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SELF-SIMILAR FLOW OF A NON-IDEAL GAS WITH INCREASING ENERGY BEHIND A MAGNETOGASDYNAMIC SHOCK WAVE UNDER A GRAVITATIONAL FIELD

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A self-similar solution for the propagation of a spherical shock wave in a non-ideal gas in the presence of an azimuthal magnetic field is investigated. The medium is assumed to be under a gravitational field due to a heavy nucleus at the origin(Roche Model). The unsteady model of Roche consists of a gas distributed with spherical symmetry around the nucleus having a large mass. It is assumed that the gravitational effect of the medium itself can be neglected compared with the attraction of the heavy nucleus. The total energy of the flow-field behind the shock is supposed to be increasing with time. Similarity solutions are obtained, and the effects of variation of the parameter of non-idealness of the gas, the shock-Mach number and the Alfven-Mach number on the flow-field behind the shock are investigated.

Key words: shock wave, non-ideal gas, self-similar flow, magnetogasdynamics, gravitational field

1. Introduction

Carrus *et al.* (1951) studied the propagation of shock waves in a gas under the gravitational attraction of a central body of the fixed mass (Roche Model) and obtained similarity solutions by numerical method. Rogers (1957) discussed a method for obtaining an analytical solution to the same problem. Ojha *et al.* (1998) discussed the dynamical behaviour of an unstable magnetic star by employing the concept of the Roche Model in an electrically conducting atmosphere. Singh (1982) studied the self-similar flow of a non-conducting perfect gas, moving under the gravitational attraction of a central body of the fixed mass, behind a spherical shock wave driven out by a propelling contact surface into a quite solar wind region. Singh and Srivastava (1989) studied the self-similar flow of a perfect gas, moving under the gravitational attraction of a central body of the fixed mass, behind a spherical shock wave moving into a conducting gas of spatially decreasing density and pervaded by a spatially decreasing magnetic field. Total energy content between the inner expanding surface and the shock front is assumed to be increasing with time. Ratkiewicz *et al.* (1994) studied similarity solutions for synchrotron emission from a supernova blast wave. In all of the works, mentioned above, the medium is taken to be a gas satisfying the equation of state of a perfect gas.

The assumption that the gas is ideal is no longer valid when the flow takes place at high temperatures. Anisimov and Spiner (1972) studied a problem of point explosion in a non-ideal gas by taking the equation of state in a simplified form, which describes the behaviour of the medium satisfactorily at low densities. Ranga Rao and Purohit (1976), Ojha (2002) and Vishwakarma and Nath (2007) also studied the propagation of shock waves in gases with the above equation of state. Roberts and Wu (1996, 2003) used an equivalent equation of state to study the shock wave theory of sonoluminescence. Vishwakarma *et al.* (2007) studied the propagation of the magnetogasdynamic cylindrical shock wave in a rotating non-ideal gas by using the equation of state taken by Roberts and Wu (1996, 2003).

In the present work, we therefore investigate the self-similar flow behind a spherical shock wave propagating in a non-ideal gas in the presence of an azimuthal magnetic field. The medium is assumed to be under a gravitational field due to a heavy nucleus at the origin (Roche Model). The unsteady model of Roche consits of a gas distributed with spherical symmetry around the nucleus having a large mass m. It is assumed that the gravitational effect of the medium itself can be neglected compared with the attraction of the heavy nucleus. The total energy of the flow-field behind the shock is supposed to be increasing with time (Freeman, 1968; Director and Dabora, 1977). This increase can be obtained by the pressure exerted on the medium by the inner expanding surface (Rogers, 1958). In order to obtain the similarity solutions of the problem, the density of the undisturbed medium is assumed to be constant. Effects of variation of the parameter of the non-idealness of the gas $\overline{b}$, the shock-Mach number M and the Alfven-Mach number M_A on the flow field behind the shock are investigated.

