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DAMAGE DETECTION IN BEAMS USING WAVELET TRANSFORM ON HIGHER VIBRATION MODES

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Technical difficulties prevented so far wider applications of higher mode shapes in damage detection. Yet these modes carry on a lot of, so much needed, information on damage inflicted to a structure. However, recent scanning laser-based vibration measurement techniques allow one to utilize these higher modes in damage detection effectively. This paper deals with the wavelet-based damage detection technique on a cantilever beam with damage in the form of a single notch of depth 20%, 10% and 5% of the beam height. The purpose of the study is to present the results of experimental and numerical analyses of damage detection based on higher order modes. The first eight modes are considered and the influence of the mode order on the effectiveness of damage detection by the continuous wavelet transform is analysed in detail.

Key words: damage detection, wavelet transform, higher vibration modes, scanning laser vibrometer

1. Introduction

The ability to damage detection and localization at the earliest possible stage becomes an important issue throughout the aerospace, mechanical or civil engineering communities. The existence of damage in a structure results in changes of global dynamic characteristics. Therefore, relatively simple vibration measurements of a structural response and extraction of information on natural frequencies, damping or mode shapes, make damage detection feasible (Ren and De Roeck, 2002a,b). Dynamic tests are successfully used to damage identification or even to damage reconstruction, especially combined with genetic algorithms (Kokot and Zembaty, 2008, 2009) or artificial neural networks (Waszczyszyn and Ziemiański, 2001; Kuźniar and Waszczyszyn, 2002; Rucka and Wilde, 2010).

A relatively recent area of research in damage detection and localization is based on the wavelet transform applied to mode shapes or static deflection line data. The wavelet transform acts as a differential operator and can be applied effectively even for noisy signals. Damage which cannot be identified directly from mode shapes, may be observed on wavelet transforms since local abnormalities in a signal lead to substantial variations of wavelet coefficients in the neighborhood of damage. The literature on wavelet transforms in damage detection is very extensive (see e.g. Wang and Deng, 1996; Hong *et al.*, 2002; Douka *et al.*, 2003; Gentile and Messina, 2003; Chang and Chen, 2003, 2004; Knitter-Piątkowska and Garstecki, 2004; Loutridis *et al.*, 2004; Messina, 2004, 2008; Knitter-Piątkowska *et al.*, 2006; Ziopaja *et al.*, 2006; Rucka and Wilde, 2006a,b; Castro *et al.*, 2007; Trentadue *et al.*, 2007; Pakrashi *et al.*, 2007). The previous studies have shown very good accuracy and effectiveness of the wavelet transform although most of the investigations were performed on numerical data without experimental verification. For a practical application of the wavelet damage detection techniques, research on experimental data is the most important. The applicability of the wavelet damage detection techniques depends on measurement precision and sampling distances. Hong *et al.* (2002), Douka *et al.* (2003), Loutridis *et al.* (2004) as well as Rucka and Wilde (2006a) performed wavelet analyses on mode shapes obtained by standard modal tests using accelerometers and a modal hammer. Trentadue *et al.* (2007) used a laser sensor for a dynamic modes measurement. Pakrashi *et al.* (2007) employed a video camera to measure the beam deflection shape. The experimental research carried out so far revealed that only relatively large defects can be detected by the wavelet transform. Hong *et al.* (2002), Pakrashi *et al.* (2007) as well as Messina (2008) considered damage of 50% depth of the beam height. Smaller depth of damage was studied by Rucka and Wilde (2006a) – 35%, Douka *et al.* (2003) – 30% and Loutridis *et al.* (2004) – 20% and 30%.

Most of the reported research based on the wavelet transform are devoted to the analysis of the first mode shape (Hong *et al.*, 2002; Chang and Chen, 2003; Douka *et al.*, 2003; Loutridis *et al.*, 2004; Rucka and Wilde, 2006a) or the first three modes (Gentile and Messina, 2003; Messina, 2004, 2008; Trentadue *et al.*, 2007; Rucka and Wilde, 2010). It is so because the lower modes are much easier to acquire than the higher ones. However, the measurements of higher order modes have recently become much simpler by applying a modern scanning laser vibrometer (Okafor and Dutta, 2000; Pai and Young, 2001; Waldron *et al.*, 2002). This raises a question if the higher order modes can also be effectively applied in damage detection using the wavelet technique. Okafor and Dutta (2000) analysed three experimental and six numerical modes of a cantilever beam by the wavelet transform. They concluded that the first and third translational modes showed the damage location clearly, but the second mode was inconclusive because the damage fell in the vicinity of zero-crossing for the second mode. Gentile and Messina (2003) indicated that damage may occur in locations having poor sensitivity for certain mode shapes. They concluded that neither the fundamental mode shape nor any other higher single mode can be *a priori* considered as particularly useful in damage detection. They also stated that all available modes or operational deflection shapes should be analysed. Castro *et al.* (2006) presented a numerical study of the quality of damage detection versus the order of vibration modes for a rod vibrating axially and formulated the conclusions about the influence of the mode order on damage detection. They stated that higher modes are more appropriate to damage detection and deduced that the modes of displacement that have a node closer to the defect of stiffness type are the best for its detection in the rod.

This paper is devoted to damage detection in a cantilever beam with damage depth of 20%, 10% and 5% of the beam height by the continuous wavelet transform (CWT) on mode shapes and operational deflection shapes (ODS). Its purpose is to present the results of experimental and numerical analyses of damage detection based on higher order modes. The first eight modes are considered and the influence of the mode order on the effectiveness of damage detection by the CWT is analysed in detail.

