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FUZZY MODEL FOR BRAKING FORCE MAXIMIZATION

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This paper shows the process of braking force realization by air brakes with brake shoes accompanied by a suitable mechanical model. The complexity of adhesion nature as a physical phenomenon as well as the limited factors on which the braking force value depends are pointed out. According to this, the model of braking force realization based on the fuzzy set theory is explained. The procedure of fuzzy controller projecting with a task to regulate the value of kidding and by that the value of braking torque through the air pressure in the braking cylinder by maximizing the braking force that can be realized according to adhesion conditions is described. The testing of the optimization model under concrete adhesion conditions of the wheels on the rails is done at the end of the paper.

Key words: railway vehicle, braking force, wheel, rail, adhesion

1. Introduction

The realization of braking force high values appears as one of the basic demands in railway exploitations based on which the braking distance is determined (Bureika and Mikaliūnas, 2008). Besides that, the maximum deceleration value depends on the braking force which is very important for city and suburban traffic vehicles from the aspect of travelling time, because short distances between stops do not allow much participation of speed limit travelling.

The braking force maximization is a difficult problem due to highly complex, non-linear and time-varying nature of adhesion characteristics between the wheel and rail (Arias-Cuevas and Li, 2008; Bureika, 2008; Lata, 2008). A study on the optimal brake control using the adhesion coefficient is given by Lee *et al.* (2007).

The maximum value of the braking force that can be realized in accordance with adhesion conditions is reached for a certain skidding value. But, because of the significant stochastic character of the adhesion nature, it is very difficult to model the dependence between these variables using classic mathematical methods. That is why the use of fuzzy logic (Zadeh, 1965) is a useful tool for solving the problem of braking force maximization, having in mind that fuzzy controllers are designed using linguistic rules without the existence of the suitable mathematical model (Zadeh, 1973; Sugeno and Tanaka, 1991).

Many successful industrial applications of the fuzzy set theory are shown by Sivinandam *et al.* (2007), and one of them is a fuzzy anti-lock system. An antiskid fuzzy controller with the emergency brake of trains is successfully applied in Japanese company Mitsubishi (Cheok and

Shiomi, 1998, 2000). FGPA realization of a fuzzy based subway train braking system is shown by Reaz and Rahman (2002).

The paper describes the process of braking force realization with air brakes with brake shoes. The adhesion phenomenon as a physical phenomenon is analysed briefly based on which the fuzzy model for braking force maximization is formed.

2. Dynamical relations in the braking regime of trains

In a general sense, the task of braking is to stop a moving train of mass m_t within a time t on a braking distance l_b (Dinić, 1986). Note that Barney *et al.* (2001) analysed the relevant parameters and shown the method for calculating the train braking distance. The braking force is a force that causes the decrease of the train speed or the stopping of the train. In a physical sense, the decrease of the train speed or stopping means the decrease of the train kinetic energy E_k to the value that is appropriate for the final speed. The work-energy theorem (Ginsberg, 2008) for the braking regime of the train reads

$$\frac{dE_k}{dt} = -(F_b + W)v \tag{2.1}$$

where F_b is the magnitude of the braking force, W is the magnitude of the resistance force, and v represents the train speed at a moment t of the braking process. Since the kinetic energy E_k is determined by

$$E_k = \frac{m_t v^2}{2} + \sum_{j=1}^{n} \frac{I_j \omega_j^2}{2} \tag{2.2}$$

where I_j and ω_j are, respectively, the moment of inertia and the angular velocity of the jth rotation body, then relation (2.1) takes the following form

$$F_b + W = -m_t(1 + \varrho)\frac{dv}{dt} \tag{2.3}$$

where ϱ is the rotation mass coefficient.

3. Braking force realization

Basically, in railway vehicles, there are two types of braking mechanisms (Hasegawa and Uchida, 1999). One type is based on using adhesion brakes (mechanical or dynamical) in which adhesion is a necessary condition for the realization of braking forces, and the second one based on using non-adhesion brakes (rail brakes or air resistance braking) in which the process of realization of braking forces is done outside the wheel-rail contact zone, that is, adhesion is not involved in the generation of braking forces. According to this, the dynamic brakes are used in the zone of higher speed (Liudvinavičius and Lingaitis, 2007) while in the zone of smaller velocity their effect weakens when mechanical brakes are used, that is, air brakes with brake shoes or discs. Having in mind that braking by air brakes is until stopping, these brakes represent the basic type of brakes on railway vehicles. The non-adhesion brakes are mostly used as additional brakes in high speed trains.