2. Fundamental equations and boundary conditions

The fundamental equations for one-dimensional adiabatic unsteady spherically symmetric flow of a perfectly conducting non-ideal gas in which an azimuthal magnetic field is permeated, in the generalized Roche Model are (Rogers, 1957; Singh and Srivastava, 1989; Vishwakarma, 2000)

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0 \\
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu h}{\rho}\frac{\partial h}{\partial r} + \frac{\mu h^2}{\rho r} + \frac{G^* m}{r^2} = 0 \\
&\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{hu}{r} = 0 \\
&\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\Big(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\Big) = 0
\end{aligned} \tag{2.1}$$

where u, p, ρ and h are the velocity, pressure, density and azimuthal magnetic field, respectively, at a radial distance r from the center of the core at time t, μ is the magnetic permeability, e is the internal energy per unit mass, m denotes the constant mass of the core and G^* is the gravitational constant. Here, it is assumed that the gravitating effect of the medium itself is negligible in comparison with the attraction of the heavy nucleus.

The above system of equations should be supplemented with an equation of state. To discover how deviations from the ideal gas can affect the solutions, we adopt a simple model. We assume that the gas obeys a simplified van der Waals equation of state of the form (Roberts and Wu, 1996, 2003; Vishwakarma *et al.*, 2007)

$$p = \frac{R^*\rho T}{1 - b\rho} \qquad\qquad e = C_v T = \frac{p(1 - b\rho)}{\rho(\gamma - 1)} \tag{2.2}$$

where R^* is the gas constant, $C_v = R^*/(\gamma - 1)$ is the specific heat at constant volume and γ is the ratio of specific heats. The constant b is the "van der Waals excluded volume"; it places a limit, $\rho_{max} = 1/b$, on the density of the gas.

We assume that the spherical shock wave is propagating outwards from the center of symmetry in a perfectly conducting non-ideal gas with constant density and a variable azimuthal magnetic field, which is at rest.

The flow variables immediately ahead of the shock front are

$$u_1 = 0 \qquad\qquad \rho_1 = \text{const} \qquad\qquad h_1 = cR^{-k} \tag{2.3}$$

where R is the shock radius, c and k are constants and the subscript 1 denotes the condition immediately ahead of the shock.

At the equilibrium state, the pressure ahead of the shock is

$$p_1 = G^* m\rho_1 R^{-1} + \mu c^2(1-k)\frac{R^{-2k}}{2k} \tag{2.4}$$

where $2k = 1$.

The jump conditions across the magnetogasdynamic shock are

$$\begin{aligned}
&\rho_2(\dot{R}-u_2) = \rho_1\dot{R} \qquad\qquad h_2(\dot{R}-u_2) = h_1\dot{R} \\
&p_2 + \frac{1}{2}\mu {h_2}^2 + \rho_2(\dot{R}-u_2)^2 = p_1 + \frac{1}{2}\mu h_1^2 + \rho_1\dot{R}^2 \\
&e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(\dot{R}-u_2)^2 + \frac{\mu h_2^2}{\rho_2} = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}\dot{R}^2 + \frac{\mu h_1^2}{\rho_1}
\end{aligned} \tag{2.5}$$

where subscript 2 denotes conditions immediately behind the shock and $\dot{R}(= dR/dt)$ denotes the velocity of the shock front.

From eqautions (2.5), we obtain

$$\begin{aligned}
&u_2 = (1-\beta)\dot{R} \qquad\qquad \rho_2 = \frac{\rho_1}{\beta} \qquad\qquad h_2 = \frac{h_1}{\beta} \\
&p_2 = \Big[\frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\Big(1-\frac{1}{\beta^2}\Big) + (1-\beta)\Big]\rho_1\dot{R}^2
\end{aligned} \tag{2.6}$$

