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IMPACT TEST ANALYSIS OF DYNAMIC MECHANICAL PARAMETERS OF A RIGID-PLASTIC MATERIAL WITH LINEAR STRAIN HARDENING

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A simple engineering method to determine dynamic mechanical parameters of a rigid-plastic material with linear strain hardening is presented in this paper. For this purpose, Taylor's impact test, i.e., perpendicular impact of a long rod on a flat rigid target has been used. The rod is made of the tested material. The transcendental equations which explicitly determinate the dynamic yield stress and the plastic strain of the rod have been derived. The general trend of the obtained results is observation of the appearance of higher strengths at higher impact velocities, which is in agreement with expectations. The applied in literature approximation of dynamic properties of metals with strain hardening by means of a perfectly plastic material is far-reaching simplification disagreeing with reality.

Key words: impact loads, Taylor impact test, dynamic yield stress

1. Introduction

Beginning from the second half of the last century, there has been much interest in dynamic initial boundary-value problems of the theory plasticity. Those problems were thoroughly addressed in monographs by Kolsky (1953), Broberg (1956), Goldsmith (1960), Shewmon and Zackay (1961), Rakhmatulin and Demyanov (1961), Zukas (1962), Perzyna (1966), Cristescu (1967), Lindholm (1968), Kinslow (1970), Nowacki (1974), Kaliski *et al.* (1992), Meyers (1994). The investigations were necessary in order to obtain reliable operation of various machine elements and special objects which are exposed to impact loadings in extreme conditions. The important place in such problems occupies the Taylor test.

Taylor published his theory in 1948 (Taylor, 1948). Originally, one-dimensional analysis of Taylor was used by Whiffin (1948) to estimate the dynamic yield stress of specimens. There has been much interest in impact testing and estimating dynamic yield stress since then. A selective review of the literature with respect to the Taylor impact test is in the papers by Jones *et al.* (1987, 1997), and is not necessary to be discussed in this paper.

The present opinion seems to be that Taylor's theory fails to provide reliable yield stress estimates, especially for tests conducted at higher velocities (Jones *et al.*, 1987, 1997). For this reason, many investigators correlate their results with sophisticated computer analyses which are capable of utilizing several complex forms of constitutive eqations. These programs can match geometry of the post-test specimen with very high accuracy and give very reliable estimates for material properties. The drawback is that these programs are expensive and often require substantial amounts of time to execute.

The Taylor impact test is a useful experiment for estimating material behaviour at high strain rates (Meyers, 1994). The test is reproducible and is reasonably economical after the initial investment has been made.

Jones *et al.* (1987) assert that simple engineering theories, such as that given by Taylor, still have considerable value. Such theories frequently give investigators insight into the interaction of physical parameters and their relationship with the outcome of the event. Most often, these interactions are difficult to ascertain from complex computer outputs. As a result, simple engineering theories often provide the basis for the design of experiments and are frequently used to refine the areas in which computing is to be done.

Bearing in mind the above-mentioned facts, the Taylor problem, for a rigid-plastic material with linear strain hardening, loaded by an impact has been solved in a closed form in this paper. The engineering transcendental equations which explicitly determine the dynamic yield stress and plastic strain have been derived.

2. Formulation of the problem

The corrected Taylor theory represented by Jones *et al.* (1987, 1997) has been used in this paper.

Consider a uniform rod of the initial dimensions: length L and cross-sectional area A_0, which impacts against a rigid boundary. Let x denote a Lagrangian coordinate aligned with the axis of the rod and having its origin at the end of the rod opposite to the impacted end. The initial velocity of the rod is denoted by V. Assuming that V is large enough, a portion of the rod will deform plastically. Let X represent the time-dependent extent of the plastic zone measured relative to the original configuration of the rod, Fig. 1a, S be the time-dependent displacement of the back end of the rod, as shown in Fig. 1b, and h – the time-dependent extent of the plastic zone measured relative to the deformed configuration of the rod. Define l as $L - X$ so that $l + X = S + l + h = L$.