In what follows, the procedure of braking force realization by air brakes with brake shoes is described. Under the influence of pressure force from the brake cylinder, which is transmitted to the brake shoes through the brake rigging, there comes to the leaning of brake lining (the changeable part of brake shoe) on the rolling surface of the wheel. The braking process is realized by friction forces between the brake linings and wheel, which leads to creating the braking torque T_b (Fig. 1) that acts in the direction opposite to the wheel rotation causinga decrease of the speed of motion.

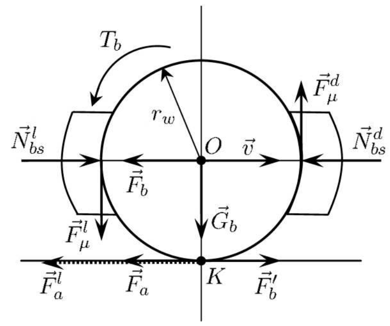

Fig. 1. The realization of braking forces on railway vehicle wheel

The magnitude of friction forces, $\vec{F}^l_\mu = -\vec{F}^d_\mu$, between the brake linings and the wheel is determined by

$$F_\mu = \mu N_{bs} \tag{3.1}$$

where μ is the coefficient of friction between the brake linings and the wheel and N_{bs} is the magnitude of the brake shoe forces, $\vec{N}^l_{bs} = -\vec{N}^d_{bs}$, that act on the wheel. The couple of friction forces creates the braking torque

$$T_b = 2r_w F_\mu \tag{3.2}$$

which can be further replaced by an equivalent couple of forces $\vec{F}_b = -\vec{F}'_b$ having the torque

$$T_b = F_b r_w \tag{3.3}$$

wherefrom, taking into account (3.2)

$$F_b = \frac{T_b}{r_w} = \frac{2r_w F_\mu}{r_w} = 2F_\mu \tag{3.4}$$

The force $\vec{F}_b$ represents the braking force that acts at the wheel center O, and $\vec{F}'_b$ is the tangential force that acts at the wheel-rail contact point K. In addition, the following inequality holds

$$F'_b \leqslant F^l_a \tag{3.5}$$

where F^l_a represents the limit value of the adhesion force determined by

$$F^l_a = G_b \Psi \tag{3.6}$$

where G_b is the weight per braked wheel and Ψ is the coefficient of adhesion. The force $\vec{F}'_b$ is balanced by force $\vec{F}_a$, i.e., $\vec{F}'_b = -\vec{F}_a$. This force represents the horizontal reaction of the rail. As a consequence, the forces $\vec{F}'_b$ and $\vec{F}_a$ vanish and the remaining force $\vec{F}_b$ causes the decrease of the vehicle speed. If the instantaneous center of zero velocity P_v coincides with the point K, then the wheel performs rolling without slipping with the speed of wheel center

$$v_0 = r_w \omega \tag{3.7}$$

where ω is the angular velocity of the wheel.

In the case of local deterioration of adhesion conditions, that is, when the force $\vec{F}_b^l$ has the magnitude $F_b^l > F_a^l$, there comes to skidding of wheels. The friction of rolling changes then into the friction of skidding so that the coefficient of adhesion is decreasing to the value $\Psi'(\Psi' < \Psi)$ and it drops with the increase of the vehicle speed. In that way, the limit value of the adhesion force is reduced, and thus the braking force magnitude which can be realized. In accordance to this, the braking couple of torque T_b can be decomposed into two couples. One, forming by friction forces of the magnitude equal to the decreased value of the adhesion force $G_b\Psi'$, and the second one, causing the decrease of the angular velocity of the wheel. The torque of this second couple is called the skidding torque T_s representing the part of the torque T_b that was not realized in the sense of braking force realization because the base did not accept it. Now, the instantaneous center of zero velocity P_v lies on the direction O-K below the point K so that the speed of the wheel center is