where $M = \sqrt{\rho_1\dot{R}^2/(\gamma p_1)}$ is the shock-Mach number referred to the frozen speed of sound $\sqrt{\gamma p_1/\rho_1}$, and $M_A = \sqrt{\rho_1\dot{R}^2/(\mu h_1^2)}$ is the Alfven-Mach number. The quantity $\beta(0<\beta<1)$ is given by the relation

$$\begin{aligned}
&\beta^3 - \beta^2\Big[\frac{2}{(\gamma+1)M^2} + \frac{\gamma\Big(\frac{1}{M_A^2}+1\Big) + 2\overline{b} - 1}{\gamma+1}\Big] + \beta\Big[\frac{\gamma - 2 + \overline{b}}{\gamma+1}\Big]\frac{1}{M_A^2} + \\
&+ \frac{\overline{b}}{(\gamma+1)M_A^2} = 0
\end{aligned} \tag{2.7}$$

where, $\overline{b} = b\rho_1$ is the parameter of non-idealness of the gas.

The shock-Mach number M_e referred to the speed of sound in non-ideal gas $\sqrt{\gamma p_1/[\rho_1(1-\overline{b})]}$ is given by

$$M_e = M\sqrt{1-\overline{b}} \tag{2.8}$$

The total energy E of the flow-field behind the shock is not constant, but assumed to be time dependent and varying as (Rogers, 1958; Freeman, 1968; Director and Dabora, 1977)

$$E = E_c t^s \tag{2.9}$$

where s is a non-negative number and E_c is a constant. The positive values of s correspond to the class in which the total energy increases with time. This increase can be achieved by the pressure exerted on the gas by the expanding surface (a contact surface or a piston). Thus the flow is headed by a shock front and has the expanding surface as the inner boundary.

3. Similarity solutions

Following the general similarity analysis of Sedov (1959), we define two characteristic parameters a and d with independent dimensions as

$$[a] = [mG^* \rho_1] \qquad\qquad [d] = [mG^*] = \left[\frac{E_c}{\rho_1}\right]^{\frac{3}{5}}$$

The single dimensionless independent variable in this case will be

$$\eta = (\alpha m G^*)^{\frac{-\delta_1}{2}} r t^{-\delta_1} \tag{3.1}$$

where

$$\delta_1 = \frac{2}{3} = \frac{2+s}{5} \tag{3.2}$$

and α is a constant to be determined by the condition that η assumes the value 1 at the shock front.

Second of equations (3.2) shows that the similarity solution of the present problem exists only when the total energy of the flow-field behind the shock increases as $t^{4/3}$, that is only when $s = 4/3$.

From (3.1), we find that

$$\dot{R}^2 = \frac{4\alpha m G^*}{9R} \qquad\qquad \frac{d\dot{R}}{dt} = -\frac{\dot{R}^2}{2R} \tag{3.3}$$

From equations (2.4) and $(3.3)_1$, we obtain the following expression for α in terms of the shock-Mach number M and Alfven-Mach number M_A

$$\frac{mG^*}{R\dot{R}^2} = \frac{9}{4\alpha} = \frac{1}{\gamma M^2} - \frac{1}{2M_A^2} \tag{3.4}$$

The quantity $9/(4\alpha)$ ($=\delta$, say) may be taken as a parameter of gravitation.

To obtain similarity solutions, we write the unknown variables in the following form (Vishwakarma and Yadav, 2003)

$$\begin{aligned} u &= \dot{R}U(\eta) & \rho &= \rho_1 g(\eta) \\ p &= \rho_1 \dot{R}^2 P(\eta) & \sqrt{\mu}h &= \sqrt{\rho_1}\dot{R}H(\eta) \end{aligned} \tag{3.5}$$

where U, g, P and H are functions of the non-dimansional variable (similarity variable) η only.

The condition to be satisfied at the inner expanding surface is that the velocity of the fluid is equal to the velocity of the surface itself. This kinematic condition, from equations (3.1) and (3.5), can be written as

$$U(\eta_p) = \eta_p \tag{3.6}$$

where η_p is the value of η at the inner expanding surface.