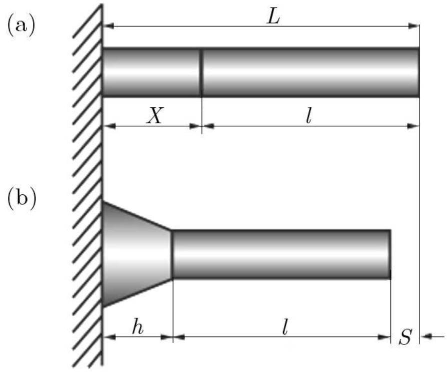

Fig. 1. Schematic illustration of the rod impacting a rigid boundary: (a) original configuration of the rod; (b) deformed configuration of the rod

Assume that the rod material is rigid-plastic with linear strain hardening, Fig. 2a, and is incompressible. The rigid-perfectly plastic material, Fig. 2b, was considered in papers by Taylor (1948) and Jones *et al.* (1987). The material model shown in Fig. 2a sufficiently approximates dynamic mechanical properties of some metals, e.g. alloy steels, especially chromium-nickel steels (Ashby and Jones, 1993; Lee and Tupper, 1951).

The influence of transverse strain and friction force between the target surface and the impacted end of the rod upon its longitudinal motion are neglected.

Consider the motion of the undeformed section of the rod. At some time t, during the deformation, suppose that the undeformed section length is $L - X(t)$ and suppose that $\dot{S}(t) = v(t)$ is its speed, see Fig. 3a. As shown in Fig. 3b, the undeformed section has lost an increment ΔX to plastic deformation at some later time $t + \Delta t$. This increment has undergone plastic deformation and now has a new cross-sectional area A_1. The mass of the plastic element, however, remains $\rho A_0 \Delta X$, where ρ is the material density which is assumed to be constant throughout the deformation process. The speed of this element is denoted by u, which will generally be different from $v + \Delta v$, i.e. the speed of the remaining rigid end. The forces that act on this new pla-

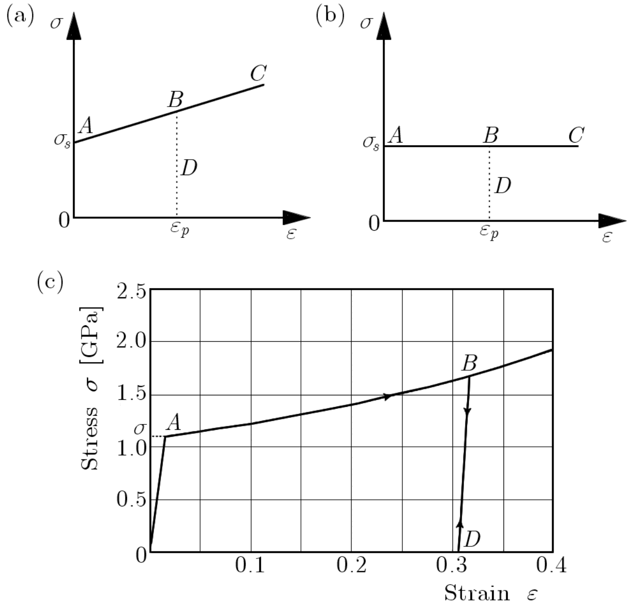

Fig. 2. Stress-strain curves for ductile materials: (a) rigid-plastic material with linear strain hardening; (b) rigid-perfectly plastic; (c) true curve for chromium-nickel steel

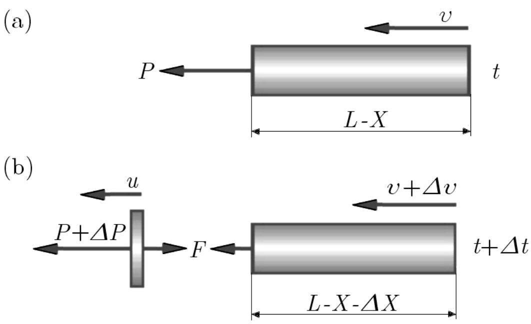

Fig. 3. Schematic illustration of the rear portion of the rod. The undeformed section shown in (a) has transferred some mass to the deformed section after a time interval Δt, as indicated in (b)

stic element are in interaction with the underformed section, denoted by F in Fig. 3b, and its interaction with the previously deformed material denoted as $P + \Delta P$ in the same figure.

