

APPARENT MASSES AND INERTIA MOMENTS OF THE PARAFOIL

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This paper presents a useful method of determination of additional forces and moments which have to be taken into account in analysis of a parafoil or paraglider flight dynamics. They are produced by apparent masses and apparent inertia moments of the air. These masses and inertia moments have strong effects on the flight dynamics of a lightly-loaded parafoil. The equations of motion for the parafoil-payload system are also shortly presented. An analytical method of calculating of the apparent masses and inertia moments is shown. Exemplary results are presented.

Keywords: apparent mass, parafoil

1. Introduction

Mutual interaction between a moving object and a fluid is a very important problem when motion is unstable – large moving objects generate motion of certain fluid mass. When the object motion is unstable and motion parameters change, motion of the fluid is disturbed.

To calculate forces acting on a moving solid body and its acceleration, it is necessary to know pressure distribution in fluid stream. This distribution depends on relative fluid velocity and also on fluid acceleration.

If the fluid is inviscid (an ideal fluid), stable motion of the solid body is continued without any external force, but this force is necessary in the case of accelerated motion. Experiments and theoretical investigations show that this force is greater than the force necessary to accelerate the same body in vacuum. This is due to the fact that any change of body motion simultaneously generates changes of fluid flow, and the acceleration of the body requires additional forces because the fluid resists this acceleration. If the fluid motion is disturbed, the inertial forces appear. They counteract these disturbances. These forces are produced by the changed pressure field on the body surface. An increase in the pressure is proportional to the body acceleration. Therefore, the additional forces (and moments) are also proportional to the acceleration and can be taken into account by increasing the body mass. This additional mass is known as the apparent mass.

The above described an unsteady aerodynamic effect, causing that the moving body can be treated as the body with greater mass and inertia moment, is called “the apparent mass effect”. It should be underlined that the additional mass and moment of inertia are not the real mass and moment of inertia of the fluid moving with the body but represents an additional energy transported to the fluid during body acceleration. For this acceleration, an additional work is executed and velocity of the body increases.

The apparent mass effect is significant for flying objects, when the mass of the disturbed air is greater than the mass of the moving object (Lissaman and Brown, 1993). The crucial parameter is the wing load factor. If this coefficient is less than 50 N/m^2 , the apparent mass effect has to be included into analysis of the body flight dynamics.

The moving body acts on various air mass depending on performed motion. Therefore, apparent masses differ depending on motion direction. A similar effect is observed for angular motions about different axes. It means that apparent masses and their moments of inertia are not

scalars. This is symbolically shown in Figs. 1 and 2 for a parafoil. They can be determined using Computational Fluid Dynamics methods (Barrows, 2002) but simplified approximate analytical methods are more popular (Barrows, 2002; Gamble, 1998; Lissaman and Brown, 1993; Ochi and Watanabe, 2011; Toglia and Vendittelli, 2010; Tweddle, 2006) – they give satisfactory accuracy. Usually, it is assumed that the parafoil has ellipsoidal or rectangular two-dimensional shape. Next, obtained results are adjusted by a series of constants for three-dimensionality.

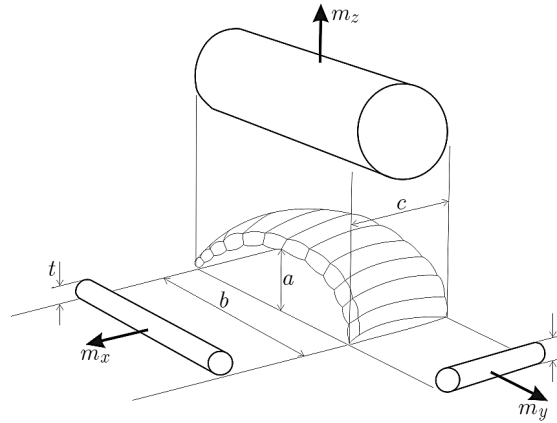


Fig. 1. Apparent masses for translational motion (Lissaman and Brown, 1993)

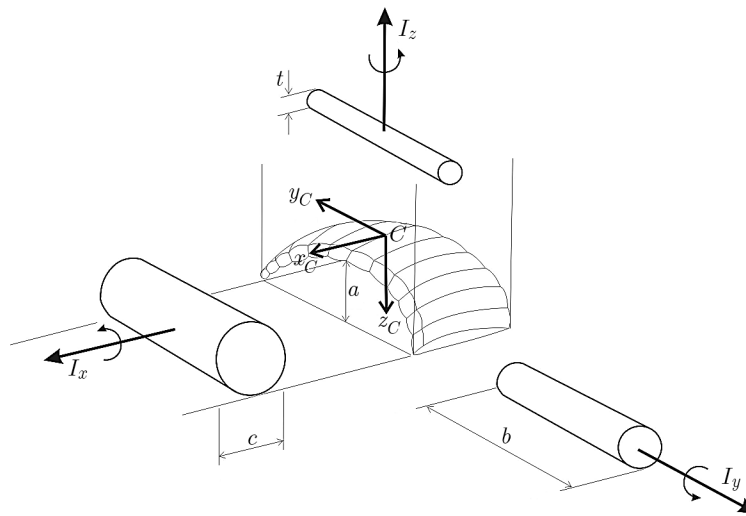


Fig. 2. Apparent masses for rotational motion (Lissaman and Brown, 1993)