Using similarity transformations (3.5), the equations of motion are transformed into

$$\begin{aligned} &-(\eta - U)\frac{dg}{d\eta} + g\Big(\frac{dU}{d\eta} + \frac{2U}{\eta}\Big) = 0 \\ &-(\eta - U)\frac{dU}{d\eta} - \frac{U}{2} + \frac{1}{g}\frac{dP}{d\eta} + \frac{H}{g}\frac{dH}{d\eta} + \frac{H^2}{g\eta} + \frac{9}{4\alpha\eta^2} = 0 \\ &-(\eta - U)\frac{dH}{d\eta} - \frac{H}{2} + H\frac{dU}{d\eta} + \frac{HU}{\eta} = 0 \\ &-(\eta - U)\frac{dP}{d\eta} + \frac{\gamma(\eta - U)}{1 - \overline{b}g}\frac{P}{g}\frac{dg}{d\eta} - P = 0 \end{aligned} \tag{3.7}$$

From equations (3.7), we have

$$\begin{aligned} \frac{dU}{d\eta} &= \Big[\frac{2\gamma PU}{1 - \overline{b}g} + \frac{(\eta - U)g}{\eta^2}\Big(\frac{1}{\gamma M^2} - \frac{1}{2M_A^2}\Big) + (\eta - U)\frac{H^2}{\eta} + H^2\Big(\frac{U}{\eta} - \frac{1}{2}\Big) + \\ &\quad -(\eta - U)\frac{Ug}{2} - P\Big]\Big[(\eta - U)^2 g - H^2 - \frac{\gamma P}{1 - \overline{b}g}\Big]^{-1} \\ \frac{dg}{d\eta} &= \frac{g}{\eta - U}\Big(\frac{dU}{d\eta} + \frac{2U}{\eta}\Big) \qquad \frac{dH}{d\eta} = \frac{H}{\eta - U}\Big(\frac{dU}{d\eta} - \frac{1}{2} + \frac{U}{\eta}\Big) \\ \frac{dP}{d\eta} &= (\eta - U)g\frac{dU}{d\eta} + \frac{Ug}{2} - \frac{H^2}{\eta - U}\Big(H\frac{dU}{d\eta} - \frac{H}{2} + \frac{HU}{\eta}\Big) - \frac{H^2}{g\eta} + \\ &\quad -\frac{g}{\eta^2}\Big(\frac{1}{\gamma M^2} - \frac{1}{2M_A^2}\Big) \end{aligned} \tag{3.8}$$

The transformed shock conditions are

$$\begin{aligned} U(1) &= 1-\beta & g(1) &= \frac{1}{\beta} \\ P(1) &= \frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\Big(1-\frac{1}{\beta^2}\Big) + 1 - \beta & H(1) &= \frac{1}{\beta M_A} \end{aligned} \tag{3.9}$$

where β is given by equation (2.7).

For exhibiting the numerical solutions, it is convenient to write the field variables in the non-dimensional form as

$$\frac{u}{u_2} = \frac{U(\eta)}{U(1)} \qquad \frac{\rho}{\rho_2} = \frac{g(\eta)}{g(1)} \qquad \frac{p}{p_2} = \frac{P(\eta)}{P(1)} \qquad \frac{h}{h_2} = \frac{H(\eta)}{H(1)} \tag{3.10}$$

Ordinary differential equations (3.8) with boundary conditions (3.9) can now be numerically integrated to obtain the solution for the flow behind the shock surface.