The change in linear momentum of the system from configuration 3a to 3b must equal the net impulse. Thus

$$\rho A_0 \Delta X u + \rho A_0 (L - X - \Delta X)(v + \Delta v) - \rho A_0 (L - X) v = \frac{2P + \Delta P}{2} \Delta t \tag{2.1}$$

where $(2P+\varDelta P)/2$ denotes the mean value of forces P and $P+\varDelta P$. Dividing both sides of this equation by $\varDelta t$ and neglecting small quantities of higher order, and taking limits $(\varDelta t \to 0)$, gives

$$(L-X)\dot{v} - \dot{X}(v-u) = \frac{P}{\rho}A_0 \tag{2.2}$$

where the superposed dots denote derivatives with rispect to time. However

$$P = \sigma A_1 = \sigma(\varepsilon_p)\frac{A_0}{1+\varepsilon_p} \tag{2.3}$$

where σ and ε_p are, respectively, the engineering stress and strain at the deformed cross section. Combining Egs. (2.2) and (2.3), gives

$$(L-X)\dot{v} - \dot{X}(v-u) = \frac{1}{\rho}f(\varepsilon_p) \tag{2.4}$$

where

$$f(\varepsilon_p) = \frac{\sigma(\varepsilon_p)}{1+\varepsilon_p} \tag{2.5}$$

In terms of the undeformed section length l, Eq. (2.4) becomes

$$l\dot{v} + \dot{l}(v-u) = \frac{1}{\rho}f(\varepsilon_p) \tag{2.6}$$

where $\dot{X} = -\dot{l}$.

This expression is the equation of motion of the undeformed (rigid) section of the rod. This equation has been solved in the following considerations.

3. General solution to simplified (u=0) equation of motion (2.6)

Previously, it was assumed that the rod material is rigid-plastic with linear strain hardening. This rod impacts the rigid target with a large enough speed V. At such a condition, the plastic wave is generated in the rod during the impact process. This wave propagates along the rod from the rigid target towards of its free end. The velocity of the wave is determined by the formula

$$a_1 = \sqrt{\frac{E_1}{\rho}} \tag{3.1}$$

where E_1 is the modulus of strain hardening.

It follows from experimental results and numerical calculations (Lee and Tupeer, 1951) as well as from analytical solutions (Włodarczyk and Jackowski, 2010) that the modulus E_1 is poorly dependent on the impact speed and one can assume that the velocity $a_1 \approx$ const.

The plastically deformed portion of the rod is placed between the rigid target and plastic wave front. That portion of the rod is motionless. The elastic strain is equal to zero $(\varepsilon_s = 0)$ in the rigid plastic model, Fig. 2a, and for that reason the velocity $u = 0$ in the deformed plastically portion of the rod contacting with rigid target.

For $u = 0$, from Eq. (2.6), we obtain

$$\frac{d(lv)}{dt} = \frac{f(\varepsilon_p)}{\rho} \tag{3.2}$$

In turn, the continuity of the Lagrangian component of displacement across the rigid-plastic interface gives

$$U(X,t) = -S$$

Differentiating this equation with respect to time, leads to

$$-\varepsilon_p \dot{X} + u = \varepsilon_p \dot{l} + u = \dot{S} = v \tag{3.3}$$

where

$$\varepsilon_p = \frac{\partial U}{\partial X} \qquad\qquad u = \frac{\partial U}{\partial t}$$

For $u = 0$, from (3.3), we have

$$\frac{dl}{dt} = \frac{v}{\varepsilon_p} \tag{3.4}$$

Next, according to Fig. 1, there is

$$S + l + h = L$$

and after differentiation, we obtain

$$\dot{S} + \dot{l} + \dot{h} = 0 \tag{3.5}$$

or

$$\dot{l} = \frac{dl}{dt} = -(a_1 + v) \tag{3.6}$$

where $\dot{h} = a_1$ and $\dot{S} = v$, $a_1 > 0$.