2. Continuous wavelet transform in damage detection

This section shortly recapitulates the theoretical bases of the continuous wavelet transform and its application to detection of singularities. For a given one-dimensional signal $f(x)$ (here in the form of a beam mode shape, as it is shown in Fig. 1) the continuous wavelet transform can be defined as (e.g. Mallat, 1998)

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int\limits_{-\infty}^{+\infty} f(x)\psi^*\Big(\frac{x-u}{s}\Big)\,dx \tag{2.1}$$

where x is the spatial variable, $Wf(u,s)$ is the wavelet coefficient for the wavelet function $\psi_{u,s}(x)$ and the real numbers s and u denote the scale and the translation parameter, respectively. In the detection of signal singularities, the vanishing moments play an important role. A wavelet has n vanishing moments if the following equation is satisfied

$$\int_{-\infty}^{+\infty} x^k \psi(x)\, dx = 0 \qquad k = 0, 1, 2, \ldots, n-1 \tag{2.2}$$

Mallat (1998) proved that for wavelets having n vanishing moments and a fast decay, there exists a smoothing function $\theta(x)$ with a fast decay such that

$$\psi(x) = (-1)^n \frac{d^n \theta(x)}{dx^n} \tag{2.3}$$

Therefore the wavelet with n vanishing moments can be rewritten as the n-th order derivative of the smoothing function $\theta(x)$, and the resulting wavelet transform can be expressed as a multiscale differential operator (Mallat, 1998)

$$Wf(u,s) = s^n \frac{d^n}{du^n} \Big(f(x) * \overline{\theta}_s(x) \Big)(u) \qquad \overline{\theta}_s(x) = \frac{1}{\sqrt{s}} \theta\Big(\frac{-x}{s}\Big) \tag{2.4}$$

where $f(x) * \overline{\theta}_s(x)$ denotes convolution of functions. Equation (2.4) reveals that the wavelet transform is the n-th derivative of the signal $f(x)$ smoothed by the function $\overline{\theta}_s(x)$ at the scale s. Singularities in a signal $f(x)$ can be detected by finding the abscissa where the maxima of the wavelet transform modulus (WTM) $|Wf(u,s)|$ converge at fine scales (Mallat, 1998). The sketch presented in Fig. 1 illustrates the scheme of operation of the wavelet-based damage detection technique.

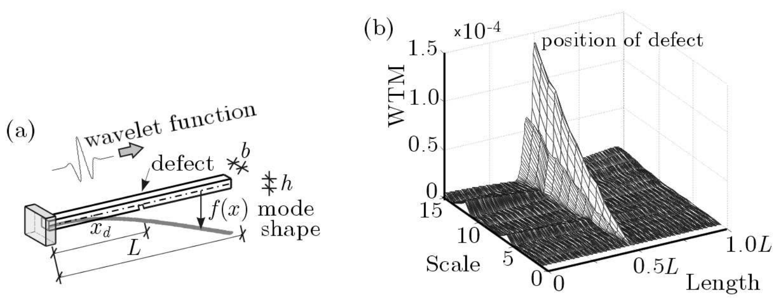

Fig. 1. (a) Geometry of the analysed beam; (b) sketch showing the wavelet-based damage detection technique

The selection of an appropriate type of the wavelet function and the choice of the number of its vanishing moments is crucial for the effective use of the wavelet analysis in damage detection. The application of wavelets which create the maximum number of wavelet coefficients that are close to zero facilitate damage identification. In this case, strong non-zero values are observed only in places where damage occurs (Fig. 1). For the first mode shape of a cantilever or simply supported beam, the wavelet function with 4 vanishing moments should be used. For structural deflection shapes, which can be approximated by polynomials of higher order than 4, the use of wavelets with higher numbers of vanishing moments is necessary. In this paper, the Gaussian wavelets gaus4, gaus6 and gaus8 (Misiti *et al.*, 2007) having four, six and eight vanishing moments, respectively, have been tested as the best candidates to damage detection with the one-dimensional continuous wavelet transform. The advantages of the Gaussian wavelets were discussed by Mallat (1998), Gentile and Messina (2003) as well as Rucka and Wilde (2006a,b).

3. Numerical analysis of damage detection on an example of a cantilever beam

A steel cantilever beam with dimensions $b = 0.01\,\mathrm{m}$, $h = 0.01\,\mathrm{m}$, $L = 0.4\,\mathrm{m}$ is considered as the testing model (Fig. 1a). The experimentally determined material parameters are: the modulus of elasticity $E = 198.25\,\mathrm{GPa}$ and the mass density $\rho = 7718.59\,\mathrm{kg/m^3}$. The beam has introduced a rectangular notch at the distance x_d measured from the fixed end to the notch centre. The notch of length 0.002 m was obtained by a high precision cut. The depth of the notch is 20% of the beam height.

Numerical simulations were performed on the first eight mode shapes (Fig. 2). The mode shapes of the beam were computed by the finite element method using the classical Euler-Bernoulli beam theory. The beam was divided into 200 elements of length 2 mm and damage was modelled as an element with a reduced height. Each mode shape line was normalized to 1 and a piecewise cubic spline data interpolation was used to decrease the sampling distance to 1 mm. Then each mode was extended outside the original beam span by a cubic spline extrapolation based on four neighbouring points to reduce the boundary effects (cf. Rucka and Wilde, 2006a,b).