2. Apparent masses and moments of inertia

2.1. Basic assumptions

Taking into account the above mentioned remarks, linear momentum and angular momentum of the air acting on a parafoil will be determined. They will be calculated in the coordinate system $Cx_Cy_Cz_C$ fixed with the parafoil (Figs. 2, 3). The important problem is to determine centres of all apparent masses. Barrows (2002) showed that *...In the more general case, it may not be possible to find a single point at which the rotational and translational motions are decoupled...* He proved that less resistance to rotational acceleration is observed for one specific point which is defined as the apparent mass centre for this rotation. For instance, for the zero thickness parafoil with circular arc spanwise camber, the confluence point of suspension lines may be

taken as the centre of apparent mass in the case of rotation about the x_C axis (rolling motion). For rotation about the perpendicular axis y_C (pitching motion) the apparent mass and its centre are different. For rotation about axis z_C (yawing motion) the centre of apparent mass is any point placed at this axis. Finally, it is assumed that (Barrows, 2002):

- motion along the x, y, z axis influences mass m_x, m_y, m_z , respectively;
- the point C_1 is the center of mass m_x ;
- the point C_2 is the center of masses m_y and m_z .

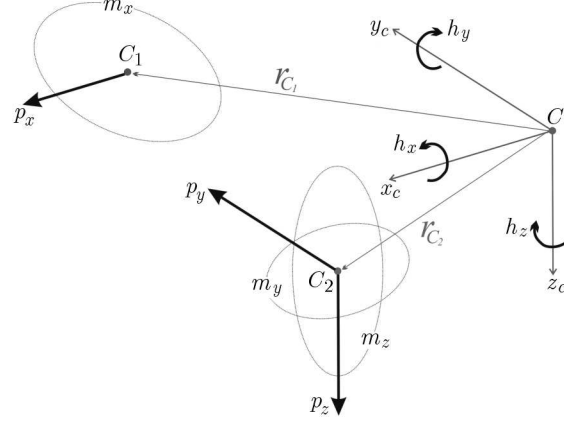


Fig. 3. Symbolic representation of the apparent mass, its linear momentum and angular momentum

2.2. Linear momentum of apparent masses

The classic formula of linear momentum is as follows

$$\mathbf{p} = m\mathbf{V} \quad (2.1)$$

and it will be used to determine the linear momentum of apparent masses.

The velocity \mathbf{V} of the i -th center of apparent mass is equal to ($i = 1, 2$)

$$\mathbf{V}_i = \mathbf{V}_C + \boldsymbol{\Omega}_C \times \mathbf{r}_{C_i} = \begin{bmatrix} U_C \\ V_C \\ W_C \end{bmatrix} + \begin{vmatrix} i & j & k \\ P_C & Q_C & R_C \\ x_{C_i} & y_{C_i} & z_{C_i} \end{vmatrix} = \begin{bmatrix} U_C + Q_C z_{C_i} - R_C y_{C_i} \\ V_C + R_C x_{C_i} - P_C z_{C_i} \\ W_C + P_C y_{C_i} - Q_C x_{C_i} \end{bmatrix} \quad (2.2)$$

where: $\mathbf{V}_C = [U_C, V_C, W_C]$ is the velocity of the parafoil mass center; $\boldsymbol{\Omega}_C = [P_C, Q_C, R_C]$ is its angular velocity. To calculate the linear momentum $\mathbf{p} = [p_x, p_y, p_z]$ one has to:

- for p_x component, take m_x mass and the first component of the vector \mathbf{V} at the point C_1 ;
- for p_y component, take m_y mass and the second component of the vector \mathbf{V} at the point C_2 ;

– for p_z component, take m_z mass and the third component of the vector \mathbf{V} at the point C_2 .
Finally, we have

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} m_x(U_C + Q_C z_{C_1} - R_C y_{C_1}) \\ m_y(V_C + R_C x_{C_2} - P_C z_{C_2}) \\ m_z(W_C + P_C y_{C_2} - Q_C x_{C_2}) \end{bmatrix} \quad (2.3)$$

The parafoil has lateral symmetry – the Cx_Cz_C plane is the symmetry plane. Therefore the points C_1 and C_2 are located at this plane. It means that

$$y_{C_1} = 0 \quad y_{C_2} = 0 \quad (2.4)$$

Formula (2.3) is reduced to

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} m_x(U_C + Q_C z_{C_1}) \\ m_y(V_C + R_C x_{C_2} - P_C z_{C_2}) \\ m_z(W_C - Q_C x_{C_2}) \end{bmatrix} = \mathbf{M}_{aV} \mathbf{V}_C + \mathbf{M}_{a\Omega} \boldsymbol{\Omega}_C \quad (2.5)$$

where the matrices are equal to

$$\mathbf{M}_{aV} = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & m_z \end{bmatrix} \quad \mathbf{M}_{a\Omega} = \begin{bmatrix} 0 & m_x z_{C_1} & 0 \\ -m_y z_{C_2} & 0 & m_y x_{C_2} \\ 0 & -m_z x_{C_2} & 0 \end{bmatrix} \quad (2.6)$$