From Eqs. (3.4) and (3.6), it follows that the plastic strain ε_p can be determined by means of the formula

$$\varepsilon_p = -\frac{v}{a_1 + v} \tag{3.7}$$

Combining Egs. (3.2), (3.4) and (3.7), gives

$$\frac{d(lv)}{dl} = -\frac{f[-v/(a_1 + v)]}{\rho(a_1 + v)} \tag{3.8}$$

Equation (3.8) has separable variables and can be immediately integrated, leading to an explicit dependence of l upon v, namely

$$\ln\frac{l}{L} = -\int\limits_V^v \frac{a_1 + y}{y(a_1 + y) + \frac{1}{\rho} f[-y/(a_1 + y)]}\,dy \tag{3.9}$$

or

$$l = l(v) = L\exp\Big(\int\limits_v^V \frac{a_1 + y}{y(a_1 + y) + \frac{1}{\rho} f[-y/(a_1 + y)]}\,dy\Big) \tag{3.10}$$

4. Dynamic mechanical parameters of the rigid-plastic material with linear strain hardening

According to Fig. 2a, we have

$$\sigma = \sigma_s + E_1\varepsilon_p = \sigma_s + \rho a_1^2\varepsilon_p \tag{4.1}$$

and then the function $f(\varepsilon_p)$ has the form

$$f(\varepsilon_p) = \frac{\sigma(\varepsilon_p)}{1 + \varepsilon_p} = \frac{\sigma_s + \rho a_1^2\Big(-\frac{v}{a_1+v}\Big)}{1 - \frac{v}{a_1+v}}$$

or

$$f\Big(-\frac{v}{a_1 + v}\Big) = \frac{\sigma_s(a_1 + v)}{a_1} - \rho a_1 v \tag{4.2}$$

where σ_s and ε_p denote, respectively, the yield stress and longitudinal plastic strain of the rod.

In order to simplify the quantitative analysis of particular dynamic mechanical parameters of the deformed rod, the following dimensionless quantities have been introduced

$$\alpha = \frac{\rho V}{\sigma_s} \qquad \alpha^* = \frac{\rho V^2}{Y_W} \qquad \beta^* = \frac{a_1}{V}$$
$$\gamma = \frac{v}{V} \qquad \xi = \frac{l}{L} \qquad \xi_f = \frac{l_f}{L} \tag{4.3}$$
$$\xi_p = \frac{h_f}{L} \qquad \xi_L = \frac{L_f}{L}$$

where the symbols Y_W and l_f denote, respectively, the dynamic yield stress of the rigid-plastic material with linear strain hardening, and the final length of the undeformed portion of the rod; L_f is the final overall length of the specimen and h_f is the final length of the deformed portion of the rod.

Substituting function (4.2) into Eq. (3.9) and using dimensionless quantities (4.3), we get

$$\ln \xi = \int\limits_{\gamma}^{1} \frac{\beta^* + \gamma_1}{\gamma_1^2 + \frac{1}{\alpha\beta^*}\gamma_1 + \frac{1}{\alpha}}\, d\gamma_1 \tag{4.4}$$

The right-hand side of Eq.(4.4) can be expressed by the following functions:
— for $\Delta > 0$

$$F_1(\alpha,\gamma) = \frac{1}{\sqrt{\Delta}}\Big(\beta^* - \frac{1}{2\alpha\beta^*}\Big) \ln \left| \left(\frac{2 + \frac{1}{\alpha\beta^*} - \sqrt{\Delta}}{2 + \frac{1}{\alpha\beta^*} + \sqrt{\Delta}} \right) \left(\frac{2\gamma + \frac{1}{\alpha\beta^*} + \sqrt{\Delta}}{2\gamma + \frac{1}{\alpha\beta^*} - \sqrt{\Delta}} \right) \right| + \tag{4.5}$$
$$+ \frac{1}{2} \ln \left| \frac{1 + \frac{1}{\alpha\beta^*} + \frac{1}{\alpha}}{\gamma^2 + \frac{1}{\alpha\beta^*}\gamma + \frac{1}{\alpha}} \right|$$