In the first simulation, the notch has been introduced at the distance $x_d = 0.201\,\mathrm{m}$. The CWT was conducted on the first eight beam mode shapes by the Gaussian wavelet family. The following wavelet functions were applied in the performed analysis: gaus4 (for the 1st mode shape), gaus6 (for the 2nd, 3rd, 4th mode shapes) and gaus8 (for the 5th, 6th, 7th, 8th mode shapes). The chosen wavelet functions create the maximum number of wavelet coeffi-

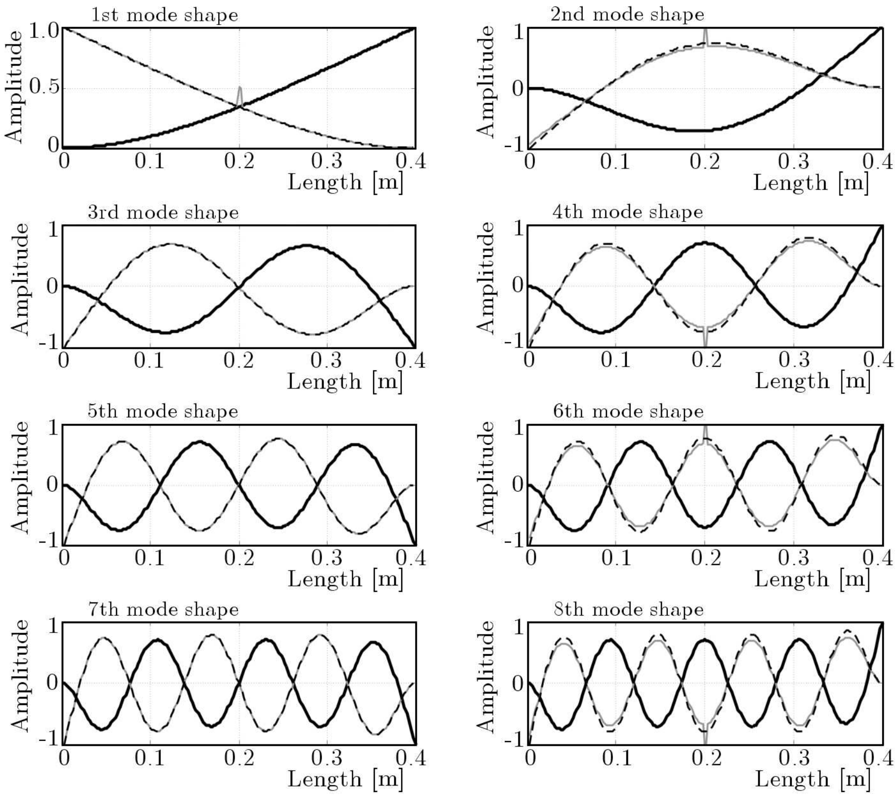

Fig. 2. Numerical mode shapes (black line), curvatures of mode shapes without damage (dashed line) and curvatures of mode shapes with damage (gray line) for the considered cantilever beam

cients that are close to zero, and non-zero values dominate at the position of damage. Figure 3 shows the computed WTM for the 1st to 8th mode shape. The modulus maximum value grows with the increasing scale s = 1-15, and the centre notch position can be very well located at $x_d = 0.201$ m for the 1st, 2nd, 4th, 6th and 8th mode shapes. However for the 3rd, 5th and 7th modes, the wavelet transform results do not allow one to identify the damage existence in this position, because the damage lies within the zero curvature part of the mode shape. It should also be noted that for the chosen wavelet (e.g. gaus8), the higher mode (see mode 6th and 8th in Fig. 3), the higher value of the wavelet transform modulus (for the 6th mode the maximum value of the WTM is 0.0106, while for the 8th mode is 0.0202). This is a result of the property of vanishing moments (Eq. (2.2)). A wavelet having n vanishing moments is orthogonal to polynomials up to degree $n-1$. Since 6th