4. Results and discussion

Distribution of the flow variables in the flow-field behind the shock front are obtained by numerical integration of equations (3.8) with boundary conditions (3.9) by the Runge-Kutta method of the fourth order. For the purpose of numerical integration, the values of constant parameters are taken to be (Roberts and Wu, 1996, 2003; Rosenau and Frankenthal, 1976; Vishwakarma *et al.*, 2007) $\gamma = 5/3$; $\overline{b} = 0$, 0.025, 0.05; $M = 5$, 10; $M_A^{-2} = 0$, 0.02, 0.1. For a fully ionized gas $\gamma = 5/3$, and therefore it is applicable to stellar medium. Rosenau and Frankenthal (1976) have shown that the effects of magnetic field on the flow-field behind the shock are significant when $M_A^{-2} \geqslant 0.01$; therefore the above values of M_A^{-2} are taken for calculations in the present problem. The value $\overline{b} = 0$ corresponds to the perfect gas case.

Figures 1-4 show the variation of the flow variables u/u_2, ρ/ρ_2, h/h_2 and p/p_2 with η at various values of the parameters $\overline{b}$, M^{-2} and M_A^{-2}. It is shown that, as we move inwards from the shock front towards the inner contact surface, the reduced fluid velocity u/u_2 and the reduced azimuthal magnetic field h/h_2 increase, and the reduced density ρ/ρ_2 decreases whereas the reduced pressure p/p_2 increases when $M_A^{-2} \neq 0$ and decreases when $M_A^{-2} = 0$ (non-magnetic case).

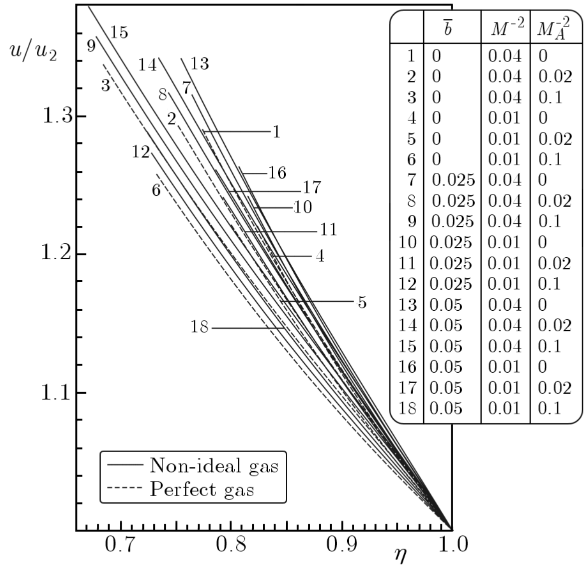


Fig. 1. Variation of reduced velocity u/u_2 in the flow-field behind the shock front for $\gamma = 5/3$