— for $\Delta < 0$

$$F_2(\alpha,\gamma) = \frac{1}{\sqrt{-\Delta}}\Big(2\beta^* - \frac{1}{\alpha\beta^*}\Big) \arctan \frac{2\gamma + \frac{1}{\alpha\beta^*}}{\sqrt{-\Delta}} + \frac{1}{2} \ln \Big| \gamma^2 + \frac{1}{\alpha\beta^*}\gamma + \frac{1}{\alpha} \Big| \tag{4.6}$$

— for $\Delta = 0$

$$F_3(\alpha,\gamma) = \frac{1}{2} \ln \left| \frac{1 + \frac{1}{\alpha\beta^*} + \frac{1}{\alpha}}{\gamma^2 + \frac{1}{\alpha\beta^*}\gamma + \frac{1}{\alpha}} \right| - \frac{\beta^*}{1 + 2\alpha\beta^*\gamma} + \frac{\beta^*}{1 + \alpha\beta^*} \tag{4.7}$$

where

$$\Delta = \Big(\frac{\sigma_s}{\rho V^2}\Big)^2 \Big(\frac{V}{a_1}\Big)^2 - \frac{4\sigma_s}{\rho V^2} = \Big(\frac{1}{\alpha\beta^*}\Big)^2 - \frac{4}{\alpha} \tag{4.8}$$

In the investigated problem, the parameter α is negative ($\alpha < 0$), and in accordance with expression (4.8) the quantity Δ is positive ($\Delta > 0$). In this context, in the following considerations only the function $F_1(\alpha,\gamma)$ will be used.

Relationships (4.4) and (4.5) lead to an explicit dependence of $l(\xi = l/L)$ upon $v(\gamma = v/v_0)$ for a rigid-plastic material with linear strain hardening, namely

$$\begin{aligned}\ln\xi &= \frac{1}{2}\ln\left|\frac{1+\frac{1}{\alpha\beta^*}+\frac{1}{\alpha}}{\gamma^2+\frac{1}{\alpha\beta^*}\gamma+\frac{1}{\alpha}}\right| + \\ &+\frac{1}{\sqrt{\Delta}}\Big(\beta^*-\frac{1}{2\alpha\beta^*}\Big)\ln\left|\left(\frac{2+\frac{1}{\alpha\beta^*}-\sqrt{\Delta}}{2+\frac{1}{\alpha\beta^*}+\sqrt{\Delta}}\right)\left(\frac{2\gamma+\frac{1}{\alpha\beta^*}+\sqrt{\Delta}}{2\gamma+\frac{1}{\alpha\beta^*}-\sqrt{\Delta}}\right)\right|\end{aligned} \tag{4.9}$$

or

$$\xi = \left|\frac{1+\frac{1}{\alpha\beta^*}+\frac{1}{\alpha}}{\gamma^2+\frac{1}{\alpha\beta^*}\gamma+\frac{1}{\alpha}}\right|^{\frac{1}{2}}\left|\frac{2+\frac{1}{\alpha\beta^*}-\sqrt{\Delta}}{2+\frac{1}{\alpha\beta^*}+\sqrt{\Delta}}\,\frac{2\gamma+\frac{1}{\alpha\beta^*}+\sqrt{\Delta}}{2\gamma+\frac{1}{\alpha\beta^*}-\sqrt{\Delta}}\right|^{\frac{1}{\sqrt{\Delta}}\left(\beta^*-\frac{1}{2\alpha\beta^*}\right)} \tag{4.10}$$