2.3. Angular momentum of apparent masses

The classic formula of angular momentum is as follows

$$\mathbf{h} = \mathbf{h}_0 + \mathbf{r}_C \times \mathbf{p} \quad (2.7)$$

where $\mathbf{h}_0 = \mathbf{I}_{app} \boldsymbol{\Omega}_C$ is the angular momentum relative to the apparent mass center, and \mathbf{r}_C is the vector between this mass center and the fixed point (the parafoil mass center). If we assume that the main inertia axes are parallel to the axes of the $Cx_c y_c z_c$ system, formula (2.7) can be written as

$$\mathbf{h} = \begin{bmatrix} I_{app-x} & 0 & 0 \\ 0 & I_{app-y} & 0 \\ 0 & 0 & I_{app-z} \end{bmatrix} \begin{bmatrix} P_C \\ Q_C \\ R_C \end{bmatrix} + \begin{vmatrix} i & j & k \\ x_{C_i} & y_{C_i} & z_{C_i} \\ p_x & p_y & p_z \end{vmatrix} = \begin{bmatrix} I_{app-x} P_C + y_{C_i} p_z - z_{C_i} p_y \\ I_{app-y} Q_C + z_{C_i} p_x - x_{C_i} p_z \\ I_{app-z} R_C + x_{C_i} p_y - y_{C_i} p_x \end{bmatrix} \quad (2.8)$$

Taking into account relation (2.4), one has

$$\mathbf{h} = \begin{bmatrix} I_{app-x} P_C - z_{C_i} p_y \\ I_{app-y} Q_C + z_{C_i} p_x - x_{C_i} p_z \\ I_{app-z} R_C + x_{C_i} p_y \end{bmatrix} \quad (2.9)$$

If we consider formula (2.5), we can obtain an expression for determining the angular momentum of apparent masses

$$\mathbf{h} = \begin{bmatrix} -z_{C_2} m_y V_C + (I_{app-x} + m_y z_{C_2}^2) P_C - z_{C_2} x_{C_2} m_y R_C \\ z_{C_1} m_x U_C - x_{C_2} m_z W_C + (I_{app-y} + m_x z_{C_2}^2 + m_z x_{C_2}^2) Q_C \\ x_{C_2} m_y V_C - x_{C_2} z_{C_2} m_y P_C + (I_{app-z} + m_y x_{C_2}^2) R_C \end{bmatrix} = \mathbf{I}_{aV} \mathbf{V}_C + \mathbf{I}_{a\Omega} \boldsymbol{\Omega}_C \quad (2.10)$$

where the matrices are as follows

$$\mathbf{I}_{aV} = \begin{bmatrix} 0 & -m_y z_{C_2} & 0 \\ m_x z_{C_1} & 0 & -m_z x_{C_2} \\ 0 & m_y x_{C_2} & 0 \end{bmatrix} \quad \mathbf{I}_{a\Omega} = \begin{bmatrix} I_{app-x} + m_y z_{C_2}^2 & 0 & -z_{C_2} x_{C_2} m_y \\ 0 & I_{app-y} + m_x z_{C_1}^2 + m_z x_{C_2}^2 & 0 \\ -x_{C_2} z_{C_2} m_y & 0 & I_{app-z} + m_y x_{C_2}^2 \end{bmatrix} \quad (2.11)$$

One can notice that $\mathbf{I}_{aV} = (\mathbf{I}_{a\Omega})^T$.

2.4. Final formulas for linear and angular momentums of apparent masses

Formulae (2.5) and (2.10) can be written in the following form

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{h} \end{bmatrix} = \begin{bmatrix} m_x & 0 & 0 & 0 & m_x z_{C_1} & 0 \\ 0 & m_y & 0 & -m_y z_{C_2} & 0 & m_y x_{C_2} \\ 0 & 0 & m_z & 0 & -m_z x_{C_2} & 0 \\ 0 & -m_y z_{C_2} & 0 & I_{app-x} + m_y z_{C_2}^2 & 0 & -z_{C_2} x_{C_2} m_y \\ m_x z_{C_1} & 0 & -m_z x_{C_2} & 0 & \mathcal{A} & 0 \\ 0 & m_y x_{C_2} & 0 & -m_y x_{C_2} z_{C_2} & 0 & I_{app-z} + m_y x_{C_2}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_C \\ \boldsymbol{\Omega}_C \end{bmatrix} \quad (2.12)$$

$$\mathcal{A} = I_{app-y} + m_x z_{C_2}^2 + m_z x_{C_2}^2$$

or

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{h} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{aV} & \mathbf{M}_{a\Omega} \\ \mathbf{I}_{aV} & \mathbf{I}_{a\Omega} \end{bmatrix} \begin{bmatrix} \mathbf{V}_C \\ \boldsymbol{\Omega}_C \end{bmatrix} \quad (2.13)$$

3. Additional forces and moments generated by apparent masses

Unstable motion of apparent masses generates additional inertia forces and moments. They act on the parafoil and have to be taken into account in its equations of motion. They can be determined on the basis of the determined above formulae for linear and angular momentums.

3.1. Inertia force of apparent masses

The inertia force of apparent masses \mathbf{F}_{c-app} can be calculated on the basis of the linear momentum (Eq. (2.5)). It is equal to the global derivative of the linear momentum \mathbf{p} taken with the minus sign

$$\begin{aligned} \mathbf{F}_{c-app} &= -\frac{d\mathbf{p}}{dt} = -\left(\frac{d'\mathbf{p}}{dt} + \boldsymbol{\Omega}_C \times \mathbf{p}\right) \\ &= -\left(\mathbf{M}_{aV} \frac{d'\mathbf{V}_C}{dt} + \mathbf{M}_{a\Omega} \frac{d'\boldsymbol{\Omega}_C}{dt}\right) - \boldsymbol{\Omega}_C \times (\mathbf{M}_{aV} \mathbf{V}_C + \mathbf{M}_{a\Omega} \boldsymbol{\Omega}_C) \end{aligned} \quad (3.1)$$

where d'/dt is the local derivative (in a moving frame system).