$$v_0 = \overline{OP_v}\omega \tag{3.8}$$

Since $\overline{OP_v} > \overline{OK}$, it is obvious that $v_0 > r_w\omega$. When the distance $\overline{OP_v}$ tends to infinity, then the angular velocity ω tends to zero causing the complete locking of wheels. The wheels will then move translationally, that is, they will skid along rails. The skidding value (Ohishi *et al.*, 2000; Allota *et al.*, 2001) can be expressed through the skidding velocity v_s as

$$v_s = v_0 - r_w\omega \tag{3.9}$$

or a skidding ratio that, for the process of braking, reads

$$s = \frac{v_s}{v_0} = \frac{v_0 - r_w\omega}{v_0} = 1 - \frac{r_w\omega}{v_0} \tag{3.10}$$

and it goes from 0 for theoretical rolling of the wheel on the rail without skidding $(v_0 = r_w\omega)$ to 1 when the locking of the wheel $(v_0 \gg r_w\omega)$ appears.

The motion of the train with completely locked wheels is an extremely unfavourable case because there comes to intensive wearing out of wheels followed by the appearance of flattened places that create hits during each spinning of the wheel, which reflects negatively in the suspension of vehicle and superstructure of railway track. Besides that, the controllability is lost, that is, the stability of motion of a vehicle on a rail which, as a consequence, may have derailment with the least of additional outside disturbances (for example, crossing the switches or any other interruption in the rail track). In order to stop the locking of wheels, it is necessary to decrease the value of braking torque, that is, the force of brake shoe that acts on the wheel or improve the conditions of adhesion of the wheels on the rail (by sanding) which means that for creating the adhesion it is necessary to satisfy condition (3.5), that is

$$2\mu N_{bs} \leqslant G_b\Psi \tag{3.11}$$

The realization of the braking force by brakes with brake discs is done in a similar way as by brakes with brake shoes. The only difference is that by brakes with discs, the force of brake lining acts normally to the discs that are firmly tied to the axle creating the friction forces that forms the braking couple. The magnitude of the braking forces produced by the brakes with discs is given by

$$F_b = 2\frac{r_{bd}}{r_w}F_\mu \tag{3.12}$$

where r_{bd} is the radius of the brake disc. The force of brake shoe in a general sense depends on the parameters of the braking system, that is, the radius of the brake cylinder d, the air pressure in the brake cylinder p_c, and the total brake rigging ratio i as follows

$$N_{bs} = \frac{d^2\pi}{4}p_c i\eta \tag{3.13}$$

where η is the brake rigging efficiency. With one chosen brake cylinder, the variables are the air pressure in the brake cylinder and the total brake rigging ratio, which means that the regulation of these two variables is necessary for managing the braking force.

The adhesion conditions vary a lot along the track due to: the wheel and rail geometry (wheel diameter and profile, rail head profile), wheel and rail material type, line configuration (curves, gradients, tunnels), rail unevenness, rail conditions in terms of presence of various pollutants (lubricants, dust, leaves, etc.), atmosphere conditions (rain, snow, dew, etc.), temperature, velocity, etc. The value of adhesion coefficient goes between 0.3-0.4 for dry and clean rails to 0.1-0.15 for moist and dirty rails (Vasic *et al.*, 2003). The most important parameters on which the value of the coefficient of friction between the brake lining and the wheel depends are the characteristics of wheel material (steel) and brake lining (chilled iron, composition and sintered materials, etc.), condition of their contact surfaces regarding the cleanliness, velocity etc. Chilled iron brake linings have the coefficient of friction that greatly depends on speed, so it takes the value of 0.05-0.1 in zones of high speed to 0.25-0.3 in zones of small speed instantaneously before stopping. As a consequence, there is a relatively low value of the braking force in zones of high speed when its value should be the highest in order to decrease the effect of braking because the length of braking distance is increasing while the locking of wheels is possible with a small speed. The use of special composition materials significantly increases the value of friction coefficient and decreases its dependence on speed. The dynamic friction coefficient of composition material brake linings goes from 0.27 to 0.33 depending on the temperature and speed. Brake linings made of composition materials are a lot lighter than the chilled and they last several times longer. The basic limitation with composition brake linings is bad thermal conductivity, and that is why there comes to overheating of wheels that causes creating of micro cracks, that is, small surface cracks on the rolling wheel surface. Thus, their use in exploitation is greatly limited. In contrast to brake shoes, linings for disc brakes are made exclusively of high friction composition materials with the dynamic coefficient of friction around 0.35 (with high speed railway vehicle brakes up to 0.42) and they practically do not depend on speed. The presence of grease decreases the value of friction coefficient while the presence of moisture decreases its value with smaller speed and it increases the value with higher speed. Brake linings made of high friction sintered materials (even up to 0.5) and that do not depend on speed are developed recently. The research with speeds more than 200 km/h have shown that brake linings made of sintered materials are a lot more high temperature and moisture resistant than composition material brake linings, so that they enable more efficiency of the brakes. Besides that, the consumption of the mentioned materials is smaller. The limitation of sintered brake linings is that they are around four times more expensive than the usual ones (Šubara, 2006). The regulation of force, by which the brake shoes work on the wheel, can be either manual or automatic. Manual regulation is done by changing the total brake rigging ratio that is applied with freight cars (empty-loaded changeover device) adjusting the value of the braking force according to vehicle gross mass. The automatic regulation is done by changing the air pressure in the brake cylinder.