At the end of the impact process, $l = l_f(\xi = \xi_f)$ and $v = 0$ ($\gamma = 0$). Then Eq. (4.10) reduces to

$$\xi_f = \Phi(\alpha) = \left|1+\alpha+\frac{1}{\beta^*}\right|^{\frac{1}{2}}\left|\frac{2+\frac{1}{\alpha\beta^*}-\sqrt{\Delta}}{2+\frac{1}{\alpha\beta^*}+\sqrt{\Delta}}\,\frac{\frac{1}{\alpha\beta^*}+\sqrt{\Delta}}{\frac{1}{\alpha\beta^*}-\sqrt{\Delta}}\right|^{\frac{1}{\sqrt{\Delta}}\left(\beta^*-\frac{1}{2\alpha\beta^*}\right)} \tag{4.11}$$

Thereby, we obtain a transcendental equation which uniquely determinates the value of the parameter $\alpha = \alpha^*$, see Fig. 4. In accordance with expression $(4.3)_2$, we have

$$Y_W = \frac{\rho V^2}{|\alpha^*|} \tag{4.12}$$

A similar equation for rigid perfectly plastic materials, Fig. 2b, was derived in te paper by Jones *et al.* (1987), namely

$$\xi_p = \frac{\beta_J(1-\xi_f)^2}{1+\beta_J(1-\xi_f)} - \Big[\frac{\beta_J\xi_f(1-\xi_f)}{1+\beta_J(1-\xi_f)^2}\Big]\ln\frac{\beta_J\xi_f}{1+\beta_J} \tag{4.13}$$

and

$$\alpha_J = \frac{1-\xi_f}{\beta_J} \qquad\qquad Y_J = \frac{\rho V^2}{\alpha_J} \tag{4.14}$$

The value of the parameter β_J is found from Eq. (4.13).

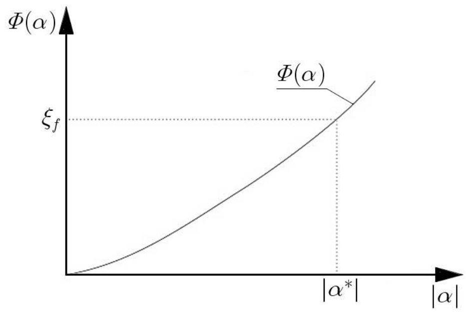

Fig. 4. Schematic illustration of determination of the parameter α^*

Taylor derived an approximate formula (Taylor, 1948) to calculate the dynamic yield stress for the rigid perfectly plastic material in the following form

$$Y_T = \frac{1-\xi_f}{2(1-\xi_f)} \frac{1}{\ln(1/\xi_f)} \rho V^2 \tag{4.15}$$

Values of the parameters Y_W, Y_J and Y_T calculated by means of Eqs. (4.11), (4.13) and (4.15) are presented in the next section.

Let us now consider the plastic strain ε_p. This parameter is determined by the formula

$$\varepsilon_p(\xi) = -\frac{\upsilon(\xi)}{a_1 + \upsilon(\xi)} = -\frac{\gamma(\xi)}{\beta^* + \gamma(\xi)} \tag{4.16}$$

where the quantity γ is the real root of the following transcendental equation

$$\xi = \left| \frac{1 + \frac{1}{\alpha^*\beta^*} + \frac{1}{\alpha^*}}{\gamma^2 + \frac{1}{\alpha^*\beta^*}\gamma + \frac{1}{\alpha^*}} \right|^{\frac{1}{2}} \left| \frac{2 + \frac{1}{\alpha^*\beta^*} - \sqrt{\Delta^*}}{2 + \frac{1}{\alpha^*\beta^*} + \sqrt{\Delta^*}} \, \frac{2\gamma + \frac{1}{\alpha^*\beta^*} + \sqrt{\Delta^*}}{2\gamma + \frac{1}{\alpha^*\beta^*} - \sqrt{\Delta^*}} \right|^{\frac{1}{\sqrt{\Delta^*}}\left(\beta^* - \frac{1}{2\alpha^*\beta^*}\right)} \tag{4.17}$$

where

$$\Delta^* = \Big(\frac{1}{\alpha^*\beta^*}\Big)^2 - \frac{4}{\alpha^*} \qquad \xi_f \leqslant \xi \leqslant 1 \qquad 0 \leqslant \gamma \leqslant 1 \tag{4.18}$$