3.2. Moment of inertia forces

The moment of inertia forces of apparent masses \mathbf{M}_{c-app} is equal to the global derivative of angular momentum \mathbf{h} (Eq. (2.7)) taken with the minus sign

$$\mathbf{M}_{c-app} = -\frac{d\mathbf{h}}{dt} = -\left(\frac{d\mathbf{h}_0}{dt} + \frac{d\mathbf{r}_C}{dt} \times \mathbf{p} + \mathbf{r}_C \times \frac{d\mathbf{p}}{dt}\right) \quad (3.2)$$

All components of this formula are determined bellow.

1. The component $d\mathbf{h}_0/dt$ is calculated as follows

$$\frac{d\mathbf{h}_0}{dt} = \frac{d'\mathbf{h}_0}{dt} + \boldsymbol{\Omega}_C \times \mathbf{h}_0 = \mathbf{I}_{app} \frac{d'\boldsymbol{\Omega}_C}{dt} + \boldsymbol{\Omega}_C \times \mathbf{I}_{app} \boldsymbol{\Omega}_C \quad (3.3)$$

2. The component $(d\mathbf{r}_C/dt) \times \mathbf{p}$ is equal to

$$\frac{d\mathbf{r}_C}{dt} \times \mathbf{p} = \left(\frac{d'\mathbf{r}_C}{dt} + \boldsymbol{\Omega}_C \times \mathbf{r}_C\right) \times \mathbf{p} = \left|\frac{d'\mathbf{r}_C}{dt} = 0\right| = (\boldsymbol{\Omega}_C \times \mathbf{r}_C) \times \mathbf{p} \quad (3.4)$$

Making calculation, one has to remember the relations between apparent masses and particular points: $m_x - C_1$, $m_y - C_2$ and $m_z - C_2$. Finally, we have

$$\frac{d\mathbf{r}_C}{dt} \times \mathbf{p} = \begin{bmatrix} \left(Q_C R_C (m_y - m_z) x_{C_2}^2 + W_C m_z (R_C x_{C_2} - P_C z_{C_2}) \right) \\ + V_C Q_C m_y x_{C_2} - P_C Q_C (m_y - m_z) x_{C_2} z_{C_2} \\ Q_C m_z z_{C_2} (Q_C x_{C_2} - W_C) - Q_C m_x x_{C_1} (Q_C z_{C_1} + U_C) \\ \left(P_C Q_C (m_x z_{C_1}^2 - m_y z_{C_2}^2) + Q_C R_C (m_y x_{C_2} z_{C_2} - m_x x_{C_1} z_{C_1}) \right) \\ + U_C m_x (P_C z_{C_1} - R_C x_{C_1}) + V_C Q_C m_y z_{C_2} \end{bmatrix} = \mathbf{M}_{r'p} \quad (3.5)$$

3. The component $\mathbf{r}_C \times (d\mathbf{p}/dt)$ is equal to:

Taking into consideration relation (3.1), one has

$$\mathbf{r}_C \times \frac{d\mathbf{p}}{dt} = \mathbf{r}_C \times \left(\frac{d'\mathbf{p}}{dt} + \boldsymbol{\Omega}_C \times \mathbf{p} \right) = \mathbf{r}_C \times \frac{d'\mathbf{p}}{dt} + \mathbf{r}_C \times (\boldsymbol{\Omega}_C \times \mathbf{p}) \quad (3.6)$$

Here, the first component is equal to

$$\begin{aligned} \mathbf{r}_C \times \frac{d'\mathbf{p}}{dt} &= \begin{vmatrix} i & j & k \\ x_{C_i} & 0 & z_{C_i} \\ \frac{d'p_x}{dt} & \frac{d'p_y}{dt} & \frac{d'p_z}{dt} \end{vmatrix} = \begin{bmatrix} -z_{C_2} \frac{d'p_y}{dt} \\ z_{C_1} \frac{d'p_x}{dt} - x_{C_2} \frac{d'p_z}{dt} \\ x_{C_2} \frac{d'p_y}{dt} \end{bmatrix} \\ &= \mathbf{I}_{aV} \frac{d'\mathbf{V}_C}{dt} + \mathbf{I}_{a\Omega'} \frac{d'\boldsymbol{\Omega}_C}{dt} \end{aligned} \quad (3.7)$$

The matrix $\mathbf{I}_{a\Omega'}$ is determined by

$$\mathbf{I}_{a\Omega'} = \begin{bmatrix} z_{C_2}^2 m_y & 0 & -x_{C_2} z_{C_2} m_y \\ 0 & z_{C_1}^2 m_x + x_{C_2}^2 m_z & 0 \\ -x_{C_2} z_{C_2} m_y & 0 & x_{C_2}^2 m_y \end{bmatrix} \quad (3.8)$$

The second component in (3.6) has the form

$$\mathbf{r}_C \times (\boldsymbol{\Omega}_C \times \mathbf{p}) = \begin{bmatrix} P_C W_0 z_{C_2} m_z - R_C U_0 z_{C_1} m_x - P_C Q_C x_{C_2} z_{C_2} m_z - Q_C R_C z_{C_1}^2 m_x \\ \left(Q_C (U_0 x_{C_1} m_x + W_0 x_{C_2} m_z) - V_0 (P_C x_{C_2} + R_C z_{C_2}) m_y \right) \\ + P_C R_C (z_{C_2}^2 - x_{C_2}^2) m_y + Q_C^2 (x_{C_1} z_{C_1} m_x \\ - x_{C_2} z_{C_2} m_z) + (P_C^2 - R_C^2) x_{C_2} z_{C_2} m_y \\ R_C U_0 x_{C_1} m_x - P_C W_0 x_{C_2} m_z + Q_C R_C x_{C_1} z_{C_1} m_x + P_C Q_C x_{C_2}^2 m_z \end{bmatrix} = \mathbf{M}_{r\Omega p} \quad (3.9)$$