4. Adhesion in the braking process

The existence of adhesion is caused by elastic deformities of the wheel and rail due to high pressure in the wheel-rail contact zone. The wheel-rail contact surface (see Fig. 2) consists of the rolling zone and the skidding zone (Dukkipati, 2000).

Under the influence of braking torque, the differences between the elements of the wheel center path and the point on the edge of wheel appear. With the increase in the braking torque, that is, in the air pressure in the brake cylinder, the braking force increases and by that the size of the skidding and rolling zones decreases while the skidding zone expands. The braking force

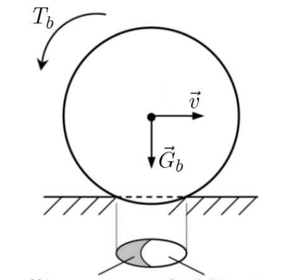

Fig. 2. Wheel-rail contact surface

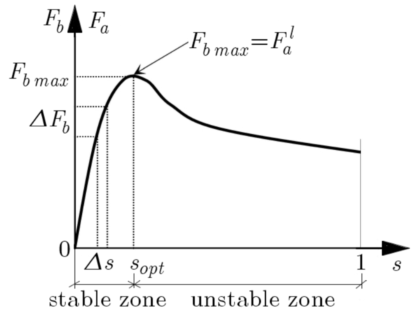

Fig. 3. Adhesion curve and $\Delta F_b - \Delta s$ analysis

shows further increase to the point of reaching the maximum value (adhesion limit) and then it starts to drop when it comes to the stopping of adhesion. This is the most important moment in the use of adhesion because the maximum braking force can be realized then $(F_{b\,max} = F_a^l = G_b\Psi_{max})$ and the skidding ratio reaches its optimal value s_{opt} (see Fig. 3). This corresponds to the point of maximum brake performance. The excess of the values s_{opt} leads to the activation of anti-skid protection and the insufficient exploiting of adhesion questions the required brake power and braking distance in the rapid braking.

The results of researches (Allota *et al.*, 2001; Yamazaki *et al.*, 2004) have shown that the adhesion limit in the process of braking is achieved for $s = 0.05$-0.25 depending on the vehicle speed and the conditions at the wheel-rail contact surface. In accordance with that, many wheel-skid prevention systems allow the maximum value of skidding ratio up to 0.2 (Department for Transport, 2007).

5. Optimization model

Let ΔF_b and Δs be, respectively, the change of the braking force and the skidding ratio in the interval Δt (see Fig. 3) determined by