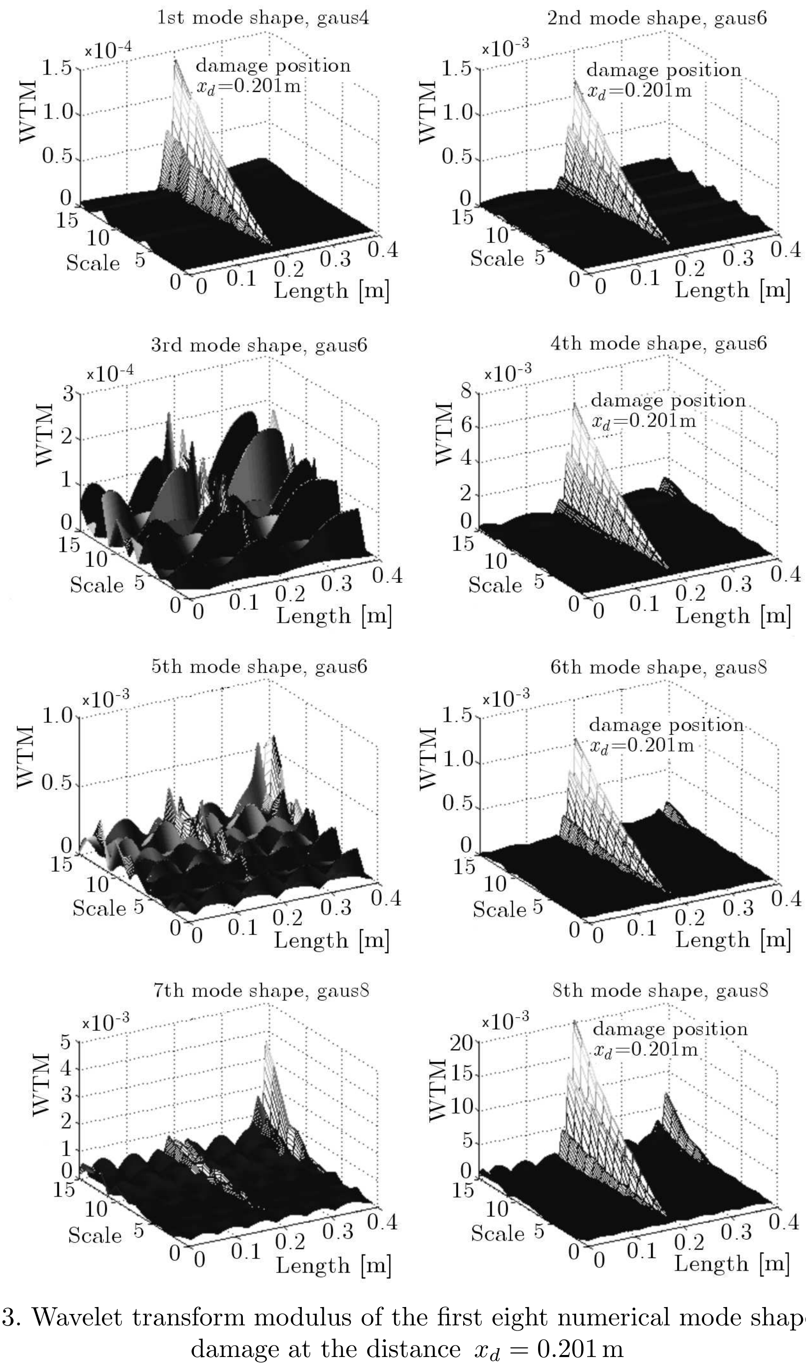

Fig. 3. Wavelet transform modulus of the first eight numerical mode shapes for damage at the distance $x_d = 0.201\,\mathrm{m}$

and 8th modes can be approximated by polynomials of higher order than 6 and 8, respectively, hence both produce some non-zero coefficients which are larger for higher order polynomials. Figure 4 presents the WTM of the 2nd and 8th mode shapes computed using gaus4 and gaus6 wavelets. Application of the wavelet function with a smaller number of vanishing moments causes that some non-zero values are observed beyond the defect position. However, the maximum value of the WTM in the defect place has a larger value for the wavelet with the smaller number of vanishing moments than for the wavelet with the larger number of vanishing moments. For the 8th mode, the maximum value of WTM is 0.0438 when gaus6 is used and 0.0202 when gaus8 is applied. This is because that the wavelet transform is proportional to the nth derivative of a function (Eq. (2.4)). Since in Eq. (2.4) a signal function is smoothed by a smoothing function, values of the WTM are smaller for higher numbers of vanishing moments n.

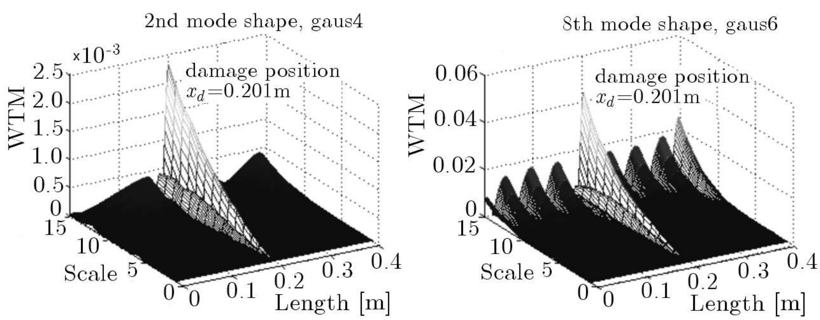

Fig. 4. Wavelet transform modulus of the 2nd and 8th numerical mode shapes for damage at the distance $x_d = 0.201\,\mathrm{m}$

In the second simulation, the notch location x_d changes from 0.009 m to 0.393 m with the step of 0.002 m, giving 193 different damage positions. Figure 5 shows the wavelet analysis on the second mode shape for the beam with the notch situated near both ends of the beam, namely at $x_d = 0.021\,\mathrm{m}$, and $x_d = 0.381\,\mathrm{m}$ as well as with the notch situated within the beam span ($x_d = 0.101\,\mathrm{m}$ and $x_d = 0.251\,\mathrm{m}$). It is visible that for the established wavelet function (here: gaus6), the maximum value of the WTM achieves different values depending on damage localization. Figure 6 shows the value of the WTM (for the scale $s = 15$) at the position of the introduced notch. In this way, we can observe localizations, where the particular mode shape is not sensitive enough to detect the damage. The 1st mode displays only one such area near the free end, whereas the 8th mode has eight such dead areas. The regions where the quality of damage detection by CWT is poor cover of course with zeros of mode shape curvatures. The maximum value of WTM for each damage position provides curves (Fig. 6) which agree with the absolute values of mode shapes curvatures (Fig. 2).