Finally, on the basis of (3.3), (3.5), (3.7) and (3.9), the moment of inertia forces of apparent masses is equal to

$$\mathbf{M}_{c-app} = - \left[\mathbf{I}_{aV} \frac{d'\mathbf{V}_C}{dt} + \mathbf{I}_{a\Omega} \frac{d'\boldsymbol{\Omega}_C}{dt} + \boldsymbol{\Omega}_C \times \mathbf{I}_{app} \boldsymbol{\Omega}_C + \mathbf{M}_{r'p} + \mathbf{M}_{r\Omega p} \right] \quad (3.10)$$

where

$$\mathbf{I}_{a\Omega} = \mathbf{I}_{app} + \mathbf{I}_{a\Omega'} \quad (3.11)$$

4. Equations of the parafoil-payload system

Dynamic properties of the parafoil-payload system can be determined on the basis of the solution to the set of differential equations describing translatory motion and rotational motion of the parafoil and of the payload. Because the main goal of this paper is to present the method of determining apparent masses and forces/moments generated by these masses, a brief description of motion equations of the system is below presented.

4.1. Equations of the parafoil motion

The basic approach is to assume that the parafoil is a rigid body. Using this assumption, the equations of motion can be determined using Newton's second law. For motion of the parafoil mass centre in the body frame, the force equation is as follows

$$m_c \left(\frac{d'\mathbf{V}_C}{dt} + \boldsymbol{\Omega}_C \times \mathbf{V}_C \right) = \mathbf{F}_{c-a} + m_c \mathbf{g} + \mathbf{F}_{c-R} + \mathbf{F}_{c-app} \quad (4.1)$$

where m_c is the mass of the canopy, \mathbf{F}_{c-a} is the total aerodynamic force acting on the parafoil, \mathbf{g} is the vector of gravity acceleration, \mathbf{F}_{c-R} is the force of suspension lines. Using Eq. (3.1), finally we have

$$\begin{aligned} (\mathbf{1}m_c + \mathbf{M}_{aV}) \frac{d'\mathbf{V}_C}{dt} + \mathbf{M}_{a\Omega} \frac{d'\boldsymbol{\Omega}_C}{dt} \\ = \mathbf{F}_{c-a} + m_c \mathbf{g} + \mathbf{F}_{c-R} - m_c \boldsymbol{\Omega}_C \times \mathbf{V}_C - \boldsymbol{\Omega}_C \times (\mathbf{M}_{aV} \mathbf{V}_C + \mathbf{M}_{a\Omega} \boldsymbol{\Omega}_C) \end{aligned} \quad (4.2)$$

where $\mathbf{1}$ is the 3×3 unit matrix.

For rotational motion about the parafoil mass centre, the moment equation has the form

$$\mathbf{I}_c \frac{d'\boldsymbol{\Omega}_C}{dt} + \boldsymbol{\Omega}_C \times (\mathbf{I}_c \boldsymbol{\Omega}_C) = \mathbf{M}_{c-a} + \mathbf{M}_{c-R} + \mathbf{M}_{c-app} \quad (4.3)$$

where \mathbf{I}_c is the inertia matrix of the canopy, \mathbf{M}_{c-a} is the total aerodynamic moment, \mathbf{M}_{c-R} is the moment generated by suspension lines. Using Eq. (3.10) we obtain

$$\mathbf{I}_{aV} \frac{d'\mathbf{V}_C}{dt} + (\mathbf{I}_c + \mathbf{I}_{a\Omega}) \frac{d'\boldsymbol{\Omega}_C}{dt} = \mathbf{M}_{c-a} + \mathbf{M}_{c-R} - \boldsymbol{\Omega}_C \times (\mathbf{I}_{app} + \mathbf{I}_c) \boldsymbol{\Omega}_C - \mathbf{M}_{r'p} - \mathbf{M}_{r\Omega p} \quad (4.4)$$

4.2. Equations of the payload motion

It is assumed that the payload is a rigid body which performs translatory and rotational motion. The equation of translation can be written in the form

$$m_p \left(\frac{d'\mathbf{V}_p}{dt} + \boldsymbol{\Omega}_p \times \mathbf{V}_p \right) = \mathbf{F}_{p-a} + m_p \mathbf{g} + \mathbf{F}_{p-R} \quad (4.5)$$

where \mathbf{V}_p is the velocity of the payload mass center; $\boldsymbol{\Omega}_p$ is its angular velocity, \mathbf{F}_{p-a} is the aerodynamic force acting on the payload, \mathbf{F}_{p-R} is the force of suspension lines ($\mathbf{F}_{p-R} = -\mathbf{F}_{c-R}$).

The balance of moments about the payload mass centre is as follows

$$\mathbf{I}_p \frac{d'\boldsymbol{\Omega}_p}{dt} + \boldsymbol{\Omega}_p \times (\mathbf{I}_p \boldsymbol{\Omega}_p) = \mathbf{M}_{p-a} + \mathbf{M}_{p-R} \quad (4.6)$$

where \mathbf{I}_p is the inertia matrix of the payload, \mathbf{M}_{p-a} is the aerodynamic moment, \mathbf{M}_{p-R} is the vector of moments generated by suspension lines.