$$\Delta F_b = F_b(t+1) - F_b(t) \qquad \Delta s = s(t+1) - s(t) \tag{5.1}$$

where $F_b(t+1)$ and $F_b(t)$ are the braking force values, and $s(t+1)$ and $s(t)$ are the values of skidding ratio at the moments $t+1$ and t, respectively. By analysing the values ΔF_b and Δs, the distance between the immediate and optimal values of the skidding ratio can be determined. If signs of ΔF_b and Δs are identical, then the immediate value of the skidding ratio is in a stable zone of the curve $F_b = f(s)$. In that case, the optimal value of the skidding ratio is higher than the immediate one. If signs of ΔF_b and Δs are different, then the immediate value of the skidding ratio is in an unstable zone of the curve $F_b = f(s)$. In that case, the optimal value of the skidding ratio is lower than the immediate one. When ΔF_b is small and Δs is large, then the immediate value of the skidding ratio is close to the optimal one. In accordance with that, the regulation of the braking torque, that is, the braking force can be done. If the immediate value of the skidding ratio is in the unstable zone of the curve $F_b = f(s)$ it is necessary to decrease the value of the braking torque quickly. Otherwise, if the immediate value of the skidding ratio is in the stable zone of the curve $F_b = f(s)$ there is a possibility to increase the value of the braking torque. Based on that, an optimization algorithm is shown in Fig. 4. The algorithm has an iterative nature. Therefore, the optimal (maximum) value of the braking force is determined by regulating the air pressure in the brake cylinder (with the fixed value of total brake rigging ratio) separately for each previously defined time interval according to the conditions of contacts between the brake linings and wheels, that is, the adhesion of rail that are of stochastic nature and subject to specific changes (random variables) having in mind the vehicle gross mass.

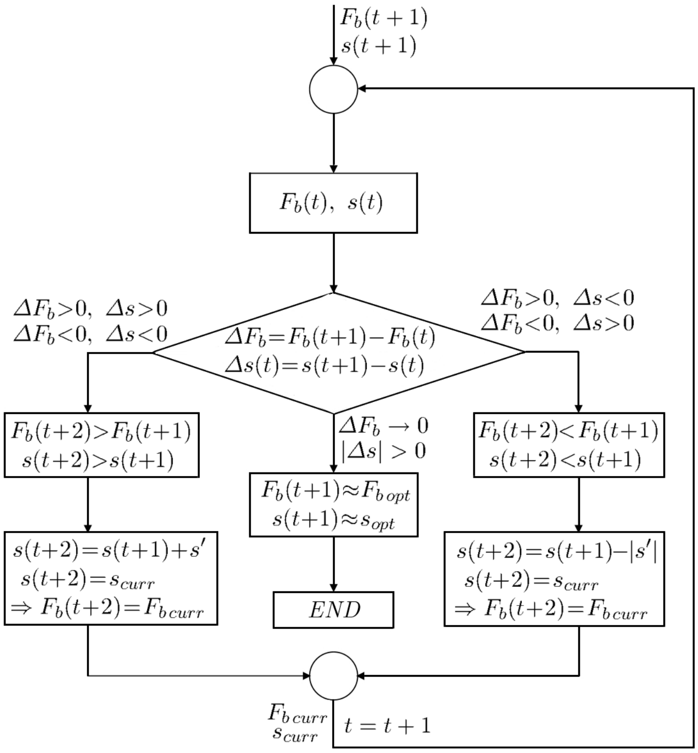

Fig. 4. An algorithm for determining the optimal (maximum) value of the braking force

The whole procedure begins at the moment t with specified values of $F_b(t)$ and $s(t)$. With the change of the air pressure in the brake cylinder there comes to a change of the braking force and by that the skidding ratio so that, at the next moment $t+1$, their values are $F_b(t+1)$ and $s(t+1)$, respectively. Based on Eqs. (5.1), the values ΔF_b and Δs are determined (the input variables of fuzzy controller) and, depending on them, the skidding ratio values are set at the next moment $t+2$

$$s(t+2) = s(t+1) \pm |s'| \tag{5.2}$$

according to which the adjustment of air pressure in the brake cylinder is done, that is, the decision about its decreasing or increasing is made. The value of the skidding ratio change s' represents the value of fuzzy controller output variable. The values $F_b(t+2)$ and $s(t+2)$ become the current values $F_{b\,curr}$ and s_{curr}, that is, the input values for the next iteration $(t=t+1)$. The procedure is repeated again and it lasts to the moment when the optimal values of the braking force and skidding ratio are reached under the given adhesion conditions. A fuzzy controller is composed of four major parts: the fuzzifier, the rule base, the inference and the defuzzifier. Furthermore, the following linguistic values of the quantities ΔF_b and Δs are chosen: negative big (NB), negative medium (NM), negative small (NS), neutral (NE), positive small (PS), positive medium (PM), and positive big (PB). Also, the following linguistic values of the quantity s' are chosen: negative big (NB), negative high medium (NHM), negative medium (NM), negative low medium (NLM), negative small (NS), neutral (NE), positive small (PS), positive low medium (PLM), positive medium (PM), positive high medium (PHM), and positive big (PB) (see Fig. 5).