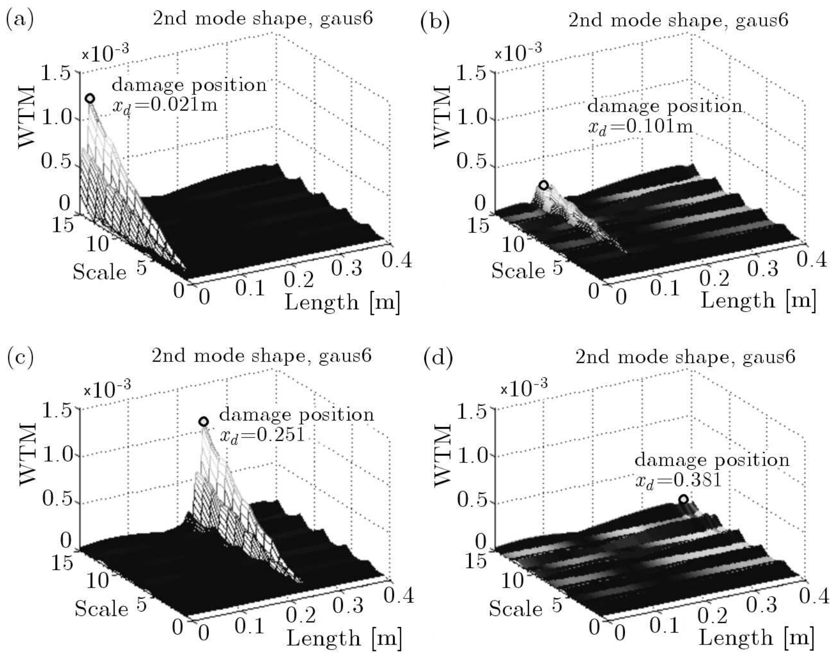

Fig. 5. Wavelet transform modulus of the 2nd numerical mode shape for damage at the distance (a) $x_d = 0.021$ m; (b) $x_d = 0.151$ m; (c) $x_d = 0.251$ m; (d) $x_d = 0.381$ m

4. Experimental verification using scanning laser vibrometer

The experimental setup is shown in Fig. 7. Measurements were performed on the previously calculated beam with the notch situated at the distance $x_d = 0.201$ m. Three different depths of the notch were considered: namely 20%, 10% and 5% of the beam height. The beam was excited using the electrodynamic vibration systems TIRA TV 50009 which consists of shaker S 503 and amplifier BAA 60. The velocity signals were measured by the Polytec Scanning Laser Vibrometer PSV-3D-400-M in 101 point distributed at the distance of 4 mm along the beam axis. The load cell PCB W208C01 was applied to register the excitation force. In the first step, resonant frequencies were found using the frequency response function (FRF). The periodic

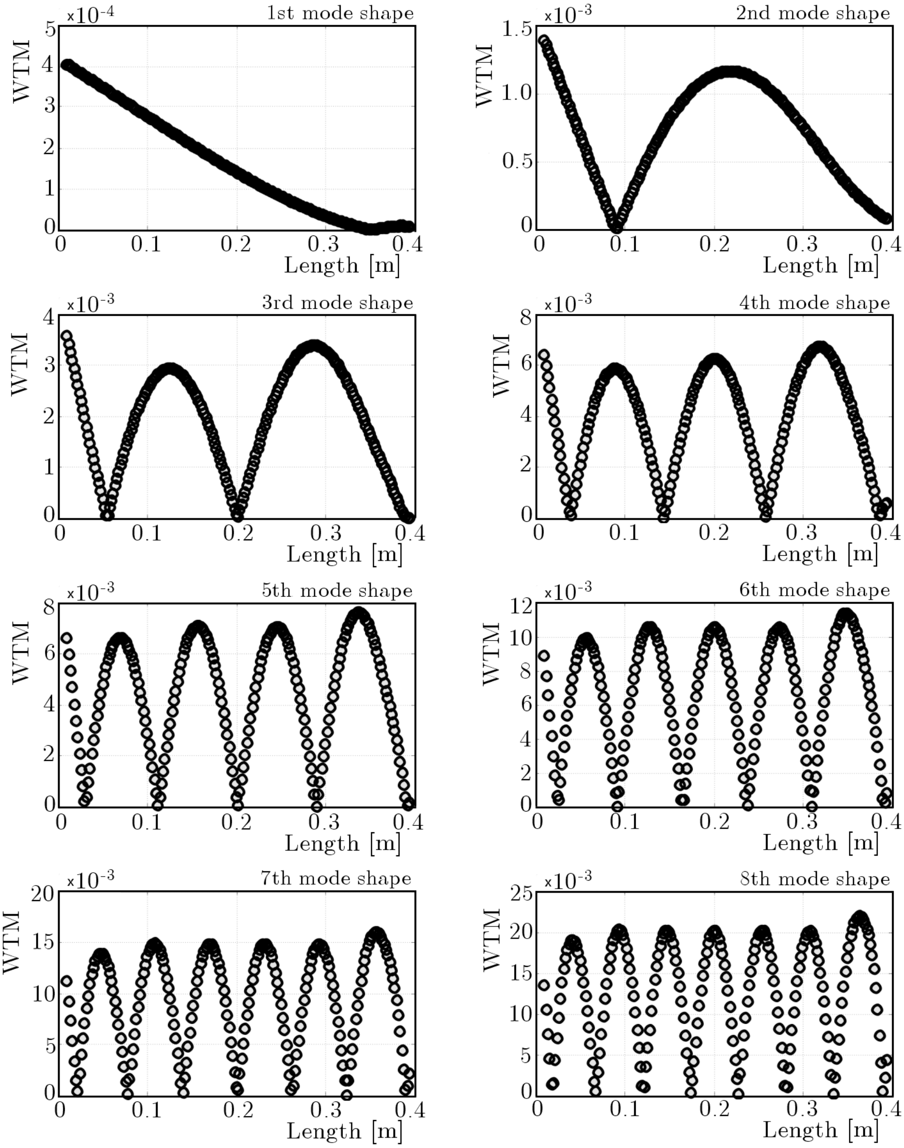

Fig. 6. Values of the wavelet transform modulus line (for the scale $s = 15$) at the position of damage (position of damage x_d from 0.009 m to 0.393 m, every 0.002 m)

chirp signal excitation was applied in the frequency range from 0 to 10 kHz. It was possible to identify eight resonant frequencies within this range with the resolution of 0.78125 Hz (for example frequencies for the beam with 20% defect were found to be: 47.65625 Hz, 285.15625 Hz, 782.8125 Hz, 1577.34375 Hz, 2709.375 Hz, 3848.4375 Hz, 5380.46875 Hz, 6796.09375 Hz). Each velocity and force measurements were repeated ten times and then the data were averaged in the frequency domain. The estimator H_1 for the FRF was used because for scanning measurements the output noise was bigger than the input noise. Next, a sine excitation at eight particular frequencies was used to measure the deflection shapes. The measured dynamic shapes for the beam with the notch of depth 20% are illustrated in Fig. 8. For the single input force and small damping, if the structure is excited at a resonance, the ODS are similar to the mode shapes (Waldron *et al.*, 2002).