5. Calculation of apparent masses and location of their centers

In a general case, for any moving body, a value of apparent mass can be calculated using CFD methods. But this way is not useful for flight dynamic problems of the parafoil, where its spatial motion is calculated on the basis of a numerical solution to the equations of parafoil motion. Therefore, more simplified methods of determining apparent mass and moments are desirable. Usually these methods are dedicated to ellipsoidal or rectangular shapes of the disturbed air. This assumption substantially simplifies the problem of finding the center of gravity of the fluid. The three-dimensional effect associated with a finite aspect ratio of the parafoil is obtained by adjusting two-dimensional results by a series of constants. One of the most popular methods has been proposed by Lissman and Brown (1993). According to this method, we have the following formulae

$$\begin{aligned}
 m_x &= 0.666\rho\left(1 + \frac{8}{3}\bar{a}^2\right)t^2b & m_y &= 0.267\rho[t^2 + 2a^2(1 - \bar{t}^2)]c \\
 m_z &= 0.785\rho\sqrt{1 + 2\bar{a}^2(1 - \bar{t}^2)}\frac{AR}{1 + AR}bc^2 \\
 I_{app-x} &= 0.0555\rho\frac{AR}{1 + AR}c^2b^3 \\
 I_{app-y} &= 0.0308\rho\frac{AR}{1 + AR}\left[1 + \frac{\pi}{6}(1 + AR)AR\bar{a}^2\bar{t}^2\right]c^2b \\
 I_{app-z} &= 0.0555\rho(1 + 8\bar{a}^2)b^3t^2
 \end{aligned} \tag{5.1}$$

where $AR = b^2/S$ is the aspect ratio, $\bar{a} = a/b$ is the arc-to-span ratio, $\bar{t} = t/c$ is the thickness-to-chord ratio.

A more precise method has been presented by Barrows (2002). It was verified numerically and it has two stages. Firstly, apparent masses are calculated for a flat parafoil. Next, the curvature of the parafoil is complied.

5.1. Stage I – the flat parafoil

Apparent masses of the flat parafoil are equal to

$$\begin{aligned}
 m_{x-fl} &= \rho k_A \frac{\pi}{4} t^2 b & m_{y-fl} &= \rho k_B \frac{\pi}{4} t^2 c & m_{z-fl} &= \rho k_C \frac{\pi}{4} c^2 b \\
 I_{app-x-fl} &= \rho k_A^* \frac{\pi}{48} c^2 b^3 & I_{app-y-fl} &= \rho k_B^* \frac{4}{48\pi} c^4 b & I_{app-z-fl} &= \rho k_C^* \frac{\pi}{48} t^2 b^3
 \end{aligned} \tag{5.2}$$

The correction factors for the three-dimensional flow effect are as follows

$$\begin{aligned}
 k_A &= 0.848 & k_B &= 0.34 - 1.24 & k_C &= \frac{AR}{1 + AR} \\
 k_A^* &= 0.84 \frac{AR}{1 + AR} & k_B^* &= 1.161 \frac{AR}{1 + AR} & k_C^* &= 0.848
 \end{aligned} \tag{5.3}$$

The factor k_B is sensitive to the tip shape. Barrows (2002) showed that $k_B = 0.33$ for an ellipsoid parafoil with the axis ratio 3:1, $k_B = 1.0$ for a rectangular parafoil with ellipsoidal end caps, $k_B = 1.24$ for a rectangular parafoil with flat end caps. Formulae (5.2) show that for the flat parafoil with thickness equal to zero we have $m_{x-fl} = 0$, $m_{y-fl} = 0$, $I_{app-z-fl} = 0$.

5.2. Stage II – the curvilinear parafoil

If the curvature of the parafoil is complex, the apparent masses and moments can be calculated taking into account the previous results obtained for the flat parafoil from the formulae

$$\begin{aligned}
 m_x &= m_{x-fl} \left(1 + \frac{8}{3} \bar{a}^2\right) & m_y &= \frac{1}{a_1^2} (R^2 m_{y-fl} + I_{app-x-fl}) \\
 m_z &= m_{z-fl} \sqrt{1 + 2\bar{a}^2(1 - \bar{t}^2)} \\
 I_{app-x} &= \frac{a_{12}^2}{a_1^2} R^2 m_{y-fl} + \frac{a_2^2}{a_1^2} I_{app-x-fl} \\
 I_{app-y} &= I_{app-y-fl} \left[1 + \frac{\pi}{6} (1 + AR) AR \bar{a}^2 \bar{t}^2\right] \\
 I_{app-z} &= (1 + 8\bar{a}^2) I_{app-z-fl}
 \end{aligned} \tag{5.4}$$

where

$$\bar{a} = \frac{R - R \cos \varepsilon_0}{2R \sin \varepsilon_0} = \frac{1 - \cos \varepsilon_0}{2 \sin \varepsilon_0} = \frac{R(1 - \cos \varepsilon_0)}{b} \tag{5.5}$$

The angle ε_0 is determined as

$$\varepsilon_0 = \arcsin \frac{b/2}{R} \tag{5.6}$$

The coefficients a_1 and a_2 define the position of the points C_1 (the center of mass m_x) and C_2 (the center of masses m_y and m_z) with respect to the joint point O , which is the point of convergence of the lines a_{12} is the distance between C_1 and C_2 . This is shown in Fig. 4. They are equal to

$$a_1 = \frac{R \sin \varepsilon_0}{\varepsilon_0} \quad a_2 = \frac{a_1 m_{y-fl}}{m_{y-fl} + \frac{I_{app-x-fl}}{R^2}} \tag{5.7}$$