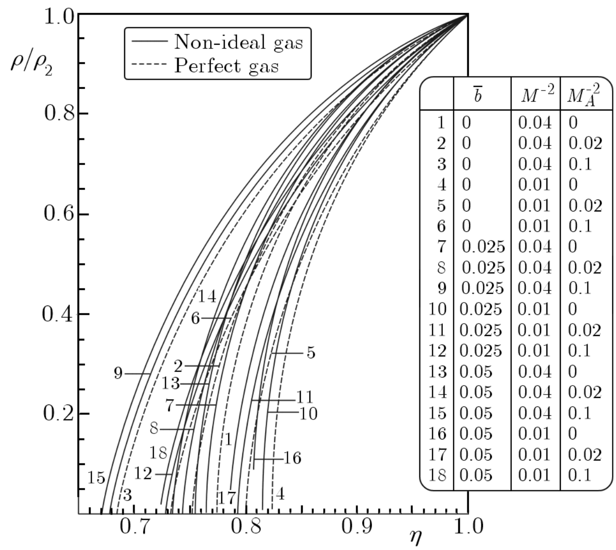
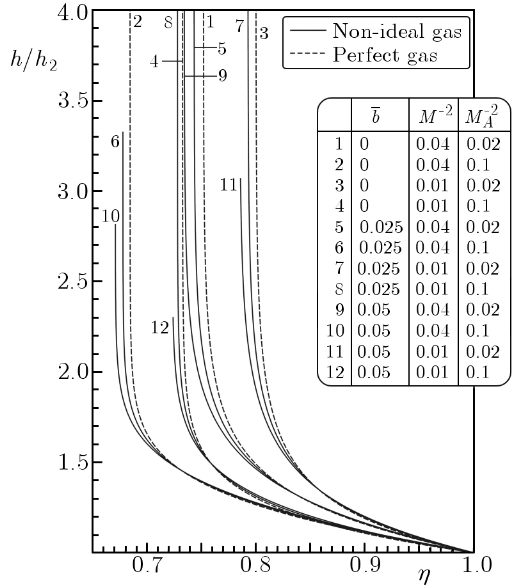
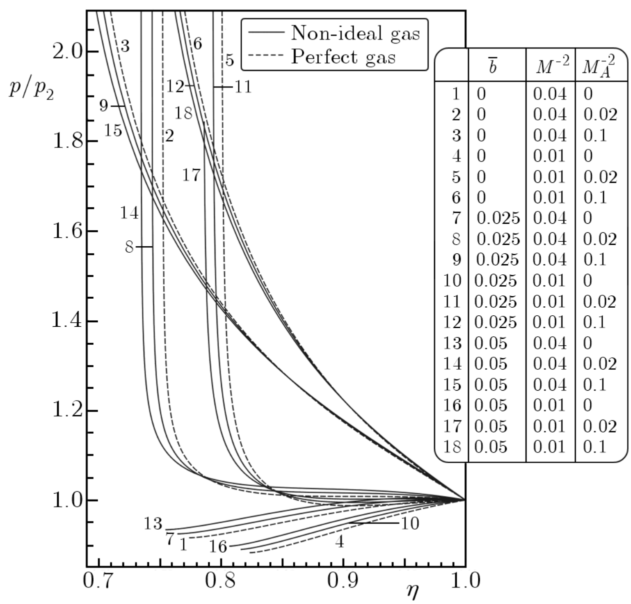


Fig. 2. Variation of reduced density ρ/ρ_2 in the flow-field behind the shock front for $\gamma = 5/3$



	$\overline{b}$	M^{-2}	M_A^{-2}
1	0	0.04	0.02
2	0	0.04	0.1
3	0	0.01	0.02
4	0	0.01	0.1
5	0.025	0.04	0.02
6	0.025	0.04	0.1
7	0.025	0.01	0.02
8	0.025	0.01	0.1
9	0.05	0.04	0.02
10	0.05	0.04	0.1
11	0.05	0.01	0.02
12	0.05	0.01	0.1

Fig. 3. Variation of reduced azimuthal magnetic field h/h_2 in the flow-field behind the shock front for $\gamma = 5/3$



	$\overline{b}$	M^{-2}	M_A^{-2}
1	0	0.04	0
2	0	0.04	0.02
3	0	0.04	0.1
4	0	0.01	0
5	0	0.01	0.02
6	0	0.01	0.1
7	0.025	0.04	0
8	0.025	0.04	0.02
9	0.025	0.04	0.1
10	0.025	0.01	0
11	0.025	0.01	0.02
12	0.025	0.01	0.1
13	0.05	0.04	0
14	0.05	0.04	0.02
15	0.05	0.04	0.1
16	0.05	0.01	0
17	0.05	0.01	0.02
18	0.05	0.01	0.1

Fig. 4. Variation of reduced pressure p/p_2 in the flow-field behind the shock front for $\gamma = 5/3$

Table 1. Density ratio β across the shock front and the position of the inner expanding surface η_p for different values of $\overline{b}$, M^{-2} and M_A^{-2} with $\gamma = 5/3$