It seems that value of the expression

$$\varphi(\gamma) = \left| \frac{1 + \frac{1}{\alpha^*\beta^*} + \frac{1}{\alpha^*}}{\gamma^2 + \frac{1}{\alpha^*\beta^*}\gamma + \frac{1}{\alpha^*}} \right|^{\frac{1}{2}} \qquad 0 \leqslant \gamma \leqslant 1 \tag{4.19}$$

is contained within the interval

$$1 \leqslant \varphi(\gamma) \leqslant \delta^* = \Big|1 + \alpha^* + \frac{1}{\beta^*}\Big|^{\frac{1}{2}}$$

In turn, $\delta^* \approx 1$ with the accuracy of several per cent (see Table 1). Therefore, with an accuracy sufficient for technical purposes, one may assume that

$$\left|\frac{1 + \frac{1}{\alpha^*\beta^*} + \frac{1}{\alpha^*}}{\gamma^2 + \frac{1}{\alpha^*\beta^*}\gamma + \frac{1}{\alpha^*}}\right|^{\frac{1}{2}} \approx 1$$

Then Eq. (4.17) reduces to

$$\gamma(\xi) = \frac{1}{2}\frac{a - bd\xi^{\frac{1}{c}}}{d\xi^{\frac{1}{c}} - 1} \tag{4.20}$$

where

$$\begin{aligned} a &= \frac{1}{\alpha^*\beta^*} + \sqrt{\varDelta^*} & \qquad b &= \frac{1}{\alpha^*\beta^*} - \sqrt{\varDelta^*} \\ c &= \frac{1}{\sqrt{\varDelta^*}}\Big(\beta^* - \frac{1}{2\alpha^*\beta^*}\Big) & \qquad d &= \frac{2 + \frac{1}{\alpha^*\beta^*} + \sqrt{\varDelta^*}}{2 + \frac{1}{\alpha^*\beta^*} - \sqrt{\varDelta^*}} \end{aligned} \tag{4.21}$$

Finally, the strain ε_p can be found by means of the formula

$$\begin{aligned} &\varepsilon_p(\xi) = \frac{a - bd\xi^{\frac{1}{c}}}{d(2\beta^* - b)\xi^{\frac{1}{c}} + a - 2\beta^*} \\ &\xi_f \leqslant \xi \leqslant 1 \qquad \varepsilon_p(\xi_f) = 0 \qquad |\varepsilon_p(1)| = |\varepsilon_{p\,max}| = \frac{1}{\beta^* + 1} \end{aligned} \tag{4.22}$$

Thereby, we obtain analytical expressions enabling analysis of the dynamic parameters Y_W and ε_p for the rigid-plastic material with linear strain hardening.

5. Example

In the examinations, uniform steel rods of the initial dimensions: length $L = 56$ mm and diameter $D = 8$ mm were used. The following mechanical parameters of steel were assumed: density $\rho = 7800\,\mathrm{kg/m^3}$, modulus of strain hardening $E_1 = 5$ GPa, engineering static yield stress $R_e = 1255$ MPa. The rods were driven by a firing gun to initial speeds contained within the interval 120-210 m/s. The rods impacted perpendicularly against a flat rigid target. Pictures of deformed rods after impact are shown in Fig. 5.