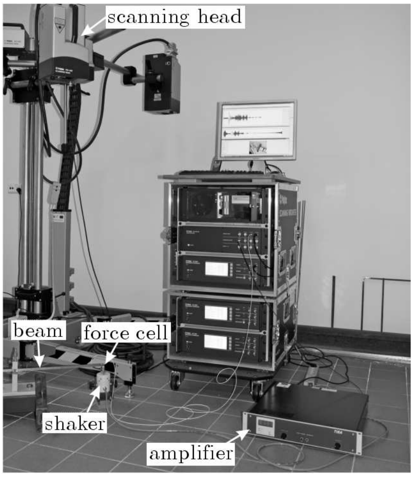

Fig. 7. Experimental setup

The wavelet transform analysis on the measured ODS for the beam with 20% defect were performed using the Gaussian wavelet family (gaus4 for the 1st and 2nd ODS, gaus6 for 3rd to 8th ODS). The wavelet transform modulus results are illustrated in Fig. 9. Detection of damage using the 1st ODS was impossible due to insufficient quality of the signal. In the case of 2nd, 4th, 6th and 8th ODS, the dominant maxima lines, corresponding to the defect positions, increase monotonically, and for larger scales they achieve the largest values. The notch presence can be easily recognized and its position determined by the wavelet analysis varies from 0.2 m to 0.201 m. The wavelet analysis

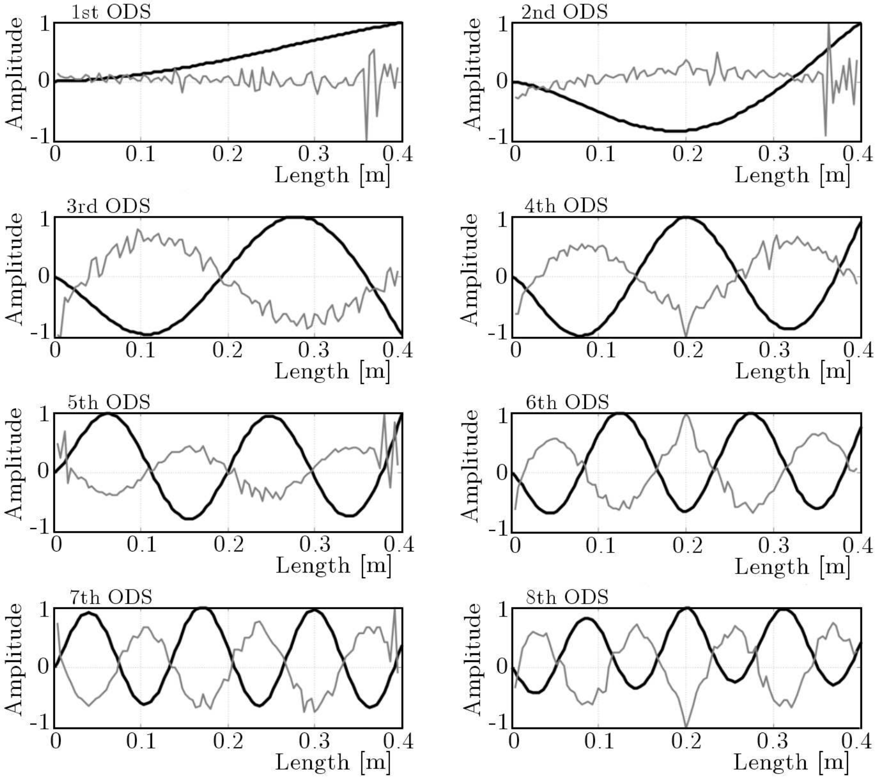

Fig. 8. Experimentally measured operational deflection shapes (black line) and their curvatures (gray line) for the beam with defect of 20% depth

of the 3rd, 5th and 7th ODS makes it impossible or ambiguous to detect damage presence and localization because these ODS have zero curvatures in the vicinity of damage location. Figure 10 shows the wavelet transform modulus of 2nd and 8th ODS performed using gaus6 and gaus8 wavelets, respectively. Note, that the application of wavelets with a higher number of vanishing moments provides worse damage identification on experimental data, because the wavelet function with smaller numbers of vanishing moments removes small single fluctuations due to measurement noise, and the detection of large variations is dominated.

Results for the beam with the defect of depth 10% are illustrated in Fig. 11. Only 6th and 8th mode enable defect localization using gaus6 wavelet. If gaus8 was applied, the identification was impossible. Defect of depth 5% of the beam height was impossible to localize, even on higher modes (Fig. 12).