The numerical values $\mathbf{x} \triangleq (x_{NB}, x_{NM}, \ldots, x_{PB}) \in \Re^{1\times 6}$, $\mathbf{y} \triangleq (y_{NB}, y_{NM}, \ldots, y_{PB}) \in \Re^{1\times 6}$, and $\mathbf{z} \triangleq (z_{NB}, z_{NHM}, \ldots, z_{PB}) \in \Re^{1\times 10}$ of the quantities ΔF_b, Δs, and s', respectively, represent the values that depend on the actual braking conditions (see Fig. 5). The control strategy is expressed as a set of linguistic "if-then" type rules. The fuzzy rule base contains 49 rules

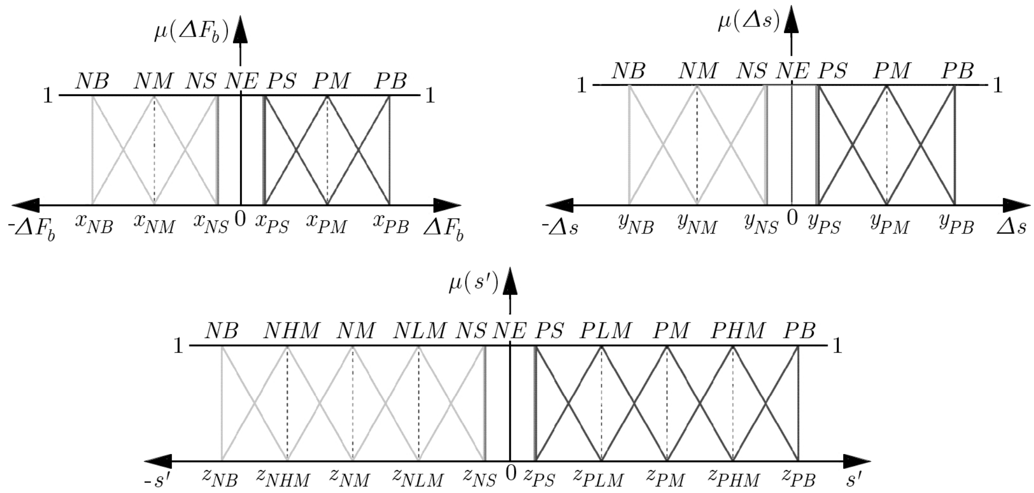

Fig. 5. Linguistic values of the input and output fuzzy variables shown with suitable fuzzy sets

as shown in Table 1. These rules were defined by a human expert and determined heuristically (Zadeh, 1973; Jang and Sun, 1995). The process of deduction, that is, the decision-making process, is defined by the algorithm of approximate or fuzzy reasoning based on the principle that each rule represents a fuzzy relation between different categories of fuzzy variables. For example: ΔF_b – negative big (NB) and Δs – negative big (NB) is a fuzzy phrase P in the Cartesian space $NB \times NB$ with the related function

$$\mu_P(\Delta F_b, \Delta s) = \min\{\mu_{NB}(\Delta F_b), \mu_{NB}(\Delta s)\} \tag{5.3}$$

The "If P, then s' is PLM" rule is also a fuzzy phrase Q in the Cartesian space $\Delta F_b \times \Delta s \times s'$ with the related function

$$\mu_Q(\Delta F_b, \Delta s, s') = \min\{\mu_P(\Delta F_b, \Delta s), \mu_{PLM}(s')\} \tag{5.4}$$

The rules are associated by means of the term "$ELSE$" and make up a fuzzy phrase S in the Cartesian space $\Delta F_b \times \Delta s \times s'$ with the related function

$$\mu_S(\Delta F_b, \Delta s, s') = \max\{\mu_{Q_1}(\Delta F_b, \Delta s, s'), \mu_{Q_2}(\Delta F_b, \Delta s, s'), \ldots, \mu_{Q_n}(\Delta F_b, \Delta s, s')\} \tag{5.5}$$