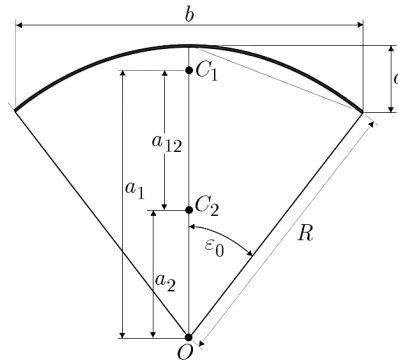


Fig. 4. Parafoil geometry and location of characteristic points

5.3. Example

To check the described above procedure, exemplary calculations for the parafoil have been done. The following initial values of parafoil geometry were used: the area $S = 21 \text{ m}^2$, the chord $c = 3 \text{ m}$, the span $b = 7 \text{ m}$, the thickness $t = 0.3 \text{ m}$. These data give the following constants: the aspect ratio $AR = 2.33$, the thickness-to-chord ratio $\bar{t} = 0.1$. The apparent masses and moments of inertia for both the flat and curvilinear parafoil are gathered in Table 1.

For the flat parafoil we have:

- apparent masses: $m_{x-fl} = 0.51 \text{ kg}$, $m_{y-fl} = 0.09 \text{ kg}$, $m_{z-fl} = 42.44 \text{ kg}$,
- apparent moments of inertia: $I_{app-x-fl} = 145.58 \text{ kgm}^2$, $I_{app-y-fl} = 14.99 \text{ kgm}^2$ and $I_{app-z-fl} = 2.1 \text{ kgm}^2$.

These data are in the first row in Table 1. Analyzing them one can notice that the most significant is the apparent mass m_{z_fl} and the apparent moments of inertia $I_{app_x_fl}$ and $I_{app_y_fl}$. Values of the rest are rather small.

For the curvilinear parafoil, calculations were performed for different lengths R of the suspension lines. The obtained results are presented in Table 1 and in Figs. 5 and 6. Comparisons between the data for the flat and curvilinear parafoil give the conclusion that the spanwise camber fundamentally decreases the apparent moment of inertia I_{app_x} and increases the apparent mass m_y . During the rolling about the confluence point, the apparent moment of inertia I_{app_x} decreases because, theoretically, the parafoil with a circular arc and with zero thickness does not disturb the surrounding air. In this case, the only disturbances are generated by thickness of the parafoil. In the case of lateral motion, the side projection of the parafoil determines the amount of the disturbed air – the apparent mass m_y . For the flat parafoil, only the thickness influences this effect, but for the curvilinear parafoil, its side projection has to be taken into account.

Table 1. Apparent masses and moments of inertia for the flat and curvilinear parafoil

R [m]	ε_0 [deg]	a_1 [m]	a_2 [m]	m_x [kg]	m_y [kg]	m_z [kg]	I_{app_x} [kgm ²]	I_{app_y} [kgm ²]	I_{app_z} [kgm ²]
Flat parafoil									
				0.51	0.26	42.44	145.58	14.99	2.10
Curvilinear parafoil									
5.0	44.4	4.51	0.19	0.57	7.46	44.16	6.22	15.02	2.80
5.5	39.5	5.07	0.26	0.56	5.96	43.78	7.08	15.01	2.64
6.0	35.7	5.62	0.34	0.55	4.91	43.52	8.26	15.01	2.53
6.5	32.6	6.16	0.43	0.54	4.13	43.33	9.49	15.01	2.46
7.0	30.0	6.68	0.54	0.54	3.54	43.19	10.77	15.00	2.40
7.5	27.8	7.21	0.66	0.54	3.08	43.08	12.07	15.00	2.36
8.0	25.9	7.73	0.79	0.53	2.72	43.00	13.40	15.00	2.32
8.5	24.3	8.25	0.94	0.53	2.42	42.93	14.73	15.00	2.29
9.0	22.9	8.76	1.11	0.53	2.17	42.87	16.07	15.00	2.27
9.5	21.6	9.28	1.29	0.53	1.96	42.82	17.40	15.00	2.25
10.0	20.5	9.79	1.48	0.53	1.79	42.78	18.71	15.00	2.24

6. Conclusions

The obtained results show that for the flat and curvilinear parafoil the apparent mass m_z is many times greater than the other two apparent masses m_x and m_y . The mass m_z may be greater than the mass of the canopy. If the parafoil has spanwise camber, the mass m_y is also crucial but smaller than m_z . These apparent masses should be included into the equations of parafoil motion.

For the flat parafoil, the apparent moment of inertia I_{app_x} is significant but in the case of the parafoil with spanwise camber its value is equal between 4 and 12 percent of the value for the flat parafoil. It means that the spanwise camber of the parafoil significantly reduces the additional effect of the apparent inertia on rolling motion. For the curvilinear parafoil, the value of apparent moment of inertia I_{app_y} is almost the same as for the flat parafoil. This moment of inertia is comparable with the value of the apparent moment of inertia I_{app_x} for the curvilinear parafoil.