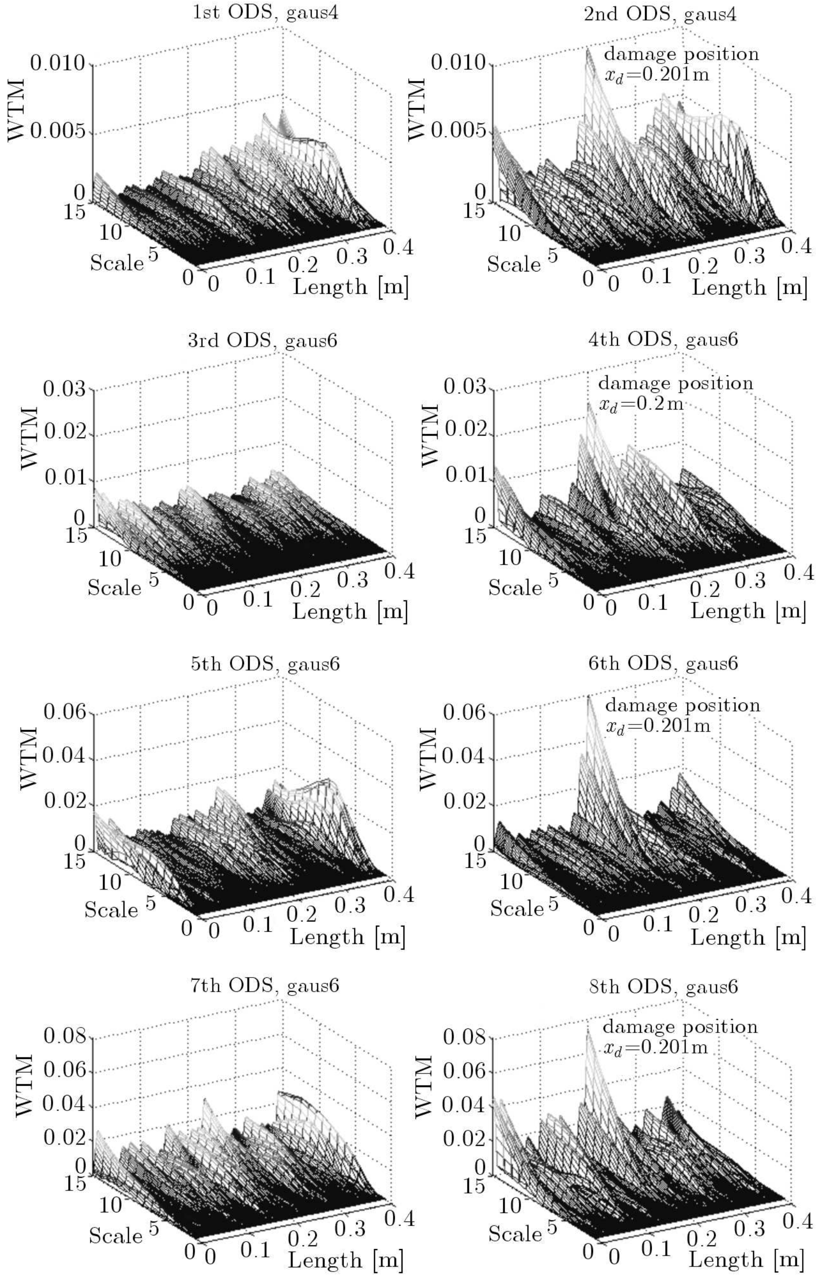

Fig. 9. Wavelet transform modulus of the first eight experimental operational deflection shapes (defect with depth of 20%)

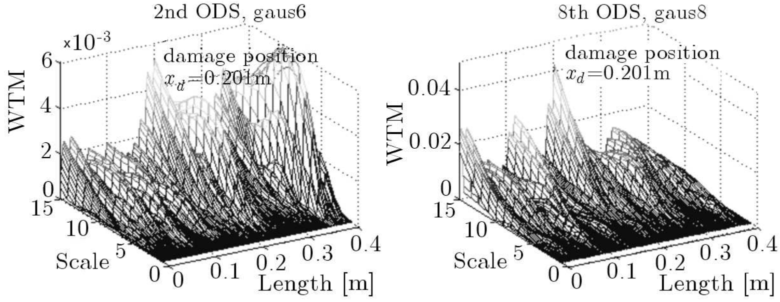

Fig. 10. Wavelet transform modulus of the 2nd and 8th experimental operational deflection shapes (defect with depth of 20%)

5. Conclusions

In this paper, the wavelet-based damage detection technique was investigated both experimentally and numerically on an example of the cantilever beam with damage in the form of the notch of depth 20%, 10% and 5% of the beam height. The analysis was performed on the first eight mode shapes. Results of the research on the effectiveness of the wavelet-base damage detection technique applied to higher vibration modes lead to the following conclusions:

- For the established wavelet function, if the mode is higher, the value of the wavelet transform modulus is also higher, what indicates that higher modes are more sensitive to the presence of the defect.
- For the established mode shape, if the number of vanishing moments of the wavelet is higher, the WTM contains a large number of zero values, what facilitates damage identification. In this case, strong non-zero values are observed only in places where the damage occurs. On the other hand, the application of the wavelet function with a smaller number of vanishing moments causes that some non-zero values are observed beyond the defect position.
- For the established mode shape, the maximum value of the WTM in the defect place is larger for the wavelet with the smaller number of vanishing moments than for the wavelet with larger numbers of vanishing moments.

Experimental investigations demonstrated that:

- Damage detection by the CWT on the first ODS was impossible, even though the ODS was determined using very precise apparatus.
- Damage detection by the wavelet analysis was more effective on higher experimentally measured ODS.