Table 1. Fuzzy rule base

s'		Δs						
		NB	*NM*	*NS*	*NE*	*PS*	*PM*	*PB*
ΔF_b	*NB*	*PLM*	*PM*	*PHM*	*NE*	*NM*	*NHM*	*NB*
	NM	*PS*	*PLM*	*PM*	*NE*	*NLM*	*NM*	*NHM*
	NS	*PS*	*PS*	*PS*	*NE*	*NS*	*NLM*	*NM*
	NE	*NE*	*NE*	*NE*	*NE*	*NE*	*NE*	*NE*
	PS	*NM*	*NLM*	*NS*	*NE*	*PLM*	*PS*	*NE*
	PM	*NHM*	*NM*	*NLM*	*NE*	*PHM*	*PM*	*PS*
	PB	*NB*	*NHM*	*NM*	*NE*	*PB*	*PM*	*PLM*

The "max-min" composition is used for the inference module, and the Center of Gravity method is used for the defuzzification module. This way, from the resulting fuzzy set, obtained via the algorithm of approximate reasoning, a single (representative) numerical value of the output variable is chosen. The non-linear function of the system associates the values of change of the braking force ΔF_b and the skidding ratio Δs with the magnitude of the next change of the skidding ratio s', shown in the form of its increase or decrease. The controller described has been tested for various values of ΔF_b and Δs, arbitrarily specified in the range of -0.08 to 0.08, and presented in Table 2 for numerical values of the input and output fuzzy variable. The obtained values of output variable s', depending on the values of input variables ΔF_b and Δs, are shown in Figs. 6a and 6b.

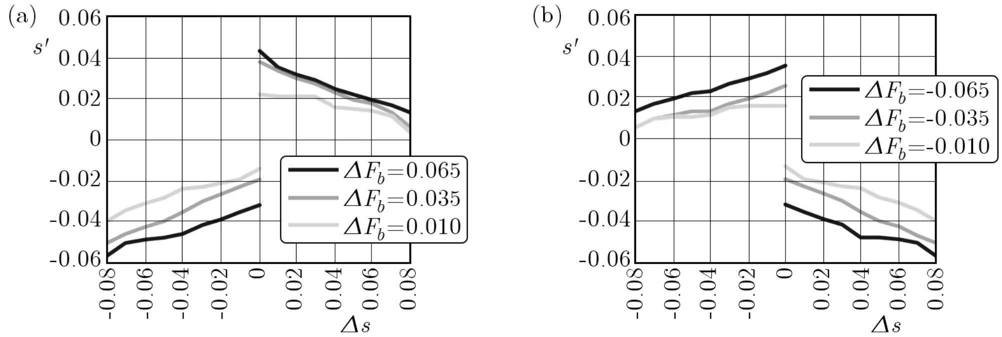

Fig. 6. Diagrams $s' = f(\Delta s)$ for (a) positive values and (b) negative values of ΔF_b

6. Realization of the optimization model

The optimization model is tested under concrete adhesion conditions between the wheel and the rail. To compare the resulting numerical values with the real ones, the results of research by Allota *et al.* (2001) are used (Fig. 7) and the modelling with the numerical values of fuzzy variables shown in Table 2 is done.

Table 2. Numeric values of fuzzy variables ΔF_b, Δs, and s'

	NB	*NHM*	*NM*	*NLM*	*NS*	*PS*	*PLM*	*PM*	*PHM*	*PB*
x	-0.07	–	-0.035	–	-0.001	0.001	–	0.035	–	0.07
y	-0.08	–	-0.04	–	-0.001	0.001	–	0.04	–	0.08
z	-0.065	-0.05	-0.035	-0.02	-0.002	0.002	0.0125	0.025	0.0375	0.05