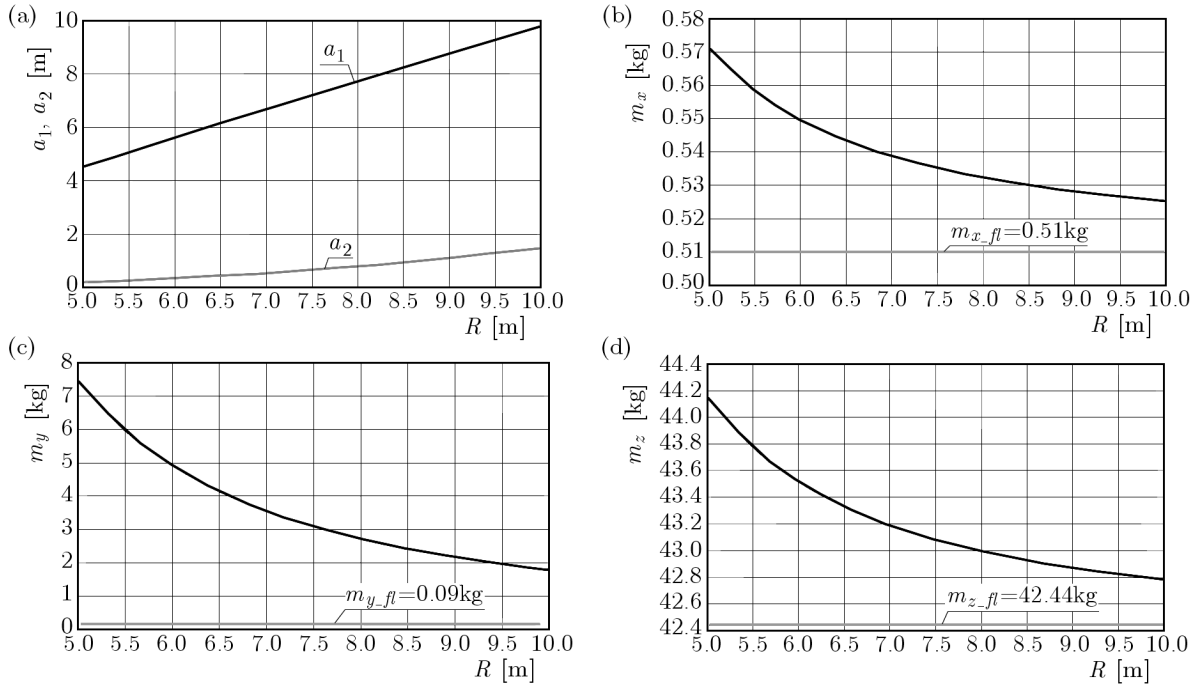


Fig. 5. (Location of apparent masses (a); apparent mass: m_x (b), m_y (c), m_z (d)

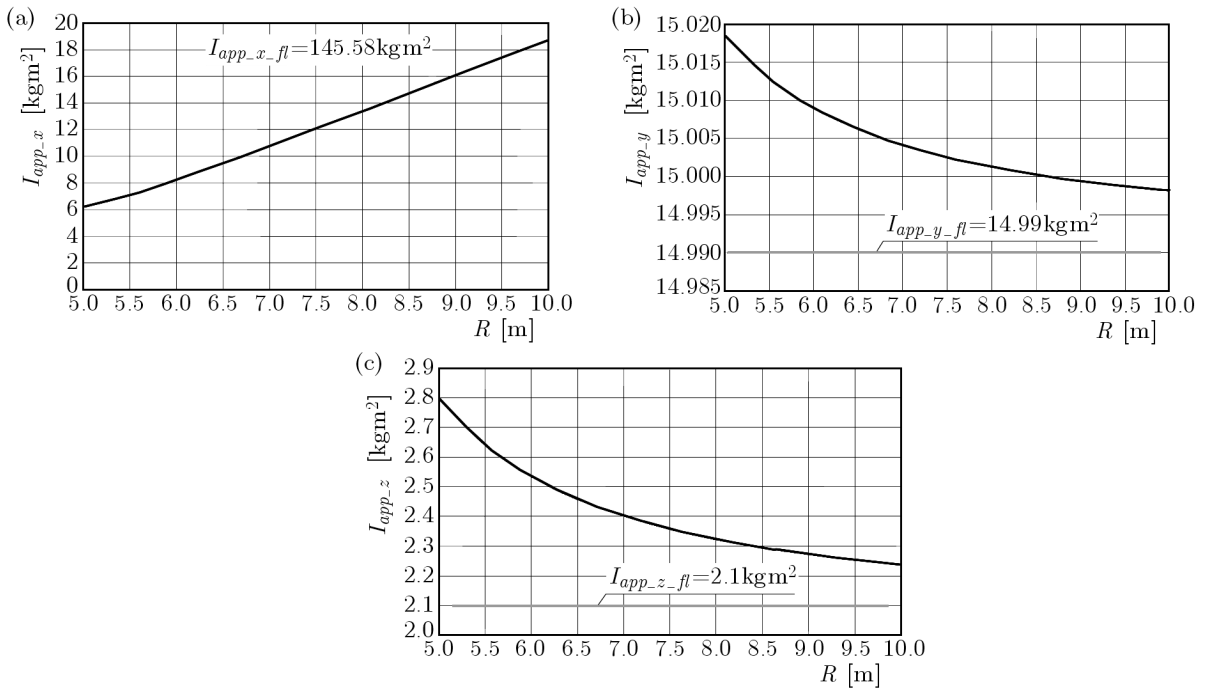


Fig. 6. Apparent inertia moment: I_{app-x} (a), I_{app-y} (b), I_{app-z} (c)

The length R of the suspension lines influences the apparent masses and moments of inertia. We can see that all of them, except one, decrease. Only the apparent moment of inertia I_{app-x} grows for increasing R , and this influence is rather important.

It should be underlined that the presented above analytical method of the apparent masses and inertia moments calculations is dedicated to the parafoil. For a parachute, one should use different formulae because of different geometry (Dobrokhodov *et al.*, 2003).

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