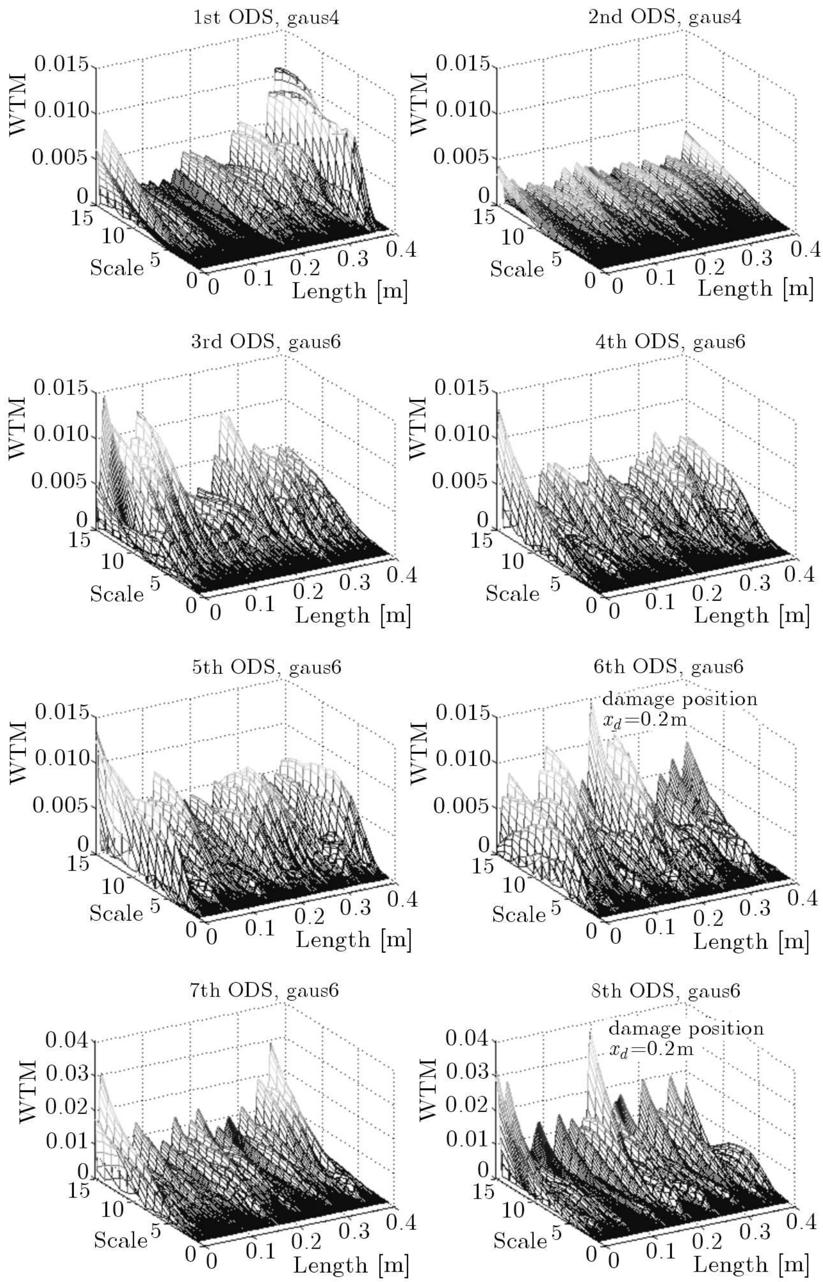

Fig. 11. Wavelet transform modulus of the first eight experimental operational deflection shapes (defect with depth of 10%)

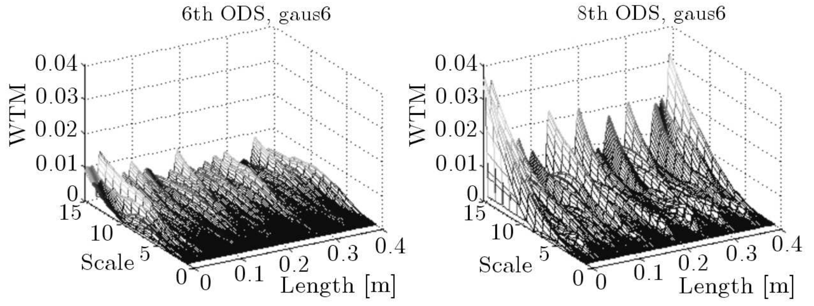

Fig. 12. Wavelet transform modulus of the 6th and 8th experimental operational deflection shapes (defect with depth of 5%)

- The smallest detectable defect was found to be of depth 10% of the beam height. Localization of this defect was only possible on higher vibration modes (on the 6th and 8th ODS).
- For experimental data, the analysis of higher operational deflection shapes by the CWT using wavelets with smaller numbers of vanishing moments appeared to be more effective. Application of wavelets with higher number of vanishing moments provided worse or even impossible damage identification.

Each mode shape has one or more regions in which it loses sensitivity to damage detection by the CWT. These regions are in agreement with the zeros of mode shapes curvatures. Higher modes are more sensitive to damage; however they contain more dead zones, where the quality of damage detection by the CWT is poor. Dead zones for particular mode shapes generally do not cover; therefore damage detection based on the CWT is possible for each damage position, if more than one mode is available. For reliable damage localization, minimum two modes are necessary. The future studies will be dedicated to develop a solution that will allow selecting which particular modes should be measured (instead of measuring all possible modes) in order to identify damage at an arbitrary position by the CWT technique.

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Wykrywanie uszkodzeń w konstrukcjach belkowych za pomocą transformaty falkowej na bazie wyższych postaci drgań

Streszczenie

Niniejsza praca poświęcona jest technice diagnostyki konstrukcji bazującej na transformacie falkowej. Badany obiekt to belka wspornikowa z uszkodzeniami w firmie nacięcia o głębokości 20%, 10% oraz 5% wysokości belki. Pomiary postaci drgań wykonano za pomocą nowoczesnego wibrometru laserowego. Celem pracy jest przedstawienie eksperymentalnych i numerycznych analiz wpływu wyższych postaci drgań na efektywność wykrywania uszkodzeń metodą ciągłej transformaty falkowej.

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