwo – metaloceramika – stal oraz metaloceramika – stal, zbadano rozkład oraz ewolucję temperatury w powyższych układach. Zbadano wpływ grubości warstwy na wartość maksymalnej temperatury na powierzchni kontaktu. Wyniki przedstawiono w formie wykresów.

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