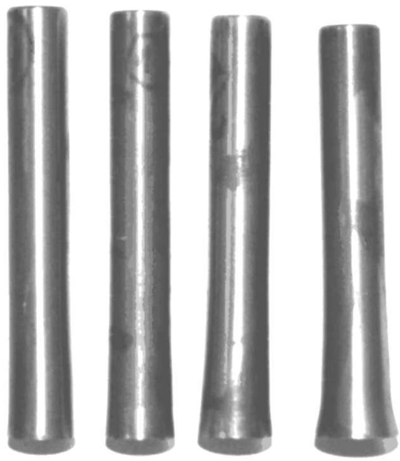

Fig. 5. Pictures of deformed rods

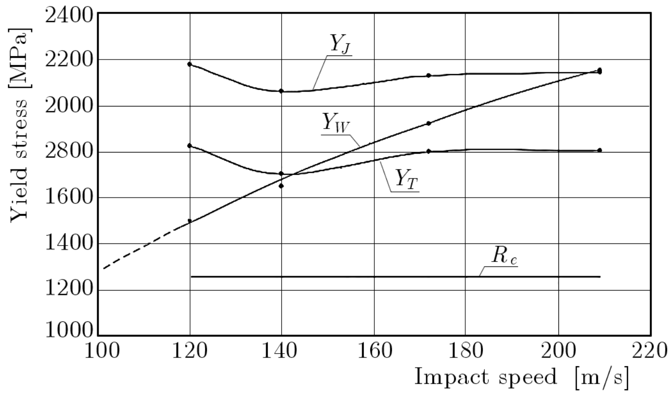

Fig. 6. Variations of parameters Y_W, Y_J and Y_T in function of the impact initial speed V; Y_T – dynamic yield stress for rigid perfectly plastic material, Y_J – dynamic yield stress according to the model by Jones *et al.* (1987), Y_W – dynamic yield stress for rigid plastic material with linear strain hardening

Experimental data obtained from the impact tests of the above-mentioned steel and results of calculations are given in Fig. 6 and listed in Table 1.

The following conclusions result from the obtained data.

- Strain hardening of a material influences to a considerable degree the dynamic yield stress (see Fig. 6 – Y_W and Table 1). The general trend of the obtained results indicate higher strengths at higher impact velocities which is in agreement with the analytical expressions. The parameter Y_W increases almost linearly together with the increase of the impact velocity in the range 100-200 m/s.

Table 1. Comparison of the experimental and analytical results from Taylor tests for chromium-nickel steel

V [m/s]	ξ_L	ξ_f	ξ_p	α^*	β^*	δ^*
120	0.973	0.60	0.373	−0.075	6.667	1.037
140	0.961	0.58	0.381	−0.093	5.714	1.040
172	0.945	0.56	0.385	−0.120	4.651	1.046
209	0.922	0.53	0.392	−0.158	3.828	1.050
$\lvert\varepsilon_{p\,max}\rvert$	Y_W [MPa]	Y_J [MPa]	Y_T [MPa]	$\dfrac{Y_W}{R_e}$	$\dfrac{Y_J}{R_e}$	$\dfrac{Y_T}{R_e}$
0.085	1500	2176	1824	1.195	1.733	1.453
0.094	1649	2059	1702	1.314	1.641	1.356
0.109	1920	2127	1800	1.530	1.695	1.434
0.124	2152	2141	1804	1.715	1.706	1.437

- In this range of the impact velocity, the parameters Y_J and Y_T nearly do not change. Due to error in the Taylor theory (Jones *et al.*, 1987) the inequality $Y_J > Y_T$ is fulfilled.
- The approximation of dynamic properties of metals with strain hardening by means of the model of a perfectly plastic material is far-reaching simplification disagreeing with the reality.

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Inżynierska analiza dynamicznych parametrów mechanicznych sztywno-plastycznego materiału z liniowym wzmocnieniem dla testu udarowego

Streszczenie

W pracy przedstawiono prostą inżynierską metodę określania dynamicznych parametrów mechanicznych sztywno-plastycznych materiałów z liniowym wzmocnieniem. Zastosowano do tego celu uderzeniowy test Taylora, tj. prostopadłe uderzenie pręta, wykonanego z testowanego materiału, w nieodkształcalną płytę. Wyprowadzono

przestępne równania, z których można określić *explicite* dynamiczną granicę plastyczności Y_W i odkształcenie plastyczne ε_p materiału pręta. Uzyskano wzrost parametru Y_W, wraz ze wzrostem prędkości uderzenia pręta, co jest zgodne z oczekiwaniem. Stosowana w literaturze aproksymacja dynamicznych właściwości metali modelem idealnie-plastycznym jest daleko idącym uproszczeniem, niezgodnym z rzeczywistością.

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