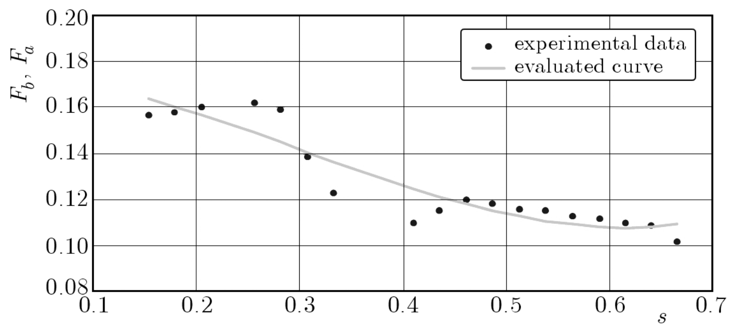

Fig. 7. $F_b = f(s)$ curve-experimental data

Note that the numerical values of the fuzzy variable ΔF_b are shown in the form of the values F_b/G_b. Using the optimization procedure, the braking force and skidding ratio grow to the moment $t = 11$ when they reach the optimal values $F_b(11) = 0.1604$ and $s(11) = 0.2217$. In the case of excessive increasing of the air pressure in the brake cylinder (or sudden deterioration of adhesion conditions), there comes to dropping of the braking force and sudden increasing of the skidding ratio. Then the immediate value of the skidding ratio is in the unstable zone of the curve $F_b = f(s)$. The regulation of the air pressure in the brake cylinder lasts to the moment $t = 6$ when the braking force and the skidding ratio reach the optimal values $F_b(6) = 0.16$ and $s(6) = 0.2077$. The testing results of optimization model through finding the optimal values of the braking force and skidding ratio are shown in Fig. 8.

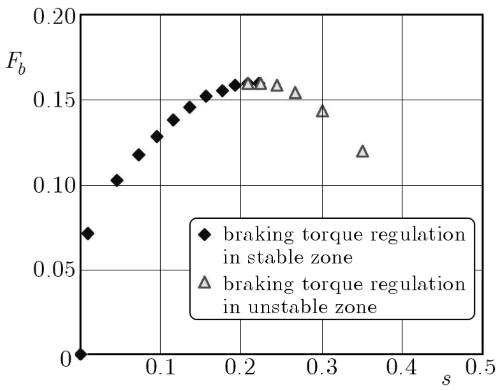

Fig. 8. Optimal values of the braking force and skidding ratio obtained by testing the optimization model

Comparing the obtained results with the measured values shown in Fig. 7, a suitable degree of coincidence is observed. The difference in the obtained optimal values of the braking force is a consequence of the defined parameter values of suitable fuzzy controller sub-systems and goes up to 1%. The parameters of suitable fuzzy controller sub-systems can be defined in some other way, which means that in order to improve it, the designing of the controller is still an open problem for further research. Besides that, it is possible to widen the model with a larger number of input variables.

7. Conclusion

Having in mind the stochastic character of adhesion that represents the complex function of many variables, mostly without the mathematical interpretation, the problem of braking force maximization is solved by fuzzy logic. The conveniences of fuzzy logic that does not demand the modelling of the existing suitable mathematical model are used. Based on the mechanical model of the braking force realization process, an optimization model is presented within which a fuzzy controller is formed and whose task is to regulate the value of skidding in order to maximize the braking force that can be realized in accordance with the conditions of adhesion. The degree of coincidence between the obtained values and the measured ones is satisfactory. This signifies the possibility of the application of the controller described. By applying the described model, it is possible to reach higher adhesion exploitation that brings to improving the braking effect in terms of increasing deceleration and shortening the braking distance length. Besides that, the model can represent the base for designing control systems for preventing the locking of wheels.

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Maksymalizacja siły hamowania za pomocą modelu z logiką rozmytą

Streszczenie

W artykule opisano proces generowania siły hamującej w pneumatycznych hamulcach szczękowych za pomocą odpowiedniego modelu mechanicznego. Szczególny nacisk położono na złożoność natury adhezji jako zjawiska fizycznego oraz ograniczoność czynników, od których zależy wartość siły hamującej. W tym kontekście zbudowano model oparty na logice rozmytej jako teoretyczne narzędzie do określania siły hamowania. Opisano procedurę projektowania sterownika rozmytego, którego zadaniem jest kontrola wartości poślizgu poprzez dobór momentu hamującego wynikającego z ciśnienia w zbiorniku układu pneumatycznego oraz aktualnych warunków adhezji. Na zakończenie przeprowadzono test modelu optymalizującego siłę hamowania dla konkretnych warunków adhezji pomiędzy szyną, a kołem poruszającego się po niej pojazdu.

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