$\overline{b}$	M^{-2}	M_A^{-2}	β	η_p
0	0.04	0	0.400000	0.7741250
		0.02	0.418474	0.7519095
		0.1	0.488109	0.6844721
	0.01	0	0.325000	0.8236720
		0.02	0.344752	0.8000735
		0.1	0.417444	0.7327128
0.025	0.04	0	0.418750	0.7645350
		0.02	0.435571	0.7432718
		0.1	0.500606	0.6779128
	0.01	0	0.343750	0.8150435
		0.02	0.361215	0.7927513
		0.1	0.428142	0.7280513
0.05	0.04	0	0.437500	0.7546676
		0.02	0.452864	0.7342914
		0.1	0.513580	0.6709548
	0.01	0	0.362500	0.8061769
		0.02	0.377997	0.7850806
		0.1	0.439469	0.7230134

The density ratio β across the shock front and the position of the inner expanding surface η_p are tabulated in table 1 for $\gamma = \frac{5}{3}$ and various values of $\overline{b}$, M^{-2} and ${M_A}^{-2}$.

It is found that the effects of an increase in the value of the parameter of non-idealness $\overline{b}$ of the gas are:

(i) to increase the value of β, i.e. to decrease the shock strength (see Table 1);

(ii) to increase the reduced velocity u/u_2 and the reduced density ρ/ρ_2 at any point in the flow field behind the shock (see Figs. 1 and 2);

(iii) to slightly increase the reduced pressure p/p_2 and the reduced magnetic field h/h_2 at any point in the flow field near the shock front and to decrease these quantities at any point in the flow field near the inner expanding surface in the magnetic case whereas to increase the reduced pressure p/p_2 at any point in the flow field behind the shock in the non-magnetic case (see Figs. 3 and 4);

(iv) to increase the distance of the inner expanding surface from the shock front (see Table 1). Physically it means that the gas behind the shock is less compressed, i.e. the shock strength is reduced, which is the same as given in (i) above.

The effects of an increase in the value of M (i.e. a decrease in the value of M^{-2}) are:

(i) to decrease the value of β, i.e. to increase the shock strength (see Table 1);

(ii) to decrease the reduced velocity u/u_2 and the reduced density ρ/ρ_2 at any point in the flow field behind the shock (see Figs. 1 and 2);

(iii) to increase the reduced pressure p/p_2 and the reduced azimuthal magnetic field h/h_2 at any point in the flow field behind the shock, in general; in the magnetic case, whereas to decrease the reduced pressure p/p_2 at any point in the flow field behind the shock in the non-magnetic case (see Figs. 3 and 4);

(iv) to decrease the distance of the inner expanding surface from the shock front (see Table 1).

The effects of an increase in the value of M_A^{-2} (i.e. the effects of an increase in the strength of ambient magnetic field) are:

(i) to increase the value of β, i.e. to decrease the shock strength (see Table 1);

(ii) to decrease the reduced velocity u/u_2 and the reduced magnetic field h/h_2, and to increase the reduced density ρ/ρ_2 and the reduced pressure p/p_2 at any point in the flow field behind the shock (see Figs. 1 to 4);

(iii) to increase the distance of the inner expanding surface from the shock front (see Table 1).

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Automorficzny przepływ gazu niedoskonałego o wzrastającej energii za magnetogazodynamiczną falą uderzeniową w obecności pola grawitacyjnego

Streszczenie

W pracy przedstawiono automorficzne rozwiązanie dla problemu propagacji sferycznej fali uderzeniowej w gazie niedoskonałym w obecności azymutalnego pola magnetycznego. W rozważaniach przyjęto, że ośrodek podlega wpływowi pola grawitacyjnego pochodzącego od jądra (model Roche'a) Nieustalony model Roche'a opisuje gaz o sferycznej symetrii dookoła jądra o dużej masie m. Założono, że efekt grawitacyjny od samego ośrodka jest pomijalny w porównaniu do przyciągania od ciężkiego jądra. Przyjęto również, że całkowita energia pola przepływu gazu za falą uderzeniową rośnie z upływem czasu. Otrzymano automorficzne rozwiązania dla tego zagadnienia oraz zbadano wpływ zmienności parametru określającego niedoskonałość gazu b, zmienności liczby Macha M oraz Alfven-Macha M_A na pole przepływu za falą uderzeniową.

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